# Stationary Analysis Fluid Model Driven by an $M^{[k]}/M/1$ Queue


**Presented by**

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**Abstract**

A Fluid queue is a mathematical model used to describe fluid level in a reservoir subject to randomly determined periods of filling and emptying the system without interruption called a buffer, according to a randomly varying rate regulated by an external stochastic environment. Such fluid queues are used as a mathematical tool for modeling, for example, to approximate discrete models, model the spread of wildfires in ruin theory and to model high speed data networks, a router, computer networks including call admission control, traffic shaping and modeling of TCP and production and inventory systems. The fluid from the first phase (i.e, fluid output of the $M^{[k]}/M/1$ queue) goes to the second phase represents a buffer with a constant leak rate c. We always assume that service rate is greater than buffer, $\mu > c$. In this paper, a fluid queue driven by infinite queue with fixed –size batch arrivals. The generating function technique is used to obtain the expressions for steady-state distribution of both the buffer content and stationary state probabilities of background birth-death process. Hence, the performance measures are computed whereas the server utilization analysis and mean buffer content are investigated. . In addition, some numerical results are provided to illustrate the effect of various parameters on the distribution of the buffer content.

***Keywords*****:** Fluid Queue; $M^{[k]}/M/1$ Queue; Fixed-size Batch Arrivals; Buffer Content; Generating Function Method.

**Mathematics Subject Classification (2000):** 90B22, 60K25, 68M20, 44A10.

## 1. Introduction

Fluid queueing systems with infinite space are considered one of the important and useful tools which contribute to modern applications. Indeed, several situations arise in which these phenomena occur and need to be examined. Traffic shaping, modeling of transport control protocol, inventory and production systems are known examples of these systems. For more details, see Adan and Resing (1996), Anick et al. (1982), Barbot (2002), Kulkarni (1997), and Mitra (1988) and references therein.

Fluid queues driven by an infinite queueing system have been studied by many authors. Closed form expressions in terms of modified Bessel functions are obtained by Sherif Ammar (2015) for the model in fluid queues driven by an $M/M/1$ queue. In addition, the spectral expansion is used to obtain the distribution of the exact buffer occupancy. In Darwiesh et al. ( 2021 ), a fluid queue having an infinite buffer capacity is considered for the cases where it is filled and depleted by a fluid at constant rates. The simple series form is applied to the joint stationary distribution of the buffer occupancy. Parthasarathy and Vijyashree ( 2002 ) gave the solutions of a fluid queue fed by an $M/M/1$ when a general boundary condition is assumed. They compared the results with those of the model studied by Adan and Resing (1996 ). Mao et al .(2010) investigated a fluid model which is driven by a simple queue with single and multiple exponential vacations. In this case, a system of first order homogeneous linear differential equations is obtained to describe the distribution of the trivariate process of external environment and buffer content. This system has been solved utilizing the standard spectral method. In order to study the buffer occupancy distribution for high-speed networks, the authors in Viswanathan et al. (2010) used two independent finite state birth-death processes in a fluid queue model. Furthermore, a plethora of studies has discussed the fluid queues driven by birth-death process and including vacations and disasters (see for example, Ammar (2014) and Mao et al. (2010, 2011, 2012).

In this paper, the authors analyze a fluid model subject to simple queue drive with fixed-size batch arrivals. More specifically, the generating function for the system is found for the steady-state distribution of buffer occupancy in Section 2. The solution is obtained using the power series in Section 3. The performance measures, such as mean buffer content and server utilization, are determined in Section 4. Finally, the numerical illustrations and conclusions are presented in Sections 5 and 6.

## 2. Model Description

Suppose that there is a fluid model that is driven by a single server queueing process with service rates and state-dependent arrival. The model is formed from an infinitely large buffer for which the fluid flow is regulated via the state of the background queueing process. Denoting the background queuing process by $\{X(t), t \geq 0\}$ which has values in $\Omega = \{0, 1, 2, ...\}$. In particular, $X(t)$ refers to the number of customers in the system at time $t$. Let $\lambda_j$ and $\mu_j$ denote the mean arrival and service rates, respectively, where there are $j$ customers in the queue. The arrivals are supposed to be of Poisson fashion whereas the service times are exponentially distributed. In addition, the service discipline is supposed to be first is first out (FIFO). Denoting by $C(t)$, the content of fluid in the buffer at time $t$. When the system is in state $j$, the buffer content changes at the net input rate $r_j = r$. Equivalently, it is the input rate minus the output rate, that implies that it can take both positive or negative values. For the case where the buffer is empty and the Markov Process is in a state $0$ with rate $r_0 < 0,$ the buffer will still empty. Assume that $\mu_0 = 0$ and $\mu_j = \lambda_j = 0,$ if $j \notin \Omega.$ It is clear that the 2-dimensional process $\{X(t), C(t), t \geq 0\}$ establishes a Markov process with unique stationary distribution under a suitable stability condition.

Therefore, the following differential equation describes the rate of change in $C(t),$

$$\frac{dC(t)}{dt} = \begin{cases} 0, & if\ C(t) = 0, and\ X(t) = 0 \\ r_0, & if\ C(t) = 0, and\ X(t) > 0, \\ r, & if\ C(t) > 0. \end{cases} \tag{1}$$

The limit distribution for $C(t)$ exists as $t \to \infty,$ and the stationary net input rate must be negative. In other words,

$$d = r_0\,\pi_0 + r\sum_{j=1}^{N} \pi_j = r_0\pi_0 + r(1-\pi_0) < 0, \tag{2}$$

and $\pi_0 = 1-\rho\,,\ \ \pi_j = (\frac{\lambda}{\mu}) \sum_{m=1}^{\min(j,k)} \pi_{j-m}\ \ ,\ \rho = \frac{k\lambda}{\mu}.$

Therefore

$$d = (r_0 - r)(1-\rho) + r \,\text{.}$$

where $\pi_j, j \in \Omega$ are the stationary state probabilities associated with the background birth-death process. Moreover, assume that the above stability conditions are satisfied.

Assuming that

$$F_j(t,x) \equiv \Pr\{X(t) = j,\ C(t) \leq x\},\ j \in \Omega, \qquad t, x \geq 0, \tag{3}$$

and

$$F_j(x) \equiv \lim_{t\to\infty} \Pr\left\{X(t)=j,\ C(t)\le x\right\},\quad j\in\Omega,\quad x\ge 0 \tag{4}$$

The, it can be confirmed that the Kolmogorov forward equations for the Markov process $\{X(t),C(t)\}$ are given by

$$\frac{\partial F_0(t,x)}{\partial t} = -r_0\frac{\partial F_0(t,x)}{\partial x} - \lambda_0 F_0(t,x) + \mu_1 F_1(t,x), \tag{5}$$

$$\frac{\partial F_j(t,x)}{\partial t} = -r\frac{\partial F_j(t,x)}{\partial x} - \left(\lambda_j+\mu_j\right)F_j(t,x) + \mu_{j+1}F_{j+1}(t,x),\ j=1,2,\ldots k-1 \tag{6}$$

$$\frac{\partial F_j(t,x)}{\partial t} = -r\frac{\partial F_j(t,x)}{\partial x} - \left(\lambda_j+\mu_j\right)F_j(t,x) + \lambda_{j-1}F_{j-1}(t,x) + \mu_{j+1}F_{j+1}(t,x),\ j=k,k+1,\ldots. \tag{7}$$

If the process is in equilibrium state, then

$$\partial F_j(t,x)/\partial t \equiv 0,\quad F_j(t,x)\equiv F_j(x).$$

Hence, the above system (5-7) can be reduced to the following system:

$$\frac{dF_0(x)}{dx} = -\frac{\lambda_0}{r_0}F_0(x) + \frac{\mu_1}{r_0}F_1(x), \tag{8}$$

$$\frac{dF_j(x)}{dx} = -\frac{\left(\lambda_j+\mu_j\right)}{r}F_j(x) + \frac{\mu_{j+1}}{r}F_{j+1}(x),\ x\ge 0,\qquad j=1,2,\ldots,k-1 \tag{9}$$

$$\frac{dF_j(x)}{dx} = -\frac{\left(\lambda_j+\mu_j\right)}{r}F_j(x) + \frac{\lambda_{j-1}}{r}F_{j-1}(x) + \frac{\mu_{j+1}}{r}F_{j+1}(x),\ x\ge 0,\qquad j=k,k+1,\ldots \tag{10}$$

The buffer contents increase for positive net input rate of fluid flow for the buffer. The buffer cannot be empty in this case. It follows that the solution to (8-10) should be satisfied by the boundary conditions

$$F_j(0)=0,\qquad j\in\{j\in\Omega : r>0\} \tag{11}$$

$Pr\{C=0\}=F_0(0)=d_0$, for some constant $d_0\ (0<d_0<1)$ (12)

The stationary probability of the empty fluid queue is found by:

$$Pr\{C=0\}=\frac{d}{r_0}=\frac{r_0\pi_0+\sum_{j=1}^{\infty}r\pi_j}{r_0}=\frac{r_0\pi_0+r(1-\pi_0)}{r_0}. \tag{13}$$

Moreover, the next relations are also satisfied

$$F_j(\infty)\equiv\lim_{z\to\infty}F_j(x)=\pi_j, \quad j\in\Omega. \tag{14}$$

## 3. Stationary Solution of Fluid Queue driven by $M^{[k]}/M/1$ Queue

The fluid model studied in previous section is investigated when it has the background process as an $M^{[k]}/M/1$ queue with fixed–size batch arrivals. Let the mean arrival and service rates be $\lambda_j=\lambda$ and $\mu_j=\mu$, respectively.

For $F_j(x)$ assume that $H(z,x)$ represents the moment generating function, $\hat{H}(z,s)$ the Laplace- Stieltjes transform of $F_j(x)$.

$$H(z,x)=\frac{r_0}{r}F_0(x)+\sum_{n=1}^{\infty}z^n F_n(x)\text{ ,with } H(z,0)=\frac{r_0\,d_0}{r}.$$

Multiplying (9) and (10) by $z^j$ and then summing over all integer values of $j$, we get

$$\begin{aligned}\frac{\partial H(z,x)}{\partial x}&=\frac{1}{r}\left[-(\lambda+\mu)+\mu z^{-1}+\lambda z^k\right]H(z,x)\\&+\frac{1}{r}\left[\frac{\mu r_0}{r}(1-z^{-1})-(1-\frac{r_0}{r})(1-z^k)\right]F_0(x)\end{aligned} \tag{15}$$

Now, the solution of (15) is determined as follows

$$\begin{aligned}H(z,x)&=\frac{d_0 r_0}{r}\exp-\frac{1}{r}(\lambda+\mu)x.\exp\frac{1}{r}(\lambda z^k+\mu z^{-1})x\\&+\frac{\mu r_0}{r^2}(1-z^{-1})\int_0^x\exp-\frac{1}{r}(\lambda+\mu)(x-\zeta).\exp\frac{1}{r}(\lambda z^k+\mu z^{-1})(x-\zeta)F_0(\zeta)\,d\zeta\\&-\frac{\lambda}{r}(1-\frac{r_0}{r})(1-z^k)\int_0^x\exp-\frac{1}{r}(\lambda+\mu)(x-\zeta).\exp\frac{1}{r}(\lambda z^k+\mu z^{-1})(x-\zeta)F_0(\zeta)d\zeta\end{aligned} \tag{16}$$

The function $\exp\frac{1}{r}(\lambda z^k + \mu z^{-1})x$ is embedded in the generating function solution (16). It can be written as

$$\exp\frac{1}{r}(\lambda z^k + \mu z^{-1})x = \sum_{n=-\infty}^{\infty} z^n \left(\frac{\lambda}{\mu}\right)^{\frac{n}{k+1}} V_n^{(k)}(\alpha x) \tag{17}$$

with $\alpha = [\mu^k \lambda]^{\frac{1}{k+1}} = \mu\left(\frac{\lambda}{\mu}\right)^{\frac{1}{k+1}}$,

$$V_{-n}^{(k)}(x) = \sum_{l=0}^{\infty} \frac{x^{l(k+1)+n}}{l!(k+n)!} \text{ and } V_n^{(k)}(x) = \sum_{l=\sigma_n}^{\infty} \frac{x^{l(k+1)-n}}{l!(k-n)!}.$$

For $n > 0$ the variable $\sigma_n$ is defined as $\sigma_n = [\frac{n}{k}]$ where the notation $[\frac{n}{k}]$ designates the smallest integer not less than $\frac{n}{k}$.

By substituting from Eq. (17) into Eq. (16) and comparing the coefficients of $z^n$ on both sides of Eq. (16) results in

$$\begin{aligned} F_n(x) &= \frac{d_0 r_0}{r} \exp -\frac{1}{r}(\lambda+\mu)x \left(\frac{\lambda}{\mu}\right)^{\frac{n}{k+1}} V_n^{(k)}(\alpha x) \\ &+ \frac{\mu r_0}{r^2}\left(\frac{\lambda}{\mu}\right)^{\frac{n}{k+1}} \int_0^x \exp\frac{-1}{r}(\lambda+\mu)\zeta \left[ V_n^{(k)}(\alpha\zeta) - \frac{\alpha}{\mu} V_{n+1}^{(k)}(\alpha\zeta)\right] F_0(x-\zeta)\,d\zeta \\ &- \frac{\lambda}{r}\left(1-\frac{r_0}{r}\right)\left(\frac{\lambda}{\mu}\right)^{\frac{n}{k+1}} \int_0^x \exp\frac{-1}{r}(\lambda+\mu)\zeta . \left[ V_n^{(k)}(\alpha\zeta) - \left(\frac{\alpha}{\mu}\right)^{-k} V_{n-k}^{(k)}(\alpha\zeta)\right] F_0(x-\zeta)\,d\zeta, \; n>0 \end{aligned} \tag{18}$$

Obtaining Laplace transform of equations (15) and doing some simplifications, we get

$$\hat{H}(z,s) = \frac{z\left[ d_0 r_0 + \left\{ \frac{\mu r_0}{r}(1-z^{-1}) - \lambda\left(1-\frac{r_0}{r}\right)(1-z^k)\right\} \hat{F}_0(s)\right]}{-\frac{\lambda}{r} z^{k+1} + \left(s + \frac{\lambda+\mu}{r}\right) z - \frac{\mu}{r}} \,. \tag{19}$$

Noting that the denominator of Eq. (19) is a polynomial of degree $k+1$ in $z$ and hence it has $k+1$ zeros. However, the Rouch`s theorem implies that only one zero lies within the unit circle ( say $z_0(s)$ ).

Hence, we get

$$\hat{F}_0(s) = \frac{d_0\, r_0}{\frac{\mu\, r_0}{r}(z_0^{-1}(s)-1) - \lambda(\frac{r_0}{r}-1)(1-z_0^{k+1}(s))} \tag{20}$$

which can be rewritten as

$$\hat{F}_0(s) = \frac{d_0\, r}{\mu} \sum_{n=0}^{\infty} \sum_{i=0}^{n} z_0^{i+1}(s)\; g^{n-i}(s) \tag{21}$$

where $g(s) = \frac{\lambda(r_0 - r)}{\mu\, r_0} \sum_{l=1}^{k} [z_0(s)]^l$

By computing the inverse of Eq. (21), it yields that

$$F_0(x) = \frac{d_0\, r}{\mu} \sum_{n=0}^{\infty} \sum_{i=0}^{n} [z_0(x)]^{*(i+1)} * [g(x)]^{*(n-i)} \tag{22}$$

and $g(x) = \frac{\lambda(r_0 - r)}{\mu\, r_0} \sum_{l=1}^{k} [z_0(x)]^{*l}$

Here *n denotes the n-fold convolution.

Now, employing the method proposed by Luchak (1956, 1958) is used to calculate the inverse Laplace transform in the way that

$$[z_0(x)]^{*\nu} = L^{-1}[z_0(s)]^{\nu} = \frac{\mu\nu}{\tau}\left\{\frac{\tau^{\nu}}{\nu!} + \sum_{n=1}^{\infty} \frac{\theta^n}{n!} \frac{\tau^{n(k+1)\nu}}{(nk+\nu)!}\right\} e^{-(1+\theta)\tau},$$

where $\tau = \frac{\mu x}{r}$ and $\theta = \frac{\lambda}{\mu}$.

Therefore, the closed form expressions for $F_n(x)$ of the two models (18) and (22) are obtained analytically. The stationary distribution of the buffer content is obtained by:

$$F(x) = \lim_{t\to\infty} \Pr(C(t) \le x) = \sum_{j=o}^{\infty} F_j(x)$$

$$F(x) = \frac{d_0 r_0}{r} + (1 - \frac{r_0}{r}) F_0(x) \tag{23}$$

In addition, all the joint steady state probabilities are computed explicitly in terms of a power series function.

*Remark*, the generating function of the fluid queue given with $M^{[k]}/M/1$ queue in steady state can be obtained from that of fluid queue having $M/E_k/1$queue (via replacing $k\mu$ by $\mu$) in Eq.(15) (with $k = 2$ ), see Vijayashree and Anjuka (2016).

## 4. Performance Measures of Fluid Model

In this section, some crucial performance measures are examined. The formulations for these measures are defined as follows:

### 4.1. Server Utilization

The probability that the buffer is non-empty is given by

$$\text{Utilization} = 1 - \sum_{j=0}^{\infty} F_j(0) = 1 - F_0(0). \tag{24}$$

$$\text{or Utilization} = 1 - d_0,\ 0 < d_0 < 1. \tag{25}$$

where

$$d_0 = \frac{d}{r_0} = \frac{(r_0 - r)(1-\rho) + r}{r_0}. \tag{26}$$

Thus, the equilibrium condition of the fluid queue is

$$\rho < 1\ ,\ d < 0 \text{ and } 0 < d_0 < 1.$$

### 4.2. Expected buffer content

The expected buffer content $(C)$ is expressed as:

$$E(C) = \int_0^{\infty} [1 - F(x)]\,dx = \int_0^{\infty} \left[1 - \frac{d_0 r_0}{r} - (1 - \frac{r_0}{r}) F_0(x)\right] dx. \tag{27}$$

## 5. Numerical Example and Observations

For $k=2$, the above equations become

$$F(x)=\frac{d_0\,r_0}{r}+(1-\frac{r_0}{r})F_0(x),$$

$$E(C)=\int_0^\infty[1-F(x)]\,\mathrm{d}x=\int_0^\infty\left[1-\frac{d_0\,r_0}{r}-(1-\frac{r_0}{r})F_0(x)\right]dx.$$

$$F_0(x)=\frac{d_0\,r}{\mu}\sum_{n=0}^{\infty}\sum_{i=0}^{n}[z_0(x)]^{*(i+1)}*[g(x)]^{*(n-i)} \text{ with } g(x)=\frac{\lambda(r_0-r)}{\mu\,r_0}\sum_{l=1}^{2}[z_0(x)]^{*l},$$

$$z_0(x)=\frac{\mu}{r}e^{-(\frac{\lambda+\mu}{r})x}+\sum_{n=1}^{\infty}\frac{\lambda^n}{\mu^n n!(2n+1)!}(\frac{\mu}{r})^{3n+1}x^{3n}\,e^{-(\frac{\lambda+\mu}{r})x},$$

and

$$d_0=\frac{(r_0-r)(\mu-2\lambda)+r\mu}{\mu r_0}.$$

To show the variations of the stationary distribution corresponding to the buffer content and the estimated buffer content for different values of parameters. Figure 1 illustrates the behavior of the buffer content distribution $F(x)$ against the buffer size $x$ for $\lambda=1$, $\mu=4$, $k=2$ and $r_0=-4$, and different values of $r$ . Figure 2 presents the corresponding behavior of the expected buffer content against $\mu$ for the same set of parameter values $k=2$.

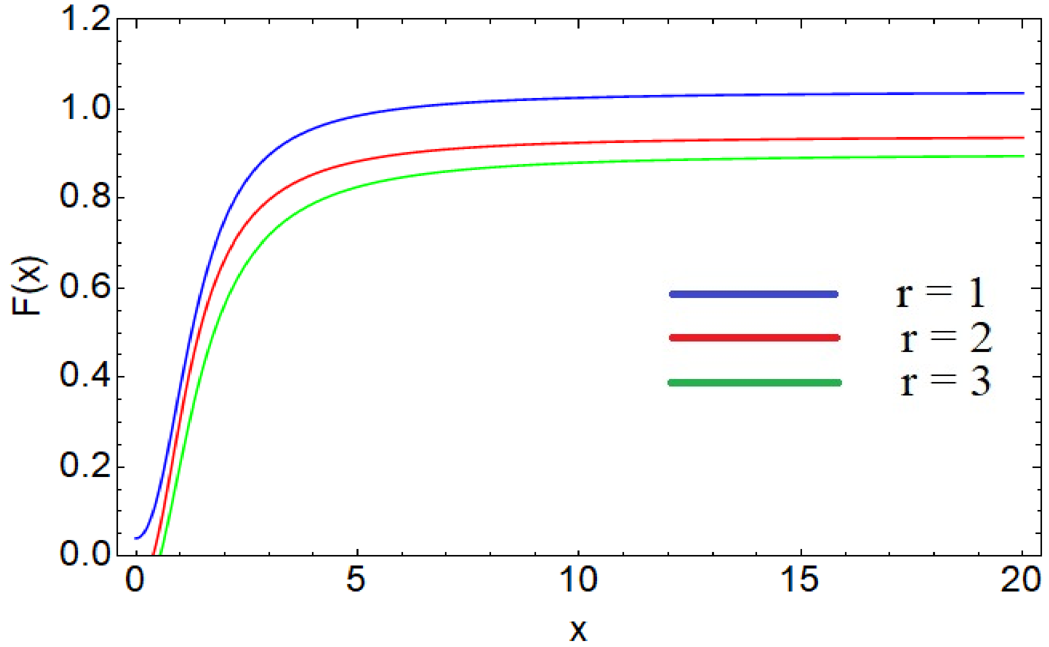


Fig. 1 The buffer content distribution, $F(x)$ vs. the buffer size $x$ for different values of $r$.

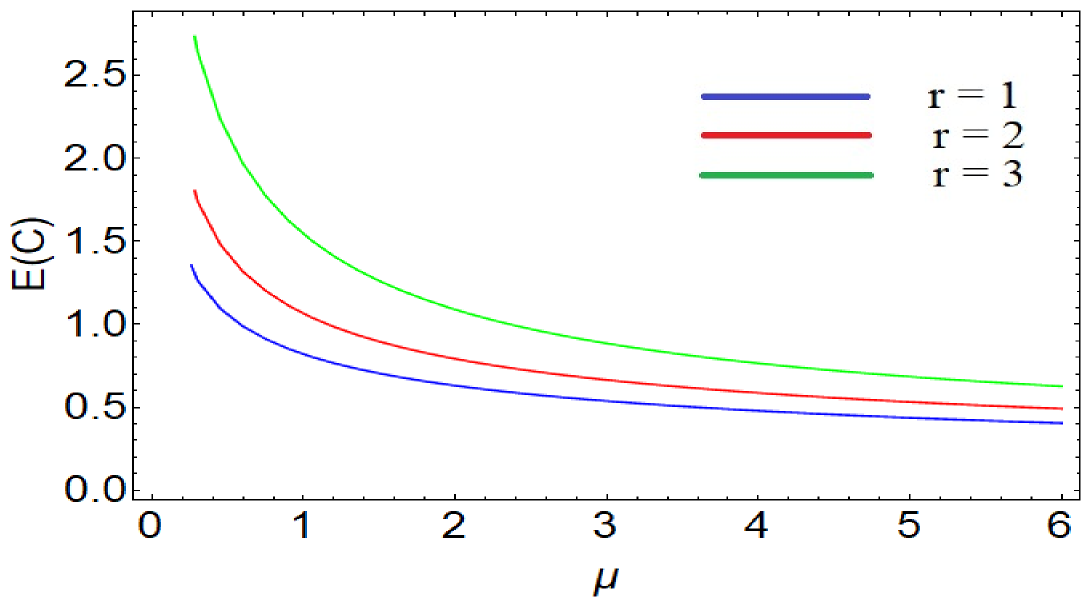


Fig. 2 The expected buffer content $E(C)$ against $\mu$.

## 6. Conclusions

This study investigates a fluid queue model driven by an $M^{[k]}/M/1$ queue with fixed-size batch Poisson arrivals and a single server with exponential service times distribution. Using power series technique, the steady-state distribution of the buffer occupancy is obtained in terms of power series As shown in Figure 1, $F(x)$ is an increasing function provided that the buffer content, $x$ is increased in the way that the distribution of the buffer content decreases with $r$. It is observed that there is a positive mass at $x \to 0$ and $F(x) \to 1$ when $x \to \infty$. Hence, this means that the buffer occupancy has mixed distribution. Also, Figure 2 shows the mean of the stationary buffer content with service rate $\mu$. Note that the curves of $E(C)$ increase as the value of $r$ and $\mu$ decrease. Finally, some performance measures involving server utilization and mean buffer content are attained.

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# Stationary Analysis Fluid Model Driven by an $M^{[k]}/M/1$ Queue

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**الملخص:**

صفوف انتظار السوائل هي نموذج رياضي يستخدم لوصف مستوى السائل في خزان يخضع لفترات محددة عشوائي لملء وتفريغ النظام دون انقطاع يسمى المخزون المؤقت ، وفقًا لمعدل متغير عشوائيًا تنظمه عملية عشوائية خارجية. تُستخدم طوابير السوائل هذه كأداة رياضية للنموذج، على سبيل المثال ، لتقريب النماذج المنفصلة ، ونموذج انتشار حرائق الغابات في نظرية الحرائق ، ونموذج شبكات البيانات عالية السرعة ، وجهاز التوجيه ، وشبكات الكمبيوتر بما في ذلك التحكم في قبول المكالمات ، وتشكيل حركة المرور و النموذج الخاص بالمخزون السلعي وأنظمة الإنتاج والجرد. يمثل السائل من المرحلة الأولى (أي خروج السائل لصف الانتظار $M^{[k]}/M/1$) إلى المرحلة الثانية مخزنًا مؤقتًا بمعدل تسرب ثابت . نفترض دائمًا أن معدل الخدمة أكبر من المخزن المؤقت ( $\mu > c.$ ). في هذا البحث ، صفوف الانتظار مائعة المولدة بطابور لانهائي مع وصول دفعة ذات حجم ثابت. يتم استخدام تقنية وظيفة التوليد للحصول على تعبيرات لتوزيع الحالة المستقرة لكل من كمية المخزون المؤقت واحتمالات حالة الاتزان لعملية الولادة والوفاة المعروفة. ومن ثم ، يتم حساب مقاييس الأداء وكذلك يمكن التحقيق من تحليل استخدام الخام ومتوسط محتوى المخزن المؤقت. . بالإضافة إلى ذلك ، تم الحصول على بعض النتائج العددية لتوضيح تأثير المعلمات المختلفة على توزيع محتوى المخزن المؤقت و كمية المخزون المتوقعة.

**الكلمات المفتاحية:** نموذج الطابورالسائل ( Fluid Queue $M^{[k]}/M/1$ ) ؛ عملية الوصول في دفعات ثابتة ؛ كمية المخزون؛ اسلوب الدالة المولدة Generating Function Method.