

# On Fuzzy Primal Exterior Point Simplex Algorithm for Solving Linear Programming Problems with Fuzzy Numbers

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**Abstract.** Linear programming problems with fuzzy variables have been investigated by many researchers in the recent literature. Some methods to solve these problems such as the primal simplex method and the dual simplex method, are based on the concept of comparison of fuzzy numbers by using ranking functions. In this paper, We first introduce a new method fuzzy exterior point simplex algorithm for solving fuzzy number linear programming problems, by use of linear ranking function. We state our recently established results and develop the fuzzy exterior point simplex algorithm. Finally, we give an illustrative example.

**Keywords :** Fuzzy linear programming, ranking function, exterior point simplex algorithm.

## 1. Introduction

Fuzzy set theory has been applied to many disciplines such as control theory, management sciences, mathematical modelling and industrial applications. The concept of fuzzy mathematical programming in the general level was first proposed by Tanaka *et al.* [1]. The first formulation of fuzzy linear Programming (FLP) was proposed by Zimmermann [2]. Afterwards, many authors considered various types of

the FLP problems and proposed several approaches for solving these problems .Many convenient methods are based on the concept of comparison of fuzzy numbers using ranking functions [3,4,5,6,7,8]. Nasser *et al.* [8] introduced a primal simplex algorithm for solving the linear programming problems with trapezoidal fuzzy variables. Mahdavi-Amiri and Nasser [6] introduced a dual simplex algorithm for solving the linear programming problems with trapezoidal fuzzy variables. Ganesan and Veeramani [9] introduced a new approach for solving a kind of linear programming involving symmetric trapezoidal fuzzy numbers without converting them to the crisp linear programming problems. Nasser and Mahdavi-Amiri [7] based on this approach defined a new concept of duality of these problems. Ebrahimnejad and Nasser [10] used the complementary slackness to solve fuzzy linear programming without the need for a simple tableau.

In this paper, We first explain the concept of the comparison of fuzzy numbers by introducing a linear ranking function. Moreover, we describe basic feasible solution for the Fuzzy Number Linear Programming (FNLP) problems and we propose the Exterior Point Simplex Algorithm (EPSA) , which was first proposed by Paparrizos [11].

## 2. Definitions and Notations

The aim of this section is to present some notations and results which are of useful in our further consideration.

### A. Fuzzy Sets

Let  $X$  be a classical set of objects, called the universe, whose generic elements are denoted by  $x$ . The membership in a crisp subset of  $X$  is often viewed as characteristic function  $\mu_A(x)$  from  $X$  to  $\{0,1\}$  such that:

$$\mu_A(x) = \begin{cases} 1 & ,if \ x \in A \\ 0 & ,otherwise \end{cases}$$

where  $\{0,1\}$  is called a valuation set.

If the valuation set is allowed to be the real interval  $[0,1]$ ,  $A$  is called a *fuzzy set* proposed by Zadeh [11].  $\mu_A(x)$  is the degree of membership of  $x$  in  $A$ . The closer the value of  $\mu_A(x)$  is to 1, the more  $x$  belong to  $A$ . Therefore,  $A$  is completely characterized by the set of ordered pairs:  $A = \{(x, \mu_A(x)) | x \in X\}$ .

The *support* of a fuzzy set  $A$  is the crisp subset of  $X$  and is presented as:

$$Supp(A) = \{x \in X | \mu_A(x) > 0\}$$

The  $\alpha$ -*level* ( $\alpha$ -*cut*) set of a fuzzy set  $A$  is a crisp subset of  $X$  and is denoted by

$$A_\alpha = \{x \in X | \mu_A(x) \geq \alpha\}$$

A fuzzy set  $A$  in  $X$  is *convex* if :

$$\mu_A(\lambda x + (1-\lambda)y) \geq \min\{\mu_A(x), \mu_A(y)\} \text{ such that } x, y \in X \text{ and } [0,1] \in \lambda.$$

Alternatively, a fuzzy set is convex if all  $\alpha$ -*level* sets are convex. Note that in this paper we suppose that  $X = R$ .

A *fuzzy number*  $A$  is a convex normalized fuzzy set on the real line  $R$  such that :

- 1- It exists at least one  $x_0 \in R$  with  $\mu_A(x) = 1$ .
- 2-  $\mu_A(x)$  is piecewise continuous.

Among the various types of fuzzy numbers, triangular and trapezoidal fuzzy numbers are of the most important. Note that, in this study we only consider trapezoidal fuzzy numbers. A fuzzy number is a *trapezoidal fuzzy number* if the membership function of it be in the following form:

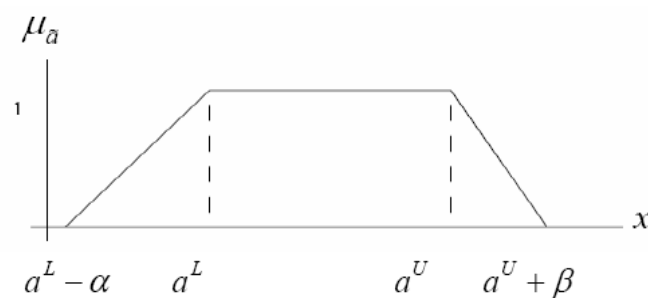


Fig. 1 Trapezoidal Fuzzy Number

We show any trapezoidal fuzzy number by,  $a = (a^L, a^U, \alpha, \beta)$  where the support of  $a$  is  $(a^L - \alpha, a^U + \beta)$ , and the modal set of  $a$  is  $[a^L, a^U]$ . Let  $F(R)$  be the set of trapezoidal fuzzy numbers. In the next subsection we describe arithmetic on  $F(R)$ .

### 3. Arithmetic on Fuzzy Numbers ( on trapezoidal fuzzy numbers )

Let  $a = (a^L, a^U, \alpha, \beta)$  and  $\tilde{b} = (b^L, b^U, \gamma, \theta)$  be two trapezoidal fuzzy numbers and  $x \in \mathbb{R}$ . Then, the results of applying fuzzy arithmetic on the trapezoidal fuzzy numbers as shown in the following:

(i) Image of  $\tilde{a}$ :  $-\tilde{a} = (-a^U, -a^L, \beta, \alpha)$

(ii) Addition:  $\tilde{a} + \tilde{b} = (a^L, a^U, \alpha, \beta) + (b^L, b^U, \gamma, \theta)$

$$= [a^L + b^L, a^U + b^U, \alpha + \gamma, \beta + \theta]$$

(iii) Subtraction:  $\tilde{a} - \tilde{b} = \tilde{a} + (-\tilde{b}) = (a^L, a^U, \alpha, \beta) + (-b^U, -b^L, \theta, \gamma)$

$$= (a^L - b^U, a^U - b^L, \alpha + \theta, \beta + \gamma)$$

(iv) Scalar Multiplication:

$$x > 0, \quad x\tilde{a} = [xa^L, xa^U, x\alpha, x\beta]$$

$$x < 0, \quad x\tilde{a} = [xa^U, xa^L, -x\beta, -x\alpha]$$

#### 4- Ranking Functions

Ranking is a viable approach for ordering fuzzy numbers. Various types of ranking functions have been introduced and some have been used for solving linear programming problems with fuzzy parameters [3, 4, 6, 7, 12]. A review of some common methods for ranking fuzzy subsets of unit interval can be seen in [13].

Here, we deal with ranking the elements of  $F(\mathbb{R})$ . In fact, an effective approach for ordering the elements of  $F(\mathbb{R})$  is to define a ranking function  $\mathfrak{R} : F(\mathbb{R}) \rightarrow (\mathbb{R})$  mapping trapezoidal fuzzy numbers into  $\mathbb{R}$ . Consider  $\tilde{a}$  and  $\tilde{b}$  in  $F(\mathbb{R})$ . Define order on  $F(\mathbb{R})$  as follows [6, 7]:

$$\tilde{a} \underset{\mathfrak{R}}{\geq} \tilde{b} \quad \text{if } \mathfrak{R}(\tilde{a}) \geq \mathfrak{R}(\tilde{b}), \quad (1)$$

$$\tilde{a} \underset{\mathfrak{R}}{>} \tilde{b} \quad \text{if } \mathfrak{R}(\tilde{a}) > \mathfrak{R}(\tilde{b}), \quad (2)$$

$$\tilde{a} \underset{\mathfrak{R}}{=} \tilde{b} \quad \text{if } \mathfrak{R}(\tilde{a}) = \mathfrak{R}(\tilde{b}), \quad (3)$$

Where  $\tilde{a}$  and  $\tilde{b}$  are in  $F(R)$ . Also we write  $\tilde{a} \leq \tilde{b}$  if  $\tilde{b} \geq \tilde{a}$ . Then, for any linear ranking  $\mathfrak{R}$  we may obtain:  $\tilde{a} \underset{\mathfrak{R}}{\geq} \tilde{b}$  if and only if  $\tilde{a} - \tilde{b} \underset{\mathfrak{R}}{\geq} \tilde{0}$ , or if and only if  $-\tilde{b} \underset{\mathfrak{R}}{\geq} -\tilde{a}$ .

Also, if  $\tilde{a} \underset{\mathfrak{R}}{\geq} \tilde{b}$  and  $\tilde{c} \underset{\mathfrak{R}}{\geq} \tilde{d}$ , then  $\tilde{a} + \tilde{c} \underset{\mathfrak{R}}{\geq} \tilde{b} + \tilde{d}$ .

One suggestion for a linear ranking function as following:

$$\mathfrak{R}(\tilde{a}) = a^L + a^U + \frac{1}{2}(\beta - \alpha) \quad (4)$$

where  $\tilde{a} = [a^L, a^U, \alpha, \beta] \in F(R)$ .

## 5- FUZZY LINEAR PROGRAMMING

In this section, we introduce fuzzy linear programming (FLP) problems. So, we first define linear programming problems.

### A. Linear Programming

A linear programming (LP) problem is defined as:

$$\begin{aligned} \text{Max} \quad & z = cx \\ \text{s.t.} \quad & Ax = b \\ & x \geq 0 \end{aligned}$$

where  $c = (c_1, c_2, \dots, c_n)$ ,  $b = (b_1, b_2, \dots, b_n)^T$  and  $A = [a_{ij}]_{m \times n}$ .

In the above problem, all of the parameters are crisp [14]. Now, if some of the parameters be fuzzy numbers we obtain a fuzzy linear programming which is defined in the next subsection.

### B. Fuzzy Linear Programming

Suppose that in the linear programming problem some parameters be fuzzy numbers. Hence, it is possible that some coefficients of the problem in the objective function, technical coefficients, the right-hand side coefficients or decision making variables be fuzzy number [4,5,15,16,17,18,19]. Here, we consider the linear programming problems with fuzzy numbers in the objective function.

## 6- FUZZY NUMBER LINEAR PROGRAMMING

A fuzzy number linear programming (FNLP) problem is defined as follows:

$$\begin{aligned} \text{Max} \quad & z = \underset{\mathfrak{R}}{\tilde{c}}x \\ \text{s.t.} \quad & Ax = b \\ & x \geq 0 \end{aligned}$$

Where  $b \in R^m$ ,  $x \in R^n$ ,  $A \in R^{m \times n}$ ,  $\tilde{c}^T \in (F(R))^n$  and  $\mathfrak{R}$  is a linear ranking function.

Now, we describe a fuzzy primal exterior point simplex algorithm ( *FPEPSA* ), which solve *FNLP* problem directly on the primal simplex tableau.

The algorithm begins with a basic feasible point . we find the set of non-basic indices  $J_- = \{j : a_{0j} < 0\}$ . If  $J_- = \emptyset$ , then the first main point is most optimal and

the algorithm stops. Otherwise, we calculate  $a_{i0} = \sum_{j \in J_-} a_{ij}$  and  $a_{i0} = \sum_{j \in J_-} a_{ij}$  where  $i =$

1, 2, . . .,  $m$  after that find the set of indices  $I_+ = \{i : a_{i0} > 0\}$ . If  $I_+ = \emptyset$ , then the problem is unbounded. Otherwise, we determine the index of entering variable  $r$  from

ratio test  $\frac{b_r}{a_{r0}} = \min \left\{ \frac{b_i}{a_{i0}} : i \in I_+ \right\}$ . In order to find the index of the leaving variable  $s$ ,

we must find the set of indices  $J_+ = \{j : a_{0j} > 0\}$  and after that we calculate  $\theta_1$  and  $\theta_2$ .

If  $\theta_1 \leq \theta_2$  put  $s = k$  otherwise  $s = l$ . the last step is the pivoting by the pivot element  $a_{rs}$ .

EPSA constructs two paths to the optimal solution. One path consists of basic but not feasible solutions; so this is an “exterior path”. The second path is feasible. It consists of line segments, the endpoints of which lie on the boundary of the feasible region.

## Exterior Point Simplex Algorithm Description

### tep0: (Initialization)

Start with a feasible basic point and construct the corresponding tableau exterior simplex.

### Step1: (Test of termination)

Find the set  $J_- = \{j : a_{0j} < 0\}$ . If  $J_- = \emptyset$ , STOP. The problem is optimal.

Otherwise, calculate  $a_{00} = \sum_{j \in J_-} a_{ij}$  and  $a_{i0} = \sum_{j \in J_-} a_{ij}$  where  $i = 1, 2, \dots, m$

### Step2: (Choice of entering variable)

Find the set  $I_+ = \{i : a_{i0} > 0\}$ . If  $I_+ = \emptyset$ , STOP. The problem is unbounded.

Otherwise, determine the index of entering variable  $r$  from the relation

$$\frac{b_r}{a_{r0}} = \min \left\{ \frac{b_i}{a_{i0}} : i \in I_+ \right\}$$

### Step3: (Choice of leaving variable)

Put  $J_+ = \{j : a_{0j} > 0\}$  and calculate

$$\theta_1 = \frac{-a_{0k}}{a_{rk}} = \min \left\{ \frac{-a_{0j}}{a_{rj}} : j \in J_-, a_{rj} > 0 \right\}$$

$$\theta_2 = \frac{-a_{0l}}{a_{rl}} = \min \left\{ \frac{-a_{0j}}{a_{rj}} : j \in J_+, a_{rj} < 0 \right\}$$

Find the index of the leaving variable  $s$ , if  $\theta_1 \leq \theta_2$  put  $s = k$  otherwise  $s = l$ .

### Step4: (Pivoting)

Form the next tableau by the pivoting variable  $a_{rs}$  i.e.

$$\text{Put } a_{rj} \leftarrow \frac{a_{rj}}{a_{rs}} \quad \text{where } j=1, 2, \dots, n, n+1$$

$$a_{ij} \leftarrow a_{ij} - \frac{a_{rj}}{a_{rs}} a_{is} \quad \text{where } i=0, 1, 2, \dots, m \ (i \neq r) \text{ and } j = 1, 2, \dots, n, n+1$$

If  $s = k$  put  $a_{0r} \leftarrow a_{0r} - 1$ .

and go to Step1.

## 7. A Numerical Example

For an illustration of the above method we solve FNLN problem by use of Fuzzy Primal Exterior Point Simplex Algorithm ( FPEPSA ).

### Example 7.1.

$$\max_{\mathfrak{R}} \tilde{z} = (2, 4, 2, 6) x_1 + (2, 6, 1, 3) x_2 + (1, 3, 1, 3) x_3$$

$$\text{s.t. } x_1 + x_2 + 2x_3 + x_4 = 2$$

$$2x_1 + 3x_2 + 4x_3 + x_5 = 3$$

$$6x_1 + 6x_2 + 2x_3 + x_6 = 8 \quad \text{where } : x_i \geq 0, i = 1, 2, 3, 4, 5, 6$$

Step (0): we construct the initial tableau of exterior simplex:

Basis		$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	R.H.S
Z	(-13, -5, 12,4)	(-4,-2, 6, 2)	(-6,-2,3,1)	(-3,-1,3,1)	0	0	0	0
$x_4$	4	1	1	2	1	0	0	2
$x_5$	9	2	3	4	0	1	0	3
$x_6$	14	6	6	2	0	0	1	8

$(\tilde{z}_1 - c_1, \tilde{z}_2 - c_2, \tilde{z}_3 - c_3) = ( (-2, -4, 6, 2), (-4, -4, 3, 1), (-2, -2, 3, 1) )$  and  $(\gamma_1, \gamma_2, \gamma_3) = (\mathfrak{R}(\gamma_1), \mathfrak{R}(\gamma_2), \mathfrak{R}(\gamma_3)) = (-8, -9, -5)$ .

Step (1):

$$J_- = \{j: a_{oj} < 0\} = \{1, 2, 3\} \neq 1 \text{ the Algorithm does not stop.}$$

Step (2):  $I_+ = \{i: a_{io} > 0\} = \{1, 2, 3\} \neq \Phi$  the problem is not unbounded

$$\frac{br}{a_{ro}} = \min \left\{ \frac{bi}{a_{io}}, i \in I_+ \right\} = \min \left\{ \frac{b_1}{a_{10}}, \frac{b_2}{a_{20}}, \frac{b_3}{a_{30}} \right\} = \min \left\{ \frac{1}{2}, \frac{1}{3}, \frac{4}{7} \right\} = \frac{1}{3} \Rightarrow r = 2$$

Step (3):  $J_+ = \{j: a_{oj} > 0\} = \Phi$

$$\theta_1 = \frac{-a_{ok}}{a_{rk}} = \min \left\{ \frac{-a_{oj}}{a_{rj}} = j \in J_-, a_{rj} > 0 \right\} = \min \left\{ \frac{-a_{o1}}{a_{21}}, \frac{-a_{o2}}{a_{22}}, \frac{-a_{o3}}{a_{23}} \right\} = \min \{(1, 2, 1, 3),$$

$$\left. \left( \frac{2}{3}, 2, \frac{1}{3}, 1 \right), \left( \frac{1}{4}, \frac{3}{4}, \frac{1}{4}, \frac{3}{4} \right) \right\}$$



$$R(\theta_1) = \min \left\{ R(2,1,1,3), R\left(\frac{4}{3}, \frac{4}{3}, \frac{1}{3}, 1\right), R\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{4}, \frac{3}{4}\right) \right\} = \min \left\{ 4, 3, \frac{5}{4} \right\} = \frac{5}{4}$$

$$\therefore k = 3$$

$$\theta_2 = \frac{-a_{0L}}{a_{rL}} = \min \left\{ \frac{-a_{0j}}{a_{rj}} : j \in J_+, a_{rj} < 0 \right\} \Rightarrow R(\theta_2) = \min \{ \Phi \} = \infty$$

$$\Rightarrow R(\theta_1) < R(\theta_2) \Rightarrow \theta_1 < \theta_2 \Rightarrow s = k = 3$$

the pivot element is  $a_{23}$

Step (4): the next tableau by pivot element:

Basis		$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	R.H.S
Z	$\left(-5, -3, \frac{42}{4}, \frac{30}{4}\right)$	$\left(-3, -1, \frac{13}{2}, \frac{7}{2}\right)$	$\left(-2, -3, \frac{15}{4}, \frac{13}{4}\right)$	$\tilde{0}$	0	$\left(0, 1, \frac{1}{4}, \frac{3}{4}\right)$	0	$\left(0, 3, \frac{3}{4}, \frac{9}{4}\right)$
$x_4$	$\frac{-1}{2}$	0	$\frac{-1}{2}$	0	1	$\frac{-1}{2}$	0	$\frac{1}{2}$
$x_3$	$\frac{5}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	0	$\frac{1}{4}$	0	$\frac{3}{4}$
$x_6$	$\frac{19}{2}$	5	$\frac{9}{2}$	0	0	$\frac{-1}{2}$	1	$\frac{13}{2}$

Step (1):  $J_- = \{j: a_{0j} < 0\} = \{1, 2\} \Rightarrow$  the Algorithm does not stop.

$$(\gamma_1, \gamma_2, \gamma_5) = (R(\gamma_1), R(\gamma_2), R(\gamma_5)) = \left(\frac{-11}{2}, \frac{-21}{4}, \frac{5}{4}\right)$$

Step (2):  $I_+ = \{i: a_{i0} > 0\} = \{2, 3\} \neq \Phi \Rightarrow$  the problem is bounded

$$\frac{br}{a_{ro}} = \min \left\{ \frac{bi}{a_{io}}, i \in I_+ \right\} = \min \left\{ \frac{b_2}{a_{20}}, \frac{b_3}{a_{30}} \right\} = \min \left\{ \frac{3}{5}, \frac{13}{19} \right\} = \frac{3}{5} \Rightarrow r = 2 <$$

Step (3):  $J_+ = \{j: a_{0j} > 0\} = \{5\}$

$$\theta_1 = \frac{-a_{0k}}{a_{rk}} = \min \left\{ \frac{-a_{0j}}{a_{rj}} : j \in J_+, a_{rj} > 0 \right\} = \min \left\{ \frac{-a_{0l}}{a_{2l}}, \frac{-a_{02}}{a_{22}} \right\} = \min \left\{ (6, 2, 7, 13), \left(\frac{10}{3}, \frac{10}{3}, \frac{13}{3}\right) \right\}$$

$$R(\theta_1) = \min \left\{ R(6,2,7,13), R\left(\frac{10}{3}, \frac{10}{3}, \frac{13}{3}, 5\right) \right\} = \min \{11, 7\} = 7 \Rightarrow k = 2$$

$$\theta_2 = \frac{-a_{ok}}{a_{rL}} = \min \left\{ \frac{-a_{oj}}{a_{rj}} : j \in J, a_{rj} < 0 \right\} \Rightarrow R(\theta_2) = \min \{ \Phi = \infty \}$$

$$\Rightarrow R(\theta_1) < R(\theta_2) \Rightarrow \theta_1 < \theta_2 \Rightarrow s = k = 2$$

the pivot element is  $a_{22}$

Step (4): the next tableau by pivot element

Basis		$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	R.H.S
Z	$\left(\frac{-4}{3}, \frac{2}{3}, \frac{26}{3}, 6\right)$	$\left(\frac{-4}{3}, \frac{2}{3}, \frac{26}{3}, 6\right)$	$\tilde{0}$	$\left(\frac{8}{3}, 4, \frac{13}{3}, 5\right)$	0	$\left(\frac{2}{3}, 2, \frac{4}{3}, 2\right)$	0	(3, 5, 4, 6)
$x_4$	$\frac{1}{3}$	$\frac{1}{3}$	0	$\frac{2}{3}$	1	$-\frac{1}{3}$	0	1
$x_2$	$\frac{2}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	0	$\frac{1}{3}$	0	1
$x_6$	2	2	0	-6	0	-2	1	2

Step (1):  $J_- = \{j : a_{oj} < 0\} = \{1\} \neq \Phi \Rightarrow$  the algorithm does not stop.

$$(\gamma_1, \gamma_3, \gamma_5) = (R(\gamma_1), R(\gamma_3), R(\gamma_5)) = (-2, 7, 3)$$

Step (2):  $I_+ = \{i : a_{io} > 0\} = \{1, 2, 3\} \neq \Phi \Rightarrow$  the problem is bounded

$$\text{Ratio test: } \frac{br}{a_{ro}} = \min \left\{ \frac{bi}{a_{io}}, i \in I_+ \right\} = \min \left\{ \frac{b_1}{a_{10}}, \frac{b_2}{a_{20}}, \frac{b_3}{a_{30}} \right\} = \min \left\{ 3, \frac{3}{2}, 1 \right\} = 1$$

The index of the entering variable is  $r = 3$

Step (3):  $J_+ = \{j : a_{oj} > 0\} = \{3, 5\}$

$$\theta_1 = \frac{-a_{ok}}{a_{rk}} = \min \left\{ \frac{-a_{oj}}{a_{rj}} : j \in J, a_{rj} > 0 \right\} = \min \left\{ \frac{-a_{o1}}{a_{31}} \right\} = \left( \frac{2}{3}, \frac{-1}{3}, 3, \frac{13}{3} \right)$$

$$R(\theta_1) = R\left(\frac{2}{3}, \frac{-1}{3}, 3, \frac{13}{3}\right) = 1$$

$$\theta_2 = \frac{-a_{oL}}{a_{rL}} = \min \left\{ \frac{-a_{oj}}{a_{rj}} : j \in J, a_{rj} < 0 \right\} = \min \left\{ \left(\frac{5}{9}, \frac{5}{9}, \frac{13}{18}, \frac{5}{6}\right), \left(\frac{2}{3}, \frac{2}{3}, \frac{2}{3}, 1\right) \right\}$$

$$R(\theta_2) = \min \left\{ R\left(\frac{5}{9}, \frac{5}{9}, \frac{13}{18}, \frac{5}{6}\right), R\left(\frac{2}{3}, \frac{2}{3}, \frac{2}{3}, 1\right) \right\} = \min \left\{ \frac{7}{6}, \frac{3}{2} \right\} = \frac{7}{6}$$

$$R(\theta_1) < R(\theta_2) \Rightarrow s = k = 1$$

the pivot element is  $a_{31}$

Step (4): the next tableau by pivot element

Basis	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	R.H.S
Z	$\tilde{0}$	0	$(0, \frac{1}{2}, 1, 2)$	0	$(0, \frac{1}{2}, 1, 2)$	0	(4, 6, 6, 8)
$x_4$	0	0	$\frac{5}{3}$	1	0	$-\frac{1}{6}$	$\frac{2}{3}$
$x_2$	0	$\frac{1}{3}$	$\frac{10}{3}$	0	1	$-\frac{1}{3}$	$\frac{1}{3}$
$x_1$	1	0	-3	0	-1	$\frac{1}{2}$	1

$$(\gamma_3, \gamma_5) = (R(\gamma_3), R(\gamma_5)) = (1, 1)$$

Step (1):  $J : \{j : a_{oj} <_R 0\} = \Phi \Rightarrow$  the Algorithm stops .

the solution is  $z = R(z) = R\left(\frac{16}{3}, \frac{10}{3}, 10, \frac{44}{3}\right) = 11, x_1 = 1, x_2 = \frac{1}{3}, x_3 = 0, x_4 = \frac{2}{3},$   
 $x_5 = x_6 = 0$

## 8. Conclusion

We considered fuzzy number linear programming problems and introduced a new method fuzzy primal exterior point simplex algorithm (FPEPSA) for solving fuzzy number linear programming problems, by use of linear ranking function. Finally, we gave an illustrative example.

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