On the Extended Finite Fourier Transform of a Firmly Fixed Continuously Time Series with Missing Data

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Dr. Amira El-Desokey; Ali. Ben Aros and Dr. Mohammed Ghazal

Abstract- This paper investigates the properties of the extended finite Fourier transform of the firmly fixed of two vector-valued continuously time series. In their own terms, both time series can be conceived of as continuous. Both of the different time series have the property of having a continuous advance throughout their entire. The two relevant time series both have the attribute of a continuous progression throughout their data. This is one of the characteristics that allow them to be flexible. The extended finite Fourier transform provides the basis for the investigations while these characteristics is studied using the framework that it provides. Following that, a data analysis is carried out, during which time both regular and irregular repeating patterns are searched for and examined. This happens while the analysis is being carried out. This process immediately follows the phase in which the data was obtained. This is done simultaneously with data analysis, so the two operations are parallel.

Keywords: Finite Fourier transforms, Missing Data, Data Window, continuously fixed time series, Wishart Distribution.

1. INTRODUCTION

THE extended finite Fourier transform discussed with many authors such that, Brillinger,[1],[2], Dahlhaus,[3], Elhassanein, [4], [5], Farag, and Ghazal[6], [7], Ghazal,[8], Ghazal et al., [9], [10], [11], [15], [16], Ghazal, and Elhassanin, [12], Ghazal, and Farag, [13], [14], Mokkadis, et al., [17]. This paper is organized as follow, Section 1, Introduction, the statistical analysis of the extended finite continuous Fourier transform with observed data (regular pattern) will discussed in Sec.2, in Sec. 3, we will study the extended finite continuous Fourier Transform with missing observations (irregular pattern), and in Sec. 4, the characteristics of the extended continuous finite Fourier transform in irregular patterns will be discussed. In section 5, we discussed the practical implications of our theoretical research for the Arab Cement Factory, focusing on the company's monthly production and quantity of concrete sales from January 2010 to December 2015.

2. THE STATISTICAL ANALYSIS OF THE EXTENDED FINITE CONTINUOUS FOURIER TRANSFORM WITH OBSERVED DATA (REGULAR PATTERN)

Let $(s+r)$ be a two vector-valued fixed continuous time series

$$\beta(t)=\begin{bmatrix}X(t)\\ \tau(t)\end{bmatrix}, \tag{2.1}$$

$t=0,\pm1,\pm2,\ldots$, With $X(t)$, s vector-valued and $\tau(t)$, r vector-valued a firmly fixed $(s+r)$ vector-valued series with components,

$$\begin{bmatrix}X_j(t)\\ \tau_i(t)\end{bmatrix}, j=1,2,\ldots,s, i=1,2,\ldots,r,$$

where moments exist, then the mean is defined as

$$EX(t)=0, E\tau(t)=0, \tag{2.2}$$

The covariances are given by

$$\begin{aligned}&E\{[X(t+h)-\ell_x][X(t)-\ell_x]^T\}=\ell_{xx}(h),\\ &E\{[X(t+h)-\ell_x][\tau(t)-\ell_\tau]^T\}=\ell_{x\tau}(h),\\ &E\{[\tau(t+h)-\ell_\tau][\tau(t)-\ell_\tau]^T\}=\ell_{\tau\tau}(h),\end{aligned} \tag{2.3}$$

with spectral densities:

$$f_{xx}(d) = (2\pi)^{-1} \int_{-\infty}^{\infty} \ell_{xx}(h) Exp(-idh) dh$$

$$f_{x\tau}(d) = (2\pi)^{-1} \int_{-\infty}^{\infty} \ell_{x\tau}(h) Exp(-idh) dh, for\ d \in R \qquad (2.4)$$

$$f_{\tau\tau}(d) = (2\pi)^{-1} \int_{-\infty}^{\infty} \ell_{\tau\tau}(h) Exp(-idh) dh$$

Let $\phi_a(t), a = 1,2, \dots, s, (t \in R)$ be an independent series of $\beta(t)$ such that every t,

$$P[\phi_a(t) = 1] = p_a,$$
$$P[\phi_a(t) = 0] = q_a \qquad (2.5)$$

and,

$$E\{\phi_a(t)\} = P, \qquad (2.6)$$

Then we can define the altered continuous series as

$$Z(t) = \phi(t)\beta(t), \qquad (2.7)$$

Where,

$$Z_a(t) = \phi_a(t)\beta_a(t), \qquad (2.8)$$

Assumption I. Let $\boldsymbol{\lambda_a^{(T)}(t)}$ be a variation that is constrained and vanishes $0 < \boldsymbol{t} < \boldsymbol{T - 1}$. Which is known as the data window and satisfies

$$\frac{1}{T} \int_0^T \lambda_a{}^{(T)}(t) dt \underset{T \to \infty}{\rightarrow} \int_0^1 \lambda_a(h) dh, a = \overline{1, s}$$

$$\alpha^{(T)}{}_{a_1, \dots, a_k}(d) = \int_0^T \left[\prod_{j=1}^{M} \lambda_{a_j}^{(T)}(t) \right] Exp\{-idt\}\, dt$$

Use an r -vector, $\underline{G}$, and $r \times s$ filter $\{b(h)\}$, so

$$\tau(t) \approx \underline{G} + \sum_{h=-\infty}^{\infty} b(t-h) X(h), \qquad (2.9)$$

Using Hermitian matrix then

$$E\{[\tau(t) - \underline{G} - \textstyle\sum_{h=-\infty}^{\infty} b(t-h)X(h)][\tau(t) - \underline{G} - \sum_{h=-\infty}^{\infty} b(t-h)X(h)]^T\}, (2.10)$$

Theorem 2.1. [2]. Supposing $f_{xx}(d)$ given by (2.4), which is nonsingular, and $d \in R$. for a $(s+r)$vector-valued of Second-order fixed continuous time series with mean (2.2) and autocovariance function (2.3). Following this, the values of, $\underline{G}$,, and b(h) that minimize (2.10) are provided

$$\underline{G} = \ell_\tau - \left(\sum_{h=-\infty}^{\infty} b(h)\right)\ell_x = \ell_\tau - H(0)\ell_x, \qquad (2.11)$$

and

$$b(h) = (2\pi)^{-1}\int_0^{2\pi} H(a)Exp\{iha\}\,da, \qquad (2.12)$$

where,

$$H(d) = f_{\tau x}(d)f_{xx}(d)^{-1}, \qquad (2.13)$$

The filter $\{b(h)\}$ is absolutely summable. The minimum achieved is

$$\int_0^{2\pi}[\,f_{\tau\tau}(a) - f_{\tau x}(a)f_{xx}(a)^{-1}f_{x\tau}(a)]da, \qquad (2.14)$$

3. The Extended Finite CONTINUOUS Fourier Transform with Missing Observations (Irregular Pattern)

Theorem 3.1. Let $\mathbf{Z_a(t) = \phi_a(t)\beta_a(t), a = 1, 2, \dots, min(s, r)}$, are stochastic fixed continuous process with missing observations, $\mathbf{X_a(t), \tau_a(t), a = 1, 2, \dots, min(s, r)}$ and $\mathbf{\phi_a(t)}$ is Bernoulli Sequences and satisfies (2.8), then,

$$E\{Z_a(t)\} = 0, \qquad (3.1)$$

$$Cov\{Z_{a_1}(t_1), Z_{a_2}(t_2)\} = \begin{bmatrix} P_{a_1a_2}\ell_{xx}(h) & P_{a_1a_2}\ell_{xx}(h)H(d)^T \\ P_{a_1a_2}H(d)\ell_{xx}(h) & P_{a_1a_2}H(d)\ell_{xx}(h)H(d)^T \end{bmatrix}, \quad (3.2)$$

Proof.

From the independence of $\beta_a(t)$ and $\phi_a(t)$, then (3.1) is obtained directly. Now we have,

$$Cov\{Z_{a_1}(t_1), Z_{a_2}(t_2)\} = Cov\{\phi_{a_1}(t)\beta_{a_1}(t), \phi_{a_2}(t)\beta_{a_2}(t)\}$$

$$= Cov\left\{\begin{bmatrix}\phi_{a_1}(t_1)X_{a_1}(t_1)\\ \phi_{a_1}(t_1)\tau_{a_1}(t_1)\end{bmatrix},\begin{bmatrix}\phi_{a_2}(t_2)X_{a_2}(t_2)\\ \phi_{a_2}(t_2)\tau_{a_2}(t_2)\end{bmatrix}^T\right\} \quad =$$

$$E\begin{bmatrix}\phi_{a_1}(t_1)X_{a_1}(t_1)\phi_{a_2}(t_2)X_{a_2}(t_2) & \phi_{a_1}(t_1)X_{a_1}(t_1)\phi_{a_2}(t_2)\tau_{a_2}(t_2)\\ \phi_{a_1}(t_1)\tau_{a_1}(t_1)\phi_{a_2}(t_2)X_{a_2}(t_2) & \phi_{a_1}(t_1)\tau_{a_1}(t_1)\phi_{a_2}(t_2)\tau_{a_2}(t_2)\end{bmatrix}$$

$$= \begin{bmatrix}E[\phi_{a_1}(t_1)\phi_{a_2}(t_2)]Cov\,[X_{a_1}(t_1),X_{a_2}(t_2)] & E\left[\phi_{a_1}(t_1)\phi_{a_2}(t_2)\right]Cov[X_{a_1}(t_1),\tau_{a_2}(t_2)]\\ E[\phi_{a_1}(t_1)\phi_{a_2}(t_2)]\;Cov\,[\tau_{a_1}(t_1),X_{a_2}(t_2)] & E[\phi_{a_1}(t_1)\phi_{a_2}(t_2)]\;Cov[\tau_{a_1}(t_1),\tau_{a_2}(t_2)]\end{bmatrix}$$

$$= \begin{bmatrix}P_{a_1a_2}Cov\,[X_{a_1}(t_1),X_{a_2}(t_2)] & P_{a_1a_2}Cov[X_{a_1}(t_1),G+H(a)X_{a_2}(t_2)]\\ P_{a_1a_2}Cov[G+H(a)X_{a_1}(t_1),X_{a_2}(t_2) & P_{a_1a_2}Cov[G+H(a)X_{a_1}(t_1),G+H(a)X_{a_2}(t_2)\end{bmatrix}$$

From (2.3) we have,

$$Cov\{Z_{a_1}(t_1),Z_{a_2}(t_2)\} = \begin{bmatrix}P_{a_1a_2}\ell_{xx}(h) & P_{a_1a_2}\ell_{xx}(h)H(d)^T\\ P_{a_1a_2}H(d)\ell_{xx}(h) & P_{a_1a_2}H(d)\ell_{xx}(h)H(d)^T\end{bmatrix}$$

Then (3.2) is obtained.

Definition: The Wishart Distribution [2], [89-90].

Assume that X and τ are independent random vectors in Rk, and that $Vec\,[X\,\tau]$ is a normal vector of dimension 2k. If this is the case, then the Wishart distribution holds for the complex random vector $\Psi = X + i\tau$. Three parameters, $G = E(\psi), \theta = E[(\psi - G)(\overline{\psi} - \overline{G})^T], and\ \ell = E[(\psi - G)(\psi - G)^T]$ characterize this distribution, where ψ^T denotes matrix transpose and signifies complex conjugate. Here, G can be any k-dimensional complex vector, must be Hermitian and non-negative definite, and ℓ should be a symmetric matrix. Moreover, matrices θ and ℓ are such that the matrix $\overline{\theta} - \overline{\ell}^T \theta^{-1}\ell$ is also non- negative definite and the matrices θ and ℓ are related to the covariance matrices of X and τ.

Theorem 3.2. Let $Z_a(t) = \phi_a(t)\beta_a(t), a = 1,2,\ldots,\min(s,r)$ are stochastic fixed continuous process with missing observations $X_a(t),\tau_a(t), a = 1,2,\ldots,\min(s,r)$ and $\phi_a(t)$ is Bernoulli sequence and satisfies (2.8), and $\lambda_a^{(T)}(t)$ satisfies Assumption I, then the continuously extended finite Fourier transform can be defined as

$$\gamma_a^{(T)}(d) = \left[2\pi\int_0^T\left(\lambda_a^{(T)}(t)\right)^2\right]^{-1/2}\int_{-\infty}^{\infty}\lambda_a^{(T)}(t)Z_a(t)\exp\{-idt\}\,dt, \text{for } d \in R \tag{3.3}$$

Which is distributed approximately as Wishart distribution (Complex normal distribution) as

$$N_{s+r}^{c}\left(\underline{0},\begin{bmatrix} v_1 & v_2 \\ v_3 & v_4 \end{bmatrix}\right), \qquad (3.4)$$

Where,

$$v_1 = P_{a_1a_2}\int_R f_{a_1a_2}(u)\Omega_{a_1a_2}{}^{(T)}(d_1-u,d_2-u)du,$$

$$v_2 = P_{a_1a_2}\int_R f_{a_1a_2}(u)H(d)^T\Omega_{a_1a_2}{}^{(T)}(d_1-u,d_2-u)du,$$

$$v_3 = P_{a_1a_2}\int_R H(d)f_{a_1a_2}(u)\Omega_{a_1a_2}{}^{(T)}(d_1-u,d_2-u)du,$$

$$v_4 = P_{a_1a_2}\int_R H(d)f_{a_1a_2}(u)H(d)^T\Omega_{a_1a_2}{}^{(T)}(d_1-u,d_2-u)du,$$

Proof.

To prove theorem (3.2), first we have to prove that

$$E\{\gamma_a(t)\} = 0,$$

Which comes directly using (3.1) and (3.3).

Now we have,

$$Cov\left\{\gamma_{a_1}^{(T)}(d_1),\gamma_{a_2}^{(T)}(d_2)\right\} = E\left\{\gamma_{a_1}^{(T)}(d_1)\overline{\gamma_{a_2}^{(T)}(d_2)}\right\}$$

$$= (2\pi)^{-1}\left[\int_{t_1=0}^{T}\int_{t_2=0}^{T}\left(\lambda_{a_1}^{(T)}(t_1)\right)^2\left(\lambda_{a_2}^{(T)}(t_2)\right)^2 dt_1dt_2\right]^{-1/2}\times$$

$$\times\int_{t_1=0}^{T}\int_{t_2=0}^{T}\lambda_{a_1}(t_1)\lambda_{a_2}(t_2)Cov(Z_{a_1}(t_1),Z_{a_2}(t_2))\,exp(-i(t_1d_1 - t_2d_2)dt_1dt_2$$

Using theorem (3.1), equation (3.2) then

$$Cov\left\{\gamma_{a_1}^{(T)}(d_1),\gamma_{a_2}^{(T)}(d_2)\right\} = (2\pi)^{-1}\left[\int_{t_1=0}^{T}\int_{t_2=0}^{T}\left(\lambda_{a_1}^{(T)}(t_1)\right)^2\left(\lambda_{a_2}^{(T)}(t_2)\right)^2\right]^{-1/2}\times$$

$$\times\int_{t_1=0}^{T}\int_{t_2=0}^{T}\lambda_{a_1}(t_1)\lambda_{a_2}(t_2)\exp(-i(t_1d_1 - t_2d_2)dt_1dt_2\times$$

$$\times\begin{bmatrix} P_{a_1a_2}\ell_{xx}(t_1-t_2) & P_{a_1a_2}\ell_{xx}(t_1-t_2)H(d)^T \\ P_{a_1a_2}H(d)\ell_{xx}(t_1-t_2) & P_{a_1a_2}H(d)\ell_{xx}(t_1-t_2)H(d)^T \end{bmatrix} \qquad (3.4)$$

substituting $t_1 - t_2 = h, t_2 = t$, then we have,

$$Cov\left\{\gamma_{a_1}^{(T)}(d_1), \gamma_{a_2}^{(T)}(d_2)\right\} =$$

$$= (2\pi)^{-1}\left[\int_{t_1=0}^{T}\int_{t_2=0}^{T}\left(\lambda_{a_1}^{(T)}(t_1)\right)^2\left(\lambda_{a_2}^{(T)}(t_2)\right)^2 dt_1 dt_2\right]^{-1/2} \times$$

$$\times \int_{t_1=-T}^{T}\int_{t_2=0}^{T}\lambda_{a_1}(t+h)\lambda_{a_2}(t)\, exp(-it(d_1-d_2)dtdh \times$$

$$\times$$

$$\begin{bmatrix} P_{a_1a_2}\ell_{xx}(h) & P_{a_1a_2}\ell_{xx}(h)H(d)^T \\ P_{a_1a_2}H(d)\ell_{xx}(h) & P_{a_1a_2}H(d)\ell_{xx}(h)H(d)^T \end{bmatrix}, \tag{3.5}$$

$$= \begin{bmatrix} P_{a_1a_2}\ell_{xx}(h) & P_{a_1a_2}\ell_{xx}(h)H(d)^T \\ P_{a_1a_2}H(d)\ell_{xx}(h) & P_{a_1a_2}H(d)\ell_{xx}(h)H(d)^T \end{bmatrix} = \begin{bmatrix} v_1 & v_2 \\ v_3 & v_4 \end{bmatrix}, \tag{3.6}$$

where,

$$\ell_{xx}(h) = E\{X(t+h)X(t)\} = \int_{-\infty}^{\infty} f_{xx}(d)\, exp(idh)\,, \tag{3.7}$$

Substituting (3.7) into (3.5) we have,

$$v_1 = p_{a_1a_2}\int_{-\infty}^{\infty} f_{a_1a_2}(u)(2\pi)^{-1}\left[\int_{t_1=0}^{T}\int_{t_2=0}^{T}\left(\lambda_{a_1}^{(T)}(t_1)\right)^2\left(\lambda_{a_2}^{(T)}(t_2)\right)^2 dt_1 dt_2\right]^{-1/2} \times$$

$$\times\left\{\int_{t_1=0}^{T}\int_{t_2=0}^{T}\lambda_{a_1}(t_1)\lambda_{a_2}(t_2)\, exp(-i[(d_1-u)t_1-(d_2-u)t_2]\, dt_1dt_2\right\}du,$$

$$v_1 = p_{a_1a_2}\int_{-\infty}^{\infty} f_{a_1a_2}(u)\, \Omega_{a_1a_2}(d_1-u, d_2-u)du,$$

Where,

$$\Omega_{a_1a_2}(d_1-u, d_2-u) =$$

$$(2\pi)^{-1}\left[\int_{t_1=0}^{T}\int_{t_2=0}^{T}\left(\lambda_{a_1}^{(T)}(t_1)\right)^2\left(\lambda_{a_2}^{(T)}(t_2)\right)^2 dt_1 dt_2\right]^{-1/2} \times$$

$$\times\left\{\int_{t_1=-T}^{T}\int_{t_2=0}^{T}\lambda_{a_1}^{(T)}(t+h)\lambda_{a_2}^{(T)}(t_2)\exp\left(-i[(d_1-u)t-(d_2-u)t]\, dtdu\right\},$$

Also

$$v_2 = p_{a_1a_2}\int_{-\infty}^{\infty} f_{a_1a_2}(u)H(d)^T\, \Omega_{a_1a_2}(d_1-u, d_2-u)du \quad ,$$

$$v_3 = p_{a_1a_2}\int_{-\infty}^{\infty} H(d)f_{a_1a_2}(u)\, \Omega_{a_1a_2}(d_1-u, d_2-u)du \quad ,$$

$$v_4 = p_{a_1a_2}\int_{-\infty}^{\infty} H(d)f_{a_1a_2}(u)H(d)^T\, \Omega_{a_1a_2}(d_1-u, d_2-u)du,$$

Then the theorem is proved.

4. THE CHARACTERISTICS OF THE EXTENDED FINITE CONTINUOUS FOURIER TRANSFORM IN IRREGULAR PATTERN

Lemma 4.1. Since (3.3) defines $\gamma_a^{(T)}(d), a = 1, \ldots, min(r, s)$, as the distribution function, we have that the dispersion of $\gamma_a^{(T)}(d)$ meets the following.

$$D\gamma_a{}^{(T)}(d) = P_{aa}\begin{bmatrix} \int_{-\infty}^{\infty} f_{aa}(d-v)\,\Omega_{aa}(v)dv & \int_{-\infty}^{\infty} f_{aa}(d-v)\,H(d)^T\Omega_{aa}(v)dv \\ \int_{-\infty}^{\infty} H(d)f_{aa}(d-v)\,\Omega_{aa}(v)dv & \int_{-\infty}^{\infty} H(d)f_{aa}(d-v)\,H(d)^T\Omega_{aa}(v)dv \end{bmatrix}, \tag{4.1}$$

Lemma 4.2. Let $\lambda_a{}^{(T)}(t), t \in R, a = 1, \ldots, min(s, r)$ is bounded by a constant N and satisfying

$$\left|\lambda_a{}^{(T)}(t+h) - \lambda_a(t)\right| \le L|h|\,,$$

Then,

$$\left|\int_0^T \lambda_{a_1}{}^{(T)}(t)\lambda_{a_2}{}^{(T)}(t)\,exp(-idt)dt\right| \le \frac{1}{\left|\frac{d}{2}\right|} + NL, \tag{4.2}$$

For some constants$N$, L and, $d \in R$, $d \neq 0$, $a_1, a_2 = 1, \ldots, min(s, r)$.

Lemma 4.3. *For* $d_1 - d_2 \neq 0, d_1, d_2 \in R$ *and* $\lambda_a{}^{(T)}(t), t \in R, a = 1, \ldots, min(s, r)$ is bounded by constant Nand satisfying (4.2), then,

$$\left|Cov\{\gamma_{a_1}{}^{(T)}(d_1), \gamma_{a_2}{}^{(T)}(d_2)\}\right| \le$$
$$\le \frac{NL}{2\pi\sqrt{\int_0^T\int_0^T(\lambda_{a_1}{}^{(T)}(t_1))^2(\lambda_{a_2}{}^{(T)}(t_2))^2dt_1dt_2}} \times$$
$$\times\left\{\frac{1}{NL|(d_1-d_2)/2|}\int_{-T}^{T}\left|\ell_{a_1a_2}(h)\right|dh + \int_{-T}^{T}\left|\ell_{a_1a_2}(h)\right|[|h|+1]dh\right\}, \tag{4.3}$$

For $a_1, a_2 = 1, \ldots, min(s, r)$.

Theorem 4.1. *For* $d_1 - d_2 \neq 0, d_1, d_2 \in R$ *and* $\lambda_a{}^{(T)}(t), t \in R$,

$a = 1, \ldots, min(s, r)$ is bounded by constant Nand satisfying (4.2) and,

$$\int_{-\infty}^{\infty}[|h|+1]\left|\ell_{a_1a_2}(h)\right|dh<\infty,$$

then,

$$\underset{T\to\infty}{Lim}\, Cov\left\{\gamma_{a_1}{}^{(T)}(d_1), \gamma_{a_2}{}^{(T)}(d_2)\right\}=0,$$

For all $a_1, a_2 = 1, \dots, min(s, r)$.

Proof.

Assumption I and Lemma (4.3) provide the foundation for the proof.

Theorem 4.2. For any $d \in R$, the function $\Omega_{aa}{}^{(T)}(d), a = 1, \dots, \min(s, r)$ is the Kernel that satisfies the following characteristics:

$$(1)\int_{-\infty}^{\infty}\Omega_{aa}{}^{(T)}(d)dd=1, a=1,\dots,min(s,r), \tag{4.4}$$

$$(2)\underset{T\to\infty}{Lim}\int_{-\infty}^{-\Theta}\Omega_{aa}{}^{(T)}(d)dd=\underset{T\to\infty}{Lim}\int_{\Theta}^{\infty}\Omega_{aa}{}^{(T)}(d)dd=0, \tag{4.5}$$

For $> 0, a = 1, \dots, min(s, r), d \in R,$,

$$(3)\ \underset{T\to\infty}{Lim}\int_{-\Theta}^{\Theta}\Omega_{aa}{}^{(T)}(d)dd=1, \tag{4.6}$$

for all $\Theta > 0, a = 1, \dots, min(s, r), d \in R$.

Theorem 4.3. For the function $\Omega_{aa}{}^{(T)}(x) a = 1, \dots, \min(s, r), x \in R$, that satisfies the characteristics of the theorem (4.2), and if the spectral density function $f_{aa}(x), a = 1, \dots, \min(s, r), x \in R$ is bounded and continuously at a point $x = d, d \in R$, then

$$\underset{T\to\infty}{\mathrm{Lim}} D\gamma_a{}^{(T)}(d)=\begin{bmatrix} f_{aa}(d) & f_{aa}(d)H(a)^T \\ H(a)f_{aa}(d) & H(a)^T f_{aa}(d)H(a)\end{bmatrix}\underset{T\to\infty}{\to}0,$$
$$a=1,\dots,\min(s,r), \tag{4.7}$$

5. **Application Study**

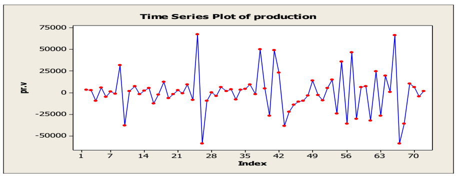
We discuss the practical implications of our theoretical research for the Arab Cement Factory, focusing on the company's monthly production and quantity of concrete sales from January 2010 to December 2015.

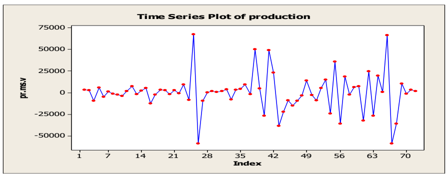
5.1.1. Analyzing the Production.

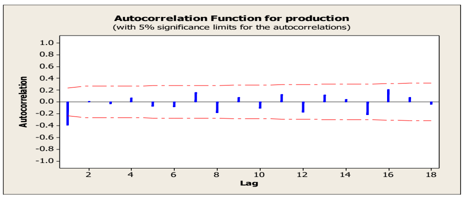
Our results, obtained from a model of firmly fixed time series with some missing observations, will be compared with the classical results, obtained when all observations are present, in this investigation.

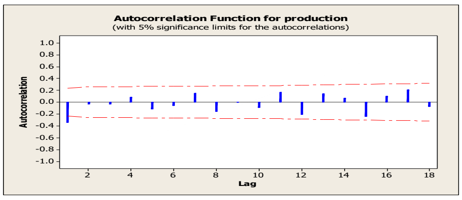
Let $\eta_a(t)=\varphi_a(t)X_a(t)$, $a=1,2,\ldots,r$, where $X_a(t),(t=0,\pm 1,\ldots)$ is a firmly fixed s-vector valued time series, and $\varphi_a(t)$ is a Bernoulli sequence of stochastically independent random variables be stochastically independent of $X_a(t)$. We assume that the data $X_a(t),(t=(1,2,\ldots,T]$, which is the average of the monthly production for which all observations are available, is available with some missing observations, $\varphi=1$, $\eta_a(t)=X_a(t)$, The table (5.1.1) compares the results with and without missing observations, assuming that some observations are randomly missing $\varphi_a(t)=0$ in the classical scenario.

Table (5.1.1) Analysis of Production Data with and without Missing data.

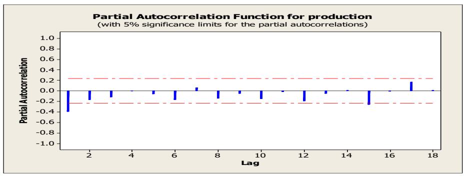
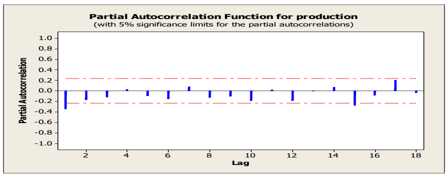
<table>
<tr><th>without missing data</th><th>with missing data</th></tr>
<tr><td>

The monthly average production</td><td>

The monthly average production</td></tr>
<tr><td>

ACF of the monthly average production</td><td>

ACF of the monthly average production</td></tr>
</table>

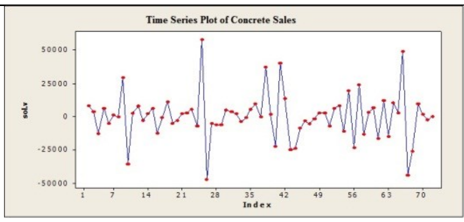
 PACF of the monthly average production	 PACF of the monthly average production
ARIMA(1,1,1) Final Parameters Estimates Type Coef SE Coef T P AR 1 0.6013 0.0987 4.39 0.000 MA 1 0.968 0.0523 20.89 0.000 Residuals: SS = 299874459104 (back forecasts excluded) MS = 43459707, DF = 68 Modified Box-Pierce (Ljung-Box) Chi-Square statistic Lag 12 24 36 48 Chi-Square 11.4 21.6 37.6 49.7 DF 9 21 33 45 P-Value 0.240 0.414 0.262 0.288	***ARIMA(1,1,1)*** Final Parameters Estimates Type Coef SE Coef T P AR 1 0.6684 0.1124 5.01 0.000 MA 1 0.9678 0.0519 21.53 0.000 Residuals: SS = 297761014349 (back forecasts excluded) MS = 367970146, DF = 68 Modified Box-Pierce (Ljung-Box) Chi-Square statistic Lag 12 24 36 48 Chi-Square 11.3 23.5 40.5 524 DF 9 21 33 45 P-Value 0.251 0.314 0.168 0.205

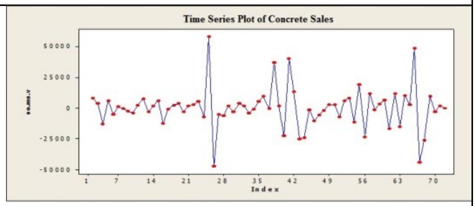
5.1.2. Analysis of concrete sales

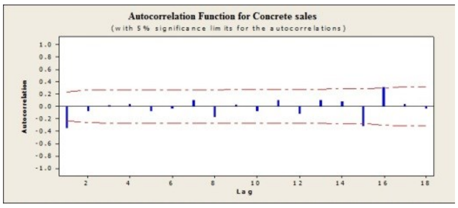
Our results, obtained from a model of firmly fixed time series with some missing observations, will be compared with the classical results, obtained when all observations are present, in this investigation.

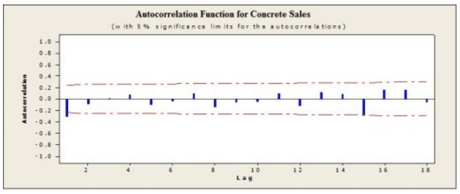
Let $\eta_a(t) = \varphi_a(t) X_a(t)$, $a = 1,2,...,r$, where $X_a(t),(t = 0,\pm 1,...)$ is a firmly fixed s-vector valued time series, and $\varphi_a(t)$ is a Bernoulli sequence of stochastically independent random variables be stochastically independent of $X_a(t)$. We assume that the data $X_a(t),(t = (1,2,...,T]$, which is the average of the monthly concrete sales for which all observations are available, is available with some missing observations, $\varphi = 1$, $\eta_a(t) = X_a(t)$, The table (5.1.2) compares the results with and without missing observations, assuming that some observations are randomly missing $\varphi_a(t) = 0$ in the classical scenario.

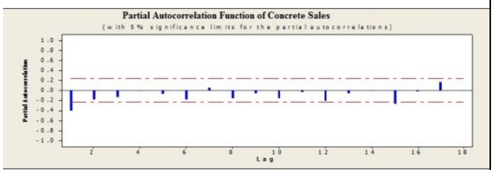
Table (5.1.2) Analysis of concrete sales with and without missing data

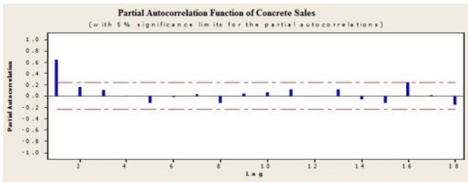
<table>
<tr><td>without missing data</td><td>with missing data</td></tr>
<tr><td>


The monthly average concrete sales
</td><td>


The monthly average concrete sales
</td></tr>
<tr><td>


ACF The monthly average concrete sales
</td><td>


ACF of The monthly average concrete sales
</td></tr>
<tr><td>


PACF of The monthly average concrete sales
</td><td>


PACF of The monthly average concrete sales
</td></tr>
<tr><td>

ARIMA(1,1,1)

Final Estimates of Parameters
Type Coef SE Coef T P
AR 1 0.5185 0.1081 4.75 0.000
AM 1 0.9826 0.0524 21.06 0.000
Residuals: SS = 18165211322 (back forecasts excluded)
MS = 224782472, DF = 68
Modified Box-Pierce (Ljung-Box) Chi-Square statistic
Lag 12 24 36 48
Chi-Square 7.01 22 37.2 50.3
DF 9 21 33 45
P-Value 0.557 0.364 0.297 0.295
</td><td>

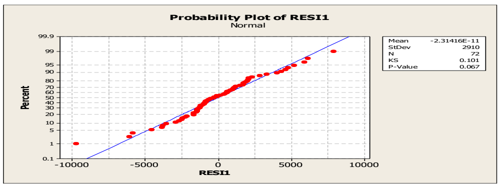
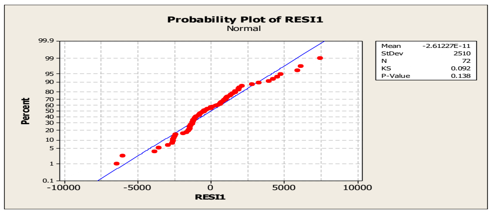
ARIMA(1,1,1)

Final Estimates of Parameters
Type Coef SE Coef T P
AR 1 0.5516 0.1700 5.42 0.000
AM 2 0.9376 0.0488 21.57 0.000
Residuals: SS = 15881423120 (back forecasts excluded)
MS = 232289248, DF = 68
Modified Box-Pierce (Ljung-Box) Chi-Square statistic
Lag 12 24 36 48
Chi-Square 7.4 19.8 35.5 47.6
DF 9 21 33 45
P-Value 0.641 0.562 0.384 0.326
</td></tr>
</table>

5.1.3. Analysis the Correlation between Concrete Sales and Monthly Production

Now, we turning to compare our results—a regression model between monthly averages of Production and monthly averages of concrete sales with some missing observations—against the classical results, where all observations are available, for the two scenarios indicated in the table below (5.1.3)

Table (5.1.3) Comparing the regression analysis with and without missing data

Without missing data	With missing data
The regression equation is Concrete Sales = 3633 + 0.773 Production Predictor Coef SE Coef T P Constant 3633 610.2 5.05 0.000 Production 0.773 0.01438 54.56 0.000 S = 2813.61 R-Sq = 96.7% R-Sq(adj) = 96.7% Analysis of Variance Source DF SS MS F P Regression 1 26511160296 26651166902 2985.7 0.000 Residual Error 70 640151719 85961 0.000 Total 71 27151312015 Durbin-watson statistic =1.67188	The regression equation is Concrete Sales = 3322 + 0.734 Production Predictor Coef SE Coef T P Constant 3322 502.6 5.91 0.000 Production 0.734 0.01027 63.845 0.000 S = 2391.61 R-Sq = 97.9% R-Sq(adj) = 99.9% Analysis of Variance Source DF SS MS F P Regression 1 27655793510 26575791530 4023.66 0.000 Residual Error 70 474238847 6893119 Total 71 28130032357 Durbin-watson statistic =1.67825
 Normal-graph for standardized Residuals	 Normal-graph for standardized Residuals

5.1.4. Results and Discussion

(1) No significant differences were found between the findings of studying classical time series and those of studying time series with missing observations.

(2) Similar to the case of missing data, the findings of the studied regression model between classical time series X(t), τ (t) satisfied all theoretical, mathematical, and least squares constraints.

6. Conclusion

The analysis of the function of the extended finite CONTINUOUS Fourier transform of the firmly fixed Continuously time series in regular patterns is approximately to the analysis of the function of the extended finite Fourier transform of the firmly fixed Continuously time series in irregular pattern.

REFERENCES

[1] Brillinger D.R., "Asymptotic Characteristics of Spectral Estimates of Second Order, Biometrika56 (2), (2001), 375-390.

[2] Brillinger, D.R., Time Series Data Analysis and Theory (2001).

[3] Dahlhaus, R., "On a Spectral Density Estimate Obtained by Averaging Periodograms", J. Appl. Probab. 22, (1985), 592-610.

[4] ELHASSAIN, A., "Nonparametric Spectral Analysis on Disjoint Segments of Observations", Journal of the Applied Mathematics, Statistics and Informatics (JAMSI), 7 (2011), No.1.

[5] ELHASSAIN, A., "On the Theory of Continuous Time series", Indian. Pure Appl. Math, 45 (3): 297-310, June (2014).

[6] Farag, E.A., and Ghazal, M. A., "Some Statistical Analysis for Continuous Time Stationary Process With Missed Data", Since Journal of Mathematics and statistics, V. 2015, Article ID: Sjms/109, 15 pages, doi: 107237/Sjms/109, 2015.

[7] Farag, E.A., and Ghazal, M. A.," Some properties of The Function Corresponding To Analysis of Time Series", Science Journal of Mathematics and Statistics, V.2016, Article ID: Sjms/112, 6 pages, doi: 10.7237/sjms/112, 2016.

[8] Ghazal, M. A., "On a Spectral Density Estimate on Non-Crossed Intervals Observation", Int. J. Appl. Math. 1(8), (1999), 875-882.

[9] Ghazal, M. A., El-Desokey , A.E., and Alargt, M. A., "Asymptotic Properties Of The Discrete Stability Time Series With Missed Observations Between Two Vector –Valued Stochastic Process", International Research Journal of engineering and technology, (IRJET), 12, pp. 2052-2063, 2017.

[10] Ghazal, M. A., El-Desokey , A.E., and Alargt, M. A., "Statistical Properties Of The Periodogram For Two Vector- Valued Stability Series With Missed Observations", Int. Research Journal of Applied Mathematics and Statistics, 4, (4), 2017.

[11] Ghazal, M. A., and El-hassanein, A., "Periodogram Analysis with Missed Observations", J. Appl. Math. Computing Vol.22, (2006), No.1-2, pp209-222.

[12] Ghazal, M. A., and Farag, E.A., "Estimating The Spectral Density, Autocovariance Function And Spectral Measure Of Continuous-Time stationary Processes, in: Annual Conference", ISSR, Cairo, Vol.33, no. part, 1998, pp.1-20.

[13] Ghazal, M. A., and Farag, E.A.," Some properties of The Expanded Finite Fourier Transform", Statistica, annoLX, no.3, 2000.

[14] Ghazal, M. A., Farag, E.A., and EL-Desokey, A.E, "Some properties of the Discrete Expanded finite Fourier transform with missed Observations", Vol.40, No.3, 30 September (2005), pp. 887-902.

[15] Ghazal, M. A., Mokaddis, G.S., and El-Desokey, , A.E., "Spectral analysis of firmly stationary Continuous time series", Journal of Mathematical Sciences, Vol.3,No.1 (2009).

[16] Mokaddis, G.S., Ghazal, M.A. and El-Desokey, A.E. "Asymptotic Properties of Spectral Estimates of Second-Order with Missed Observations", Journal of Mathematics and statistics 6(1):10-16, (2010).

حول تحويل فوريير المحدود الموسعة لسلسلة زمنية مستمرة ثابتة مع وجود بيانات مفقودة

د. أميرة إبراهيم الدسوقى؛ أ. على محمد بن عروس؛ د. محمد أبو الفتوح غزال

الملخص: يبحث هذا البحث في خصائص تحويل فوريير المحدودة الموسعة لسلسلتين زمنيتين ذات قيمة متجهية ثابتة. يمكن تصور كلتا السلسلتين الزمنيتين على أنهما مستمرتان. هذه إحدى الخصائص التي تسمح لهم بالمرونة. يوفر تحويل فوريير المحدودة الموسعة الأساس لهذه الدراسة حيث انه أثناء دراسة هذه الخصائص للسلسلتين الزمنيتين باستخدام تحويل فوريير الموسعة، يتم إجراء تحليل للبيانات، يتم خلال الدراسة استخدام أنماط السلاسل الزمنية الاعتيادية والغير اعتيادية ودراسة خصائصها أثناء إجراء التحليل.

الكلمات المفتاحية: تحويل فوريير المحدودة- البيانات المفقودة- السلاسل الزمنية المستمرة الثابتة- توزيع ويشارت- نوافذ البيانات