Numerical Study of Unsteady MHD Pulsating Flow of Couple Stress Fluid through Porous Medium between Permeable Beds Using Generalized Differential Quadrature Method (GDQM)

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Abstract—A numerical study of unsteady MHD pulsating flow of an incompressible Newtonian electrically conducting couple stress fluid through porous medium between permeable beds under the influence of periodic body acceleration. The couple stress fluid is injected into the channel from the lower permeable bed with a certain velocity and is sucked into the upper permeable bed with the same velocity. The flow between the permeable beds is assumed to be governed by couple stress fluid flow equations of Navier-Stokes and that in the permeable regions by Darcy's law. The slip condition plays an important role in shear skin, spurt, and hysteresis effects. The fluids that exhibit boundary slip have important technological applications such as in polishing valves of artificial heart and internal cavities. A numerical solution of the equation of motion is obtained by a new algorithm "modified generalized differential quadrature method (MGDQM)" which applying a generalized differential quadrature method (GDQM), to derivatives with respect to space variables of differential equations and for the time derivative applying 4th order Runge Kutta Method. This combination of DQM and

4^{th} order RK method gives very good numerical technique for solving time dependent problems. The numerical results and the effects of the material parameters are presented and discussed through graphs.

Keywords— Pulsatile flow, MHD, couple stress fluid, permeable beds, body acceleration, porous medium, Differential quadrature method, Runge-Kutta method.

1. INTRODUCTION

The MHD fluid flow in channel is an interesting area in the study of fluid mechanics because of its relevance to various engineering applications. The MHD effects are widely exploited in different industrial processes ranging from metallurgy to the production pure crystals. A field in which MHD will play an essential role is nuclear fusion, where it is involved in at least two different problems: the confinement and dynamics of plasma, and the behaviour of the liquid metal alloys employed in some of the currently considered designs of tritium breeding blankets.

A fluid flow driven by a pulsatile pressure gradient through porous media in a channel or a pipe are very much pervalent in nature and hence their study is of principal interest in many scientific and engineering applications. This type of flows are of great importance in chemical engineering (for filteration and water purification processes) and petroleum engineering (for stuyding the movement of natural gas, oil and water through the oil resevoirs). Also, this type of flows are of great importance in physiology, biomedical engineering and it has biological applications in relation to hemodynamics, industrial applications in relation to heat exchange efficiency, applications in natural systems like circulatory systems, respiratory systems, vascular diseases, in engineering systems like reciprocating pumps, IC engines, combustors and applications in MEMS micro fluidic engineering applications [1-3].

Also, the fluid flow driven by a pulsatile pressure gradient through porous media in a channel or a pip has application in the dialysis of blood through artificial kidneys or blood flow in the lung alveolar sheet. Afifi and Gad [4] studied the flow of a Newtonian, incompressible fluid under the effect of transverse magnetic field through a porous medium between infinite parallel walls on which a sinusoidal traveling wave is imposed. Ramamurthy and Shanker [5] studied magneto-hydrodynamic effects on blood flow through a porous channel, they considered the blood as a Newtonian fluid and conducting fluid. Arterial MHD pulsatile flow of blood under periodic body acceleration has been studied by Das and Saha [6]. The effect of uniform transverse magnetic field on its pulsatile motion through an axi-symmetric tube is analyzed by Dulal and Ananda [7]. Chaturani and Palanisamy [8] discussed the flow characteristics of blood under external body acceleration. Eldesoky [9] studied the unsteady pulsatile flow of blood through porous medium in an artery under the influence of periodic body acceleration and slip condition in the presence of magnetic field considering blood as an incompressible electrically conducting fluid. An analytical solution of the equation of motion is obtained by applying the Laplace transform. Eldesoky [10] studied the influence of slip on the peristaltic flow in an axisymetric cylindrical tube. A compressible Maxwell fluid saturates the homogenous porous medium. Modified Darcy's law has been used to model the governing equation. The analytical solutions of the equation of motion are obtained by applying a perturbation analysis. Recently, Eldesoky, Kamel, Hussien and Abumandour [11] studied the unsteady pulsatile flow through porous medium in an artery under the influence of periodic body acceleration in the presence of magnetic field numerically using Generalized Differential

Quadrature Method (GDQM). Eldesoky [12] studied the influence of relaxation time on MHD pulsatile flow of blood through porous medium in an artery under the effect of periodic body acceleration in the presence of magnetic field considering blood as an incompressible electrically conducting fluid. An analytical solution of the equation of motion is obtained by applying the Laplace Transform. Wang [13] studied the pulsatile flow of a viscous fluid in a porous channel. Vajravelu et al. [14] studied the pulsatile flow of a viscous fluid between permeable beds.

The boundary conditions for the flow through porous beds need special attention. Generally the no-slip condition is valid on the boundary when a fluid flows between impermeable surfaces. But when it flows between permeable surfaces, the no-slip condition is no longer valid since there will be a migration of fluid, tangential to the boundary within the permeable surfaces. The velocity within the permeable beds will be different from the velocity of the fluid in the channel and we have to match the two velocities at the interface. A simple theory presented by Beavers and Joseph in [15] based on replacing the effect of the boundary layer with a slip velocity proportional to the exterior velocity gradient is proposed and shown to be in reasonable agreement with experimental results.

The theory of couple stress fluids initiated by Stokes [16] is a generalization of the classical theory of viscous fluids, which allows for the presence of couple stresses and body couples in the fluid medium, which studied a series of boundary-value problems are solved to indicate the effects of couple stresses. A striking feature of this model is that it results in equations that are similar to the Navier Stokes equations, there by facilitating a comparison with the results for the classical non polar case. An excellent introduction to this theory written by Stokes [17] in which he has presented a detailed account of couple stress fluids. This theory has several industrial and scientific applications as well, which comprise pumping fluids such as synthetic fluids, polymer thickened oils, liquid crystal, animal blood, synovial fluid present in synovial joints and the theory of lubrication [18-23]. Some salient references to couple stress flows through tubes and channels can be seen in Stokes [17]. Several flows past axisymmetric bodies dealing with couple stress fluids have been studied by Rao and Iyengar [24] and Srinivasacharya [25]. Rao and Iyengar [24] have made analytical and computational studies of diverse couple stress fluid flows dealing with a class of axisymmetric problems. EL-Dabe et.al [26] discussed the effects of couple stresses on pulsatile hydro magnetic Poiseuille flow, using perturbation technique. Srivastava [27] studied the effects of an axially symmetric mild stenosis on the flow of blood, when blood is represented by a couple stress fluid. He also studied the peristaltic transport of a couple stress fluid under a zero Reynolds number and long wavelength approximation [28]. Naduvinamani et al. studied a number of problems with couple stress fluid between porous journal bearings and porous rectangular plates [29-31]. Devakar and Iyengar [32, 33] studied Stokes Problems for an incompressible couple stress fluid under isothermal conditions and run up flow of a couple stress fluid between parallel plates using Laplace transform technique. Radhika and Iyengar [34] studied the Stokes flow of an incompressible couple stress fluid past a porous spheriodal shell.

Numerical approximation methods for solving partial differential equations have been widely used in various engineering fields. Classical techniques such as finite elements and finite differences methods are well developed and well known. These methods can provide very accurate results by using a large number of grid points. In seeking an alternate numerical method using fewer grid points to find results with acceptable accuracy, the method of DQM was introduced by [36-45].

In this paper, modified GDQM used for studying the unsteady pulsating flow of an incompressible couple stress fluid between permeable beds through Porous Medium in channel under the influence of periodic body acceleration in the presence of magnetic field. Modified GDQM is a new algorithm which applying a generalized differential quadrature method (GDQM), to derivatives with respect to space variables of differential equations and for the time derivative applying 4th orders Runge Kutta Method. This combination of DQM and 4th order RK method gives very good numerical technique for solving time dependent problems. Stability of 4$^{\mathbf{th}}$ order RKM criterias are controlled with several values of time increment Δt and number of grid points N in space region. The fluid is driven by an unsteady pressure gradient through the permeable beds. The flow through the permeable beds is assumed to be governed by Darcy's law and the flow between permeable beds by couple stress fluid flow equations of Stokes. The equations are solved numerically and the numerical results are presented and discussed through graphs. We observe that our results are in good agreement with those obtained by Vajravelu et al. [14] in the respective special cases considered by them.

2. MATHEMATICAL FORMULATION

We consider the pulsatile flow of an incompressible couple stress fluid between two permeable beds. The fluid is injected into the channel from the lower permeable bed with a velocity V and is sucked into the upper permeable bed with the same velocity. The flow between the permeable beds is assumed to be governed by couple stress fluid flow equations Stokes [17].

Let the *x*-axis be taken along the interface and the *y*-axis perpendicular to it. Let *y* = *0* and *y* = *h* represent the interfaces of the permeable beds under consideration (see figure 1).

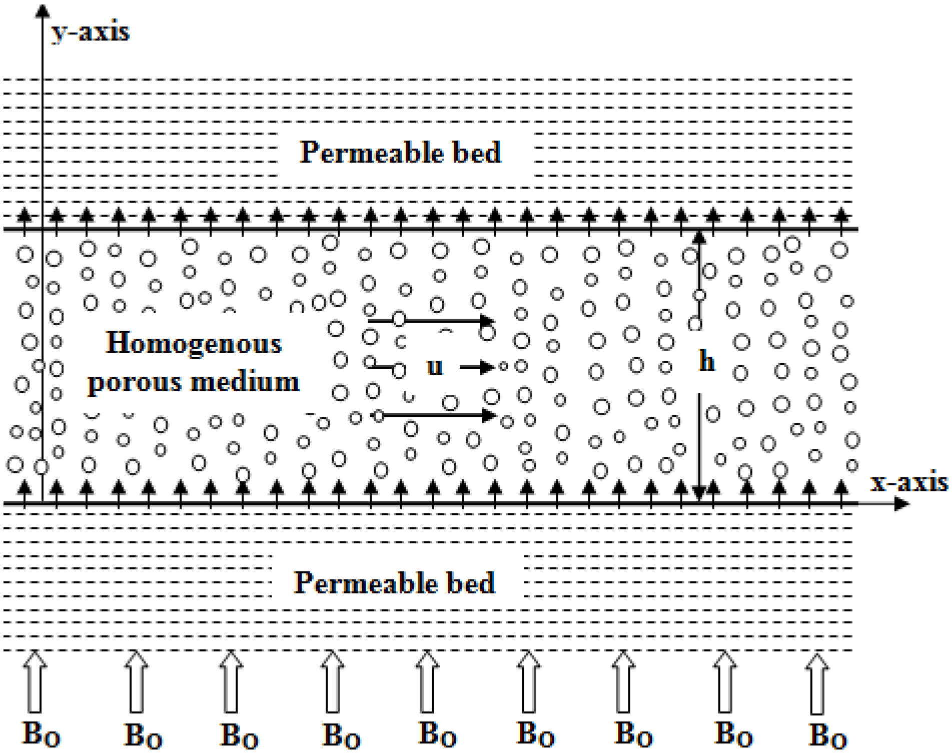

Figure 1: Flow diagram for the flow

The following assumptions are made in the analysis of the problem; the permeable beds are homogeneous, the flow as axially symmetric, laminar, pulsatile and fully developed, and the pressure gradient and body acceleration *G* are given by:

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = A_O + A_1 \cos(\omega_p t), \qquad t \geq 0, \tag{1}$$

$$G = a_O \cos(\omega_b t), \qquad t \geq 0 \tag{2}$$

where A_O and A_1 are pressure gradient of steady flow and amplitude of oscillatory part respectively, a_O is the amplitude of the body acceleration, $\omega_p = 2\pi f_p$, $\omega_b = 2\pi f_b$ with f_p is the pulse frequency, and f_b is the body acceleration frequency and *t* is time.

Under the above mentioned assumption, the field equations describing a couple stress fluid flow [17] are:

$$\frac{\partial \rho}{\partial t}+div\left(\rho \bar{q}\right)=0 \tag{3}$$

$$\rho \frac{\partial \bar{q}}{\partial t}=\rho \bar{f}+\frac{1}{2}curl\left(\rho \bar{c}\right)-grad\left(p\right)-\mu curl\left(curl\left(\bar{q}\right)\right)$$
$$-\eta curl\left(curl\left(curl\left(curl\left(\bar{q}\right)\right)\right)\right)+\left(\lambda+2\mu\right)grad\left(div\left(\bar{q}\right)\right) \tag{4}$$

where ρ is the density of the fluid, $\bar{q}$ is the velocity vector, p is the fluid pressure and $\bar{f}, \bar{c}$ are the body force per unit mass and body couple per unit mass respectively. The quantities λ and μ are the viscosity coefficients and η, η' are the couple stress viscosity coefficients satisfying the constraints

$$\mu \geq 0,\ \ 3\lambda+2\mu \geq 0,\ \ \eta \geq 0,\ \ \left|\eta'\right| \leq \eta \tag{5}$$

There is a length parameter $l=\sqrt{\frac{\eta}{\mu}}$ which is a characteristic measure of the polarity of the couple stress fluid and this parameter is identically zero in the case of non polar fluids.

Under the assumption made, we have $\bar{q}=\left(u(y,t),V,0\right)$.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{6}$$

$$\frac{\partial u}{\partial t}+V\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\bar{G}+\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}-\frac{\eta}{\rho}\frac{\partial^4 u}{\partial y^4}-\left(\frac{\mu}{\rho k}\right)u+\frac{\bar{J}\times\bar{B}}{\rho} \tag{7}$$

$$0=\frac{\partial p}{\partial y} \tag{8}$$

Maxwell's generalized electromagnetic field equations are

$$\bar{\nabla}\cdot\bar{B}=0,\quad \bar{\nabla}\cdot\bar{D}=\rho,\quad \bar{\nabla}\times\bar{B}=\mu_O\bar{J},\quad \bar{\nabla}\times\bar{E}=-\frac{\partial \bar{B}}{\partial t}. \tag{9}$$

Ohm's law is

$$\bar{J}=\sigma\left(\bar{E}+\bar{q}\times\bar{B}\right). \tag{10}$$

where μ_O the fluid magnetic permeability, $\bar{B}=\left(0,B_O,0\right)$the magnetic field vector, $\bar{E}$ the electric field vector, $\bar{J}$ the current density vector, k is the permeability parameter of porous medium, σ the fluid electric conductivity and D the electric flux density. For small magnetic Reynolds number, the linearlized magnetohydrodynamic force $\overline{J}\times\overline{B}$ can be put into the following form:

D and B are the electric and magnetic flux densities

$$\overline{J}\times\overline{B}=-\sigma B_O^2 u. \tag{11}$$

Under the above assumptions the equation of motion (7) is

$$\frac{\partial u}{\partial t}+V\frac{\partial u}{\partial y}=A_O+A_1\cos(\omega_p t)+a_O\cos(\omega_b t)+\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}-\frac{\eta}{\rho}\frac{\partial^4 u}{\partial y^4}-\left(\frac{\mu}{\rho k}\right)u+\frac{\sigma B_O^2 u}{\rho} \tag{12}$$

For the slip condition $u_t=A_p\,\partial u_t/\partial n$, where u_t is the tangential velocity, n is normal to the surface, and A_p is a coefficient close to the mean free path of the molecules of the fluid [35]. Than the boundary conditions that must be satisfied by the fluid on the wall of channel are:

$$\left.\begin{array}{lll} u=A_p\dfrac{\partial u}{\partial y} & \text{at} & y=0 \\ u=A_p\dfrac{\partial u}{\partial y} & \text{at} & y=h \end{array}\right\}(\text{Slip conditions}) \tag{13a}$$

$$\left.\begin{array}{lll} \dfrac{\partial^2 u}{\partial y^2}=0 & \text{at} & y=0 \\ \dfrac{\partial^2 u}{\partial y^2}=0 & \text{at} & y=h \end{array}\right\}(\text{Vanishing of couple stresses}) \tag{13b}$$

Let us introduce the following dimensionless quantities:

$$u^*=\frac{u}{\omega h},\quad t^*=\frac{t\nu}{h^2},\quad A_O^*=\frac{\rho h}{\mu\omega}A_O,\quad A_1^*=\frac{\rho h}{\mu\omega}A_1,$$
$$a_O^*=\frac{\rho h}{\mu\omega}a_O,\quad k^*=\frac{k}{h^2},\quad b=\frac{\omega_b}{\omega_p},\quad y^*=\frac{y}{h}. \tag{14}$$

In terms of these variables, equation (12) after dropping the stars becomes:

$$\frac{\partial u}{\partial t} = A_O + A_1 \cos(t) + a_O \cos(bt) - \frac{1}{\beta^2}\frac{d^4u}{dy^4} + \frac{d^2u}{dy^2} -$$

$$R\frac{du}{dy} - \left(H_a{}^2 + \frac{1}{k}\right)u. \tag{15}$$

Also, the boundary conditions equation (13) becomes:

$$\left.\begin{array}{lll} u = k_n \dfrac{du}{dy} & \text{at} & y = 0 \\ u = k_n \dfrac{\partial u}{\partial y} & \text{at} & y = 1 \end{array}\right\}\left(\text{Slip conditions}\right) \tag{16a}$$

$$\left.\begin{array}{lll} \dfrac{d^2u}{dy^2} = 0 & \text{at} & y = 0 \\ \dfrac{d^2u}{dy^2} = 0 & \text{at} & y = 1 \end{array}\right\}\left(\text{Vanishing of couple stresses}\right) \tag{16b}$$

And the initial condition is

$$u(y, 0) = 1 \qquad \text{at} \qquad t = 0 \tag{17}$$

Where the Hartmann number, *Ha*, the Reynolds number, *R*, the Couple stress parameter β and the Knudsen number k_n are defined respectively by

$$H_a = B_O h\sqrt{\frac{\sigma}{\mu}}, \qquad R = \frac{\rho V h}{\mu}, \qquad \beta^2 = \frac{h^2\mu}{\eta}, \qquad k_n = \frac{A_p}{h}. \tag{18}$$

3. Generalized Differential Quadrature Method (GDQM)

The DQM is a numerical solution technique for initial and/or boundary value problems. This technique has been successfully employed in a variety of problems in engineering and physical sciences. The DQM approximates the derivative of a function at any location by a linear summation of all the functional values along a mesh (grid) line. In order to overcome the deficiencies which appears in classical DQM, Bellman et al. [36, 37], a generalized differential quadrature (GDQ), which was recently proposed by Shu and Richards [39-42] for solving partial differential equations in fluid mechanics, vibration analysis and structural analysis. The technique of GDQM for the solution of partial differential equations extended and generalized. Numerical examples have shown the super accuracy, efficiency, convenience and the great potential of this method. The GDQM, which was recently proposed by [39-41] for solving partial differential equations. For the discretization of the first and higher order derivatives, the following linear constrained relationships are applied

$$f_x^{(n)}(x_i, t) = \sum_{j=1}^{N} C_{ij}^{(n)}.f(x_j, t), \qquad n = 1,2,...,N-1, \tag{19}$$

for $i = 1,2,...,N$; where $f_x^{(n)}$ indicate n^{th} order derivatives of *f(x,t)* with respect to *x* at x_i, *N* is the number of grid points in the whole dominant $C_{ij}^{(n)}$ are the weighting coefficients. The key to DQ is to determine the weighting coefficients for the discretization of a derivative of any order. In order to find a simple algebraic expression for calculating the weighting coefficients without restricting the choice of grid meshes, [39-41] gave a convenient and recurrent formula for determining the derivative weighting coefficients.

To determine the weighting coefficients of the GDQ method as:

Weighting coefficients for the first order derivative

$$C_{ij}^{(1)} = \frac{M_N^{(1)}(x_i)}{(x_i - x_j).M_N^{(1)}(x_j)}, \qquad i, j = 1, 2, \cdots, N \quad \text{and} \quad i \neq j \tag{20}$$

$$C_{ii}^{(1)} = -\sum_{j=1, j\neq i}^{N} C_{ij}^{(1)}, \qquad i = 1, 2, \cdots, N \tag{21}$$

Where

$$M(x) = (x - x_1)(x - x_2)\cdots(x - x_N) \tag{22a}$$

$$M^{(1)}(x_i) = \prod_{k=1, k\neq i}^{N} (x_i - x_k) \tag{22b}$$

Weighting coefficients for the second and higher order derivatives

$$C_{ij}^{(n)} = n\left(C_{ij}^{(1)} . C_{ii}^{(n-1)} - \frac{C_{ij}^{(n-1)}}{x_i - x_j} \right),$$

$$\text{for} \quad j \neq i, \quad i, j = 1,2,...,N; \quad n = 2,3,...,N-1. \qquad (23)$$

$$C_{ii}^{(n)} = -\sum_{j=1.j\neq i}^{N} C_{ij}^{(n)}, \quad \text{for} \quad i, j = 1,2,...,N; \quad n = 2,3,...,N-1. \quad (24)$$

Where $C_{ij}^{(n)}$ and $C_{ij}^{(n-1)}$ are the weighting coefficients of the n^{th} and the $(n-1)^{th}$ derivatives. Thus equations (23) and (24) together with equations (20) and (21) give a convenient and general form for determining the weighting coefficients for the derivatives of orders one through *N–1*.

4. Numerical discretization and stability of the scheme

In the present study, substituting the DQ derivative approximations given in equation (19) in the governing equation (15) and boundary conditions equation (16). The coordinates of the grid points are chosen according to Chebyshev-Gauss-Lobatto by using *N* sampling as:

$$X(i) = \frac{1}{2}\left[1 - \cos\left(\frac{i-1}{N-1}\pi\right)\right], \qquad i = 1,2,3,...,N;$$

The GDQM is applied for the discretization of space derivatives of the unknown function *u*.

$$\frac{\partial u(y_i,t)}{\partial t} = A_O + A_1\cos(t) + a_O\cos(bt) - \frac{1}{\beta^2}\sum_{j=1}^{N} C_{i,j}^{(4)} \cdot u_j + \sum_{j=1}^{N} C_{i,j}^{(2)} \cdot u_j -$$
$$R\sum_{j=1}^{N} C_{i,j}^{(1)} \cdot u_j - \left(H_a{}^2 + \frac{1}{k}\right) u_i. \qquad i = 1,2,\cdots,N \qquad (25)$$

Where u_i, $i = 1,2,...,N$; is the velocity value at the grid y_i, $C_{ij}^{(1)}$, $C_{ij}^{(2)}$ and $C_{ij}^{(4)}$ are the weighting coefficient matrixes of the first, second and forth order derivatives. Similarly, the derivatives in the boundary conditions can be discretized by the GDQM. As a result, the numerical boundary conditions can be written as:

$$\left.\begin{aligned} u_1 &= k_n \sum_{k=1}^{N} C_{1,k}^{(1)} \cdot u_k \\ u_N &= k_n \sum_{k=1}^{N} C_{N,k}^{(1)} \cdot u_k \end{aligned}\right\} \quad (\text{Slip conditions}) \qquad (26a)$$

$$\left.\begin{aligned} \sum_{k=1}^{N} C_{1,k}^{(2)} \cdot u_k &= 0 \\ \sum_{k=1}^{N} C_{1,k}^{(2)} \cdot u_k &= 0 \end{aligned}\right\} \quad (\text{Vanishing of couple stresses}) \qquad (26b)$$

Equation (26) can't be easily substituted into the governing equation. However, we can give solutions, $u_1, u_2,\ u_{N-1}$ and u_N as:

$$u_1 = R_{36} \cdot \sum_{k=3}^{N-2} C_{1,k}^{(1)} \cdot u_k + R_{37} \cdot \sum_{k=3}^{N-2} C_{N,k}^{(N)} \cdot u_k + R_{38} \cdot \sum_{k=3}^{N-2} C_{1,k}^{(2)} \cdot u_k + R_{39} \cdot \sum_{k=3}^{N-2} C_{N,k}^{(2)} \cdot u_k \quad (27a)$$

$$u_2 = R_{24} \cdot \sum_{k=3}^{N-2} C_{1,k}^{(1)} \cdot u_k + R_{26} \cdot \sum_{k=3}^{N-2} C_{N,k}^{(N)} \cdot u_k + R_{25} \cdot \sum_{k=3}^{N-2} C_{1,k}^{(2)} \cdot u_k + R_{27} \cdot \sum_{k=3}^{N-2} C_{N,k}^{(2)} \cdot u_k \quad (27b)$$

$$u_{N-1} = R_{20} \cdot \sum_{k=3}^{N-2} C_{1,k}^{(1)} \cdot u_k + R_{21} \cdot \sum_{k=3}^{N-2} C_{N,k}^{(N)} \cdot u_k + R_{22} \cdot \sum_{k=3}^{N-2} C_{1,k}^{(2)} \cdot u_k + R_{23} \cdot \sum_{k=3}^{N-2} C_{N,k}^{(2)} \cdot u_k \quad (27c)$$

$$u_N = R_{48} \cdot \sum_{k=3}^{N-2} C_{1,k}^{(1)} \cdot u_k + R_{49} \cdot \sum_{k=3}^{N-2} C_{N,k}^{(N)} \cdot u_k + R_{50} \cdot \sum_{k=3}^{N-2} C_{1,k}^{(2)} \cdot u_k + R_{51} \cdot \sum_{k=3}^{N-2} C_{N,k}^{(2)} \cdot u_k \quad (27d)$$

According to equation (27), $u_1, u_2,\ u_{N-1}$ and u_N is expressed in terms of $u_3, u_4, \cdots, u_{N-2}$, and can be easily substituted into the governing equation (25). It should be noted that equation (26) provides four boundary equations. In total we have N unknowns $u_1, u_2, \cdots, u_N$. In order to close the system, the discretized governing equation (25) has to be applied at $N-4$ mesh points. This can be done by applying equation (25) at grid points $y_3, y_4, \ldots, y_{N-2}$. Substituting equations (27) into equation (25) gives:

$$\frac{\partial u(y_i, t)}{\partial t} = A_O + A_1 \cos(t) + a_O \cos(bt) - \frac{1}{\beta^2} \sum_{j=3}^{N-2} C_{i,j}^{(4)} \cdot u_j + \sum_{j=3}^{N-2} C_{i,j}^{(2)} \cdot u_j -$$

$$R\sum_{j=3}^{N-2} C_{i,j}^{(1)} \cdot u_j - \left(H_a{}^2 + \frac{1}{k}\right) u_i . \quad i = 3,4,\cdots,N-2 \qquad (28)$$

It is noted that equation (28) has $N-4$ equations with $N-4$ unknowns.

Now, the discretization for time derivative will be performed by using Runge-Kutta 4th order Method. Now, $\frac{\partial u}{\partial t}$ is also considered discretized as $\frac{\partial u_{ij}}{\partial t}$, thus equation (28) is a set of DQ algebraic equations which can be written in a matrix form

$$[A]\{u\} = \{b\}, \qquad (29)$$

Where $\{u\} = \{u_3, u_4, \cdots, u_{N-2}\}$ is a vector of unknown $N-4$ functional values at all discretized points of the region, $[A]$ is the $(N-4)\times(N-4)$ coefficient matrix, and the right hand side vector $\{b\}$ of size $(N-4)\times 1$ contains first order time derivatives of the function u at the same discretized points. Therefore a numerical scheme is necessary for handling these time derivatives. Equation (29) can be solved by several time integration schemes such as Euler, Modified Euler, and Runge-Kutta 4th order Methods. Here, Runge-Kutta Method is going to be used since it is a one step method obtained from the Taylor series expansion of u up to and including the terms involving $(\Delta t)^4$ where Δt is the step size with respect to time. The 4th order RKM since its stability region is larger comparing to the other time integration methods and simple for the computations.

The resulting algebraic system of equation (29) can originally be considered as an initial value problem in the form (a set of ordinary differential equations in time)

$$\{b\} = \frac{d\{u\}}{dt} = [A]\{u\} \qquad (30)$$

Thus the 4th order RKM gives for the governing equation the following vector equation

$$\{u_{n+1}\} = \{u_n\} + \frac{\Delta t}{6}\left[\{K_1\} + 2\{K_2\} + 2\{K_3\} + \{K_4\}\right] \tag{31}$$

Where;

$$K_1 = f\,(t_n, u_n),$$

$$K_2 = f\left(t_n + \frac{\Delta t}{2}, u_n + \frac{\Delta t}{2}K_1\right),$$

$$K_3 = f\left(t_n + \frac{\Delta t}{2}, u_n + \frac{\Delta t}{2}K_2\right),$$

$$K_4 = f\,(t_n + \Delta t, u_n + \Delta t\, K_3),$$

Applying 4th order RKM equation (31) in equation (30). Thus, we can easily write by taking $[A]\{u\}$ as the vector function $\{f\,(t,\{u\})\}$ in the sample initial value problem $\dot{u} = f\,(t,u)$ So,

$$\{f\,(t,\{u\})\} = [A]\{u\} \tag{32}$$

The Matlab program has been used to solve this problem and get the velocity distribution.

5. Numerical results and discussion

We studied the unsteady pulsating flow of an incompressible couple stress fluid between permeable beds through Porous Medium in channel under the influence of periodic body acceleration in the presence of magnetic field numerical using new algorithm modified DQM. The algorithm is coded in Matlab 7.14.0.739 and the simulations are run on a Pentium 4 CPU 900 MHz with 1 GB memory capacity. We have shown the relation between the different parameters of motion such as Hartmann number *Ha*, Couple stress parameter *β*, Reynolds number *R*, Knudsen number k_n, the permeability parameter of porous medium *k* with the velocity distribution to investigate the effect of changing these parameters on the flow. Hence, we can be controlling the process of flow.

A numerical code has been written to calculate the velocity distribution according to equation (29).

Figs. 2, shows the variation of the velocity distribution with respect to Couple stress parameter β. It is seen that as β increases (i.e., as Coefficient of couple stress viscosity η decreases) the unsteady velocity increases. As $\beta \to \infty$ (i.e., as $\eta \to 0$), the velocity corresponds to non polar fluid.

Figs. 3, shows the variation of the velocity distribution with respect to the Hartmann number *Ha*. It is seen that as *Ha* increases the unsteady velocity decreases.

Figs. 4, shows the variation of the velocity distribution with respect to the permeability parameter of porous medium *k*. It is seen that as *k* increases the unsteady velocity increases.

Figs. 5, shows the variation of the velocity distribution with respect to the Reynolds number *R* have less influence on the unsteady velocity. It is seen that as *R* increases the unsteady velocity decreases. The decrease in velocity for each *R* is clearly visible for the region $y = 0$ to $y = 0.8$. As $y = 0.8$ to $y = 1.0$, the velocity profile are almost coinciding for each *R*.

Figs. 6, shows that the variation of the velocity distribution with respect to the Knudsen number *kn*. It is clearly visible that the velocity increases with increasing *kn* for the region $y = 0$ to $y = [0.4 : 0.5]$. As $y = 0.5$ to $y = 1.0$, the velocity decrease with increasing *kn*.

Figs. 7, shows the effects of Reynolds number *R* with the velocity distribution for non polar viscous fluid ($\beta \to \infty$ i.e., $\eta \to 0$). As Reynolds number *R* is increasing the velocity at any *y* is decreasing. For $R = 0$, the maximum velocity is attained exactly midway between the lower and upper permeable beds. As *R* increases, the maximum velocity is attained nearer to the upper permeable bed.

Figs. 8, shows the effects of the permeability parameter of porous medium *k* with the velocity distribution for non polar viscous fluid ($\beta \to \infty$ i.e., $\eta \to 0$). As *k* is increasing the velocity at any *y* is increasing. The velocity profile corresponding to $R = 2$. the maximum velocity nearer to the upper plate slightly (slowly).

Figs. 9, shows the effects of the Knudsen number *kn* with the velocity distribution for non polar viscous fluid ($\beta \to \infty$ i.e., $\eta \to 0$). As the Knudsen number *kn* is increasing for the region $y = 0$ to $y = [0.4 : 0.5]$. As As $y = 0.5$ to $y = 1.0$, the velocity decrease with increasing *kn*. The velocity profile corresponding to $R = 2$. Here again the maximum velocity nearer to the upper plate slightly (slowly).

Figs. 10, 11, shows the effects of the Hartmann number *Ha* with the velocity distribution for non polar viscous fluid ($\beta \to \infty$ i.e., $\eta \to 0$). As *Ha* is increasing the velocity at any *y* is decreasing. The velocity profile corresponding to $R = 2$. The maximum velocity region increase with increase the Hartmann number *Ha*.

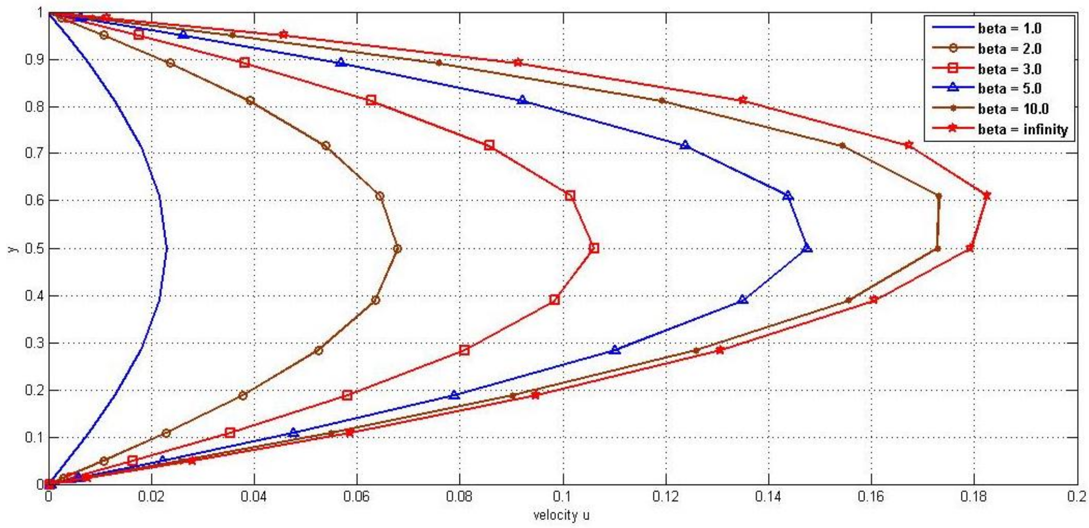

Fig. 2 The effect of Couple stress parameter on velocity distribution [Ha=1, kn=0.001, k=0.5, R=2, A_O=2, A_1=1, a_O=1, b=2].

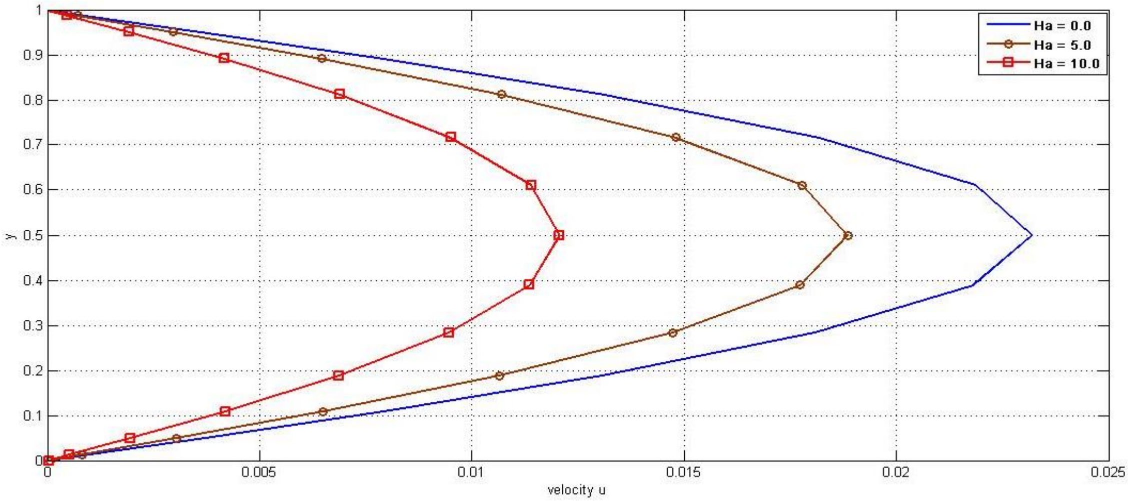

Fig. 3 The effect of Hartman number on velocity distribution [k=0.5, R=2, kn=0.001, β=1, A_O=2, A_1=1, a_O=1, b=2].

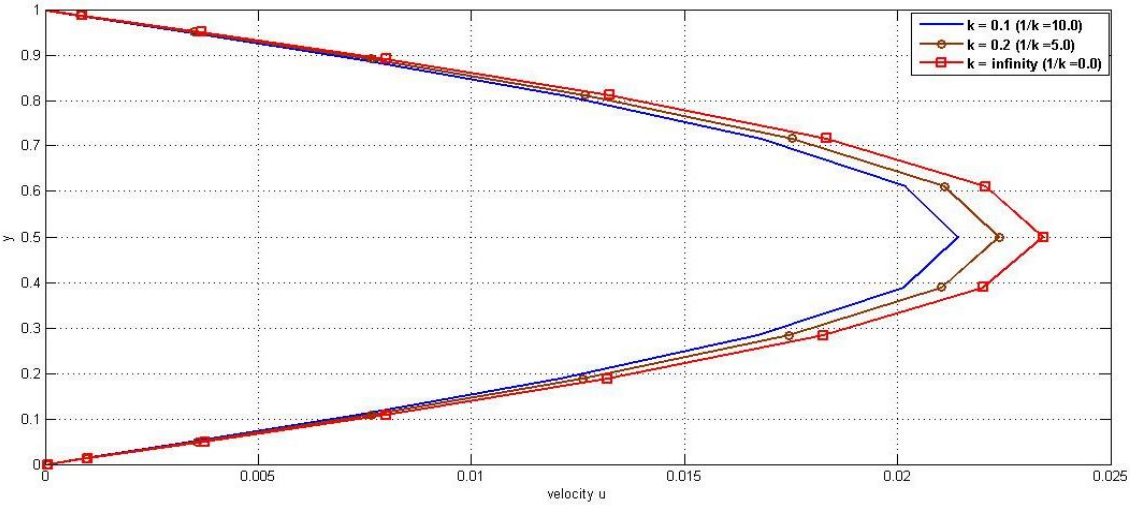

Fig. 4 The effect of Porosity parameter on velocity distribution [Ha=1, R=2, kn=0.001, β=1, A_O=2, A_1=1, a_O=1, b=2].

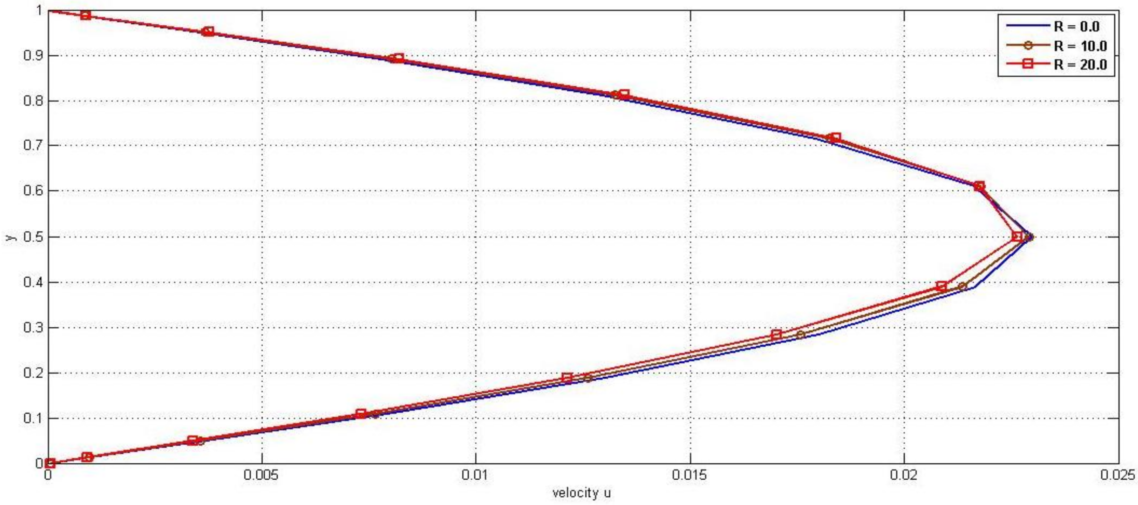

Fig. 5 The effect of Reynolds number on velocity distribution [Ha=1, k=0.5, kn=0.001, β=1, AO=2, A1=1, aO=1, b=2].

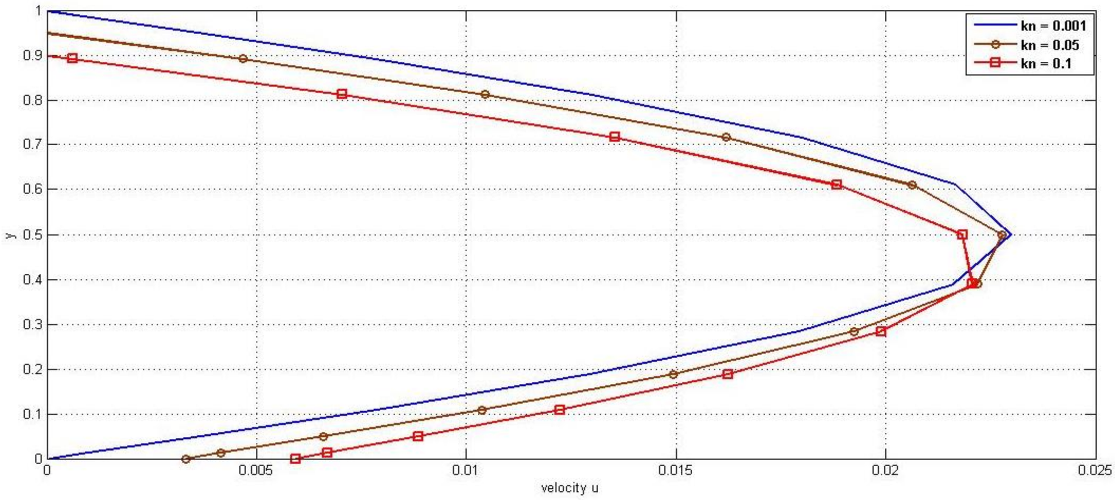

Fig. 6 The effect of Knudsen number on velocity distribution [Ha=1, k=0.5, R=2, β=1, A_O=2, A_1=1, a_O=1, b=2].

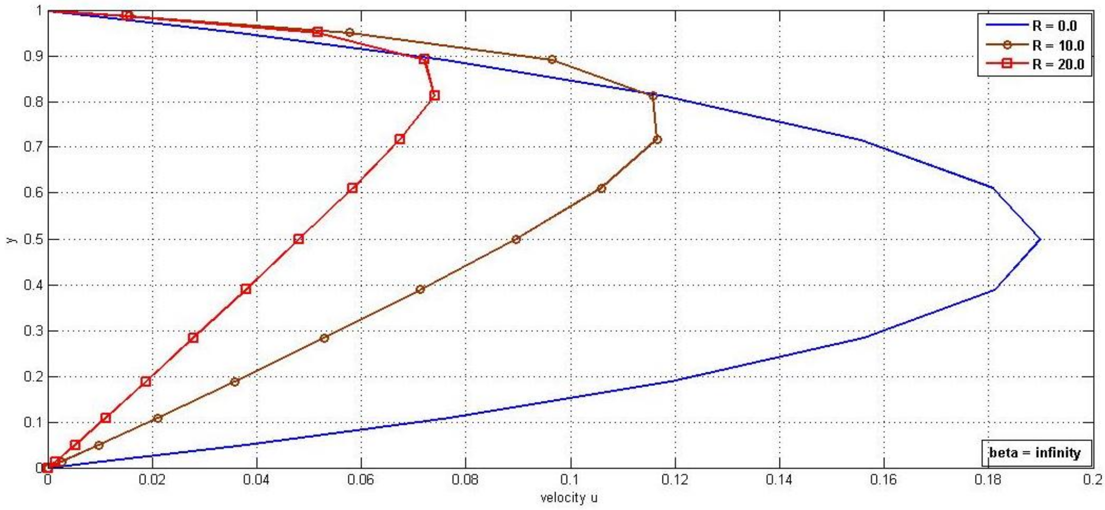

Fig. 7 The effect of Reynolds number on velocity distribution [Ha=1, k=0.5, kn=0.001, A_O=2, A_1=1, a_O=1, b=2, $\beta\to\infty$ ($\eta\to 0$)].

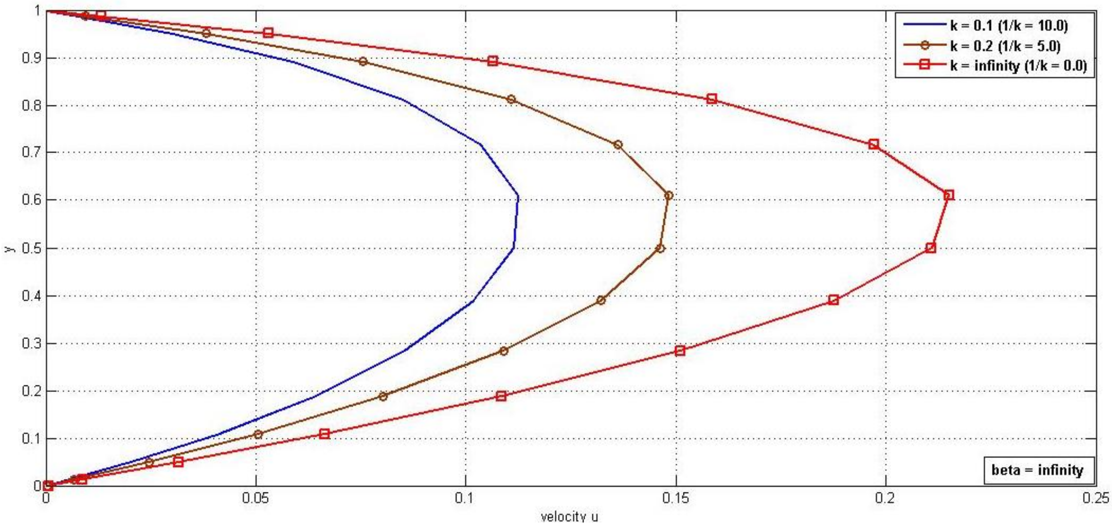

Fig. 8 The effect of Porosity parameter on velocity distribution [Ha=1, R=2, kn=0.001, A_O=2, A_1=1, a_O=1, b=2, $\beta\to\infty$ ($\eta\to 0$)].

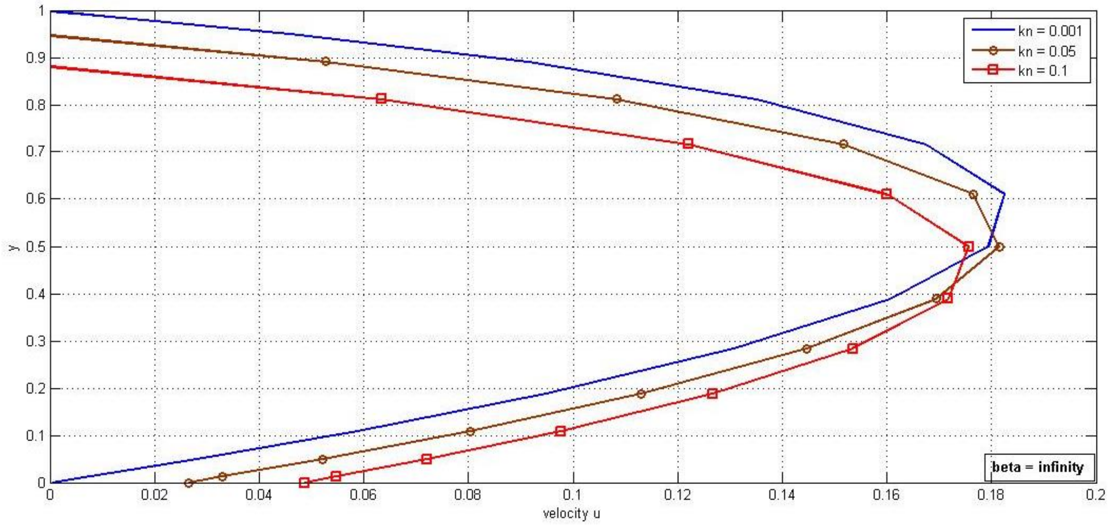

Fig. 9 The effect of Knudsen number on velocity distribution [Ha=1, R=2, k=0.5, A_O=2, A_1=1, a_O=1, b=2, β→∞ (η→0)].

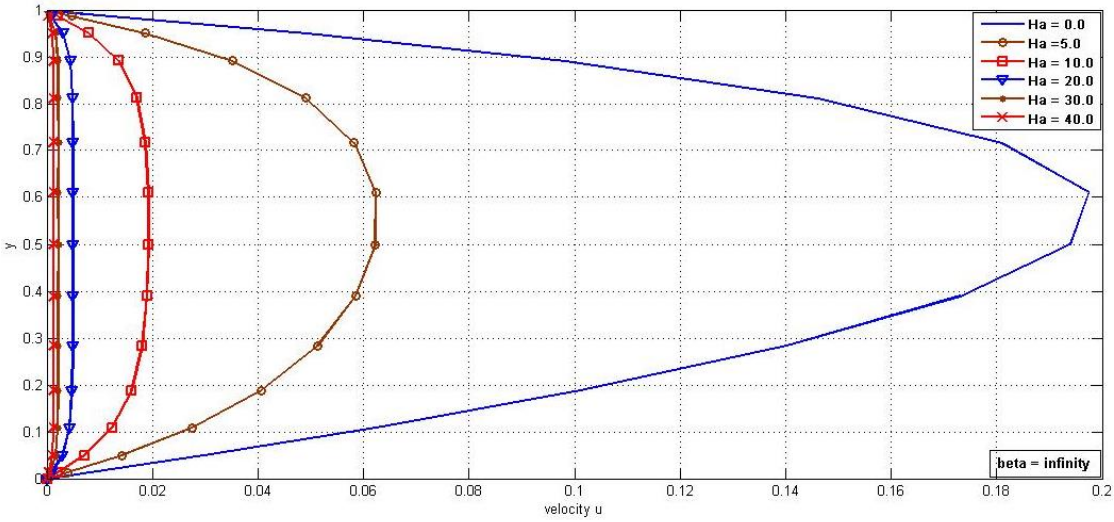

Fig. 10 The effect of Hartman number on velocity distribution [k=0.5, R=2, kn=0.001, A_O=2, A_1=1, a_O=1, b=2, β→∞ (η→0)].

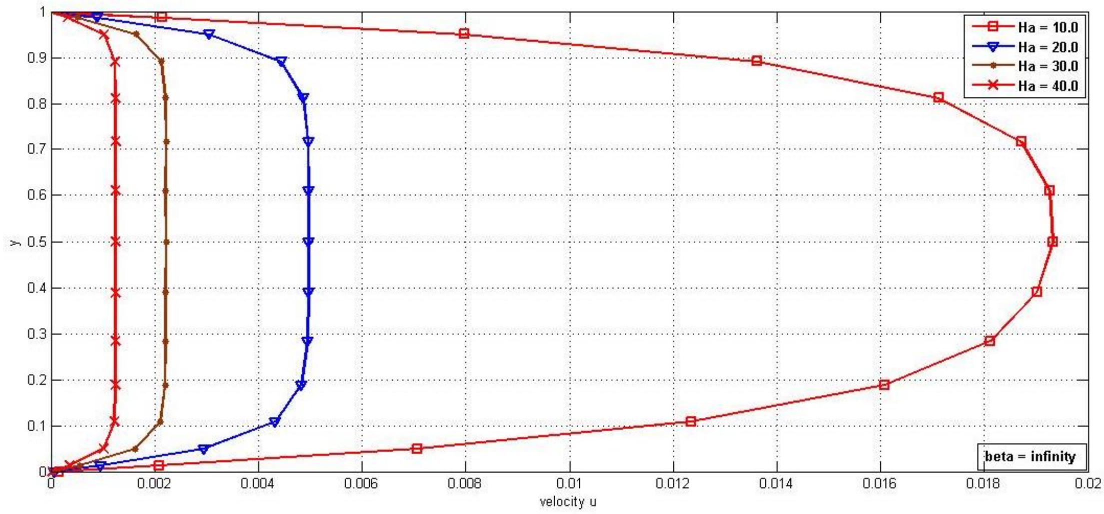

Fig. 11 The effect of Hartman number on velocity distribution [k=0.5, R=2, kn=0.001, A_O=2, A_1=1, a_O=1, b=2, β→∞ (η→0)].

6. CONCLUSIONS

We have studied the unsteady pulsating flow of an incompressible couple stress fluid between permeable beds through Porous Medium in channel under the influence of periodic body acceleration in the presence of magnetic field using a new algorithm modified GDQM. The slip condition on the wall artery has been considered.

The velocity has been obtained numerically. It is of interest to note that the unsteady velocity increases with increasing of the Couple stress parameter β and the permeability parameter of porous medium k whereas it decreases with increasing the Reynolds number R and the Hartmann number Ha. Also the velocity increases with increasing the Knudsen number kn for the region $y = 0$ to $y = [0.4 : 0.5]$. As $y = 0.5$ to $y = 1.0$, the velocity decrease with increasing kn.

It is observed that the presence of couple stresses results in a decrease in the velocity. Also it is observed that when the couple stresses are present, the Reynolds number seems to have no influence on the unsteady velocity component. This is in contrast with the disturbance we see in the absence of couple stresses.

It is observed that when the couple stresses are present:

- As the Reynolds number R increases, the maximum velocity is attained nearer to the upper permeable bed plate.
- As the Knudsen number kn decreases, the maximum velocity is attained nearing to the upper permeable bed plate slightly (slowly).
- As the Hartmann number Ha increases, the maximum velocity region increase.
- As the permeability parameter of porous medium k increases, the maximum velocity region increase.

APPENDIX

$$R_1 = 1 - k_n A_{1,1}, \qquad R_2 = k_n^2 A_{N,1}$$

$$R_3 = 1 - k_n A_{N,N} - k_n A_{1,1} + k_n^2 A_{1,1} A_{N,N} - k_n^2 A_{N,1} A_{1,N}$$

$$R_4 = A_{N,1} k_n,$$

$$R_5 = \frac{R_2}{R_3}, \qquad R_6 = \frac{R_1 R_2}{R_3 R_4}$$

$$R_7 = \frac{k_n + k_n A_{1,N} R_5}{R_1}, \qquad R_8 = \frac{k_n A_{1,N} R_6}{R_1}$$

$$R_9 = B_{1,1} R_7 A_{1,2} + B_{1,1} R_8 A_{N,2} + B_{1,2} + B_{1,N} R_5 A_{1,2} + B_{1,N} R_6 A_{N,2}$$

$$R_{10} = B_{1,1} R_7 + B_{1,N} R_5, \qquad R_{11} = B_{1,1} R_8 + B_{1,N} R_6$$

$$R_{12} = B_{1,1} R_7 A_{1,N-1} + B_{1,1} R_8 A_{N,N-1} + B_{1,N-1} + B_{1,N} R_5 A_{1,N-1} + B_{1,N} R_6 A_{N,N-1}$$

$$R_{13} = B_{N,1} R_7 A_{1,2} + B_{N,1} R_8 A_{N,2} + B_{N,2} + B_{N,N} R_5 A_{1,2} + B_{N,N} R_6 A_{N,2}$$

$$R_{14} = B_{N,1} R_7 + B_{N,N} R_5, \qquad R_{15} = B_{N,1} R_8 + B_{N,N} R_6$$

$$R_{16} = B_{N,1} R_7 A_{1,N-1} + B_{N,1} R_8 A_{N,N-1} + B_{N,N-1} + B_{N,N} R_5 A_{1,N-1} + B_{N,N} R_6 A_{N,N-1}$$

$$R_{17} = R_{13} R_{12} - R_{16} R_9, \qquad R_{18} = R_{14} R_9 - R_{13} R_{10}$$

$$R_{19} = R_{15} R_9 - R_{13} R_{11}$$

$$R_{20} = \frac{R_{18}}{R_{17}}, \qquad R_{21} = \frac{R_{19}}{R_{17}}, \qquad R_{22} = -\frac{R_{13}}{R_{17}}, \qquad R_{23} = -\frac{R_9}{R_{17}}$$

$$R_{24} = -\frac{R_{10} + R_{12} R_{20}}{R_9}, \qquad R_{25} = \frac{-1 + R_{12} R_{22}}{R_9}$$

$$R_{26} = -\frac{R_{11} + R_{12} R_{21}}{R_9}, \qquad R_{27} = -\frac{R_{12} R_{23}}{R_9}$$

$$R_{28} = R_{24}\left[R_7 A_{12} + R_8 A_{N,2}\right], \qquad R_{29} = R_{25}\left[R_7 A_{12} + R_8 A_{N,2}\right]$$

$$R_{30} = R_{26}\left[R_7 A_{12} + R_8 A_{N,2}\right], \qquad R_{31} = R_{27}\left[R_7 A_{12} + R_8 A_{N,2}\right]$$

$$R_{32} = R_{20}\left[R_7 A_{1,N-1} + R_8 A_{N,N-1}\right], \qquad R_{33} = R_{21}\left[R_7 A_{1,N-1} + R_8 A_{N,N-1}\right]$$

$$R_{34} = -R_{22}\left[R_7 A_{1,N-1} + R_8 A_{N,N-1}\right], \qquad R_{35} = R_{23}\left[R_7 A_{1,N-1} + R_8 A_{N,N-1}\right]$$

$$R_{36} = R_{28} + R_7 + R_{32}, \qquad R_{37} = R_{30} + R_8 + R_{33}$$

$$R_{38} = R_{29} + R_{34}, \qquad R_{39} = R_{31} + R_{35}$$

$$R_{40} = R_{24}\left[R_5 A_{1,2} + R_6 A_{N,2}\right], \qquad R_{41} = R_{25}\left[R_5 A_{1,2} + R_6 A_{N,2}\right]$$

$$R_{42} = R_{26}\left[R_5 A_{1,2} + R_6 A_{N,2}\right], \qquad R_{43} = R_{27}\left[R_5 A_{1,2} + R_6 A_{N,2}\right]$$

$$R_{44} = R_{20}\left[R_5 A_{1,N-1} + R_6 A_{N,N-1}\right], \qquad R_{45} = R_{21}\left[R_5 A_{1,N-1} + R_6 A_{N,N-1}\right]$$

$$R_{46} = -R_{22}\left[R_5 A_{1,N-1} + R_6 A_{N,N-1}\right], \qquad R_{47} = R_{23}\left[R_5 A_{1,N-1} + R_6 A_{N,N-1}\right]$$

$$R_{48} = R_{40} + R_5 + R_{44}, \qquad R_{49} = R_{42} + R_6 + R_{45}$$

$$R_{50} = R_{41} + R_{46}, \qquad R_{51} = R_{43} + R_{47}$$

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