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Taylor Series Approach to Grey Multiobjective Integer Linear Fractional Programming Problem and A case Study for Environmental Economic Energy Dispatch

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ABSTRACT

This paper presents the use of a Taylor series for multiobjective integer linear fractional programming problem having grey parameters in the right-side of the constraints (GMOILFP). To deal with the grey parameters in the right-side of the constraints the positioned programming should be used. An equivalent grey multiobjective linear fractional programming problem (GMOLFP) is formulated using Gomory's cutting plane method. The Taylor series, which is a series expansion that a representation of a function, is applied to convert the fractional functions into polynomials. In the proposed approach a white value of each grey parameter is determined, Taylor series is applied and the functions are unified by using the nonnegative weighted sum method. Thus, the problem is reduced to a single linear objective with grey parameter in the right side of the constrains. An algorithm for solving GMOILGP problem with grey interval coefficients using positioned programming and Taylor series polynomials is proposed. A numerical example is provided to demonstrate the efficiency and feasibility of the proposed approach. A special case study for handling the environmental economic energy dispatch problem also is included in this paper, the main goal of this problem is how to schedule committed generators to meet the load required to minimize the pollution emissions and fuel cost. The model formulation for a special case study problem is presented, the mathematical model will be considered as (GMOLFP) and the problem is solved according to the proposed solution algorithm.

1. INTRODUCTION

Multiobjective optimization problems (MOPs) are come across in many fields, such as energy systems [1-5], scheduling [6-8], management [9], and structural optimum design [10,11]. Generally, many problems can be modelled as multiobjective optimization problems, in which multiple conflicting objectives are to be optimized simultaneously. Optimization problems, which frequently appear in scientific research and engineering, are often MOPs. Yin et al. [12] studied the crashworthiness and reliability of a foam-filled bionic thin-walled structure based on bio inspired design. The optimization of a vehicle door structure with a hybrid material was investigated to achieve lightweight design [13]. To obtain better performance, various multiobjective optimization strategies have been proposed and widely applied to engineering problems [14-15].

Numerous existent problems of human society are expressed by Mathematical modelling of several objectives, and these objectives are connected and so interfered to one another. Usually, they are fractional functions and need simultaneous optimization under some equality or inequality constraints. The multiobjective linear fractional programming problems consist of linear constraints and linear fractional objective functions which contain affine functions in its numerators and denominators. The multiobjective linear fractional programming problems are widely used in many real-life situations [16]. Usually, there is no single feasible solution which optimizes allthe objective simultaneously, so the notion of Pareto optimal solution was considered [17]. The Pareto optimal solutions of a multiobjective geometric programming problem were derived by Ojha and Biswal [18, 19].

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In 1960, to convert the fractional objectives into linear ones, a transformation technique was sophisticated by Charnes and Cooper [20]. For solving the linear fractional programming problems, a method which is greatly used, was suggested by Bitran and Novaes [21], the method of Dinkelbach approach [22] was considered, and the method of conjugate gradient was used by Tantawy [23]. The non-linear fractional programming problem was solved by Wolf [24]. The multiobjective linear fractional programming problems were solved by Costa [25], Valipour et al. [26], Chakrobarty and Gupta [27] and Dangwal [28]. The multiobjective linear fractional programming problem with interval coefficients was suggested by Pal and Sen [29]. The single objective linear fractional programming problem was considered by Borza et al. [30] and by Almogy and Levin [31]. Miettinen [32] clarified several methods to solve multiobjective optimization problems [17]. The mixed integer fractional programming problems were solved by Zhixia and Fengqi [33]. The fuzzy linear fractional programming problems were considered by Das et al. [34] and by Chinnadurai and Muthukumar [35]. The multilevel multiobjective fractional programming problems were solved by Osman et al. [36]. The fuzzy multiobjective linear fractional programming problems were considered by Chang [37] and Toksari [38]. The geometric programming with fuzzy parameters, was suggested by Liu [39].

The theory of grey systems was established by professor Julong Deng which has been studying problems, with uncertain systems, that involved partially known information [40,41]. Two papers on grey systems were published by professor Julong Deng in 1982 (Deng 1982a, b) [42,43]. After that date, a lot of publications, that involved the theory of grey systems, were published around the world [44]. The grey systems curriculums have been set up for undergraduate, Master and PhD programs in many universities around the world. As a result of uncertainty in the real world, so the expressing of the crisp coefficients, in mathematical programming problems, is impossible. The theory of the grey systems has been used to screen uncertainty and currently it is used for decision-making, forecast modelling, control, and appraisal [45-49].

In order to world population grows and developing countries, the electric energy became one of the most environmental economic energy resources in the world. The electric energy, in any one location, can be generated by the fossil fuelled energy plants which are plentiful on the earth [50,51]. The aim for the generation of the economic electric energy dispatch [52, 53] is the scheduling for the output units of the generating, such that the load demand is satisfied at minimum pollutant emissions [54] and minimum operating cost, while satisfying all system constraints. Thus, the environmental economic energy dispatch (EEED) problem can be handled as a Multiobjective optimization problem with multiple conflicting objectives [55, 56].

In this work, a new framing is sophisticated to address with multiobjective integer linear fractional programming problem that have grey parameters (GMOILFP) in the right-hand side of the constraints. The problem is formulated and the method with the proposed solution algorithm to solve (GMOILFP) is introduced. The positioned programming [57,58] is used to deal with the grey parameters in the right-side of the constraints. Gomory's cutting plane method [59] is considered, therefore, an equivalent grey multi-objective linear fractional programming problem (GMOLFP) is formulated. The proposed method applied 1st order Taylor series polynomial [60,61] to obtain the polynomial objective function that equivalent to the fractional objective function. The nonnegative weighted sum [62] is used and the multiple objective linear programming problem (GMOLPP) with grey is reduced to a single-objective linear programming problem with grey that could be solved [63]. The paper is divided into 7 sections. Section 2 contains some basic definitions for the theory of grey systems. Section 3 focused on a definition of multiobjective integer linear fractional programming problem that have grey parameters (GMOILFP) in the right-hand side of the constraints. The proposed approach is presented in Section 4. An illustrative example is solved by using the proposed procedure in Section 5. A special case study for handling the environmental economic energy dispatch problem is given in section 6. Finally, conclusions are presented in section 7.

2. THEORY OF GREY SYSTEMS

The information in the theory of the grey systems is distributed into three types, which are totally known, incomplete and completely uncertain information that are denoted by white, grey and black respectively [64]. The basic concepts for the theory of grey systems are the interval grey numbers.

Definition 1: (Interval grey number $a(\otimes)$)

$a(\otimes) \in [\underline{a}, \overline{a}], \underline{a} < \overline{a}$ is called an interval grey number, where $\overline{a}$and $\underline{a}$ are the upper and lower limits. It has an unknown exact value, but its upper and lower limits are defined [65,66]. It has just singular number in that interval $[\underline{a}, \overline{a}]$, which differ to the meaning of the interval value but the arithmetic of both are the same [67].

Definition 2: (The length of interval grey number $L\ (a(\otimes))$

Supposing that interval grey number$a(\otimes)$ has the upper limit of $\overline{a}$ and the lower limit of $\underline{a}$, the length of $a(\otimes)$ interval grey number is defined as the following Equation [68].

$$\mathrm{L}(a(\otimes)) = \overline{a} - \underline{a}$$

Definition 3: (The "kernel" of interval grey number$\hat{a}(\otimes)$)

Suppose an interval grey number$a(\otimes)$,$\underline{a} < \overline{a}$, then$\hat{a}(\otimes) = 0.5(\underline{a} + \overline{a})$ is called the "kernel" of grey number $a(\otimes)$.

Definition 4: (White number $a(\otimes) = a$).

The white number $a(\otimes) = a$ is an interval grey number$a(\otimes)$,$\underline{a} < \overline{a}$ when $\underline{a} = \overline{a} = a$.

Definition 5: **(**The degree of greyness**)**

The degree of greyness of $a(\otimes)$is calculated as in the following equation. If Ω represents the background of grey numbers, while μ represents the measurement of grey numbers [69, 70].

$$g^0(a(\otimes)) = \mu(a(\otimes)) / \mu\ (\Omega)$$

3. PROBLEM FORMULATION

The basic definition for the problem of multiobjective integer linear fractional programming that have grey numbers (GMOILFP) in the right-hand side constraints is shown here. The problem formulation is stated as the below statements

$$\max\ (f_1(x), f_2(x), \dots, f_k(x)) \qquad (3.a)$$

Subject to

$x \in M$where, $M = \{x \in R^n | Ax \leq b(\otimes), x \geq 0 \text{ and integer}\}$ (3.1b)

where $$f_i(x) = \frac{c_i^T x + \alpha_i}{d_i^T x + \gamma_i} \quad \text{for i = 1, 2, ...,k} \qquad (3.1c)$$

In addition, c_i *and* $d_i$are n-vectors, and α_i *and* $\gamma_i$are scalar constants, A is an $(m \times n)$ matrix, x is an n-vector of the decision variables, $b(\otimes)$ is an n-vector of grey constraints right hand side for resources, and $d_i^T x + \gamma_i > 0$ for all $x \in M$.

The *n*-vector of grey right hand side constraints for resources $b(\otimes)$ were given [58] and could be written as:

$$b(\otimes) = [b_1(\otimes), b_2(\otimes), \dots, b_m(\otimes)]^T \qquad (3.2)$$

where $b_i(\otimes)$ are the interval grey numbers and

$$b_i(\otimes) \in [\underline{b_i}, \overline{b_i}], \underline{b_i} \geq 0, i = 1,2, \dots, m \qquad (3.3)$$

4. THE PROPOSED ALGORITHM FOR SOLVING PROBLEM (GMOILFP)

In an earlier work, the solution algorithm to the problem of multiobjective integer linear fractional programming (MOILFPP) was presented [71]. The 1st order Taylor series polynomial was applied, and then the polynomial objective function that is equivalent to the fractional objective function was obtained. Then, the nonnegative weighted sum method was used and the multiobjective integer linear programming problem (MOILPP) was reduced to a single-objective problem. Thus, an optimal integer solution has been be found via the branch and bound method [72].

In this section, we suggest a proposed method for solving problem (GMOILFP) with grey numbers in the right hand -side constraints. In the first step the positioned programming should be used to deal with the grey parameters in the right-side of the constraints. Then by using Gomory's cutting plane method, an equivalent grey multi-objective linear fractional programming problem (GMOLFP) is formulated. The proposed method applied 1st order Taylor series polynomial to obtain the polynomial objective function that equivalent to the fractional objective function. After that, the nonnegative weighted sum is used and the multiple objective linear programming problem (GMOLPP) with grey is reduced to a single-objective linear programming problem that could be solved.

Definition 6: ***(***White values of grey numbers)

The white values of grey numbers were considered and given by

$$\tilde{b}_i(\otimes) = \beta_i \overline{b_i} + (1 - \beta_i)\underline{b_i}\ ;\ i = 1,2, \dots, m \qquad (4.1)$$

Such that β_i is the positioned coefficients of the constraint vector for resources. The coefficient β_i reflects market supplies of the *i*th resource. Where low and high $\beta_i$express short and enough supplies respectively. Such that $0 \le \beta_i \le 1$ and i *take any value* $1, 2, \dots, m$. And $\tilde{b}_i(\otimes)$ are the whitened values of the vector of constraints for resources.

Then, the positioned grey multiobjective integer linear fractional programming (GMOILFP) is formulated as:

$$\text{Max } \left(f_1(x), f_2(x), \dots, f_k(x)\right) \quad (4.2a)$$

Subject to

$$AX \le \tilde{b}(\otimes), X \ge 0 \text{and integer} \quad (4.2b)$$

Now, by using (Eq. 4.1) the problem (GMOILFP) (4.2 a) - (4.2b) can be rewritten in the following:

$$\text{Max } f_1(x) = \frac{c_1^T x + \alpha_1}{d_1^T x + \gamma_1}$$

$$\text{Max } f_2(x) = \frac{c_2^T x + \alpha_2}{d_2^T x + \gamma_2} \quad (4.3a)$$

$$\vdots$$

$$\text{Max } f_k(x) = \frac{c_k^T x + \alpha_k}{d_k^T x + \gamma_k}$$

Subject to

$x \in M$ where , $M = \left\{x \in R^n \middle| \sum_{j=1}^n a_{ij} x_j \le \tilde{b}_i(\otimes), (i = 1, \dots, m), x_j \ge 0 \text{ and integer}\right\}$ (4.3b)

As was mentioned [73] that the cutting-plane method was used, and then the equivalent (GMOLFP) can be rewritten in the following form:

$$\text{Max } f_1(x) = \frac{c_1^T x + \alpha_1}{d_1^T x + \gamma_1}$$

$$\text{Max } f_2(x) = \frac{c_2^T x + \alpha_2}{d_2^T x + \gamma_2} \quad (4.4a)$$

$$\vdots$$

$$\text{Max } f_k(x) = \frac{c_k^T x + \alpha_k}{d_k^T x + \gamma_k}$$

Subject to

$x \in [M]$ where, $M = \left\{x \in R^n \middle| \sum_{j=1}^n a_{ij} x_j \le \tilde{b}_i(\otimes), (i = 1, \dots, m), x_j \ge 0 \text{ and integer}\right\}$ (4.4b)

Where $[M]$ is the convex hull of the feasible region M. If the problem (GMOLFP) (4.4) is solved for each objective function one by one then the suitable values of the variables are resulted

At that point, the 1st order Taylor series is used then, the objective functions in (4.4a) are transformed to the polynomial objective functions that equivalent to the fractional objective functions. After that, the nonnegative weighted sum is used and the multiple objective linear programming problem (GMOLPP) with grey is reduced to a single-objective linear programming problem with grey that could be solved [58,63].

The proposed Solution Algorithm:

The position programming [58], Gomory's cutting plane method [59], the algorithm that have been used [73] to solve (MOLFP) and the method that have been used [63] to solve linear programming with grey parameters are applied here. The steps of the proposed approach to solve (GMOILFP) are described in the following manner:

1) The positioned programming is used to deal with the grey parameters in the right-side of the constraints. Gomory's cutting plane method is considered, therefore, an equivalent grey multi-objective linear fractional programming problem (GMOLFP) is formulated as follows : Max $\left(f_1(x), f_2(x), \dots, f_k(x)\right)$ s.t $x \in [M]$ where, $M = \left\{x \in R^n \middle| \sum_{j=1}^n a_{ij} x_j \le \beta_i \overline{b_i} + (1 - \beta_i)\underline{b_i}, (i = 1, \dots, m), x_j \ge 0 \text{ and integer}\right\}$ (4.5)
2) Determine $x_r^* = (x_{r1}^*, \dots, x_{rn}^*)$ which is value when the r^th objective function, $f_r(x), (r = 1, 2, \dots, k)$, are maximized where n is number of the variables.
3) Transform the objective functions $f_r(x), (r = 1, 2, \dots, k)$ by using the following 1st order Taylor series to polynomial functions in the following that was stated [60,61] as:

$$f_r(x) \simeq \overline{f_r(x)} = f_r(x_r^*) + \sum_{j=1}^n (x_j - x_{rj}^*) \frac{\partial f_r(x_r^*)}{\partial x_j}, (r = 1, 2, \dots, k) \quad (4.6)$$

4) Use the nonnegative weighted sum method [62] to convert the multiobjective linear programming problems with grey parameters in the right hand side of the constraints to single objective.
5) So the problem (4.4a) and (4.4b) will become Max $\sum_{r=1}^k w_r f_r(x)$ s.t $x \in [M]$ where, , $M = \{x \in R^n | Ax \le b(\otimes), x \ge 0 \text{ and integer}\}$, Which is a single-objective linear programming problem with grey parameter in the right side of the constraints that could be solved.
6) Find the satisfactory solution by using any applicable software.
7) While changing the weights, a new optimal solution appeared.

5. EXAMPLE

Now, an example of (GMOILFP) with grey numbers in the right- hand side constraints is provided then solved using the proposed algorithm. The problem to be solved here is the multiobjective integer linear fractional programming problem that has grey parameters in the right side of the constraints (GMOILFP):

(GMOILFP): $$Max\ f_1(x_1,x_2) = \left(\frac{2x_1+3x_2}{x_1+4x_2+6}\right)$$

$$Max\ f_2(x_1,x_2) = \left(\frac{3x_1+4x_2}{6x_1+4x_2+3}\right)$$

$$Max\ f_3(x_1,x_2) = (-x_1 - x_2)$$

Subject to

$$2x_1 - x_2 \le [8,11]$$

$$-x_1 + 3x_2 \le [10,13]$$

$$x_1, x_2 \ge 0 \quad \text{and integer.}$$

According to the proposed solution algorithm by conducting the following steps:

With the help of the position and the Gomory cutting-plane method, an equivalent problem of multiobjective linear fractional programming that have grey numbers (GMOLFP) in the right-hand side constraints corresponding to the problem (GMOILFP) can be formulated as follows:

(GMOLFP): $$Max\ f_1(x_1,x_2) = \left(\frac{2x_1+3x_2}{x_1+4x_2+6}\right)$$

$$Max\ f_2(x_1,x_2) = \left(\frac{3x_1+4x_2}{6x_1+4x_2+3}\right)$$

$$Max\ f_3(x_1,x_2) = (-x_1 - x_2)$$

Subject to

$$2x_1 - x_2 \le 11\beta_1 + 8(1-\beta_1)$$

$$-x_1 + 3x_2 \le 13\beta_2 + 10(1-\beta_2)$$

$$x_1 \le 4$$

$$x_2 \le 3$$

$$0 \le \beta_1 \le 1$$

$$0 \le \beta_2 \le 1$$

$$x_1, x_2 \ge 0\,.$$

If the problem (GMOLFP) is solved for each objective function one by one, then $f_1(4,0) = 0.8$, $f_2(0,3) = 0.8, f_3(0,0) = 0$. Then the 1st order Taylor series is used to transform the objective functions into polynomial functions as following:

$$f_1(x_1,x_2) \simeq \overline{f_1}(x_1,x_2) = 0.12x_1 - 0.02x_2 + 0.32$$

$$f_2(x_1,x_2) \simeq \overline{f_2}(x_1,x_2) = -0.12x_1 + 0.053x_2 + 0$$

$$f_3(x_1,x_2) \simeq \overline{f_3}(x_1,x_2) = -x_1 - x_2$$

If The weighted sum method is used with $w_1 = 0.6, w_2 = 0.4\ and\ w_3 = 0.0$, $w_1 + w_2 + w_3 = 1$, then the problem of multiobjective linear fractional programming that have grey numbers (GMOLFP) in the right-hand side constraints can be rewritten as a grey single objective linear problem as follows:

(GLP):

$$Max\left[0.6\overline{f_1}(x_1,x_2) + 0.4\overline{f_2}(x_1,x_2) + 0.0\overline{f_3}(x_1,x_2)\right] = 0.024\left[\underline{x_1},\overline{x_1}\right] + 0.0092\left[\underline{x_2},\overline{x_2}\right] + 0.448$$

Subject to

$$2\left[\underline{x_1},\overline{x_1}\right] - \left[\underline{x_2},\overline{x_2}\right] \le [8,11]$$

$$-\left[\underline{x_1},\overline{x_1}\right] + 3\left[\underline{x_2},\overline{x_2}\right] \le [10,13]$$

$$\left[\underline{x_1},\overline{x_1}\right] \le [4,4]$$

$$\left[\underline{x_2},\overline{x_2}\right] \le [3,3] \le$$

$$\left[\underline{x_1},\overline{x_1}\right], \left[\underline{x_2},\overline{x_2}\right] \ge 0.$$

Therefore, the solution can be obtained as $\left[\underline{x_1},\overline{x_1}\right] = [3.9587, 4]\ and\ \left[\underline{x_2},\overline{x_2}\right] = [2.9990, 3]$ with the optimum objective value functions:

$$f_1^*\left(\left[\underline{x_1},\overline{x_1}\right],\left[\underline{x_2},\overline{x_2}\right]\right) = [0.768836, 0.77432167],$$
$$f_2^*\left(\left[\underline{x_1},\overline{x_1}\right],\left[\underline{x_2},\overline{x_2}\right]\right) = [0.6121051282, 0.6193836101]$$
$$, f_3^*\left(\left[\underline{x_1},\overline{x_1}\right],\left[\underline{x_2},\overline{x_2}\right]\right) = [-7, -6.9577],$$

And the crisp solution can be obtained as (x_1^*, x_2^*)=(4,3) with the optimum objective value functions:

$$f_1^*(x_1^*,x_2^*) = 0.74, \quad f_2^*(x_1^*,x_2^*) = 0.319, f_3^*(x_1^*,x_2^*) = -7$$

Now, changing the weights with the values $w_1 = 0.5, w_2 = 0.5\ and\ w_3 = 0.0$, the solution can be obtained as $\left[\underline{x_1},\overline{x_1}\right] = [0.0, 0.04128]\ and\ \left[\underline{x_2},\overline{x_2}\right] = [2.9990, 3]$ with the optimum objective value functions:

$$f_1^*\left(\left[\underline{x_1},\overline{x_1}\right],\left[\underline{x_2},\overline{x_2}\right]\right) = [0.4986896717, 0.504698822],$$
$$f_2^*\left(\left[\underline{x_1},\overline{x_1}\right],\left[\underline{x_2},\overline{x_2}\right]\right) = [0.7867426389, 0.8084715924]$$
$$, f_3^*\left(\left[\underline{x_1},\overline{x_1}\right],\left[\underline{x_2},\overline{x_2}\right]\right) = [-3.04128, -2.999],$$

And the crisp solution can be obtained as (x_1^*, x_2^*) =(0,3) with the optimum objective value functions:

$$f_1^*(x_1^*, x_2^*) = 0.26, \quad f_2^*(x_1^*, x_2^*) = 0.799, f_3^*(x_1^*, x_2^*) = -3$$

Again, choosing the weights with the values $w_1 = 0.7, w_2 = 0.25\ and\ w_3 = 0.05$, the solution can be obtained as $\left[\underline{x_1}, \overline{x_1}\right] = [3.958717, 4]\ and\ \left[\underline{x_2}, \overline{x_2}\right] = [0.0, 0.001]$ with the optimum objective value functions:

$$f_1^*\left(\left[\underline{x_1}, \overline{x_1}\right], \left[\underline{x_2}, \overline{x_2}\right]\right) = [0.79142682939, 0.8036175744],$$
$$f_2^*\left(\left[\underline{x_1}, \overline{x_1}\right], \left[\underline{x_2}, \overline{x_2}\right]\right) = [0.43979229, 0.4487090494],$$
$$f_3^*\left(\left[\underline{x_1}, \overline{x_1}\right], \left[\underline{x_2}, \overline{x_2}\right]\right) = [-4.001, -3.958717],$$

And the crisp solution can be obtained as (x_1^*, x_2^*) =(4,0) with the optimum objective value functions:

$$f_1^*(x_1^*, x_2^*) = 0.8, \quad f_2^*(x_1^*, x_2^*) = 0.16, f_3^*(x_1^*, x_2^*) = -4$$

And, also choosing the weights with the values $w_1 = 0.4, w_2 = 0.3\ and\ w_3 = 0.3$, the solution can be obtained as $\left[\underline{x_1}, \overline{x_1}\right] = [0.0, 0.04128]\ and\ \left[\underline{x_2}, \overline{x_2}\right] = [0.0, 0.001]$ with the optimum objective value functions:

$$f_1^*\left(\left[\underline{x_1}, \overline{x_1}\right], \left[\underline{x_2}, \overline{x_2}\right]\right) = [0.0, 0.01426],$$
$$f_2^*\left(\left[\underline{x_1}, \overline{x_1}\right], \left[\underline{x_2}, \overline{x_2}\right]\right) = [0.0, 0.042613],$$
$$f_3^*\left(\left[\underline{x_1}, \overline{x_1}\right], \left[\underline{x_2}, \overline{x_2}\right]\right) = [-0.04228, 0.0],$$

And the crisp solution can be obtained as (x_1^*, x_2^*) =(0,0) with the optimum objective value functions:

$$f_1^*(x_1^*, x_2^*) = 0.32, \quad f_2^*(x_1^*, x_2^*) = 0.64, f_3^*(x_1^*, x_2^*) = 0.$$

Again, choosing the weights with the values $w_1 = 0.8, w_2 = 0.2\ and\ w_3 = 0.0$, the solution can be obtained as $\left[\underline{x_1}, \overline{x_1}\right] = [3.999, 4]\ and\ \left[\underline{x_2}, \overline{x_2}\right] = [0.0, 0.1061]$ with the optimum objective value functions:

$$f_1^*\left(\left[\underline{x_1}, \overline{x_1}\right], \left[\underline{x_2}, \overline{x_2}\right]\right) = [0.7672384022, 0.8319131913],$$
$$f_2^*\left(\left[\underline{x_1}, \overline{x_1}\right], \left[\underline{x_2}, \overline{x_2}\right]\right) = [0.4374571549, 0.4602311454],$$
$$f_3^*\left(\left[\underline{x_1}, \overline{x_1}\right], \left[\underline{x_2}, \overline{x_2}\right]\right) = [-4.1061, -3.999],$$

6. A CASE STUDY FOR THE ENVIRONMENTAL ECONOMIC ENERGY DISPATCH PROBLEM

6.1 A survey of model formulation of the environmental economic energy dispatch [74].

The main goal of the problem of environmental economic energy dispatch (EEED) is to schedule committed generators to meet the load required to minimize the total pollution emissions and fuel cost of the thermal energy system. To achieve an environmental economic energy dispatch scheme, Chen et al [75] considered a thermal energy dispatch system that include N fossil fuelled energy plants. In order to simply, the N fossil fuelled power plants were referred to by generators. The total fuel cost of the thermal energy system (TC), in dollar per hour, was expressed by [76]

$$Tc = \sum_{i=1}^{N}\left(a_i + b_i x_i + c_i x_i^2\right) \tag{6.1}$$

The total pollutant emissions (TM) caused by the fossil fuelled generators, in ton per hour, was given by [77]

$$\mathrm{TM} = \sum_{i=1}^{N} \left[0.01(\alpha_i + \beta_i x_i + \nu_i x_i^2) + \xi_i \exp(\lambda_i x_i)\right] \tag{6.2}$$

Such that N was the number of generators, $a_i, b_i\ and\ c_i$ were the coefficients for the cost of the ith generator, and so, $\alpha_i, \beta_i, \nu_i, \xi_i$ and λ_i were the coefficients for the pollutant emissions of the ith generator's emission characteristics. x_i was the ith component of the vector x and it was the energy output of the ith generator, then the vector x could be written as

$$x = [x_1, x_2, \ldots x_N]^T \tag{6.3}$$

The system constraints, that were considered in that study, were only two constrains. The first one could be written as an equation to describe the balance of the energy and the second one could be written as an inequality to describe the capacity of the generation. At the first, the equality constraint of energy balance demonstrated that the total energy was given as the following

$$\sum_{i=1}^{N} x_i = E_d + E_l \tag{6.4}$$

Such that E_d and E_l were the total demand and the energy transmission loss in the transmission lines, respectively. E_d was expressed by number mentioned to the total demand and E_l was expressed as the following

$$E_l = \sum_{i=1}^{N}\sum_{j=1}^{N} x_i \mathrm{B}_{ij} x_j + \sum_{i=1}^{N} \mathrm{B}_{oi} x_i + \mathrm{B}_{00} \tag{6.5}$$

Such that $\mathrm{B}_{ij}, \mathrm{B}_{oi}, and\ \mathrm{B}_{00}$, were Kron's loss coefficients of transmission lines [78]. In the second, the inequality constraint of Generation capacity indicated that the energy output of ith generator was limited as the following:

$$x_i^{min} \leq x_i \leq x_i^{max} \qquad \forall i \qquad (6.6)$$

Such that the minimum and maximum values of the energy output of the ith generator were denoted by $x_i^{min}$and x_i^{max}, respectively.

Table (1): The Definitions of the used symbols

The symbols	Definitions
N	The number of generators
a_i, b_i *and* c_i	The coefficients for the cost of the ith generator
$\alpha_i, \beta_i, \nu_i, \xi_i$ and λ_i	The coefficients for the pollutant emissions of the ith generator's emission characteristics
x_i	The ith component of the vector x and it is the energy output of the ith generator
$x = [x_{1,}x_2, \dots x_N]^T$	The vector x
E_d	The total demand
E_l	The energy transmission loss in the transmission lines
$\mathrm{B}_{ij}, \mathrm{B}_{oi}, and\ \mathrm{B}_{00},$	Kron's loss coefficients of transmission lines
$x_i^{min}$and x_i^{max}	The minimum and maximum values of the energy output of the ith generator

6.2 The model formulation for a special case study Problem

In the current section, the model formulation for a special case study problem is presented. In equations (6.1), (6.2) and (6.5), which were given in the last section, suppose that $c_i, \nu_i, \xi_i, \lambda_i$, and B_{ij} are considered equal zero in this special case study. This consideration can be written as the following:

$$c_i = \nu_i = \lambda_i = \mathrm{B}_{ij} = 0 \qquad (6.7)$$

Therefore, the mentioned equations will become as the following:

$$Tc = \sum_{i=1}^{N}(a_i + b_i x_i) \qquad (6.8)$$

$$\mathrm{TM} = \sum_{i=1}^{N} [0.01(\alpha_i + \beta_i x_i)] \qquad (6.9)$$

$$E_l = \sum_{i=1}^{N} \mathrm{B}_{oi}\, x_i + \mathrm{B}_{00} \qquad (6.10)$$

In case the right hand side of the constrains include grey interval parameters, then the mathematical model for the special case study will be considered as (GMOLFP) and the problem will be formulated as follows:

$$\text{Min } Z_1 = \frac{\mathrm{TM}}{Tc} = \frac{\sum_{i=1}^{N} [0.01(\alpha_i+\beta_i x)]}{\sum_{i=1}^{N}(a_i+b_i x)} \qquad (6.11)$$

$$\text{Min } Z_2 = Tc = \sum_{i=1}^{N}(a_i + b_i x) \qquad (6.12)$$

$$\text{Min } Z_3 = \mathrm{TM} = \sum_{i=1}^{N} [0.01(\alpha_i + \beta_i x)] \qquad (6.13)$$

Subject to

$$\sum_{i=1}^{N} x_i = E_d + \sum_{i=1}^{N} \mathrm{B}_{oi}\, x_i + \mathrm{B}_{00} \qquad (6.14)$$

$$x_i^{min}(\otimes) \leq x_i \leq x_i^{max}(\otimes) \quad \forall i \quad \text{or}$$

$$\left[\underline{x_i^{min}}, \overline{x_i^{min}}\right] \leq x_i \leq \left[\underline{x_i^{max}}, \overline{x_i^{max}}\right] \quad \forall i \qquad (6.15)$$

where the minimum and maximum grey interval values of the energy output of the ith generator are denoted by $x_i^{min}(\otimes)$and $x_i^{max}(\otimes)$, respectively. The position programming and the 1st order Taylor series polynomial is applied, and then the polynomial objective functions that is equivalent to the objective functions of the problem are obtained. So, the (GMOLFP) is converted into (GMOLP). Then, the non-negative weighted sum method is used and the multiobjective linear programming problem (GMOLP) is reduced to a single-objective problem. After that the position linear programming is applied thus, an optimal solution has been be found.

6.3 Example

The mathematical model for the special case study, that is given in equations (6.11) to (6.15), is applied here for the standard IEEE 30 bus six generators test system, this system has six generators connected through 41 tranmission lines and supplies power for 21 load buses [79], for more information see [75]. The nonnegative weighted for the three objective functions Z_1, Z_2 *and* Z_3 are 0.3, 0.3 and 0.4 respectively. The minimum and maximum grey interval values of the energy output of the ith generator are shown in Table 3. Using equation (6.7), the total system demand is $E_d = 2.834\, p.u$ that was given in [75], and the coefficients of the system that were reported in [53,75]. The coefficients of cost, pollutant emissions and the Kron's transmission, that were given in [75], are presented in Table 2 and 4.

Table (2): The coefficients of the cost and pollutant emissions of the ith generators

	x_1	x_2	x_3	x_4	x_5	x_6
Cost						
a	10	10	20	10	20	10
b	200	150	180	100	180	150
Emission						
α	4.091	2.543	4.258	5.326	4.258	6.131
β	-5.554	- 6.047	-5.094	- 3.550	- 5.094	- 5.555

Table (3): The minimum and maximum grey interval values of the energy output of the ith generator (in p.u)

	x_1	x_2	x_3	x_4	x_5	x_6
x_i^{min}	[0.04,0.06]	[0.045,0.055]	[0.045,0.05]	[0.0455,0.0505]	[0.046,0.062]	[0.044,0.061]
x_i^{max}	[0.45,0.55]	[0.54,0.7]	[0.93,1.1]	[1.12,1.3]	[0.91,1.1]	[0.56,0.7]

Table (4): The coefficients of Kron's loss transmission

	1	**2**	**3**	**4**	**5**	**6**
B_{oi}	-0.0107	0.006	-0.0017	0.0009	0.0002	0.003
B_{oo}	0.000986					

Table (5): The solution of the mathematical model for the special case study.

The energy outputs of the generators (in p.u)	$\overline{x_1}$	0.04
	$\overline{x_2}$	0.7
	$\overline{x_3}$	0.05594183
	$\overline{x_4}$	1.3
	$\overline{x_5}$	0.05
	$\overline{x_6}$	0.7
$\overline{Z_1}$ Total emission/ Total fuel-cost (ton/$)		0.000286408
$\overline{Z_2}$ Total fuel-cost ($/h)		446.349529 4
$\overline{Z_3}$ Total environmental-emission (ton/h)		0.127838076

The main goal of this problem is to schedule committed generators to meet the load required to minimize the pollution emissions and fuel cost. The model formulation for a special case study problem is presented, the mathematical model will be considered as (GMOLFP) and the problem 6.11-6.15 will be

$$\text{Min } Z_1 = \frac{\text{TM}}{Tc} = \frac{[0.01(26.607-5.54x_1-6.047x_2-5.094x_3-3.550x_4-5.094x_5-5.555x_6)]}{(80+200x_1+150x_2+180x_3+100x_4+180x_5+150x_6)} \quad (6.16)$$

$$\text{Min } Z_2 = Tc = (80 + 200x_1 + 150x_2 + 180x_3 + 100x_4 + 180x_5 + 150x_6) \quad (6.17)$$

$$\text{Min } Z_3 = \text{TM} = [0.01(26.607 - 5.54x_1 - 6.047x_2 - 5.094x_3 - 3.550x_4 - 5.094x_5 - 5.555x_6)] \quad (6.18)$$

Subject to

$$1.0107\,x_1 + 0.994x_2 + 1.0017x_3 + 0.9991x_4 + 0.9998x_5 + 0.997x_6 = 2.83498573 \quad (6.19)$$

$$[0.04,0.06] \le x_1 \le [0.45,0.55] \quad (6.20)$$

$$[0.045,0.055] \le x_2 \le [0.54,0.7] \quad (6.21)$$

$$[0.045,0.05] \le x_3 \le [0.93,1.1] \quad (6.22)$$

$$[0.0455,0.0505] \le x_4 \le [1.12,1.3] \quad (6.23)$$

$$[0.046,0.062] \le x_5 \le [0.91,1.1] \quad (6.24)$$

$$[0.044,0.061] \le x_6 \le [0.56,0.7] \quad (6.25)$$

The mathematical model for the special case study is analysed and solved according to the proposed solution algorithm. Therefore, the solution of the mathematical model for the special case study that applied here for the standard IEEE 30 bus six generators test system is presented in Table 5.

7. CONCLUSION

In this paper, we proposed an algorithm for solving GMOILGP problem with grey interval coefficients using positioned programming and Taylor series polynomials. In this method, we modified the shortcomings of a solving method, in which some variables solutions have the interval with lower bounds greater than their upper bounds. In addition, in our method, one can determine different weights for each objective function. To deal with the grey parameters in the right-side of the constraints the positioned programming should be used. An equivalent grey multi-objective linear fractional programming problem (GMOLFP) is formulated using Gomory's cutting plane method. The Taylor series is applied to convert the fractional functions into polynomials. In the proposed approach a white value of each grey parameter is determined, Taylor series is applied and the functions are unified by using the nonnegative weighted sum method. Thus, the problem is reduced to a single linear objective with grey parameter in the right side of the constrains. Numerical example is provided to demonstrate the efficiency and feasibility of the proposed approach. According to the mentioned examples, in the current work, the proposed method is both simple in use and suitable for solving different problems. The mathematical model for the special case study is applied here for the standard IEEE 30-bus test system at our special case study. The main goal of this problem is to schedule committed generators to meet the load required to minimize the pollution emissions and fuel cost. The model formulation for a special case study problem is presented. In case the right hand side of the constrains include grey interval parameters, then the mathematical model is considered as (GMOLFP) and the problem is solved according to the proposed solution algorithm.

It should be noted that the decision variable of the solution of (GMOILFP) with the grey parameters in the right- hand side of the constraints is interval grey number. It is nearly as the same solution of (MOILFP) if the interval grey number contains the crisp number. When the interval grey number not includes the crisp number then the deferent solution is resulted, and it is interval grey number.

There are many other points of research in the field of grey multi-objective optimization. Some of this research work can be summarized as follows:

(i)An algorithm is needed to solve large-scale grey multi-objective integer linear and integer linear fractional programming problems.

(ii)An algorithm is required to deal with multilevel multi-objective integer linear and integer linear fractional programming problems.

(iii)It is a promising point of research to study some real-life applications in nature of grey system.

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