

# Uncertainty Evaluation in calibration of Engineer's Steel Rulers and Measuring Tapes; GUM and Monte Carlo Methods

A. M. ElMelegy Engineering & Surface Metrology Lab., National Institute of Standards (NIS) Corresponding author's email: ahmedelmelegy@nis.sci.eg

#### Article Type: - Research article.

#### Abstract

Engineer's steel rulers and measuring tapes are considered versatile dimensional measuring tools for many industrial applications. Calibrations of these tools are carried out by comparing their scales with reference length standards. The reference standards can be geodetic baselines provided by a laser interferometer system or linear encoder that integrated in well-designed guideway. This work aims to study not only the calibration process for steel rulers and measuring tapes but also the affecting parameters on this process and the assessment of associated uncertainty by different evaluation methods. In this paper, a 1000 mm engineer's steel ruler and 5000 mm measuring tape are calibrated. A calibration system of precise guideway provided by linear encoder of 1 µm resolution are used. Associated uncertainties have been estimated based on GUM and Monte Carlo Simulation (MCS) methods. The parameters that may affect the calibration processes are carefully studied through the uncertainty evaluation. Expanded uncertainties of about 20 µm and 80 µm for calibration of 1000 mm engineer's steel ruler and 5000 mm measuring tape respectively are achieved. This study presents two issues. (1) it is first documented work in the study of uncertainty assessment and related parameters in this calibration type. (2) application of Monte Carlo Simulation (MCS) method in uncertainty evaluation in calibration of these tools.

Keywords: Steel Rulers, Measuring Tapes, Calibration, Monte Carlo, GUM

#### 1 Introduction

Length measuring tools i.e. engineer's steel rulers and measuring tapes are versatile and fast measuring tools that are used for dimensional measurements [1]. These tools are used in many applications fields i.e. industry, construction, surveying services and adjusting distances for fringe projection systems [2]. Although these tools have limited resolution of about 1 mm or 0.5 mm in best cases. It is still suitable for the nature of applications that the rulers and tapes are used for. The calibration of such tools becomes a necessary demand. The calibrations are common performed at steps of 100 mm and 1000 mm for rulers and tapes respectively. It is preferred to carry out the calibration in the same setup. This requires to use reference length instruments of long ranges [3, 4]. Many reference instruments are used in this calibration type i.e. geodetic baselines with laser interferometers and tape calibrators. Geodetic baselines have capability to measure lengths upto 40 m with accuracy upto 0.05  $\mu$ m. Tape calibrators are calibration system that have linear encoders integrated with well-designed guideways. These

calibrators can measure lengths upto 5 m with accuracy upto few micrometers. Although, geodetic baselines have higher accuracy in comparison to tape calibrators; the calibrators have low costs in their design and establishment [5]. The affecting parameters and associated uncertainty in such calibration processes should be studied and evaluated [6]. There are different uncertainty sources that affect the total combined uncertainty. Line width of scale divisions, non-sharpness degree of scale divisions, resolving power and accuracy of reference instruments are examples for the most effective contributors in evaluation of associated uncertainty [7–9]. Line width of scale divisions differs from steel rulers to measuring tapes. Steel rulers have higher sharpness in shape of scale divisions than measuring tapes. Guide to the Expression of Uncertainty in Measurement (GUM) and Monte Carlo simulation (MCS) are common methods for uncertainty evaluation in calibrations and measurements [10, 11]. GUM is published by Joint Committee for Guides in Metrology (JCGM). It provides the estimation guidelines of measurement uncertainty using law of propagation. The output quantity is characterized using normal distribution or t-distribution. The law of propagation provides a means for propagating uncertainties through a mathematical model. MCS is a numerical method that is used in calculating the uncertainty in many fields i.e. engineering, physical, biological and industrial systems based on simulation of random numbers. Evaluation of uncertainty by MCS is based on generating of random numbers for all the input parameters that affect the calibration process. Depending on individual PDFs of inputs, the probability density function (PDF) of the output is obtained. In this paper, 1000 mm engineer's steel rulers and 5000 mm measuring tape are calibrated with step of 100 mm. The associated uncertainty in each calibration type is evaluated applying two evaluation methods; GUM and MCS. The effects uncertainty sources are studied and expanded uncertainties at full range are evaluated.

# Methods and Procedure Calibration system

In this study, a Measuring Scale and Tape calibration system (MSTC2000, Octagon) is used, figure 1. It is a one dimension (1D) measuring Instrument of 2000 mm measuring range and 1  $\mu$ m resolution. This calibration system is traceable to SI units through its calibration by reference He-Nu heterodyne laser interferometer system. The system has a movable head that moves along 2000 mm linear encoder, Figure 2. This movable head has large display screen and digital camera. The screen has a crosshair that is used to be aligned with scale lines of ruler or tape at start measurements and interval steps



Figure 1: Measuring Scale & Tape Calibration System

The linear encoder that integrated in calibration system is considered in this system as the reference for length measurements in all measurements types that can be performed by it. The calibration using this system can be done with or without pressure loads. For both rulers' and tape calibration, no loads are applied. The calibration system is provided two pressure loads of 20 and 40 kg. These loads are applied only for depth tapes and tapes of plastic materials



Figure 2: Movable head with digital screen

# 2.2. Length Measuring Tools

Two length measuring tools of 1000 mm engineer's steel ruler and 5000 mm measuring tape are calibrated. Both tools have scale division of 1 mm. The calibration is carried out at interval of 100 mm.

## 2.3. Measurement Procedure

In calibration of steel ruler and measuring tape, length errors are measured based on the standards; BS 4372:2012 [12] and ISO 8322-2:1989 [13] respectively. There are four steps for calibration process, (1) the ruler or tape should be aligned and fixed on the guideway of the calibration system. (2) the starting point should be selected where you should check if the ruler-start side is clear or damaged. (3) press zero at this start side. (4) move the system head to the next length interval and check the resulted measured distance against the nominal length to determine the length deviation.

The method of measuring length deviation depends on measuring travel distance from specified point to a similar one at next specified scale division line, Figures 3–4.



Figure 3: Cross line is adjusted at the beginning of scale division line.



Figure 4: Cross line is adjusted at similar point in the next interval the beginning of scale division line.

# 3. Experimental Results

The engineer's steel ruler is calibrated along its full length. The measurements at each step are repeated 10 times. The calibration is done for scale range and scale division, Table 1–2. In similar way, the measuring tape is calibrated. The measuring tapes are calibrated commonly each 1 m but for this study it is calibrated each 100 mm, Table 3–4. The calibration results for both engineer's steel ruler and measuring tape are shown in Figures 5–6.

Table 1: Calibration	of 1000 mm Engineer's steel ruler
Nominal length, mm	Average measured errors, mm
100	0.249
200	0.170
300	0.150
400	0.131
500	0.101
600	0.059
700	0.043
800	-0.022
900	-0.063
1000	-0.067

Table 2: Calibration of Scale div	ision of 1000 mm Engineer's steel ruler
Nominal Thickness, mm	Average measured width, mm
0.25	0.245

	Table 3: Calibration of	5000 mm measuri	ng tape
Nominal	Average measured	Nominal	Average measured
length, m	errors, mm	length, m	errors, mm
100	0.125	2600	0.758
200	0.250	2700	0.753
300	0.284	2800	0.756
400	0.315	2900	0.750
500	0.342	3000	0.754
600	0.385	3100	0.766
700	0.450	3200	0.783
800	0.556	3300	0.795
900	0.680	3400	0.800
1000	0.755	3500	0.803
1100	0.760	3600	0.820

1200	0.755	3700	0.830	
1300	0.780	3800	0.860	
1400	0.775	3900	0.885	
1500	0.750	4000	0.895	
1600	0.745	4100	0.897	
1700	0.763	4200	0.899	
1800	0.755	4300	0.895	
1900	0.756	4400	0.894	
2000	0.757	4500	0.900	
2100	0.750	4600	0.901	
2200	0.745	4700	0.903	
2300	0.755	4800	0.902	
2400	0.760	4900	0.900	
2500	0.745	5000	0.904	

Table 4: Calibration of Scale	division of 5000 mm measuring tape
Nominal Thickness, mm	Average measured width, mm
0.25	0.223



# Calibration of 1000 mm Engineer's steel ruler



\_



Figure 6: Errors in calibration of 5000 mm measuring tape

#### 4. Uncertainty Evaluation based on GUM

The associated uncertainties are evaluated based on GUM [11]. In order to evaluate the associated uncertainty, the most affecting factors on the calibration process should be clearly determined. For this type of calibration, uncertainties due to instrument calibration, resolution, accuracy, graduations of ruler scale, sharpness of scale lines and temperature effect are most common factors. The calibration process of Engineer's steel ruler/tapes is expressed by mathematical model;

True length (L) = Measured length (x)+correction( $\Delta$ )+other factors ( $\varepsilon_1$ ;  $\varepsilon_2$ ; ......  $\varepsilon_n$ )

$$\mathbf{L} = \mathbf{x} + \mathbf{\Delta} + \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4 + \varepsilon_5 + \varepsilon_6 + \varepsilon_7 \tag{1}$$

Where;

L	Length of Ruler / Tape	<b>E</b> 3	Correction due to line width of scale division of Ruler / Tape
X	Measured length of Ruler / Tape	<b>E</b> 4	Correction due to non-sharpness of scale lines of Ruler / Tape.
Δ	Correction due to calibration of reference instrument	85	Correction due to miss alignment of scale of Ruler / Tape with reference instrument scale.
<b>E</b> 1	Correction due to reference instrument resolution	86	Correction due to temperature difference between Ruler / Tape and instrument.
<b>E</b> 2	Correction due to maximum errors in reference instrument	<b>E</b> 7	Correction due to temperature difference between Ruler / Tape and environmental standard temperature.

Assuming a linear model and sensitivity coefficients equal 1. by differentiation of equation (1); the contributory variances are

 $u^{2}(L) = u^{2}(x) + u^{2}(\Delta) + u^{2}(\varepsilon_{1}) + u^{2}(\varepsilon_{2}) + u^{2}(\varepsilon_{3}) + u^{2}(\varepsilon_{4}) + u^{2}(\varepsilon_{5}) + u^{2}(\varepsilon_{6}) + u^{2}(\varepsilon_{7})$ (2)

Where: u(L) uncertainty in Length (L) u(£3) uncertainty due to line width of scale division of Ruler / Tape uncertainty due to repeatability (x) uncertainty due to non-sharpness of scale lines u(x) u(£4) of Ruler / Tape. u(Δ) uncertainty in calibration of reference u(£5) uncertainty due to miss alignment of scale of instrument calibration Ruler / Tape with reference instrument scale. uncertainty in reference instrument uncertainty due to temperature difference **u(**ɛ<sub>1</sub>) u(£6) between Ruler / Tape and instrument. resolution

uncertainty due to maximum errors in u(22) reference instrument

the associated calibration uncertainty will be;

 $u(L) = [u^{2}(x) + u^{2}(\Delta) + u^{2}(\varepsilon_{1}) + u^{2}(\varepsilon_{2}) + u^{2}(\varepsilon_{3}) + u^{2}(\varepsilon_{4}) + u^{2}(\varepsilon_{5}) + u^{2}(\varepsilon_{6}) + u^{2}(\varepsilon_{7})]^{0.5}$ (3)

u(E7)

the expanded uncertainty of calibration can be determined by;

$$U(L) = K.u(L) \tag{4}$$

standard temperature.

uncertainty due to temperature difference

between Ruler / Tape and environmental

Where, K is a coverage factor which related the confidence level. It depends on the effective degree of freedom of all contributors and number of repetition of measurement results.

These factors or contributors in equation (3) that affect the calibration of either Engineer's steel ruler or Measuring tape will be described in details.

#### 4.1. uncertainty due to repeatability u(x)

The calibration for engineer's steel ruler and measuring tape is repeated 10 times at each point. The standard uncertainty due to repeatability of calibration results  $u(x_i)$  is determined by;

$$u(x_i) = \frac{\delta(xi)}{\sqrt{n}} \tag{5}$$

where;  $\delta(xi)$  is standard deviation of calibration results at point xi. n is number of repetitions. The values of u(xi) for steel ruler and measuring tape is represented in tables 5 and 6.

	Table 5: u(xi) for 1000 mm Eng	gineer's steel ruler
at worst	$\delta(xi)$ , mm	u(xi), mm
case	0.0042	0.0013
	Table 6: u(xi) for 5m me	asuring tape
at worst	$\delta(xi)$ , mm	u(xi), mm
case	0.004	0.0013

#### 4.2. uncertainty due to calibration of reference instrument $u(\Delta)$

U

The expanded uncertainty in calibration of this reference instrument is determined by;

 $U_{(instrument)} = (0.70+0.96L) \ \mu m, L is nominal length in meter$ 

at a coverage factor K = 2 and confidence level of 95% assuming normal distribution.

The standard uncertainty  $\mathbf{u}(\Delta)$  due to calibration uncertainty of reference instrument will be determined by;

$$u(\Delta) = \frac{U(reference)}{K}$$
(6)  
$$u(\Delta) = \frac{(0.70 + 0.96L)}{2} \ \mu m$$
$$u(\Delta) = (0.35 + 0.48L) \ \mu m, L \text{ is length in meter.}$$

then:

70

#### 4.3. uncertainty due to resolution of reference instrument u(ε<sub>1</sub>)

The calibration System that used for calibration has a resolution of 1  $\mu$ m. The standard uncertainty u( $\epsilon_1$ ) due to calibration uncertainty of reference instrument will be determined by;

$$u(\mathbf{\epsilon}_1) = \frac{half \ of \ resolution}{\sqrt{3}} \tag{7}$$
$$u(\mathbf{\epsilon}_1) = 0.0003 \ mm$$

where 
$$\sqrt{3}$$
 is the devisor factor assuming a rectangular uncertainty distribution.

#### 4.4. uncertainty due to maximum errors in measuring instrument $u(\epsilon_2)$

The maximum error in calibration of measuring instrument "Measuring Scale & Tape Calibration System (MTSC2000)" is found to be 3.43  $\mu$ m. The standard uncertainty u( $\epsilon_2$ ) due to this maximum error will be determined by;

$$u(\varepsilon_{2}) = \frac{0.00343}{\sqrt{3}}$$

$$u(\varepsilon_{2}) = 0.002 \ mm$$
(8)

where  $\sqrt{3}$  is the devisor factor for rectangular uncertainty distribution.

#### 4.5. uncertainty due to line width of scale lines of Ruler / Tape $u(\epsilon_3)$

The line width of scale divisions of both engineer's steel ruler and measuring tape are measured with 10 times repetition, tables 2 and 4. The standard uncertainty  $\mathbf{u}(\boldsymbol{\epsilon}_3)$  due to line width of scale division is determined by equation 1;

$$u(\mathbf{\epsilon}_3) = \frac{\delta(xi)}{\sqrt{n}} \tag{9}$$

The values of  $u(\varepsilon_3)$  for steel ruler and measuring tape are represented in table 7.

<b>Table 7:</b> u(a	3) for Engineer's steel ruler and N	Measuring tape
Uncertainty factor	Engineer's steel ruler	Measuring tape
и( <b>ɛ</b> з), тт	0.0005	0.0022

#### 4.6. uncertainty due to non-sharpness of scale lines of Ruler / Tape $u(\epsilon_4)$

The uncertainty due to non-sharpness of scale divisions of Ruler and Tape is determined experimentally and found to be about 5  $\mu$ m. The standard uncertainty **u**( $\epsilon$ 4) value with rectangular distribution is represented in table 8.

Table 8: u(ɛ4) for Engineer's steel rule	r and Measuring tape
Uncertainty factor	standard uncertainty <b>u(ε</b> 4), mm
non-sharpness of scale divisions of Ruler / Tape	0.00289

**Table 8:**  $u(\epsilon 4)$  for Engineer's steel ruler and Measuring tape

#### 4.7. uncertainty due to miss alignment of Ruler/Tape scale with instrument scale u(ɛ5)

The standard uncertainty  $\mathbf{u}(\epsilon_5)$  due to miss alignment of Ruler/Tape scale with instrument scale is determined experimentally and found to be about 5 µm. The standard uncertainty  $\mathbf{u}(\epsilon_5)$  value with rectangular distribution is represented in table 9.

	wieasuning tape
Uncertainty factor	standard uncertainty u(E4), mm
miss alignment of Ruler/Tape scale with instrument scale	0.00289

Table 9:  $u(\epsilon 5)$  for Engineer's steel ruler and Measuring tape

#### 4.8. uncertainty due to temperature difference between Ruler / Tape and instrument u(E6)

The ruler/tape has a measured temperature difference of 0.5 °C from the actual temperature of the Calibration System. The material of both calibration system standard uncertainty  $u(\varepsilon_6)$  due to this temperature difference will be determined by;

$$u(\varepsilon_6) = \frac{L \times \alpha \times \Delta}{\sqrt{3}} \tag{10}$$

where; L is measured length,  $\alpha$  is thermal expansion coefficient of ruler/tape and calibration system materials (12×10<sup>-06</sup>/°C),  $\Lambda$  is temperature difference between ruler/tape and the calibration system (0.5 °C) and  $\sqrt{3}$  is the devisor factor for rectangular uncertainty distribution.

then;

$$u(\varepsilon_6) = \frac{L \times 12 \times 10 - 6 \times 0.5}{\sqrt{3}}$$

 $u(\varepsilon_6) = 0.0035L mm$  where L is nominal length in m.

# **4.9.** uncertainty due to temperature difference between Ruler / Tape and standard temperature u(ε7)

The standard calibration temperature for ruler/tape is  $20^{\circ}C \pm 1^{\circ}C$ . The standard uncertainty **u**( $\epsilon_{6}$ ) due to this temperature fluctuation ( $\pm 1^{\circ}C$ ) will be determined by;

$$u(\varepsilon_7) = \frac{L \times \alpha \times \Delta}{\sqrt{3}}$$

where; L is measured length,  $\alpha$  is thermal expansion coefficient of ruler/tape and calibration system materials (12×10<sup>-06/°</sup>C),  $\Lambda$  is temperature deviation from standard temperature (±1°C) and  $\sqrt{3}$  is the devisor factor for rectangular uncertainty distribution.

then;

$$u(\varepsilon_7) = \frac{L \times 12 \times 10 - 6 \times 1}{\sqrt{3}}$$
$$u(\varepsilon_7) = 0.0069L \text{ mm} \qquad where L is nominal length in m.$$

The combined uncertainty budget for calibration of Engineer's Steel Rulers and Measuring Tape will be as represented in tables 10 and 11.

			distribution	degree	Contribution,
Uncertainty	denoted			of	%
sources	by	standard uncertainty		freedom	
uncertainty due to			Normal	9	2.0
repeatability (x)	u(x)	0.0013 mm (worst)			
uncertainty in			Normal	ω	0.8
calibration of					
measuring					
instrument	$u(\Lambda)$	$(0.35+0.48L) \times 10^{-3} \text{ mm}$			
uncertainty in	u (Δ)		Rectangular	ω	0.1
instrument					
resolution	$u(\varepsilon_1)$	0.000289 mm			
maximum error in			Rectangular	ω	4.7
calibration of					
measuring					
instrument	$u(\varepsilon_2)$	0.00198 mm			

|--|

uncertainty due to			Rectangular	ω	0.4
line width of scale					
lines of Ruler.	(-)	0.000577			
	$u(\varepsilon_3)$	0.000577 mm	D ( 1		10.0
uncertainty due to			Rectangular	00	10.0
non-sharpness of					
scale divisions of					
Ruler.	$u(\epsilon_4)$	0.00289 mm			
uncertainty due to			Rectangular	00	10.0
miss alignment of					
scale of Ruler with					
instrument scale.	(-)	0.00280			
	u(£5)	0.00289 mm	D ( 1		144
uncertainty due to			Rectangular	ω	14.4
l'emperature					
Dular and					
Kuler and					
instrument.	$u(\varepsilon_6)$	0.0035L mm			
uncertainty due to			Rectangular	ω	57.6
temperature			U		
difference from					
standard					
temperature.		0.00.001			
	$u(\varepsilon_7)$	0.0069L mm	<b>NT</b> 1.11.	••	100.0
Combined Uncertainty u(L)		F(0,00,40) <sup>2</sup> (0,00701) <sup>2</sup> 1	Normal distribution, 100.0		100.0
		$[(0.0048)^2 + (0.0078L)^2]$	effective degree of		
		mm	treedom = $\infty$		
Expanded uncertainty U(L)		$\sqrt{(0.0096)^2 + (0.0156L)^2}$	at Coverage factor $K = 2$ & confidence		
		mm, L in m	level 95%		

				0	
Uncertainty			distribution	degree of	Contributio
sources	denoted by	standard uncertainty		freedom	n, %
uncertainty due to			Normal	9	1.0
repeatability (x)		0.0012			
	u(x)	0.0013  mm (worst)			
uncertainty in			Normal	00	0.5
calibration of					
measuring					
instrument	$u(\Delta)$	$(0.35+0.48L) \times 10^{-3} \text{ mm}$			
uncertainty in			Rectangular	$\infty$	0.1
instrument			U		
resolution	11(81)	0.000289 mm			
maximum error in	u(ci)	0.000207 1111	Rectangular	m	03
maximum error m			Rectaliguiai	ω	0.5
calibration of					
measuring					
instrument	<b>v</b> (a)	0.00108			
	$u(\varepsilon_2)$	0.00198 mm			

uncertainty due to			Rectangular	00	0.1
line width of scale					
division of Tape.		0.000577			
uncertainty due to	$u(\varepsilon_3)$	0.000577 mm	Rectangular	m	0.5
non-sharpness of			Rectangular	w w	0.5
scale divisions of					
Tape.					
un containter due to	$u(\epsilon_4)$	0.00289 mm	Destan avlan		0.5
miss alignment of			Rectangular	ω	0.5
scale of Tape with					
instrument scale.					
	11(85)	0 00289 mm			
uncertainty due to	<b>u</b> (03)	0.00207 11111	Rectangular	ω	19.5
temperature					
difference					
instrument.	$u(\mathbf{c}_{\mathbf{c}})$	0.0035L mm			
uncertainty due to	u(86)	0.003312 11111	Rectangular	ω	78.4
temperature			0		
fluctuations in					
standard		0.00.001			
Combined Uncer	$u(\varepsilon_7)$	0.0069L mm	Normal distribution 1		100.0
Comonica Oncertainty u(L)		$[(0.0048)^2 + (0.0078L)^2]$	effective degree of freedom		100.0
		mm	$= \infty$		
Expanded uncertainty U(L)		$\sqrt{(0.0096)^2 + (0.0156L)^2}$	at Coverage factor $K = 2$ & confidence		
		mm, L in m	level 95%		

#### 5. Uncertainty Evaluation by Monte Carlo Simulation

The associated uncertainties in calibration of Engineer's steel ruler and measuring tape are evaluated once more by Monte Carlo [10–11], Figures 7–8. The evaluation process in calibration of 1000 mm Engineer's steel ruler is run with the software specifications:

GUM Workbench Edu Simulator: OMCE V:1.2.3nMean Value: 999.9676 mm\n Standard Uncertainty: 0.0092 mm\n Coverage Interval (p=0.9545): [999.9498, 999.9853] mm (Probabilistically Symmetric) \n Expanded Uncertainty Interval (p=0.9545): (+0.018, -0.018) mm (Probabilistically Symmetric) \n Number of Monte Carlo Trials: 400000\n Block size: 10000 runs\n The evaluation process in calibration of 5000 mm Measuring tape is run with the software specifications: GUM Workbench Edu Simulator: OMCE V:1.2.3\n Mean Value: 5001.015 mm\n Standard Uncertainty: 0.039 mm\n Coverage Interval (p=0.9545): [5000.942, 5001.087] mm (Probabilistically Symmetric) \n Expanded Uncertainty Interval (p=0.9545): (+0.073, -0.073) mm (Probabilistically Symmetric) \n Number of Monte Carlo Trials: 400000\n Block size: 10000 runs\n



Simulator: OMCE V:1.2.3\n Mean Value: 999.9676 mm\n Standard Uncertainty: 0.0092 mm\n Coverage Interval (p=0.9545): [999.9498, 999.9853] mm (Probabilistically Symmetric) \n Expanded Uncertainty Interval (p=0.9545): (+0.018, -0.018) mm (Probabilistically Symmetric) \n Number of Monte Carlo Trials: 4000000\n Block size: 10000 runs\n

Figure 7: Result of the Monte Carlo Simulation for uncertainty evaluation in calibration of 1000 mm Engineer's steel

ruler



Simulator: OMCE V:1.2.3\n

Mean Value: 5001.015 mm\n

Standard Uncertainty: 0.039 mm  $\n$ 

Coverage Interval (p=0.9545): [5000.942, 5001.087] mm (Probabilistically Symmetric) \n

Expanded Uncertainty Interval (p=0.9545): (+0.073, -0.073) mm (Probabilistically Symmetric) \n

Number of Monte Carlo Trials: 400000\n

Block size: 10000 runs\n

Figure 8: Result of the Monte Carlo Simulation for uncertainty evaluation in calibration of 5 m Measuring Tape

# 6. Discussion

There are many factors that act as sources for uncertainty in calibration process of Engineer's steel rulers and Measuring Tapes. The most common factors are (1) repeatability in measurement results, (2) calibration uncertainty of the reference instrument that used in calibration process, (3) the resolution of reference instrument, (4) maximum error in measuring instrument, (5) non-sharpness of scale divisions, (5) line width of scale division, (6) non-sharpness of scale divisions, (7) miss-alignment of Ruler/Tape scale with instrument scale, (8) temperature effect due to difference in temperature between Ruler/Tape and instrument and (9) temperature fluctuations in standard temperature for calibration. The relative effects of each factor are represented in Pie and Bar charts in Figures 9–10.



Figure 9: Relative effects of uncertainty sources in total combined uncertainty at full scale for calibration of 1000 mm Engineer's steel ruler.

For calibration of Engineer's steel rulers, the highest uncertainty sources that have percentage effects in total combined uncertainty are non-sharpness of scale division about 10%, miss-

alignment of ruler scale with instrument scale about 10%, difference in temperature between ruler and measuring instrument about 14% and temperature differences from standard temperature about 58%. The total effect of these four factors is about 92%. The precise control of temperature conditions and differences may reduce the effect due to these two factors. The factor of miss-alignment can be improved through precise adjustment. The factor of non-sharpness depends on the nature of under-calibration steel ruler. In general, the total expanded uncertainty at full scale of 1000 mm Engineer's steel ruler is about 0.018 mm (18  $\mu$ m).



Figure 10: Relative effects of uncertainty sources in total combined uncertainty at full scale for calibration of 5m Measuring tape.

For calibration of measuring tapes, the highest uncertainty sources that have percentage effects in total combined uncertainty are temperature fluctuations in standard temperature about 78% and difference in temperature between tape and measuring instrument about 20%. The total effect of these four factors is about 98%. The precise control of temperature conditions and differences may reduce the effect due to these two factors. In general, the total expanded uncertainty at full scale of 5000 mm measuring tape is about 0.080 mm (80  $\mu$ m).

In a comparative way to the uncertainty evaluation based on GUM approach. The Monte Carlo simulation method is used to evaluate the associated uncertainty in calibration of steel ruler and measuring tape. The mean and standard deviation of calibration results for simulations of 4000000\n (Number of Monte Carlo Trials) provide an estimate of the true value and its coverage interval. The Monte Carlo simulations resulted in expanded uncertainty of 0.018 mm and 0.078 mm in calibration of ruler and measuring tape respectively.

### 7. Conclusions

A 1000 mm steel ruler and 5000 mm measuring tape are calibrated using reference calibration system at intervals of 100 mm. The associated uncertainty in each calibration type is evaluated in details. For ruler calibration, the highest uncertainty sources that have percentage effects in total combined uncertainty are non-sharpness of scale division, miss-alignment of ruler scale with instrument scale, difference in temperature between ruler and measuring instrument and temperature fluctuation in standard temperature. For calibration of measuring tape, the highest uncertainty sources that have percentage effects in total combined uncertainty are temperature fluctuations in standard temperature, difference in temperature between tape and measuring instrument. The expanded uncertainty at full scale of 1000 mm Engineer's steel ruler is about 0.018 mm (~20  $\mu$ m). The uncertainties in calibrations are revaluated by Monte Carlo simulation method and found to be 0.018 mm in calibration of ruler and tape respectively.

#### Acknowledgement

The author would like to thank National Institute for standards (NIS) for metrological infrastructure and financial support. Thanks to our colleagues in Surface and Engineering Metrology Lab., Length and Precision Engineering Division at (NIS) for their technical support.

### References

- [1] Liyanawaduge N. P. and Sooriyaarchchi A.,"A method to calibrate steel length measuring tapes by mechanical comparison", Sri Lankan Journal of Technology (SLJoT), Vol. 3(01), p. 1-7 (2022).
- [2] Alia H. A., Amera M. A. and Omara A. A., New Simple Large Depth of Field Fringe Projection Profilometry System Using Laser Projector", Journal of Measurement Science & Applications, JMSA. Vol. 2(2), p. 28-39 (2022).
- [3] Judson L. V., "Calibration of Line Standards of Length and Measuring Tapes at The National Bureau of Standards", Supersedes Circular, p. 572 (1960).
- [4] Wang P., Zhao H. and Ren G., "Development and Application of Standard Device for Calibrating Steel Measuring Tape Based on Machine Vision", Applied Science, Vol. 12(7262), p. 1–13 (2022).
- [5] Godina A. and Acko B., "Calibration of Tape Measures with Small Measurement Unceratinty", Daam International Scientific Book, Chap. 16, p. 187–196 (2012).
- [6] JCGM 100:2008, "Evaluation of measurement data Guide to the expression of uncertainty in measurement'.
- [7] Ted Doiron and John Stoup, "Uncertainty and Dimensional Calibrations", Journal of Research of the National Institute of Standards and Technology, Vol 102(6), (1997).
- [8] Wu, J.; Liu, T., "Analysis and evaluation of uncertainty of measuring result of indication error of steel tape with automatic verification device", Chin. Insp. Bod. Lab., Vol. 27, p. 34–35 (2019).
- [9] ISO/IEC Guide 98-3,"Uncertainty of Measurement Part 3: Guide to the Expression of Uncertainty in Measurement (GUM: 1995)", 2008.
- [10] Rachakonda P., Ramnath V. and Pandey V. S., "Uncertainty Evaluation by Monte Carlo Method", MAPAN-Journal of Metrology Society of India, Vol 34(3), p.295–298 (2019).

- [11] Rachakonda P., Ramnath V. and Pandey V. S., "Monte Carlo Simulation in Uncertainty Evaluation: Strategy, Implications and Future Prospects", MAPAN-Journal of Metrology Society of India, Vol 34(3), p.299–304 (2019).
- [12] BS 4372, "Specifications for engineer's steel measuring rules", (2012).
- [13] ISO 8322-2, "Building construction Measuring instruments Procedures for determining accuracy in use — Part 2: Measuring tapes", (1989).