Electronic Journal of Mathematical Analysis and Applications
Vol. 10(2) July 2022, pp. 305-312.
ISSN: 2090-729X(online)
http://math-frac.org/Journals/EJMAA/

---

q-QUASI-2-ISOMETRIC COMPOSITION OPERATORS

E. SHINE LAL, T. PRASAD AND V.DEVADAS

Abstract. In this paper we characterize q-quasi-2-isometric and $(2, q)$-partial-isometric composition operators on L^2 space.

1. Introduction and Preliminaries

Let $\mathcal{H}$ be an infinite dimensional separable complex Hilbert space and $B(\mathcal{H})$ denote the algebra of all bounded linear operators acting on $\mathcal{H}$. An operator $T \in B(\mathcal{H})$ is said to be m-isometric if

$$\sum_{k=0}^{m}(-1)^k \binom{m}{k} T^{*m-k}T^{m-k} = 0$$

for some integer $m \geq 1$ ([1]). Inparticular 2-isometric operators has been studied extensively by Agler and Stankus ([1]), Richter ([17]) and Hillings ([8]). An operator $T \in B(\mathcal{H})$ is said to be a q-quasi-m-isometry if

$$T^{*q}\left(\sum_{k=0}^{m}(-1)^k \binom{m}{k} T^{*m-k}T^{m-k}\right)T^q = 0,$$

where q is a positive integer ([11, 12, 13]). It is evident that if T is an m-isometry, then T is a q-quasi-m-isometry.

An operator $T \in B(\mathcal{H})$ is called (m, q)-partial isometry or q-partial-m-isometry if

$$T^{q}\left(\sum_{k=0}^{m}(-1)^k \binom{m}{k} T^{*m-k}T^{m-k}\right) = 0.$$

A detailed study of this class can be found in ([10]). Let $(X, \mathcal{F}, \mu)$ be a σ finite measure space and T be a function from X into itself such that $T^{-1}(S) \in \mathcal{F}$, for all $S \in \mathcal{F}$. If T is a nonsingular measurable transformation on $(X, \mathcal{F}, \mu)$ and if the Radon-Nikodym derivative $d\mu T^{-1}/d\mu$ denoted by h is essentially bounded, then the composition operator C on $L^2(\mu)$ induced by T is given by $Cf = (f \circ T)$, $f \in L^2(\mu)$. Let $L^\infty(\mu)$ denote the space of all essentially bounded complex valued

---

2000 *Mathematics Subject Classification.* 47B20, 47B38.
Key words and phrases. quasi-2 isometric operator, (2,q) partial isometry, weighted composition operator, conditional expectation.
Submitted Jan. 31, 2022. Revised April 6, 2022.

measurable functions on X. For $\phi \in L^\infty(\mu)$, the multiplication operator M_ϕ on $L^2(\mu)$ is given by $M_\phi f = \phi f,\ \ f \in L^2(\mu)$.

Let π be an essetially bounded complex valued measurable function on X. The weighted composition operator W on $L^2(\mu)$ induced by T and π is given by $Wf = \pi(f \circ T),\ \ f \in L^2(\mu)$. Let $\pi_k = \pi(\pi \circ T)(\pi \circ T^2).....(\pi \circ T^{k-1})$. Then we have $W^k f = \pi_k (f \circ T)^k,\ \ f \in L^2(\mu)$. We refer the reader to ([15]) and ([21]) for general properties of composition operators.

Let T be a nonsingular measurable transformation on $(X, \mathcal{F}, \mu)$. Then $T^{-1}\mathcal{F}$ is a σ-subalgebra of $\mathcal{F}$ and $L^2(X, T^{-1}\mathcal{F}, \mu)$ is a closed subspace of the Hilbert space $L^2(X, \mathcal{F}, \mu)$. The conditional expectation operator associated with $T^{-1}\mathcal{F}$ is an operator defined for all non-negative measurable functions f on X and $f \in L^2(X, \mathcal{F}, \mu)$. For each f in the domain of E, $E(f)$ is the unique $T^{-1}\mathcal{F}$ measurable function satisfying

$$\int_S f d\mu = \int_S E(f) d\mu, \text{ for all } S \in T^{-1}\mathcal{F}.$$

Note that E is an orthogonal projection of $L^2(X, \mathcal{F}, \mu)$ onto $L^2(X, T^{-1}\mathcal{F}, \mu)$. We denote the conditional expectation associated with $T^{-n}\mathcal{F}$ by E_n. If $T^{-n}\mathcal{F}$ is purely atomic σ-subalgebra of $\mathcal{F}$ generated by the atoms $\{A_k\}_{k \geq 0}$, then

$$E_n(f|T^{-n}\mathcal{F}) = \sum_{k=0}^{\infty} \frac{1}{\mu(A_k)} \left(\int_{A_k} f d\mu \right) \chi_{A_k}.$$

We refer the reader to ([3, 7, 9, 16]) for more details on the properties of conditional expectation.

Measure-theoretic characterizations for some non-normal class of composition operators have been studied by Burnap et al. ([2]) and also by Emamalipour et al. ([5]). In this paper, we focus on q-quasi-2-isometric composition operators and (2,q)-partial isometric composition operators on $L^2(\mu)$.

2. q-quasi-2-isometric composition operators

In this section, we study q-quasi-2-isometric composition operators on $L^2(\mu)$. Let h_k denote the Radon-Nikodym derivative of the measure $\mu(T^k)^{-1}$ with respect to μ and $\mathcal{R}(C)$ denote the range of the composition operator C.

Proposition 2.1. *([6]) Let P denote the projection of $L^2(\mu)$ onto $\overline{\mathcal{R}(C)}$. Then*
(a) $C^*Cf = hf$ and $CC^*f = (h \circ T)Pf$, for all $f \in L^2(\mu)$.
(b) $\overline{\mathcal{R}(C)} = \{f \in L^2(\mu) : f$ is $T^{-1}\mathcal{F}$ measurable$\}$.

Theorem 2.2. *C is q-quasi-2-isometry if and only if $h_{q+2} - 2h_{q+1} + h_q = 0$.*

Proof. By the definition, C is a q-quasi-2-isometry if and only if

$$C^{*q+2}C^{q+2} - 2C^{*q+1}C^{q+1} + C^{*q}C^q = 0.$$

By Proposition (2.1), we have $C^*Cf = M_h f$ and $C^{*q}C^q f = M_{h_q} f$. Since $C^{*q+2}C^{q+2} f = M_{h_{q+2}} f$ and $C^{*q+1}C^{q+1} f = M_{h_{q+1}} f$, it follows that C is q-quasi-2 isometry if and only if

$$M_{h_{q+2}} f - 2M_{h_{q+1}} f + M_{h_q} f = 0,$$

for all $f \in L^2(\mu)$. Hence C is q-quasi-2-isometry if and only if $h_{q+2} - 2h_{q+1} + h_q = 0$. □

Burnap et al. ([2]) studied some examples to show that composition operators can separate almost all weak hyponormality classes. Now we give an example in a similar manner for q- quasi-2-isometric composition operators.

Example 2.3. Let $X = \mathbb{N} \cup \{0\}$, $\mathcal{F} = P(X)$. For any fixed $n \in \mathbb{N}$, μ is a measure defined by

$$\mu(A) = \sum_{k \in A} m_k, \ A \in \mathcal{F}$$

where m_k is the k-th term of $m = (\underbrace{1, 1, 1, 1, \ldots, 1}_{(n+1) \text{ terms}}, c_1, \ldots, c_n, c_1^2, \ldots, c_n^2, c_1^3, \ldots, c_n^3, \ldots)$, a sequence of nonnegative real numbers. Let $T{:}X \to X$ defined by

$$T(k) = \begin{cases} 0, & k = 0, 1, 2, 3, \ldots, n \\ k - n, & k \geq n + 1. \end{cases}$$

Then

$$T^q(k) = \begin{cases} 0, & k = 0, 1, 2, \ldots, qn \\ k - qn, & k \geq qn + 1. \end{cases}$$

$$T^{q+1}(k) = \begin{cases} 0, & k = 0, \ldots, (q+1)n \\ k - (q+1)n, & k \geq (q+1)n + 1. \end{cases}$$

$$T^{q+2}(k) = \begin{cases} 0, & k = 0, \ldots, (q+2)n \\ k - (q+2)n, & k \geq (q+2)n + 1. \end{cases}$$

Note that $T^{-q}\mathcal{F}$ is generated by $\{0, 1, 2, 3, \ldots, qn\}, \{qn+1\}, \{qn+2\}, \ldots$, $T^{-(q+1)}\mathcal{F}$ is generated by $\{0, 1, 2, 3, \ldots, (q+1)n\}, \{(q+1)n+1\}, \{(q+1)n+2\}, \ldots$, and $T^{-(q+2)}\mathcal{F}$ is generated by $\{0, 1, 2, 3, \ldots, (q+2)n\}, \{(q+2)n+1\}, \ldots$.
Now

$$h(0) = \frac{\sum_{i=0}^{n} m_i}{m_0} = n + 1, h(1) = \frac{m_{n+1}}{m_1} = c_1, \ldots, h(n) = \frac{m_{2n}}{m_2} = c_n.$$

Therefore, $h(k) = \frac{\mu T^{-1}(\{k\})}{\mu\{k\}} = \begin{cases} n+1, & k = 0 \\ c_1, & k = mn + 1, m \geq 0. \\ c_2, & k = mn + 2, m \geq 0. \\ \vdots & \\ c_n, & k = mn + n, m \geq 0. \end{cases}$

Similarly we can find h_q, h_{q+1} and h_{q+2} as follows:

$$h_q(k) = \begin{cases} (n+1) + \sum_{i=1}^{n} c_i + \sum_{i=1}^{n} c_i^2 + \ldots + \sum_{i=1}^{n} c_i^{q-1}, & k = 0. \\ c_1^q, & k = mn + 1, m \geq 0. \\ c_2^q, & k = mn + 2, m \geq 0. \\ \vdots & \\ c_n^q, & k = mn + n, m \geq 0. \end{cases}$$

$$h_{q+1}(k) = \begin{cases} (n+1) + \sum_{i=1}^{n} c_i + \ldots + \sum_{i=1}^{n} c_i^q, & k = 0. \\ c_1^{q+1}, & k = mn + 1, m \geq 0. \\ c_2^{q+1}, & k = mn + 2, m \geq 0. \\ \vdots & \\ c_n^{q+1}, & k = mn + n, m \geq 0. \end{cases}$$

$$h_{q+2}(k) = \begin{cases} (n+1) + \sum_{i=1}^{n} c_i + \ldots + \sum_{i=1}^{n} c_i^{q+1}, & k = 0. \\ c_1^{q+2}, & k = mn+1, m \geq 0. \\ c_2^{q+2}, & k = mn+2, m \geq 0. \\ \vdots & \\ c_n^{q+2}, & k = mn+n, m \geq 0. \end{cases}$$

Therefore, C is a q-quasi-2-isometry if and only if

$$\begin{aligned} (n+1) + \sum_{i=1}^{n} c_i + \ldots + \sum_{i=1}^{n} c_i^{q+1} - 2\left((n+1) + \sum_{i=1}^{n} c_i + \ldots + \sum_{i=1}^{n} c_i^{q}\right) + \\ \left[(n+1) + \sum_{i=1}^{n} c_i + \ldots + \sum_{i=1}^{n} c_i^{q-1}\right] = 0 \\ c_1^3 - 2c_1^2 + c_1 = 0 \\ c_2^3 - 2c_2^2 + c_2 = 0 \\ \vdots \\ c_n^3 - 2c_n^2 + c_n = 0 \end{aligned}$$

Hence C is a q-quasi-2 isometry if and only if $c_1 = 0, 1$; $c_2 = 0, 1; \ldots; c_n = 0, 1$.

Note that if $n = 2$ and $(m_k) = (1, 1, 1, c_1, c_2, c_1^2, c_2^2, c_1^3, c_2^3, \ldots)$, it is evident that C is 2-isometry for $c_1 = 1, c_2 = 1$. But C is not a 2-isometry for $c_1 = 0, c_2 = 0$; $c_1 = 0, c_2 = 1$; $c_1 = 1, c_2 = 0$.

An operator T on a Hilbert space is q-quasi-2-expansive if $T^{*(q+2)}T^{(q+2)} - 2T^{*(q+1)}T^{(q+1)} + T^{*q}T^q \leq 0$ ([19, 18, 20]). Example (2.3) is a q-quasi-2-expansive composition operator for $c_1 = 0, 1$; $c_2 = 0, 1 \ldots$; $c_n = 0, 1$.

3. q-quasi-2-isometric weighted composition operators

In this section, we characterize q-quasi-2-isometric weighted composition operators. Let T be a nonsingular measurable transformation on X.

Proposition 3.1. *([3]) If W is the weighted composition operator induced by T and π on $L^2(\mu)$, then the following statements hold.*
(i) $W^*W(f) = hE(\pi^2) \circ T^{-1}(f),\ f \in L^2(\mu)$.
(ii) For each $k \in \mathbb{N},\ W^{*k}W^k(f) = h_k E_k(\pi_k^2) \circ T^{-k}(f),\ f \in L^2(\mu)$, where $\pi_k = \pi(\pi \circ T)(\pi \circ T^2) \ldots (\pi \circ T^{k-1})$.

Theorem 3.2. *W is q-quasi-2-isometry if and only if*
$h_{q+2}E_{q+2}(\pi_{q+2}^2) \circ T^{-(q+2)} - 2h_{q+1}E_{q+1}(\pi_{q+1}^2) \circ T^{-(q+1)} + h_q E_q(\pi_q^2) \circ T^{-q} = 0.$

Proof. Suppose that W is a weighted composition operator induced by π and T on $L^2(\mu)$. Then W is q-quasi-2-isometry if and only if

$$W^{*(q+2)}W^{q+2} - 2W^{*(q+1)}W^{q+1} + W^{*q}W^q = 0.$$

By proposition (3.1), we get $W^{*(q+2)}W^{q+2} = h_{q+2}E_{q+2}(\pi_{q+2}^2) \circ T^{-(q+2)}$, $W^{*(q+1)}W^{q+1} = h_{q+1}E_{q+1}(\pi_{q+1}^2) \circ T^{-(q+1)}$ and $W^{*q}W^q = h_q E_q(\pi_q^2) \circ T^{-q}$. Hence W is q-quasi-2-isometry if and only if

$$h_{q+2}E_{q+2}(\pi_{q+2}^2) \circ T^{-(q+2)} - 2h_{q+1}E_{q+1}(\pi_{q+1}^2) \circ T^{-(q+1)} + h_q E_q(\pi_q^2) \circ T^{-q} = 0.$$

□

Example 3.3. Let $\pi = (1, 0, 1, 0, 1, \ldots) \in L^\infty(\mu)$ and $X = \mathbb{N} \cup \{0\}$. If $\mathcal{F} = P(X)$ and μ is a measure defined by

$$\mu(A) = \sum_{k \in A} m_k, \ A \in \mathcal{F}$$

where m_k is the k-th term of $m = (1, 1, 1, c, d, c^2, d^2, c^3, d^3, \ldots)$, a sequence of non-negative real numbers, then $(X, \mathcal{F}, \mu)$ is a σ-finite measure space. Let T be a measurable non singular transformation on X defined by

$$T(k) = \begin{cases} 0, & k = 0, 1, 2 \\ k - 2, & k \geq 3. \end{cases}$$

Then weighted composition operator $W(f) = \pi(f \circ T)$ is of 1-quasi-2-isometry if and only if

$$h_3 E_3(\pi_3^2) \circ T^{-3} - 2h_2 E_2(\pi_2^2) \circ T^{-2} + hE(\pi^2) \circ T^{-1} = 0.$$

Given that $\pi = (1, 0, 1, 0, \ldots)$. Then $\pi^2 = (1, 0, 1, 0, \ldots)$. Now, $\pi_2 = \pi(\pi \circ T) = (1, 0, 1, 0, \ldots)$, $\pi_2^2 = (1, 0, 1, 0, \ldots)$, $\pi_3 = \pi(\pi \circ T)(\pi \circ T^2) = (1, 0, 1, 0, \ldots)$ and $\pi_3^2 = (1, 0, 1, 0, \ldots)$. Hence

$$E(\pi^2 | T^{-1}\mathcal{F}) = \sum_{n=0}^{\infty} \frac{1}{\mu(S_n)} \left(\int_{S_n} \pi^2 d\mu \right) \chi_{S_n},$$

where S_n denote the atoms of $T^{-1}(\mathcal{F})$. Thus

$$E(\pi^2 | T^{-1}\mathcal{F}) = (\frac{m_0 + m_2}{m_0 + m_1 + m_2}, \frac{m_0 + m_2}{m_0 + m_1 + m_2}, \frac{m_0 + m_2}{m_0 + m_1 + m_2}, 0, 1, 0, 1, \ldots).$$

Hence

$$E(\pi^2) = (\frac{2}{3}, \frac{2}{3}, \frac{2}{3}, 0, 1, 0, 1, \ldots).$$

Now

$$E_2(\pi_2^2 | T^{-2}\mathcal{F}) = \sum_{n=0}^{\infty} \frac{1}{\mu(S_n)} \left(\int_{S_n} \pi_2^2 d\mu \right) \chi_{S_n},$$

where S_n denote the atoms of $T^{-2}(\mathcal{F})$. Then

$$E_2(\pi_2^2 | T^{-2}\mathcal{F}) = (p, p, p, p, p, 0, 1, 0, 1, \ldots),$$

where

$$p = \frac{m_0 + m_2 + m_4}{m_0 + m_1 + m_2 + m_3 + m_4} = \frac{2 + d}{3 + c + d}$$

and

$$E_3(\pi_3^2 | T^{-3}\mathcal{F}) = (q, q, q, q, q, q, q, 0, 1, 0, 1, \ldots,)$$

where

$$q = \frac{m_0 + m_2 + m_4 + m_6}{m_0 + m_1 + m_2 + m_3 + m_4 + m_5 + m_6} = \frac{2 + d + d^2}{3 + c + d + c^2 + d^2}.$$

Therefore W is a quasi -2-isometry if and only if it satisfies the following system of equations

$$\begin{aligned} 2+d+d^2-2(2+d)+2=0 \\ \frac{c^3(2+d+d^2)}{3+c+d+c^2+d^2}-\frac{2c^2(2+d)}{3+c+d}+\frac{2c}{3}=0 \\ \frac{d^3(2+d+d^2)}{3+c+d+c^2+d^2}-\frac{2d^2(2+d)}{3+c+d}+\frac{2d}{3}=0. \end{aligned} \qquad (1)$$

From (1), W is a quasi-2- isometry if and only if $c=0, d=0$ and $c=1.5, d=0$

4. $(2,q)$-PARTIAL ISOMETRIC COMPOSITION OPERATORS

In this section, we characterize $(2,q)$-partial isometric composition operators and give an example.

Theorem 4.1. *C is $(2,q)$-partial isometry if and only if $h_2 f\circ T^q-2hf\circ T^q+f\circ T^q=0$, for all $f\in L^2(\mu)$.*

Proof. By the definition, C is $(2,q)$-partial isometry if and only if

$$C^q\left(C^{*2}C^2-2C^*C+I\right)(f)=0.$$

By Proposition (2.1), we obtain $C^{*2}C^2-2C^*C+I=h_2-2h+1$. Therefore, C is a $(2,q)$-partial isometry if and only if $C^q\left(h_2-2h+1\right)(f)=0$ and hence C is a $(2,q)$-partial isometry if and only if $h_2 f\circ T^q-2hf\circ T^q+f\circ T^q=0$, for all $f\in L^2(\mu)$. □

.

Example 4.2. Let $X=\mathbb{N}\cup\{0\}$ and $\mathcal{F}=P(X)$. If μ is a measure defined by

$$\mu(A)=\sum_{k\in A} m_k,\ A\in\mathcal{F}$$

where m_k is the k-th term of $m=(1,1,1,c,d,c^2,d^2,c^3,d^3,\ldots)$, a sequence of non-negative real numbers, then $(X,\mathcal{F},\mu)$ is a σ-finite measure space. Let T be a measurable non singular transformation on X defined by

$$T(k)=\begin{cases} 0, & k=0,1,2 \\ k-2, & k\geq 3. \end{cases}$$

Note that $T^{-1}\mathcal{F}$ is generated by $\{0,1,2\},\{3\},\{4\},\ldots$ and $T^{-2}\mathcal{F}$ is generated by $\{0,1,2,3,4\},\{5\},\{6\},\ldots$ Then the Radon -Nykodym derivatives are given by

$$h(k)=\frac{\mu T^{-1}(\{k\})}{\mu\{k\}}=\begin{cases} 3, & k=0 \\ c, & k=2m+1, m\geq 0 \\ d, & k=2m+2, m\geq 0. \end{cases}$$

$$h_2(k)=\frac{\mu T^{-2}(\{k\})}{\mu\{k\}}=\begin{cases} 3+c+d, & k=0 \\ c^2, & k=2m+1, m\geq 0 \\ d^2, & k=2m+2, m\geq 0. \end{cases}$$

From Theorem (4.1), C is a $(2,1)$-partial isometry if it satisfies the following system of equations

$$3 + c + d - 5 = 0$$
$$c^2 - 2c + 1 = 0$$
$$d^2 - 2d + 1 = 0.$$

Hence C is a $(2,1)$-partial isometry if $c = 1$ and $d = 1$.

Acknowledgement: The authors would like to express sincere thanks to the referees for helpful comments and suggestions. The second author is supported by seed money project grant UO.No. 11874/2021/Admn, University of Calicut.

References

[1] J. Agler and M. Stankus, “m-isometric transformations of Hilbert Space I,” Integral Equations Operator Theory 21, 4, 383-429, 1995.
[2] C. Burnap, IL Bong Jung and A. Lambert, “Separating Partial Normality Classes with Copmostion Operators,” J. Operator Theory, 53, 2, 381-397, 2005.
[3] J. Campbell and J. Jamison, “On some classes of weighted composition operators,” Glasgow Math. J. 32, 87-94, 1990.
[4] H. Emamalipour, M. R. Jabbarzadeh and Z. Moayyerizadeh, “Separating Partiality Normality Classes with Weighted Composition Operators on L^2,” Bull. Iranian Math. Soc., Vol. 43, No. 2, pp. 561-574, 2017.
[5] H. Emamalipour, M. R. Jabbarzadeh and M.S. Chegeni, “Some weak p-hyponormal classes of weighted composition operators,” Filomat. 31, 9, 2643-2656, 2017.
[6] D. J. Harrington and R. Whitley, “Seminormal composition operators,” J. Operator Theory. 11, 125-135, 1984.
[7] J. Herron, “Weighted conditional expectation operators,” Oper. Matrices. 5, 1, 107-118, 2011.
[8] C. Hillings, “Two-isometries on Pontryagin spaces,” Integral Equations Operator Theory. 61, 211-239, 2008.
[9] A. Lambert, “Hyponormal composition operators,” Bull. Lond. Math. Soc. 18, 395-400, 1986.
[10] O. A. Mahmoud Sid Ahmed, “Generalization of m-partial isometries on a Hilbert spaces,” Internat. J. Pure Appl. Math. Volume 104, No.4, 599-619, 2015.
[11] S. Mecheri and S. M. Patel, “On quasi-2-isometric Operators,” Linear Multilinear Algebra, Vol. 66, No. 5, 1019-1025, 2018.
[12] S. Mecheri and T. Prasad, “On n-quasi-m-isometric operators,” Asian-Eur. J. Math, 9, 4, 8 Pages, 2016.
[13] T. Prasad, “Spectral properties of some extensions of isometric operators,” Ann. Funct. Anal., 11, 626-633, 2020.
[14] T. Prasad, “Class p-wA(s,t) composition operators,” Asian-Eur. J. Math, Vol. 13, 2050086, 10 pages, 2020.
[15] E. Nordgren, “Composition operators,” Hilbert Space Operators Proceedings 1977, Lecture Notes in Math. No. 693, Springer Verlag, Berlin, 1978.
[16] M. M. Rao, “Conditional measure and applications,” Monographs and Textbooks in Pure and Applied Mathematics, Marcel Dekker, NewYork, 1993.
[17] S. Richter, “A representation theorem for cyclic analytic two isometries,” Trans. Amer. Math. Soc. 238, 325-349, 1991.
[18] S. Richter, “Invariant subspaces of the Dirichlet shift,” J. Reine Angew. Math. 386, 205220, 1988.
[19] J. Shen and G. Ji, “Spectral properties and the dynamics of quasi-2-expansive operators,” J. Spectr. Theory 10, 323-335, 2020.
[20] S. M. Shimorin, “Wold-type decompositions and wandering subspaces for operators close to isometries,” J. Reine Angew. Math. 531, 147189, 2001.
[21] R.K. Singh and J.S. Manahas, “Composition Operators on Function Spaces,” North-Holland Mathematics Studies 179.

E. Shine Lal
Department of Mathematics, University College, Thiruvananthapuram, Kerala, India-695034
E-mail address: shinelal.e@gmail.com

T.Prasad
Department of Mathematics, University of Calicut, Calicut University PO, Malappuram, Kerala, India-673635
E-mail address: prasadvalapil@gmail.com

V Devadas
Department of Mathematics, Sree Narayana College, Alathur, Kerala, India-678682
E-mail address: v.devadas.v@gmail.com