



## On L(2,1) and Prime cordial labelings

M. A. Seoud <sup>a</sup>, G. M. Abd ElHamid <sup>b</sup> and M. S. Abo Shady <sup>c</sup>

<sup>a</sup> Department of Mathematics, Faculty of Science, Ain Shams University, Abbassia, Cairo, Egypt  
<sup>b,c</sup> Department of Mathematics, M.T.C., Kobry ElKobba, Cairo, Egypt

### Abstract

An L(2, 1)-labeling of a graph  $G$  is a function  $f$  from the vertex set  $V(G)$  into the set of nonnegative integers such that  $|f(x) - f(y)| \geq 2$  if  $d(x, y) = 1$  and  $|f(x) - f(y)| \geq 1$  if  $d(x, y) = 2$ , where  $d(x, y)$  denotes the distance between  $x$  and  $y$  in  $V(G)$ . The L(2, 1)-labeling number,  $\lambda(G)$ , of  $G$  is the minimum  $k$  where  $G$  has an L(2, 1)-labeling  $f$  with  $k$  being the absolute difference between the largest and smallest image points of  $f$ . A prime cordial labeling of a graph  $G$  with vertex set  $V$  is a bijection  $f$  from  $V$  to  $\{1, 2, \dots, |V|\}$  such that if each edge  $uv$  is assigned the label 1 if  $\gcd(f(u), f(v)) = 1$  and 0 if  $\gcd(f(u), f(v)) > 1$ , then the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. In this paper we find the labeling number  $\lambda(G)$  for some families of graphs, and we give an upper bound for the number of edges of any graph on  $n$  vertices to have a prime cordial labeling, and we compare this upper bound with the number of edges of two families of graphs.

*Keywords:* L(2, 1)-labeling, prime cordial labeling, C<sup>++</sup> programming Language

## 1 Introduction

By a graph  $G$  we mean a finite, undirected, connected graph without loops or multiple edges. We denote by  $n$  and  $m$  the order and size of the graph  $G$ . Terms not defined here are used in the sense of Harary [2].

## 2 L(2,1) labeling

### 2.1 Background [4]

In 1991, Roberts proposed a variation of the frequency assignment problem in which “close” transmitters must receive different frequencies and “very close” transmitters must receive frequencies that are at least two units apart. To translate the problem into the language of graph theory, the transmitters are represented by the vertices of a graph. Two vertices are “very close” if they are adjacent and “close” if they are at distance 2 in the graph. Along this direction, Yeh and afterwards Griggs and Yeh proposed that the  $L(2, 1)$ -labeling of a simple graph  $G$  is a function  $f$  from the vertex set  $V(G)$  into the nonnegative integers such that  $|f(x) - f(y)| \geq 2$  if  $d(x, y) = 1$  and  $|f(x) - f(y)| \geq 1$  if  $d(x, y) = 2$ , where  $d(x, y)$  denotes the distance between  $x$  and  $y$  in  $V(G)$ . The span of  $f$  is the absolute difference between the largest assigned label and the smallest assigned label of  $V(G)$ . The  $L(2, 1)$ -labeling number of  $G$ , denoted



as  $\lambda(G)$ , is the minimum span taken over all  $L(2, 1)$ -labelings of  $V(G)$ . Without loss of generality, we consider only those  $L(2, 1)$ -labelings of  $G$  with image that contains 0, and we define a  $k$ - $L(2, 1)$ -labeling of  $G$  to be an  $L(2, 1)$ -labeling of  $G$  which assigns no label greater than  $k$ . Thus,  $\lambda(G)$  is the minimum  $k$  such that  $G$  has a  $k$ - $L(2, 1)$ -labeling.

An optimal solution has been provided by Griggs and Yeh [1], who labeled each vertex  $i$  of a circuit of order  $n$  as follows:

$$f(v_i) = \begin{cases} 0 & \text{if } i \equiv 0 \pmod{3} \\ 2 & \text{if } i \equiv 1 \pmod{3} \\ 4 & \text{if } i \equiv 2 \pmod{3} \end{cases}$$

However the above labeling is redefined depending on whether  $n \equiv 1 \pmod{3}$  or  $n \equiv 2 \pmod{3}$ . In the first case,  $f(n-4), \dots, f(n-1)$  become

$$f(v_i) = \begin{cases} 0 & \text{if } i = n-4 \\ 3 & \text{if } i = n-3 \\ 1 & \text{if } i = n-2 \\ 4 & \text{if } i = n-1 \end{cases}$$

In the second case,  $f(n-2)$  and  $f(n-1)$  are modified as

$$f(v_i) = \begin{cases} 1 & \text{if } i = n-2 \\ 3 & \text{if } i = n-1 \end{cases}$$

## 2.2 New results

**Theorem 2.2.1:** The graph  $G$  with the number of vertices  $n$  and the distance between any two vertices is at most two must be labeled such that at least  $\lambda(G) = n - 1$ .

**Proof:** Since the maximum distance between any two vertices is two, then using  $L(2,1)$ -labeling, we cannot repeat any label in this graph. So, making the minimum label “0”, then for  $n$  vertices, the maximum label will be at least  $n - 1$ , i.e.  $\lambda(G) = n - 1$ . An example of a graph with  $n = 10$  vertices and has  $\lambda = 9$  is shown in the following Figure.

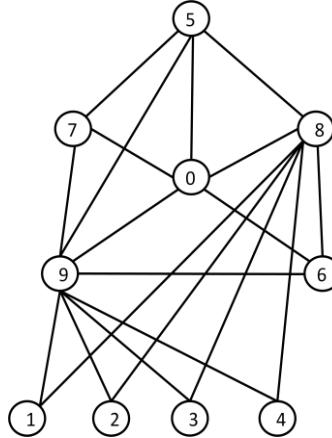


Figure 2.1: A graph with 10 vertices and  $\lambda = 9$

**Theorem 2.2.2:** The conjunction  $P_n \wedge P_m$  has a labeling number  $\lambda = 7$ .

**Proof:** These graphs has a number of vertices  $nm$  and a number of edges  $2(n - 1)(m - 1)$ . We define the following labeling function, and we see that  $\lambda(P_n \wedge P_m) = 7$  as follow:

$$f(v_i^j) = \begin{cases} 0, & i \equiv 0,1 \pmod{4} \\ 1, & i \equiv 2,3 \pmod{4} \end{cases} : j \equiv 0 \pmod{3},$$

$$f(v_i^j) = \begin{cases} 3, & i \equiv 0,1 \pmod{4} \\ 4, & i \equiv 2,3 \pmod{4} \end{cases} : j \equiv 1 \pmod{3},$$

$$f(v_i^j) = \begin{cases} 6, & i \equiv 0,1 \pmod{4} \\ 7, & i \equiv 2,3 \pmod{4} \end{cases} : j \equiv 2 \pmod{3}.$$

**Example:**  $\lambda(P_5 \wedge P_8) = 7$  as shown in the following Figure.

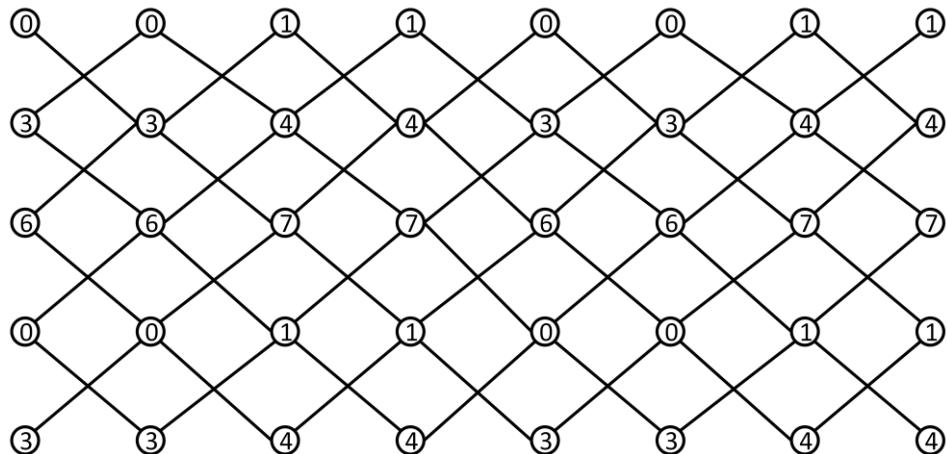


Figure 2.2: L(2,1)- labeling of  $P_5 \wedge P_8$  with  $\lambda = 7$



**Definition:** The triangular snake  $T_n$  is the graph which is obtained from a path  $P_n$  with vertices  $\{v_1, v_2, \dots, v_n\}$  by joining the vertices  $v_i$  and  $v_{i+1}$  to a new vertex  $u_i$  for  $i = 1, 2, \dots, n-1$ .

**Theorem 2.2.3:** The corona  $T_n \odot K_1$  has the labeling number  $\lambda(T_n \odot K_1) = 7$ .

**Proof:** This graph has the set of vertices  $\{v_1, v_2, \dots, v_n; u_1, u_2, \dots, u_{n-1}; a_1, a_2, \dots, a_n; b_1, b_2, \dots, b_{n-1}\}$ . By labeling the path  $P_n$  with the labels "0", "2" and "4", and since each vertex of the path  $P_n$  has degree 5, so, we see that  $\lambda(T_n \odot K_1)$  is at least 6, and the vertex labeled "4" in  $P_n$  has also five adjacent vertices which will make  $\lambda(T_n \odot K_1) = 7$ . Then using the following labeling function we label the graph  $T_n \odot K_1$ .

$$f(v_i) = \begin{cases} 0, & i \equiv 1 \pmod{3} \\ 2, & i \equiv 2 \pmod{3} \\ 4, & i \equiv 0 \pmod{3} \end{cases}$$

$$f(u_i) = \begin{cases} 7, & i \equiv 1 \pmod{2} \\ 6, & i \equiv 0 \pmod{2} \end{cases}$$

$$f(a_i) = \begin{cases} 3, & i \equiv 1 \pmod{3} \\ 5, & i \equiv 2 \pmod{3} \\ 1, & i \equiv 0 \pmod{3} \end{cases}$$

$$f(b_i) = \begin{cases} 4, & i \equiv 1 \pmod{3} \\ 0, & i \equiv 2 \pmod{3} \\ 2, & i \equiv 0 \pmod{3} \end{cases}$$

**Example :**  $\lambda(T_7 \odot K_1) = 7$  as shown in the following Figure.

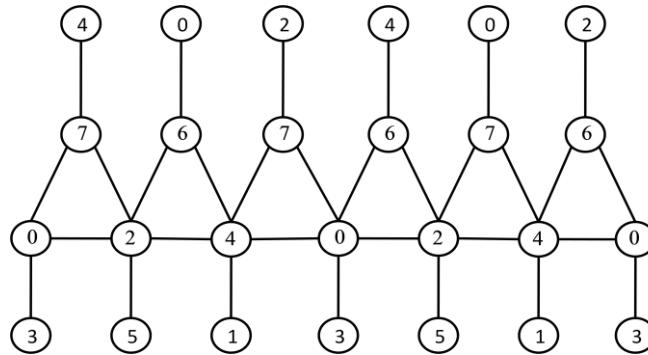


Figure 2.3: L(2,1)- labeling of  $T_7 \odot K_1$  with  $\lambda = 7$

**Definition:** The Sun Flower  $SF(n)$  is the graph obtained from the cycle  $C_n$  with vertices  $\{v_1, v_2, \dots, v_n\}$  and creating new vertices  $\{u_1, u_2, \dots, u_n\}$  such that  $u_i, i = 1, 2, \dots, n-1$ , is connected to  $v_i$  and  $v_{i+1}$ , and  $u_n$  is connected to  $v_n$  and  $v_1$ .

**Theorem 2.2.4:** The Sun Flower  $SF(n)$  has the labeling number  $\lambda(SF(7)) = 7$ .



**Proof:** This graph has  $2n$  vertices and  $3n$  edges, the set of vertices are  $\{v_1, v_2, \dots, v_n; u_1, u_2, \dots, u_n\}$ . Using the labeling function of circuits as defined in section 1.1 to label the cycle  $C_n$ , we have to label the vertices  $\{u_1, \dots, u_n\}$  with the labels  $\{5, 6, 7\}$  as follow:

If  $n$  is odd:

$$\begin{aligned} f(u_1) &= 5, \\ f(u_{2i}) &= 6 \quad 1 \leq i \leq \frac{n-1}{2}, \\ f(u_{2i+1}) &= 7 \quad 1 \leq i \leq \frac{n-1}{2}. \end{aligned}$$

If  $n$  is even:

$$\begin{aligned} f(u_{2i-1}) &= 6 \quad 1 \leq i \leq \frac{n}{2}, \\ f(u_{2i}) &= 7 \quad 1 \leq i \leq \frac{n}{2}. \end{aligned}$$

**Example :**  $\lambda(SF(7)) = 7$  and  $\lambda(SF(8)) = 7$  as shown in the following Figure.

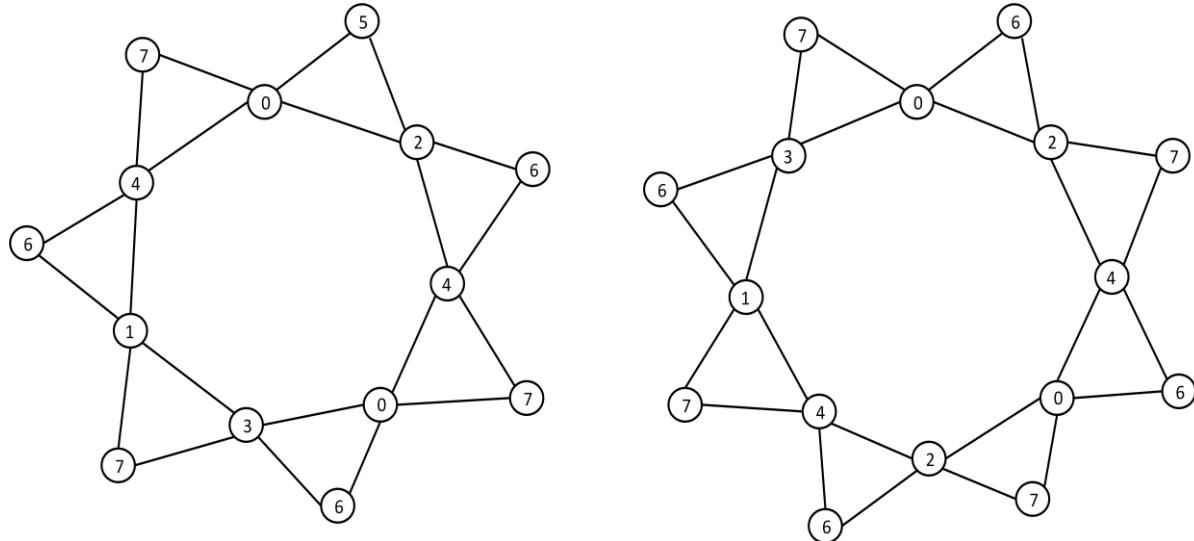


Figure 2.4: L(2,1)- labeling of  $SF(7)$  and  $SF(8)$  with  $\lambda = 7$

**Theorem 2.2.5:** The Cartesian Product  $P_n \times C_4$  has the labeling number  $\lambda(P_n \times C_4) = 7$ .

**Proof:** This graph has  $|V| = 4n$  vertices and  $|E| = 4(2n - 1)$  edges. Let the set of vertices  $V(P_n \times C_4) = \{v_i^j : 1 \leq i \leq n, 1 \leq j \leq 4\}$ . Using the following labeling function, we see that  $\lambda(P_n \times C_4) = 7$ .

$$f(v_i^j) = \begin{cases} 0, & i \equiv 1 \pmod{4} \\ 2, & i \equiv 2 \pmod{4} \\ 5, & i \equiv 3 \pmod{4} \\ 3, & i \equiv 0 \pmod{4} \end{cases}, \quad j \equiv 1 \pmod{4}$$



$$f(v_i^j) = \begin{cases} 6, & i \equiv 1 \pmod{4} \\ 4, & i \equiv 2 \pmod{4} \\ 7, & i \equiv 3 \pmod{4} \\ 1, & i \equiv 0 \pmod{4} \end{cases}, \quad j \equiv 2 \pmod{4}$$

$$f(v_i^j) = \begin{cases} 2, & i \equiv 1 \pmod{4} \\ 0, & i \equiv 2 \pmod{4} \\ 3, & i \equiv 3 \pmod{4} \\ 5, & i \equiv 0 \pmod{4} \end{cases}, \quad j \equiv 3 \pmod{4}$$

$$f(v_i^j) = \begin{cases} 4, & i \equiv 1 \pmod{4} \\ 6, & i \equiv 2 \pmod{4} \\ 1, & i \equiv 3 \pmod{4} \\ 7, & i \equiv 0 \pmod{4} \end{cases}, \quad j \equiv 0 \pmod{4}.$$

**Example:**  $\lambda(P_6 \times C_4) = 7$  as shown in the following Figure.

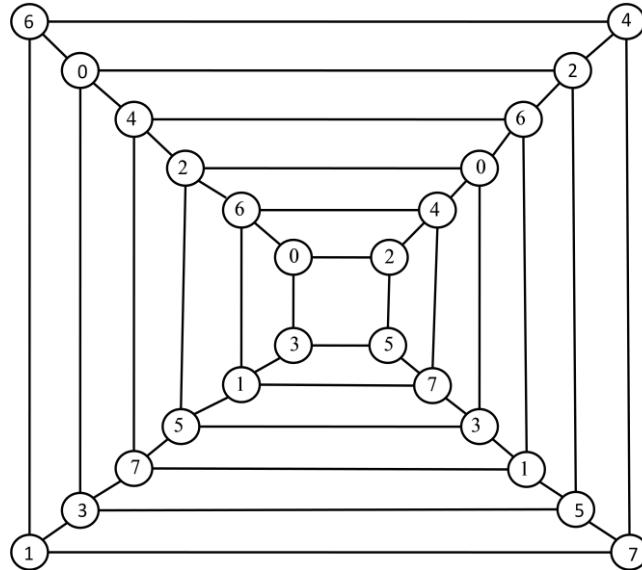


Figure 2.5: L(2,1)- labeling of graph  $P_6 \times C_4$  with  $\lambda = 7$

**Theorem 2.2.6:** The graph which is obtained from  $C_m$  by attaching a pendent path  $P_n$  to every vertex of  $C_m$  has a labeling number  $\lambda = 5$ .

**Proof:** This graph has  $nm$  vertices and  $nm$  edges. So, using the following labeling function, we see that the labeling number for this graph is  $\lambda = 5$ .

We have three cases for this graph, the first case when  $m \equiv 0 \pmod{3}$ , the second case when  $m \equiv 1 \pmod{3}$  and the third one when  $m \equiv 2 \pmod{3}$

1-  $m \equiv 0 \pmod{3}$ :



In this case we will use the following labeling function,

$$f(v_i^j) = \begin{cases} 0, & j=1 \\ 3, & j=2 \\ 1, & j=3 \\ 4, & j \equiv 1 \pmod{3} \\ & j \neq 1 \\ 0, & j \equiv 2 \pmod{3} \\ & j \neq 2 \\ 2, & j \equiv 0 \pmod{3} \\ & j \neq 3 \end{cases}, \quad i \equiv 1 \pmod{3}.$$

$$f(v_i^j) = \begin{cases} 2, & j \equiv 1 \pmod{3} \\ 5, & j = 2 \\ 0, & j \equiv 0 \pmod{3} \\ & i \equiv 2 \pmod{3}, \\ 4, & j \equiv 2 \pmod{3} \\ & j \neq 2 \end{cases}$$

$$f(v_i^j) = \begin{cases} 4, & j=1 \\ 1, & j=2 \\ 3, & j=3 \\ 0, & j \equiv 1 \pmod{3} \\ & j \neq 1 \\ 2, & j \equiv 2 \pmod{3} \\ & j \neq 2 \\ 4, & j \equiv 0 \pmod{3} \\ & j \neq 3 \end{cases}, \quad i \equiv 0 \pmod{3}.$$

**Example:** The graph which is obtained from  $C_6$  by attaching a pendent path  $P_6$  to every vertex of  $C_6$  is shown in the following Figure with a labeling number  $\lambda = 5$ .

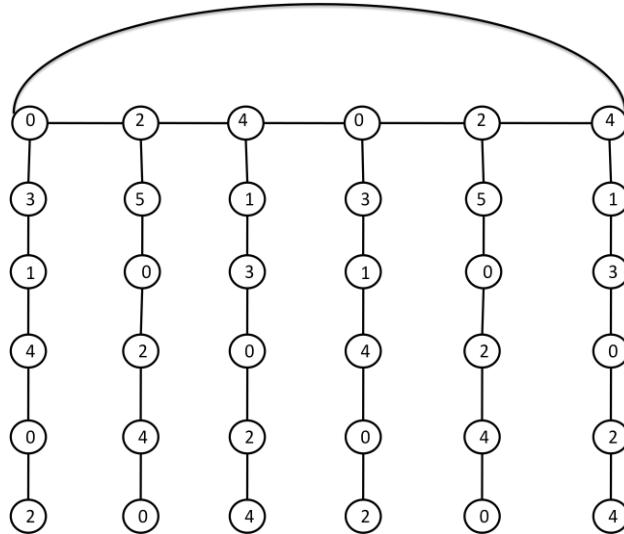


Figure 2.6: L(2,1)- labeling of graph which is obtained from  $C_6$  by attaching a pendent path  $P_6$  to every vertex of  $C_6$  with  $\lambda = 5$

### 2- $m \equiv 1 \pmod{3}$ :

In this case we will use the following labeling function,



$$f(v_i^j) = \begin{cases} 4, & j=1 \\ 1, & j=2 \\ 3, & j=3 \\ 0, & j \equiv 1 \pmod{3} \\ & j \neq 1 \\ 2, & j \equiv 2 \pmod{3} \\ & j \neq 2 \\ 4, & j \equiv 0 \pmod{3} \\ & j \neq 3 \end{cases}, \quad i \equiv 0 \pmod{3},$$

$$f(v_i^j) = \begin{cases} 0, & j=1 \\ 3, & j=2 \\ 1, & j=3 \\ 4, & j \equiv 1 \pmod{3} \\ & j \neq 1 \\ 0, & j \equiv 2 \pmod{3} \\ & j \neq 2 \\ 2, & j \equiv 0 \pmod{3} \\ & j \neq 3 \end{cases}, \quad i \equiv 1 \pmod{3},$$

$$f(v_i^j) = \begin{cases} 2, & j \equiv 1 \pmod{3} \\ 5, & j = 2 \\ 0, & j \equiv 0 \pmod{3} \\ 4, & j \equiv 2 \pmod{3} \end{cases}, \quad i \equiv 2 \pmod{3},$$

and the last four vertices are modified as follow:

$$f(v_{m-3}^j) = \begin{cases} 0, & j \equiv 1 \pmod{3} \\ 2, & j \equiv 2 \pmod{3} \\ 4, & j \equiv 0 \pmod{3} \end{cases}$$

$$f(v_{m-2}^j) = \begin{cases} 3, & j = 1 \\ 5, & j = 2 \\ 0, & j \equiv 0 \pmod{3} \\ 2, & j \equiv 1 \pmod{3} \\ & j \neq 1 \\ 4, & j \equiv 2 \pmod{3} \\ & j \neq 2 \end{cases},$$

$$f(v_{m-1}^j) = \begin{cases} 1, & j = 1 \\ 5, & j = 2 \\ 0, & j \equiv 0 \pmod{3} \\ 2, & j \equiv 1 \pmod{3} \\ & j \neq 1 \\ 4, & j \equiv 2 \pmod{3} \\ & j \neq 2 \end{cases},$$

$$f(v_m^j) = \begin{cases} 4, & j \equiv 1 \pmod{3} \\ 2, & j \equiv 2 \pmod{3} \\ 0, & j \equiv 0 \pmod{3} \end{cases}.$$

**Example:** The graph which is obtained from  $C_7$  by attaching a pendent path  $P_6$  to every vertex of  $C_7$  is shown in the following Figure with a labeling number  $\lambda = 5$ .

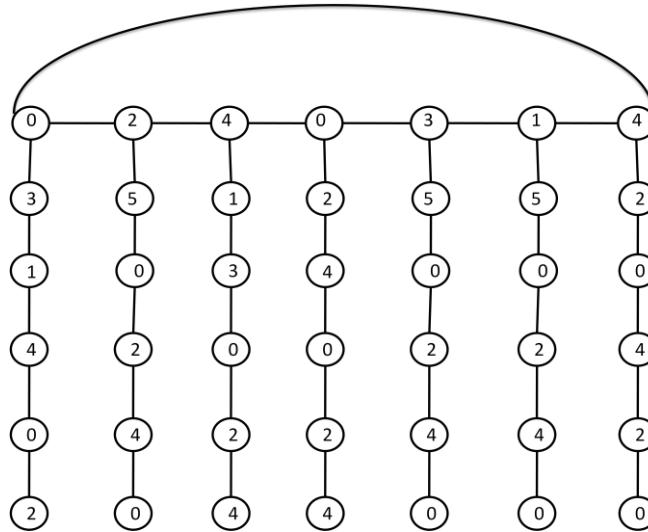


Figure 2.7: L(2,1)- labeling of graph which is obtained from  $C_7$  by attaching a pendent path  $P_6$  to every vertex of  $C_7$  with  $\lambda = 5$

### 3- $m \equiv 2 \pmod{3}$ :

In this case we will use the following labeling function,

$$f(v_1^j) = \begin{cases} 0, & j \equiv 1 \pmod{3} \\ 4, & j \equiv 2 \pmod{3} \\ 2, & j \equiv 0 \pmod{3} \end{cases}$$

$$f(v_i^j) = \begin{cases} 2, & j \equiv 1 \pmod{3} \\ 5, & j = 2 \\ 0, & j \equiv 0 \pmod{3}, \quad i \equiv 2 \pmod{3}, \\ 4, & j \equiv 2 \pmod{3} \end{cases}$$

$$f(v_i^j) = \begin{cases} 4, & j=1 \\ 1, & j=2 \\ 3, & j=3 \\ 0, & j \equiv 1 \pmod{3} \\ 2, & j \equiv 2 \pmod{3} \\ 4, & j \equiv 0 \pmod{3} \end{cases}, \quad i \equiv 0 \pmod{3},$$

$$f(v_i^j) = \begin{cases} 0, & j=1 \\ 3, & j=2 \\ 1, & j=3 \\ 4, & j \equiv 1 \pmod{3} \\ 0, & j \equiv 2 \pmod{3} \\ 2, & j \equiv 0 \pmod{3} \end{cases}, \quad i \equiv 1 \pmod{3},$$

and the last three vertices are modified as follow:



$$f(v_{m-2}^j) = \begin{cases} 4, & j \equiv 1 \pmod{3} \\ 0, & j \equiv 2 \pmod{3}, \\ 2, & j \equiv 0 \pmod{3} \end{cases}$$

$$f(v_{m-1}^j) = \begin{cases} 1, & j = 1 \\ 5, & j = 2 \\ 0, & j \equiv 0 \pmod{3} \\ 2, & j \equiv 1 \pmod{3} \\ & j \neq 1 \\ 4, & j \equiv 2 \pmod{3} \\ & j \neq 2 \end{cases}$$

$$f(v_m^j) = \begin{cases} 3, & j = 1 \\ 0, & j \equiv 2 \pmod{3} \\ 2, & j \equiv 0 \pmod{3} \\ 5, & j \equiv 1 \pmod{3} \\ & j \neq 1 \end{cases}$$

**Example:** The graph which is obtained from  $C_8$  by attaching a pendent path  $P_6$  to every vertex of  $C_8$  is shown in the following Figure with a labeling number  $\lambda = 5$ .

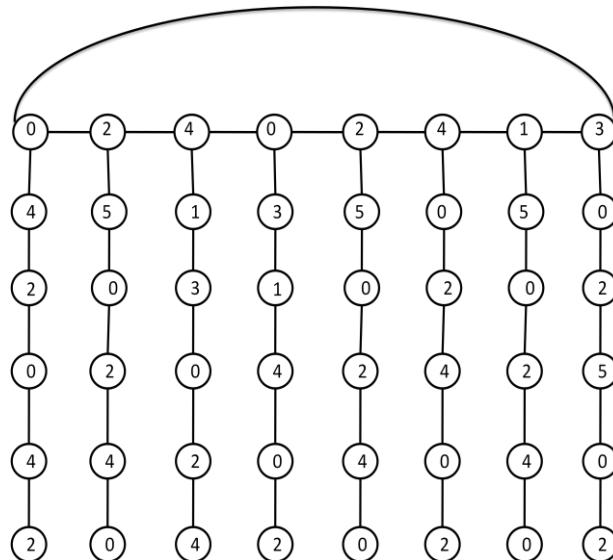


Figure 2.8: L(2,1)- labeling of graph which is obtained from  $C_8$  by attaching a pendent path  $P_6$  to every vertex of  $C_8$  with  $\lambda = 5$



### 3 Prime Cordial labeling

#### 3.1 Back ground

Sundaram, Ponraj, and Somasundaram [3] have introduced the notion of prime cordial labelings. A prime cordial labeling of a graph  $G$  with vertex set  $V$  is a bijection  $f$  from  $V$  to  $\{1, 2, \dots, |V|\}$  such that if each edge  $uv$  is assigned the label 1 if  $\gcd(f(u), f(v)) = 1$  and 0 if  $\gcd(f(u), f(v)) > 1$ , then the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. In [3] Sundaram, Ponraj, and Somasundaram prove the following graphs are prime cordial:  $C_n$  if and only if  $n \geq 6$ ;  $P_n$  if and only if  $n \neq 3$  or 5;  $K_{1,n}$  ( $n$  odd); the graph obtained by subdividing each edge of  $K_{1,n}$  if and only if  $n \geq 3$ ; bistars; dragons; crowns; triangular snakes if and only if the snake has at least three triangles; ladders;  $K_{1,n}$  if  $n$  is even and there exists a prime  $p$  such that  $2p < n + 1 < 3p$ ;  $K_{2,n}$  if  $n$  is even and if there exists a prime  $p$  such that  $3p < n + 2 < 4p$ ; and  $K_{3,n}$  if  $n$  is odd and if there exists a prime  $p$  such that  $5p < n + 3 < 6p$ . They also prove that if  $G$  is a prime cordial graph of even size, then the graph obtained by identifying the central vertex of  $K_{1,n}$  with the vertex of  $G$  labeled with 2 is prime cordial, and if  $G$  is a prime cordial graph of odd size, then the graph obtained by identifying the central vertex of  $K_{1,2n}$  with the vertex of  $G$  labeled with 2 is prime cordial. They further prove that  $K_{m,n}$  is not prime cordial for a number of special cases of  $m$  and  $n$ . Sundaram and Somasundaram and Youssef observed that for  $n \geq 3$ ,  $K_n$  is not prime cordial provided that the inequality  $\varphi(2) + \varphi(3) + \dots + \varphi(n) > n(n - 1)/4 + 1$  is valid for  $n \geq 3$ . This inequality was proved by Yufei Zhao.

#### 3.2 New results

**Theorem 3.2.1:** A necessary condition for a graph  $G$  of order  $n$  to be a prime cordial graph, is that its number of edges  $|E(G)| \leq m + 1$ , where

$$m = \sum_{\substack{i,j=1 \\ i \neq j}}^n a(v_i, v_j) : a(v_i, v_j) = 1 \text{ iff } \gcd(f(v_i), f(v_j)) > 1,$$

$$\text{and } a(v_i, v_j) = 0 \text{ iff } \gcd(f(v_i), f(v_j)) = 1.$$

**Proof:** we count the all edges could be labeled 0; that is counting edges whose adjacent vertices labels' are not relatively prime.

By using next algorithm, we count these edges and make a table representing the upper bound  $m + 1$  for the number of vertices  $2 \leq n \leq 600$  as shown in table 1.



**Algorithm 3.2.2:**

We make an algorithm to make a table for the upper bound for the number of edges of any graph with the number of vertices equal to  $n$  to be prime cordial graph.

**INPUT** The size of the table according to the number of vertices.

**OUTPUT** The upper bound for the number of edges corresponding to the number of vertices  $n$ .

**Step 1:** Set  $p$ ; (size of the table)

$x$ ; (multiples of the prime divisors)

$m$ ; (used to count the number of labels that are not relatively prime with each other)

$a$ ; (array to store the prime divisors of the composite divisors)

$b$ ; (array to store the multiples of the prime divisors of the composite divisors)

**Step 2:** Enter the size of the table in  $p$ ;

**Step 3:** FOR index  $i = 2:p$

**Step 4:** Initialize count = 0;

**Step 5:** FOR index  $j = 2:i$

**Step 6:** Check if  $j$  is prime or not using a SUBFUNCTION prime( $j$ );

**Step 7:** If  $j$  is prime

**Step 8:** Get the multiples of  $j$  and store in  $x$  and count them;

**Step 9:** Store the number of multiples in  $m$ ;

**Step 10:** If  $j$  is not prime

**Step 11:** Get the prime divisors of  $j$  without repetitions and store in array  $a$ ;

**Step 12:** Get the multiples for each prime divisor stored in array  $a$  and store in array  $b$  without repetition;

**Step 13:** Count the labels stored in array  $b$  and store in  $m$ ;

**Step 14:** count = count +  $m$ ;

**Step 15:** OUTPUT ( $i$  and  $m+1$ );

**Step 13:** STOP;

We implement this algorithm using C++ programming language.

**Result 3.2.3:** In table 2 we present the number of edges of the graphs  $G_i$  (where  $G_i$  is a regular graph with degree  $n - i$ ) compared with the upper bound calculated in table 1 and deduce that  $G_i, i = 3,4 \dots, 10$  are not prime cordial graphs for the underlined bold numbers.



**Result 3.2.4:** In table 3 we present the number of edges of the graphs  $C_n^i$  (where  $E_n^i = |E(C_n^i)|$ ) compared with the upper bound calculated in table 1 and deduce that  $C_n^i, i = 2,3,4 \dots, 12$  are not prime cordial graphs for the underlined bold numbers.

## 4 References

- [1] A.A. Bertossi and M. Cristina Pinotti, Wireless Ad Hoc Networking, Taylor & Francis group, LLC, 2007.
- [2] F. Harary, Graph Theory (Addison-Wesley, Reading, MA 1969).
- [3] M. Sundaram, R. Ponraj, and S. Somasundram, Prime cordial labeling of graphs, J. Indian Acad. Math., 27 (2005) 373-390.
- [4] Z. Shao, Roger K. Yeh, Kin Keung Poon, Wai Chee Shiu, The L(2, 1)-labeling of K1,n-free graphs and its applications, Applied Mathematics Letters 21 (2008) 1188–1193.

Table 1: Upper bound for the number of edges of a graph with  $n$  vertices to be prime cordial.

$n$	$m + 1$								
2	1	59	1253	116	5087	173	11371	230	20365
3	1	60	1339	117	5175	174	11605	231	20585
4	3	61	1339	118	5293	175	11713	232	20823
5	3	62	1401	119	5337	176	11903	233	20823
6	9	63	1453	120	5511	177	12023	234	21145
7	9	64	1515	121	5531	178	12201	235	21245
8	15	65	1547	122	5653	179	12201	236	21483
9	19	66	1637	123	5737	180	12463	237	21643
10	29	67	1637	124	5863	181	12463	238	21925
11	29	68	1707	125	5911	182	12681	239	21925
12	43	69	1755	126	6089	183	12805	240	22275
13	43	70	1845	127	6089	184	12995	241	22275
14	57	71	1845	128	6215	185	13075	242	22537
15	69	72	1939	129	6303	186	13325	243	22697



16	83	73	1939	130	6465	187	13377	244	22943
17	83	74	2013	131	6465	188	13567	245	23095
18	105	75	2081	132	6647	189	13727	246	23425
19	105	76	2159	133	6695	190	13961	247	23485
20	127	77	2191	134	6829	191	13961	248	23739
21	143	78	2297	135	6953	192	14215	249	23907
22	165	79	2297	136	7095	193	14215	250	24205
23	165	80	2391	137	7095	194	14409	251	24205
24	195	81	2443	138	7281	195	14605	252	24563
25	203	82	2525	139	7281	196	14827	253	24627
26	229	83	2525	140	7463	197	14827	254	24881
27	245	84	2643	141	7559	198	15101	255	25133
28	275	85	2683	142	7701	199	15101	256	25387
29	275	86	2769	143	7745	200	15339	257	25387
30	317	87	2829	144	7935	201	15475	258	25733
31	317	88	2923	145	7999	202	15677	259	25817
32	347	89	2923	146	8145	203	15745	260	26143
33	371	90	3053	147	8269	204	16023	261	26327
34	405	91	3089	148	8419	205	16111	262	26589
35	425	92	3183	149	8419	206	16317	263	26589
36	471	93	3247	150	8637	207	16465	264	26955
37	471	94	3341	151	8637	208	16687	265	27067
38	509	95	3385	152	8795	209	16743	266	27381
39	537	96	3511	153	8907	210	17065	267	27561
40	583	97	3511	154	9093	211	17065	268	27831
41	583	98	3621	155	9161	212	17279	269	27831
42	641	99	3697	156	9375	213	17423	270	28225
43	641	100	3815	157	9375	214	17637	271	28225
44	687	101	3815	158	9533	215	17729	272	28511
45	727	102	3953	159	9641	216	18015	273	28767
46	773	103	3953	160	9831	217	18087	274	29041
47	773	104	4063	161	9887	218	18305	275	29189
48	835	105	4175	162	10101	219	18453	276	29563
49	847	106	4281	163	10101	220	18731	277	29563
50	905	107	4281	164	10267	221	18787	278	29841
51	941	108	4423	165	10435	222	19085	279	30037
52	995	109	4423	166	10601	223	19085	280	30403
53	995	110	4561	167	10601	224	19339	281	30403
54	1065	111	4637	168	10839	225	19547	282	30781
55	1093	112	4763	169	10863	226	19773	283	30781
56	1155	113	4763	170	11073	227	19773	284	31067
57	1195	114	4917	171	11197	228	20083	285	31347
58	1253	115	4969	172	11371	229	20083	286	31677

$n$	$m+1$	$n$	$m+1$								
287	31769	345	46219	402	62785	459	81849	516	103727	573	127711
288	32151	346	46565	403	62869	460	82415	517	103839	574	128377
289	32183	347	46565	404	63275	461	82415	518	104441	575	128645
290	32537	348	47035	405	63651	462	83097	519	104789	576	129411
291	32733	349	47035	406	64125	463	83097	520	105443	577	129411



292	33027	350	47493	407	64217	464	83575	521	105443	578	130021
293	33027	351	47761	408	64775	465	84023	522	106149	579	130409
294	33445	352	48143	409	64775	466	84489	523	106149	580	131119
295	33569	353	48143	410	65273	467	84489	524	106675	581	131295
296	33871	354	48617	411	65549	468	85135	525	107243	582	132073
297	34103	355	48765	412	65963	469	85279	526	107769	583	132197
298	34401	356	49123	413	66091	470	85849	527	107861	584	132787
299	34469	357	49451	414	66653	471	86165	528	108595	585	133379
300	34907	358	49809	415	66825	472	86643	529	108639	586	133965
301	35003	359	49809	416	67271	473	86747	530	109281	587	133965
302	35305	360	50335	417	67551	474	87381	531	109645	588	134803
303	35509	361	50371	418	68025	475	87609	532	110275	589	134899
304	35827	362	50733	419	68025	476	88175	533	110379	590	135613
305	35955	363	51017	420	68671	477	88503	534	111093	591	136009
306	36373	364	51455	421	68671	478	88981	535	111313	592	136615
307	36373	365	51607	422	69093	479	88981	536	111855	593	136615
308	36747	366	52097	423	69385	480	89683	537	112215	594	137441
309	36955	367	52097	424	69815	481	89779	538	112753	595	137861
310	37333	368	52479	425	70023	482	90261	539	112989	596	138459
311	37333	369	52735	426	70593	483	90697	540	113779	597	138859
312	37763	370	53185	427	70725	484	91223	541	113779	598	139525
313	37763	371	53301	428	71155	485	91423	542	114321	599	139525
314	38077	372	53803	429	71531	486	92069	543	114685	600	140403
315	38417	373	53803	430	72053	487	92069	544	115259		
316	38735	374	54229	431	72053	488	92563	545	115483		
317	38735	375	54577	432	72627	489	92891	546	116285		
318	39161	376	54959	433	72627	490	93533	547	116285		
319	39237	377	55039	434	73133	491	93533	548	116835		
320	39619	378	55577	435	73553	492	94195	549	117211		
321	39835	379	55577	436	73991	493	94283	550	117909		
322	40213	380	56047	437	74071	494	94837	551	118001		
323	40281	381	56303	438	74657	495	95345	552	118751		
324	40711	382	56685	439	74657	496	95855	553	118919		
325	40879	383	56685	440	75215	497	96007	554	119473		
326	41205	384	57195	441	75591	498	96673	555	120005		
327	41425	385	57483	442	76089	499	96673	556	120563		
328	41759	386	57869	443	76089	500	97271	557	120563		
329	41863	387	58137	444	76687	501	97607	558	121317		
330	42361	388	58527	445	76871	502	98109	559	121425		
331	42361	389	58527	446	77317	503	98109	560	122159		
332	42695	390	59113	447	77617	504	98827	561	122639		
333	42927	391	59189	448	78127	505	99035	562	123201		
334	43261	392	59635	449	78127	506	99605	563	123201		
335	43401	393	59899	450	78785	507	99993	564	123959		
336	43879	394	60293	451	78885	508	100503	565	124191		
337	43879	395	60457	452	79339	509	100503	566	124757		
339	44469	396	61007	453	79643	510	101265	567	125241		
340	44891	397	61007	454	80097	511	101421	568	125815		
341	44971	398	61405	455	80429	512	101931	569	125815		
342	45437	399	61769	456	81051	513	102307	570	126665		
343	45533	400	62247	457	81051	514	102821	571	126665		
344	45883	401	62247	458	81509	515	103033	572	127327		

Table 2: A table giving the number of edges of  $G_i$   $|E(G_i)|$  compared with  $m + 1$ .



$n$	$n$	$ E(G_3) $	$ E(G_4) $	$ E(G_5) $	$ E(G_6) $	$ E(G_7) $	$ E(G_8) $	$ E(G_9) $	$ E(G_{10}) $
2	1								
3	1								
4	3	2							
5	9	5							
6	9	9	6	3					
7	9	<u>14</u>		7					
8	15	<u>20</u>	<u>16</u>	12	8	4			
9	19	<u>27</u>		18		9			
10	29	<u>35</u>	<u>30</u>	25	20	15	10	5	
11	29	<u>44</u>		<u>33</u>		22		11	
12	43	<u>54</u>	<u>48</u>	42	36	30	24	18	12
13	43	<u>65</u>		<u>52</u>		39		26	
14	57	<u>77</u>	<u>70</u>	<u>63</u>	56	49	42	35	28
15	69	<u>90</u>		<u>75</u>		60		45	
16	83	<u>104</u>	<u>96</u>	<u>88</u>	80	72	64	56	48
17	83	<u>119</u>		<u>102</u>		<u>85</u>		68	
18	105	<u>135</u>	<u>126</u>	<u>117</u>	<u>108</u>	99	90	81	72
19	105	<u>152</u>		<u>133</u>		<u>114</u>		95	
20	127	<u>170</u>	<u>160</u>	<u>150</u>	<u>140</u>	<u>130</u>	120	110	100
21	143	<u>189</u>		<u>168</u>		<u>147</u>		126	
22	165	<u>209</u>	<u>198</u>	<u>187</u>	<u>176</u>	165	154	143	132
23	165	<u>230</u>		<u>207</u>		<u>184</u>		161	
24	195	<u>252</u>	<u>240</u>	<u>228</u>	<u>216</u>	<u>204</u>	192	180	168
25	203	<u>275</u>		<u>250</u>		<u>225</u>		200	
26	229	<u>299</u>	<u>286</u>	<u>273</u>	<u>260</u>	<u>247</u>	<u>234</u>	221	208
27	245	<u>324</u>		<u>297</u>		<u>270</u>		243	
28	275	<u>350</u>	<u>336</u>	<u>322</u>	<u>308</u>	<u>294</u>	<u>280</u>	266	252
29	275	<u>377</u>		<u>348</u>		<u>319</u>		<u>290</u>	
30	317	<u>405</u>	<u>390</u>	<u>375</u>	<u>360</u>	<u>345</u>	<u>330</u>	315	300
31	317	<u>434</u>		<u>403</u>		<u>372</u>		<u>341</u>	
32	347	<u>464</u>	<u>448</u>	<u>432</u>	<u>416</u>	<u>400</u>	<u>384</u>	<u>368</u>	<u>352</u>
33	371	<u>495</u>		<u>462</u>		<u>429</u>		<u>396</u>	
34	405	<u>527</u>	<u>510</u>	<u>493</u>	<u>476</u>	<u>459</u>	<u>442</u>	<u>425</u>	<u>408</u>
35	425	<u>560</u>		<u>525</u>		<u>490</u>		<u>455</u>	
36	471	<u>594</u>	<u>576</u>	<u>558</u>	<u>540</u>	<u>522</u>	<u>504</u>	<u>486</u>	468
37	471	<u>629</u>		<u>592</u>		<u>555</u>		<u>518</u>	
38	509	<u>665</u>	<u>646</u>	<u>627</u>	<u>608</u>	<u>589</u>	<u>570</u>	<u>551</u>	<u>532</u>
39	537	<u>702</u>		<u>663</u>		<u>624</u>		<u>585</u>	
40	583	<u>740</u>	<u>720</u>	<u>700</u>	<u>680</u>	<u>660</u>	<u>640</u>	<u>620</u>	<u>600</u>
41	583	<u>779</u>		<u>738</u>		<u>697</u>		<u>656</u>	



Table 3: A table giving the number of edges ( $|E(C_n^i)|$ ) compared with  $m + 1$ .

$n$	$m + 1$	$E_n^2$	$E_n^3$	$E_n^4$	$E_n^5$	$E_n^6$	$E_n^7$	$E_n^8$	$E_n^9$	$E_n^{10}$	$E_n^{11}$	$E_n^{12}$
2	1											
3	1											
4	3	<b>6</b>										
5	9	<b>10</b>	<b>10</b>									
6	9	<b>12</b>	<b>15</b>									
7	9	<b>14</b>	<b>21</b>									
8	15	<b>16</b>	<b>24</b>	<b>28</b>								
9	19	18	<b>27</b>	<b>36</b>								
10	29	20	<b>30</b>	<b>40</b>	<b>45</b>							
11	29	22	<b>33</b>	<b>44</b>	<b>55</b>							
12	43	24	36	<b>48</b>	<b>60</b>	<b>66</b>						
13	43	26	39	<b>52</b>	<b>65</b>	<b>78</b>						
14	57	28	42	56	<b>70</b>	<b>84</b>	<b>91</b>					
15	69	30	45	60	<b>75</b>	<b>90</b>	<b>105</b>					
16	83	32	48	64	80	<b>96</b>	<b>112</b>	<b>120</b>				
17	83	34	51	68	85	<b>102</b>	<b>119</b>	<b>136</b>				
18	105	36	54	72	90	<b>108</b>	<b>126</b>	<b>144</b>	<b>153</b>			
19	105	38	57	76	95	<b>114</b>	<b>133</b>	<b>152</b>	<b>171</b>			
20	127	40	60	80	100	120	<b>140</b>	<b>160</b>	<b>180</b>	<b>190</b>		
21	143	42	63	84	105	126	<b>147</b>	<b>168</b>	<b>189</b>	<b>210</b>		
22	165	44	66	88	110	132	154	<b>176</b>	<b>198</b>	<b>220</b>	<b>231</b>	
23	165	46	69	92	115	138	161	<b>184</b>	<b>207</b>	<b>230</b>	<b>253</b>	
24	195	48	72	96	120	144	168	192	<b>216</b>	<b>240</b>	<b>264</b>	<b>276</b>
25	203	50	75	100	125	150	175	200	<b>225</b>	<b>250</b>	<b>275</b>	<b>300</b>
26	229	52	78	104	130	156	182	208	<b>234</b>	<b>260</b>	<b>286</b>	<b>312</b>
27	245	54	81	108	135	162	189	216	243	<b>270</b>	<b>297</b>	<b>324</b>
28	275	56	84	112	140	168	196	224	252	<b>280</b>	<b>308</b>	<b>336</b>
29	275	58	87	116	145	174	203	232	261	<b>290</b>	<b>319</b>	<b>348</b>
30	317	60	90	120	150	180	210	240	270	300	<b>330</b>	<b>360</b>
31	317	62	93	124	155	186	217	248	279	310	<b>341</b>	<b>372</b>
32	347	64	96	128	160	192	224	256	288	320	<b>352</b>	<b>384</b>
33	371	66	99	132	165	198	231	264	297	330	363	<b>396</b>
34	405	68	102	136	170	204	238	272	306	340	374	<b>408</b>
35	425	70	105	140	175	210	245	280	315	350	385	420
36	471	72	108	144	180	216	252	288	324	360	396	432
37	471	74	111	148	185	222	259	296	333	370	407	444
38	509	76	114	152	190	228	266	304	342	380	418	456
39	537	78	117	156	195	234	273	312	351	390	429	468

Military Technical College  
Kobry Elkobbah,  
Cairo, Egypt  
May 29-31,2012



6<sup>th</sup> International Conference  
on Mathematics and  
Engineering Physics (ICMEP-  
6)

40	583	80	120	160	200	240	280	320	360	400	440	480
41	583	82	123	164	205	246	287	328	369	410	451	492
42	641	84	126	168	210	252	294	336	378	420	462	504
43	641	86	129	172	215	258	301	344	387	430	473	516