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ACO: Decisive Artificial Ants

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Abstract

The Ant Colony Optimization was inspired by the foraging behavior of real ant colonies. The main determinants of an ant algorithm are the way of *pheromone update* and the *transition probability* of an ant's travel from a position to another. This paper proposes adopting the *decision systems* to develop the transition probability function and make other frequent decisions such as switching between ways of pheromone update. This increases the possibility of deriving and improving a variety of ant algorithms. The original transition probability function is investigated and other three formulas have been developed as general frameworks for the problems of *multiple objective*. The Analytic Hierarchy Process is followed as a base for this contribution; thus, unlimited number of factors can be involved. Furthermore, paradoxical views are discussed to synthesize different types of *artificial stigmergy* to energize the artificial ants with more robust interaction.

KEYWORDS

Ant Colony Optimization; Transition Probability; AHP; Multiple Objective; Cognitive/ Emotional / Hybrid Stigmergy; Pseudo-pheromone; Sub-pheromone; Sub-visibility

1. INTRODUCTION

The Ant Colony Optimization (ACO) was first introduced by Marco Dorigo and colleagues in the early 1990s (Dorigo et Al. [1]) as a novel metaheuristic for solving NP-hard problems. The ACO is a natural way to distribute tasks between autonomous agents (ants). The ACO is a class of *swarm intelligence* techniques. The ACO is inspired by the shortest path foraging behavior of various ant species. While the members of an ant colony move between food sources and their nest, the ants deposit a chemical called *pheromone* (an odorous substance) on the ground to mark their paths. Thus, the ants form in this way different *pheromone trails*. The members of the ant colony have a tendency to follow such trails through probabilistic decisions biased by the *pheromone intensity*. However, ACO is based on the artificial ant which has some major differences with the real ant (see Cordón et Al. [2]). The artificial ant is a simple agent that builds feasible solutions using artificial pheromone trails (*stigmergic information*) and *heuristic information*. For more details, refer to Dorigo et Al. [1; 3; 4].

To apply ACO, the problem should be encoded by a construction graph fully connecting a specified number of nodes (Dorigo et Al. [4]). A connection between two nodes represents an edge. Generally, ACO is an iterative constructive technique. A number of artificial ants are assigned and distributed over the nodes of the graph and initial *pheromone trails* are set on edges. Thereafter, each ant builds a solution by walking from node to node on the graph with the constraint of not revisiting any node in the same tour. An ant chooses the next node to be visited according to a stochastic mechanism that is mainly biased by the pheromone intensity: when in node r, the next node s is chosen stochastically among those available and unvisited nodes. In particular, a node s can be chosen with a probability that is proportional to the *pheromone intensity* and the *heuristic information* associated with edge rs. (Review Dorigo et Al. [1; 4].)

Several ACO algorithms have been developed in the literature, such as Ant System (AS), Ant Colony System (ACS), MAX–MIN Ant System (MMAS), Rank-based Ant System (AS_{rank}), Best-Worst Ant System (BWAS), Approximate Nondeterministic Tree Search (ANTS), and AntNet. For a detailed review about these algorithms and others, refer to Dorigo et Al. [1; 3; 4], Stützle and Hoos [5], Cordón et Al. [2], Dorigo and Stützle [6], Dorigo and Blum [7], Blum [8], Maniezzo and Roffilli [9], and Mullen et Al. [10]. It is found that ACO algorithms and their variants differ in some characteristics, such as the way of *pheromone update*, the form of *transition probability*, the *heuristic information* adopted, the number of ants distributed in the system, and initializing, reinitializing, terminating and boundary conditions.

Recently, attention to ACO is much increased. The larger part of research on ACO is concerned with the area of application (Dorigo et Al. [4]). (See Shtovba [11]; Fox et Al. [12]; Azzag et Al. [13]; Hani et Al. [14].) The attention is also directed to more challenging problems that needs to more theory such as those involve multiple objective, dynamic modifications of data, and stochastic nature of the objective function and of the constraints (Dorigo et Al. [4]). Furthermore, the theory of ACO spotlights other directions such as continuous ACO (Socha and Blum [15]; Socha and Dorigo [16]) and parallel implementation of ACO (Randall and Lewis [17]; Manfrin et Al. [18]). The contribution of this paper focuses on the extension of ACO to multiple objective problems. A decision system called Analytic Hierarchy Process (AHP) is adopted for this purpose.

In the remainder of this paper, AS (§2), Multiple Objective ACO (§3) and AHP (§4) are summarized. In §5.1, the *transition probability* function of original AS is investigated in perspective of *multicriteria*. In §5.2, three models for transition probability function are presented based on AHP. Stigmergy in the context of social life is discussed in §6. In §7, some conclusions are drawn. Some formulations are moved to Appendices A, B and C.

2. THE ANT SYSTEM

The AS is the first ACO algorithm proposed in the literature, which was developed by Dorigo et Al. in 1991 based on the Travelling Salesman Problem (Dorigo et Al. [1; 4]). Three variants, ant-cycle, ant-density, and ant-quantity algorithms, were developed, differing only in the way the pheromone trails are updated. In the latter two variants, each ant deposits *pheromone* at each step (travel from node r to node s) while building its tour (*online step-by-step pheromone update*). In ant-cycle, each ant deposits pheromone once a tour is completed (*online delayed pheromone update*). In AS, the pheromone trail intensity on the edge rs is updated as

$$\tau_{rs} \leftarrow (1-\rho)\tau_{rs} + \sum_{k=1}^{K} \Delta\tau_{rs}^{k}, \quad \text{if ant } k \text{ used edge } rs, \tag{1}$$

where $\rho \in (0,1]$ is the *pheromone evaporation* rate, K is the total number of ants in the system, and $\Delta\tau_{rs}^{k}$ is the quantity of the *pheromone* laid on edge rs by ant k.

For *ant-cycle*, at end of each tour,

$$\Delta\tau_{rs}^{k} = \begin{cases} Q/L_k, & \text{if ant } k \text{ used edge } rs \text{ in its tour} \\ 0, & \text{otherwise} \end{cases}, \tag{2}$$

where Q is a constant and L_k is the tour length of ant k.

For *ant-density*, at end of each step,

$$\Delta\tau_{rs}^{k} = \begin{cases} Q, & \text{if ant } k \text{ used edge } rs \text{ in its step} \\ 0, & \text{otherwise} \end{cases}. \tag{3}$$

For *ant-quantity*, at end of each step,

$$\Delta\tau_{rs}^{k} = \begin{cases} Q/d_{rs}, & \text{if ant } k \text{ used edge } rs \text{ in its step} \\ 0, & \text{otherwise} \end{cases}, \tag{4}$$

where d_{rs} is the distance between nodes r and s.

In AS, at each step, an ant k at node r chooses to travel to next node s with a *transition probability* that is computed as

$$P_{rs}^{k} = \begin{cases} \dfrac{[\tau_{rs}]^{\alpha}[\eta_{rs}]^{\beta}}{\sum_{u\in\mathcal{N}_k(r)}[\tau_{ru}]^{\alpha}[\eta_{ru}]^{\beta}}, & s \in \mathcal{N}_k(r) \\ 0, & \text{otherwise} \end{cases}, \tag{5}$$

where $\mathcal{N}_k(r)$ is the feasible neighborhood of ant k when located at node r, and $\alpha, \beta \geq 0$ are the parameters that weight the relative importance of the *pheromone trail*, τ_{rs}, versus the *heuristic information*, η_{rs}, which is given by

$$\eta_{rs} = 1/d_{rs}, \tag{6}$$

where d_{rs} is the distance between nodes r and s. This heuristic information is called *visibility*. Notice that Formula (5) can be explained by Bayes' Formula. Iourinski et Al. [19] have discussed several biologically inspired formulas including the *transition probability*.

Gagné et Al. [20] explored the addition of a *look-ahead* mechanism to such transition probability that allows incorporating additional information about the potential of the current partial solution (looking beyond the immediate choice horizon). That extends Formula (5) to more than two kinds of information. Another formula found similar in structure exists in Rahman et Al. [21], having three parameters as exponents.

3. MULTIPLE OBJECTIVE ACO

In general, a multiple objective optimization problem is the problem of simultaneously optimizing a set of several objectives while satisfying an active set of constraints. The most commonly used approaches to deal with such problems are four: objectives weighting, distance functions, Min–Max formulation, and Lexicographic approach (Garcìa-Martínez et Al. [22]). These approaches join the principal of reducing to a single objective problem. The ACO becomes one of the paradigms that used to solve this problem (Angus and Woodward [23]). The Multiple Objective ACO (MOACO) will be discussed in brief as follows. Table 1 names some of MOACO algorithms (Garcìa-Martínez et Al. [22]; Lezcano et Al. [24]).

Most of MOACO algorithms are basically extensions of the established single objective ACO algorithms such as AS, MMAS, and ACS (Angus and Woodward [23]). For instance, Doerner et

Al. [25] introduced an algorithm called COMPETants to deal with bi-objective transportation problems based on AS_{rank} with two ant colonies. Each colony is set with each own *pheromone trail matrix* and the objectives are combined in a weighted sum. T'Kindt et Al. [26] proposed an MMAS algorithm to solve bi-objective two-machine flowshop scheduling problem by ordering the objectives. They also used features of simulated annealing search and local search algorithms.

Table 1. Some of MOACO algorithms.

Algorithm	Ellipsis	Ellipsis	Ellipsis
Multi-objective Ant System	MAS	COMPETants	COMP
Bi-criterion Ant	BIANT	Multi-objective Ant Colony System	MOACS
Bi-criterion Multi Colony	BIAMC	Multi-objective Max–Min Ant System	M3AS
Pareto-Ant Colony Optimization	P-ACO	Multiple Objective Ant-Q	MOAQ
Multi-objective Omicron ACO	MOA	Elitist TA	e-TA
Multi-objective Network ACO	MONACO	Multi-criteria population-based ACO	MO-PACO

From military applications, a MOACO algorithm, called CHAC, has been designed to solve what is called military unit pathfinding problem (route speed and safety). Mora et Al. [27] have developed a version of CHAC called hCHAC, which models the scenarios as a grid of hexagons. The hCHAC uses two pheromone matrices and two heuristic functions (each pair is dedicated to one objective) and a single ant colony. Furthermore, two different state transition rules have been implemented: the first one combines heuristic and pheromone information of two objectives and the second one is based on dominance over neighbors; and ACS is integrated to have better control in the balance between *exploration* (of unvisited edges) and *exploitation* (of learned knowledge).

Pareto Ant Colony Optimization (P-ACO) is an important approach that is applied to extend ACO to MOACO. It resolves the absence of information flow through its pheromone values. Doerner et Al. [28] developed the P-ACO algorithm for solving multiple objective portfolio selection problem. They defined *multiple pheromone* vectors (one pheromone vector for each objective), random objective weights for each objective, and the lifespan concept. (See also Doerner et Al. [29].) Thereafter, P-ACO is followed in similar and other applications. For instance, Stummer and Sun [30] adopted the model of Doerner et Al. [28] to develop P-ACO heuristic procedure to find efficient portfolios in capital investment planning. They added a neighborhood search routine to the P-ACO algorithm to improve its performance. Pasia et Al. [31] used P-ACO in solving a bi-objective capacitated vehicle routing problem with route balancing. In this approach, the individuals of each pool share information via the local *pheromone update* of P-ACO. This update allows the individuals to explore the regions of the search space that are not yet visited by the previous individuals. On the other hand, pools share information indirectly by the *local pheromone update* and directly by the *global pheromone*

update. The global pheromone update allows the current pool to lead next pool towards a better region.
The set of parameters that used in MOACO algorithms differ from one to one and some of them are common. These include number of ants, *visibility* and *pheromone* relative weight, learning factor (MOAQ), optimality policy (MOAQ), *pheromone evaporation* rate, initial pheromone level, omicron factor (MOA), re-initiation factor (MAS), and *exploration* versus *exploitation* probability (MOACS) as presented in Lezcano et Al. [24].

Garcìa-Martínez et Al. [22] have offered a review and analysis for many MOACO algorithms. They summarized the existing algorithms and proposed a taxonomy that categorizes MOACO algorithms based on the number of *pheromone matrices* and the *heuristic matrices* involved. In addition, an empirical analysis is developed by analyzing their performance based on several instances of the bi-criteria Traveling Salesman Problem in comparison with two well known multi-objective genetic algorithms. Angus and Woodward [23] also have reviewed many existing algorithms and proposed another taxonomy that based on features common to ACO: choice of pheromone model, solution construction process, solutions are evaluated in terms of individual objectives or all objectives, how solutions are used to update the multiple or individual pheromone matrices, and how Pareto optimal solutions are treated. Both papers are found useful for intended modifications of MOACO. See also Lezcano et Al. [24].

From this review, a view is that modeling the factors involved in MOACO problems can be done easily aided by more flexible ways. Known *decision systems* can be nominated, for that purpose, such as AHP, ANP, ELECTRE, PROMETHEE, etc. (Figueira et Al. [32]). The author, as a start, proposes AHP (Saaty [33]), because of its simplicity and flexibility in building and weighting the relationships between the problem factors. In addition, AHP facilitates synthesizing *composite rules* from the given information and constructing linear combinations of the objectives.

4. THE ANALYTIC HIERARCHY PROCESS

The AHP is a weighting multicriteria decision system. It was developed by Saaty in 1980 (Saaty [33, 34]). The AHP arranges the problem factors (goal, criteria, subcriteria, and alternatives) in a descending hierarchic structure. The AHP becomes popular in solving complex decision problems. The AHP can be exhibited in a structured form as follows:

Step 1—Hierarchy
Create a hierarchy comprising all decision factors of the problem.
Step 2—Pairwise comparisons
At each level of the hierarchy, for each related group of factors, with respect to its grouping factor in the direct higher level, construct a pairwise comparison matrix.
Comments: A set of factors is grouped by only one factor in the directly higher level except the alternatives. The alternatives are grouped by each factor in the direct higher level (criterion or subcriterion). The levels can be handled forward or backward. The quantitative (measurement) factors don’t necessitate using pairwise comparison matrices. All comparisons and measurements must be unified in direction in the sense that larger value is superior and smaller value is inferior

or vice versa. A numerical scale should be used for the qualitative judgments. Each matrix is designated to relate the factors in rows to those in columns.

Step 3—Consistency

At each level, test the consistency of all pairwise comparison matrices. If a matrix isn't consistent, verify step 2.

Step 4—Weight vectors

At each level of the hierarchy, for each related group of factors, calculate the weight vector (relative weights) with respect to its grouping factor.

Comments: The relative weights of a group of factors with respect to a qualitative factor are calculated using the corresponding pairwise comparison matrix. That with respect to a quantitative factor can be calculated with suitable mathematics.

Step 5—Normalizations

Use a mathematical method to normalize the weight vectors (optional according to the problem size and conditions).

Step 6—Criteria combined weight matrix

Construct a matrix, the criteria combined weight matrix, to collect all weight vectors except that of alternatives.

Step 7—Criteria composite weight vector

At the level directly higher than the alternatives, calculate the composite weight of each factor to yield a vector (say row wise), the criteria composite weight vector.

Comments: A composite weight of a factor means its weight relative to the goal. Therefore, a composite weight of a factor, at a specific level, equals to multiplication of weights of the factors on the hierarchical path starting from this factor upward up to the second level. This can be done directly on the hierarchy.

Step 8—Alternative weight vectors

For each alternative, construct a vector (say column wise), the alternative weight vector, to collect the weights of this alternative with respect to the factors on the directly higher level (criteria or subcriteria).

Step 9—Alternatives combined weight matrix

Construct a matrix, the alternatives combined weight matrix, to collect the alternative weight vectors, say column wise (each alternative in a column).

Step 10—Overall weight matrix

Construct a matrix, the overall weight matrix, to collect the criteria composite weight vector, and the alternatives weight matrix, arranged all in columns/rows.

Step 11—Alternatives score vector (Global decision vector)

Calculate the alternatives score vector by multiplying the criteria composite weight vector (row wise) by the alternatives combined weight matrix (column wise).

The majority often use the judgment scale {1/9, 1/8, …, 1/2, 1, 2, …, 9} of Saaty for pairwise comparisons. The weight vector V of a reciprocal pairwise comparison matrix A is estimated by the matrix equation $A_{n\times n}V_{n\times 1} = \lambda_{max}V_{n\times 1}$. Where n is the number of compared factors, λ_{max} is the principal eigenvalue of A, and V is the principal eigenvector of A. To make V unique, its entries are normalized by dividing by their sum. V can be estimated by normalizing the columns of A and averaging the rows of the normalized A. Notice that if A is perfectly consistent, all columns of the normalized A become identical to V. A is said consistent if $(CR = CI/RI) \leq 0.10$, where $CI = (\lambda_{max} - n)/(n-1)$ is the consistency index and RI is an experimental random consistency index.

5. THE AHP ANT SYSTEM

5.1. Transition Probability Logic

The transition probability of AS can be analyzed in the following simple way. Suppose that τ and η are the vectors of *pheromone trails* and *visibility*, respectively, of currently available edges. The values of τ_{rs} and η_{rs} can be considered as two different weights for edge rs. Each entry in τ can be normalized dividing by the sum of all entries as

$$P_{rs}^{k}(\tau) = \begin{cases} \dfrac{\tau_{rs}}{\sum_{u \in \mathcal{N}_k(r)} \tau_{ru}}, & s \in \mathcal{N}_k(r) \\ 0, & \text{otherwise} \end{cases}, \tag{7}$$

which weights the value of τ_{rs} amongst the others and gives the sense of probability that an event occur. Also, each entry in η can be normalized as

$$P_{rs}^{k}(\eta) = \begin{cases} \dfrac{\eta_{rs}}{\sum_{u \in \mathcal{N}_k(r)} \eta_{ru}}, & s \in \mathcal{N}_k(r) \\ 0, & \text{otherwise} \end{cases}, \tag{8}$$

which weights the value of η_{rs} amongst the others and gives the sense of probability that another event occur. Then, each entry $P_{rs}^{k}(\tau)P_{rs}^{k}(\eta)$ becomes

$$P_{rs}^{k}(\tau,\eta) = \begin{cases} \left[\dfrac{\tau_{rs}}{\sum_{u \in \mathcal{N}_k(r)} \tau_{ru}}\right]\left[\dfrac{\eta_{rs}}{\sum_{u \in \mathcal{N}_k(r)} \eta_{ru}}\right], & s \in \mathcal{N}_k(r) \\ 0, & \text{otherwise} \end{cases}, \tag{9}$$

which can be renormalized as

$$P_{rs}^{k}(\tau,\eta) = \begin{cases} \dfrac{\tau_{rs}\eta_{rs}}{\sum_{u \in \mathcal{N}_k(r)} \tau_{ru}\eta_{ru}}, & s \in \mathcal{N}_k(r) \\ 0, & \text{otherwise} \end{cases}, \tag{10}$$

which gives the sense that both events occur. Formula (10) can be obtained directly by normalizing the entry $\tau_{rs}\eta_{rs}$. If the parameters α and β are used as exponents, Formula (10) becomes exactly as Formula (5). This analysis finds some relationship between the evaluation of *transition probability* and multicriteria analysis by explaining the *pheromone* and *visibility* as two different criteria with weights of τ_{rs} and η_{rs}, respectively. Thus, such analysis can be extended easily to more than two criteria by incorporating additional heuristic and/or *stigmergic information*.

5.2. AHP Transition Probability

5.2.1. Model I

Suppose that the transition state of an ant k at node r is explained by a set $C = \bigcup C_i$ of n active maximization criteria and a set $E = \bigcup E_x$ of m available unvisited edges. The goal, G, is that the ant should choose the best edge for the next travel. Following AHP, this situation can be represented by the hierarchy shown in Fig. 1. The *criteria* become *pheromone* in addition to unlimited number of *visibilities*. (A visibility may be related to distance, cost, time, profit, etc.) The alternatives become the currently available unvisited edges. (This hierarchy can be enlarged by merging subcriteria if that is necessary.) Referring to Appendix A, the corresponding *transition probability* function is

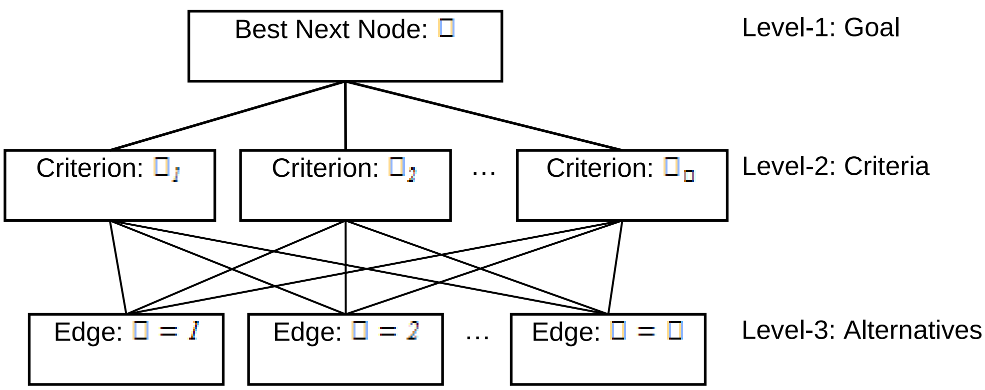

Fig. 1. A hierarchy expresses the ant's transition state.

$$P_{rs}^{k} = \begin{cases} \dfrac{\sum_{i=1}^{n} w_{C_i} w_{rs}(C_i)}{\sum_{u\in \mathcal{N}_k(r)} \sum_{i=1}^{n} w_{C_i} w_{ru}(C_i)}, & s \in \mathcal{N}_k(r), \\ 0, & \text{otherwise} \end{cases} \qquad (11)$$

where, w_{C_i} is weight of criterion i relative to G and $w_{rs}(C_i)$ is weight of edge rs relative to criterion i. Model I for *transition probability*, Formula (11), has the advantage of flexible operations for combining several criteria. Furthermore, it can be adopted as a general formula for the transition probability in ACO with *multiple objective*; that in turn generates other algorithms.

If we reduce the number of criteria to only two criteria (say *pheromone* and *visibility*) with parameters α and β, Formula (11) reduces to

$$P_{rs}^{k} = \begin{cases} \dfrac{\alpha\tau_{rs} + \beta\eta_{rs}}{\sum_{u\in \mathcal{N}_k(r)}[\alpha\tau_{ru} + \beta\eta_{ru}]}, & s \in \mathcal{N}_k(r) \\ 0, & \text{otherwise} \end{cases}, \tag{12}$$

which becomes exactly as that used in ANTS algorithm (see Dorigo and Stützle [6]) if we impose $\alpha + \beta = 1$. With rearrangement, Formula (12) becomes

$$P_{rs}^{k} = \begin{cases} \dfrac{\alpha\tau_{rs} + \beta\eta_{rs}}{\alpha\sum_{u\in \mathcal{N}_k(r)}[\tau_{ru}] + \beta\sum_{u\in \mathcal{N}_k(r)}[\eta_{ru}]}, & s \in \mathcal{N}_k(r) \\ 0, & \text{otherwise} \end{cases}. \tag{13}$$

Model I for *transition probability* sets all criteria on the same level under the goal including the pheromone as shown in Fig.1. Other models can be derived by reconstructing the hierarchical relationships between the ant's decision criteria such as merging *subcriteria*. In other words, different hierarchies can be constructed to express the ant's transition state as exhibited in the next models, Model II and Model III.

5.2.2. Model II

Suppose that the transition state of an ant k at node r is explained by *pheromone* (F) and *visibility* (V), and a set $E = \bigcup E_x$ of m available unvisited edges. Consider the visibility as a combined set $V = \bigcup v_i$ of n active *sub-visibilities*. (A *sub-visibility* may be related to distance, cost, time, profit, etc.) The goal, G, is that the ant should choose the best edge for the next travel. Following AHP, this situation can be represented by the hierarchy shown in Fig. 2.

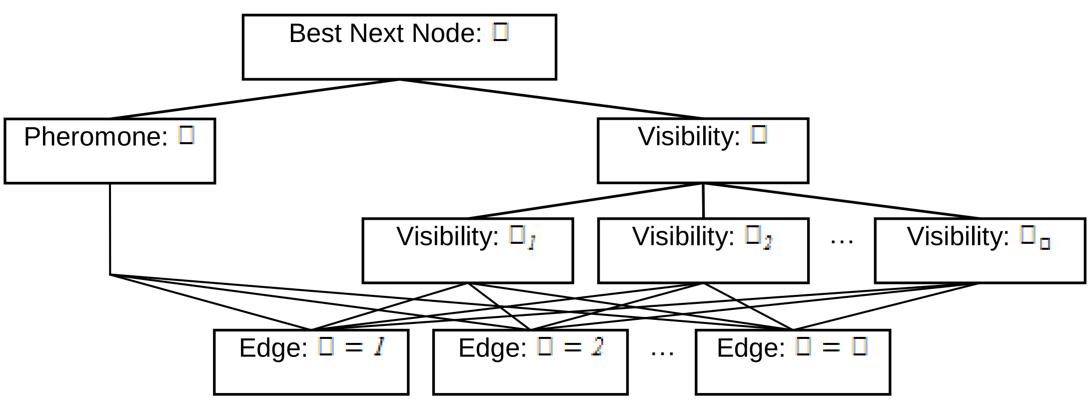

Fig. 2. Visibility enlarged hierarchy: expressing the ant's transition state.

Notice that this structure, Fig.2, places the pheromone (*stigmergic information*) and the *visibility* (heuristic information) on the same level and moves the other criteria down one level grouped under visibility. Referring to Appendix B,

$$P_{rs}^{k}=\begin{cases}\dfrac{\left(\sum_{i=1}^{n} w_V w_{v_i}(V) w_{rs}(v_i)\right)+w_F w_{rs}(F)}{\sum_{u\in\mathcal{N}_k(r)}\left[\left(\sum_{i=1}^{n} w_V w_{v_i}(V) w_{ru}(v_i)\right)+w_F w_{ru}(F)\right]}, & s\in\mathcal{N}_k(r),\\ 0, & \text{otherwise}\end{cases}\tag{14}$$

where, w_V and w_F are weights of visibility and pheromone, respectively, relative to G; $w_{v_i}(V)$ is weight of sub-visibility i relative to V; $w_{rs}(v_i)$ is weight of edge rs relative to sub-visibility i; and $w_{rs}(F)$ is weight of edge rs relative to F. Model II for *transition probability*, Formula (14), can be also adopted as a general formula in ACO with *multiple objective*. Model II sets *stigmergic information* in one group and *heuristic information* in another group. Therefore, Model II proves some degree of *weighting homogeneity* more than Model I.

5.2.3. Model III

Suppose that the transition state of an ant k at node r is explained by *pheromone* (F), *visibility* (V), and a set $E=\cup E_x$ of m available unvisited edges. Consider the visibility as a combined set $V=\cup v_i$ of n active *sub-visibilities* and the pheromone as a combined set $F=\cup f_j$ of l active *sub-pheromones*. The goal, G, is that the ant should choose the best edge for the next travel. Following AHP, this situation can be represented by the hierarchy shown in Fig. 3. (The term sub-pheromone is a paradox will be discussed later in §6.) Referring to Appendix C,

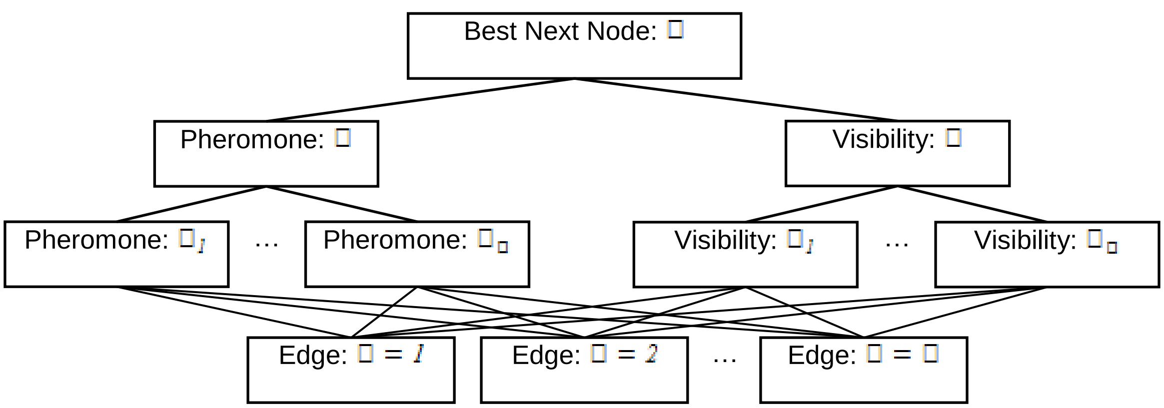

Fig. 3. Enlarged hierarchy expressing the ant's transition state.

$$P_{rs}^{k}=\begin{cases}\dfrac{\left(\sum_{i=1}^{n} w_V w_{v_i}(V) w_{rs}(v_i)\right)+\left(\sum_{j=1}^{l} w_F w_{f_j}(F) w_{rs}(f_j)\right)}{\sum_{u\in\mathcal{N}_k(r)}\left[\left(\sum_{i=1}^{n} w_V w_{v_i}(V) w_{ru}(v_i)\right)+\left(\sum_{j=1}^{l} w_F w_{f_j}(F) w_{ru}(f_j)\right)\right]}, & s\in\mathcal{N}_k(r),\\ 0, & \text{otherwise}\end{cases}\tag{15}$$

where, w_V and w_F are weights of visibility and pheromone, respectively, relative to G; $w_{v_i}(V)$ is weight of sub-visibility i relative to V; $w_{rs}(v_i)$ is weight of edge rs relative to *sub-visibility* i; and $w_{rs}(f_j)$ is weight of edge rs relative to *sub-pheromone* j. Model III for *transition probability*, Formula (15), is more general than Model II.

5.3. Remarks

The literature showed that AHP is used combined with ACO to construct a hybrid algorithm. Nevertheless, this approach isn't the same as that proposed in this paper. Up to this point, the approach proposed in this paper is by applying AHP to modify AS itself to have room for multiple objective, specifically modifying and extending the *transition probability* function. Two papers are found combining AHP with ACO. Fischer et Al. [35] introduced a decision support approach for selection of the competence cells, in a virtual enterprise, by a combination of ACO and AHP. It includes economic factors as well as social factors as soft-facts by applying AHP. The ACO is used for selection of the variant of manufacture and the according competence cells and AHP is used for the computation of the objective function value. The same approach of Fischer et Al. [35] has been followed by Kang et Al. [36] to solve a very similar problem.

6. STIGMERGY REVISITED

The concept of *stigmergy* was first introduced in 1959 by French entomologist Pierre-Paul Grassé to describe a form of *indirect communication* mediated by some *local modifications* of *shared environment* that he observed in two species of termites (Dorigo et Al. [37]). Grassé's original definition of stigmergy was "stimulation of workers by the performance they have achieved." In other words, he observed that these species react to what he called "significant stimuli" (Dorigo et Al. [4]). The term stigmergy has later been used to describe indirect communication in other social insects. See Johnson and Rossi [38] for recent quantification for ant's foraging trails.

Stigmergy helped researchers to understand the connection between the level of individual and the level of colony, showing that an alternative theory could explain the "paradox" of coordination in social insects: although the behavior of the colony as a whole looks wonderfully organized and coordinated, it seems that every insect is pursuing its own agenda without paying much attention to its nestmates (Theraulaz and Bonabeau [39]).

Two main *stigmergic mechanisms* were identified: quantitative stigmergy and qualitative stigmergy. Quantitative stigmergy is a *self-organization-based* mechanism, where the stimulus response sequence comprises stimuli that don't differ qualitatively (such as pheromone fields and gradients) and only modify the probability of response of the individuals to these stimuli. Qualitative stigmergy is based on a discrete set of stimulus types with different responses: for example, a type-1 stimulus triggers action A by individual I1; action A transforms the type-1 stimulus into a type-2 stimulus that triggers action B by individual I2. (See Theraulaz and Bonabeau [39].) Some species use *multiple pheromone* flavors (Van Dyke Parunak [40]); thus, quantitative and qualitative *stigmergy* occur. Table 2 distinguishes four varieties of stigmergy with examples.

Self-organization is a set of dynamical mechanisms whereby structures appear at the global level of a system from interactions of its lower-level components. The main four features that govern *self-organization* in the insect colonies are positive feedback (amplification), negative feedback (for counter-balance and stabilization), amplification of fluctuations (randomness, errors, random walks), and multiple interactions. (This is stated from Das et Al. [41].) Tummolini and Castelfranchi [42] defined *stigmergy* as the process of indirect communication of behavioral messages using traces as implicit signals and exploiting the ability of others to explain such signals. They stated that the *indirect interaction* or the interaction through the environment is necessary to understand self-organizing systems but self-organization is just a function of indirect interaction and not a defining feature. (Notice that ant's pheromone is *explicit* signal left *implicit*.)

Table 2. Varieties of stigmergy (Van Dyke Parunak [40]).

	Marker-Based (based on other agents' traces)	Sematectonic (based on current solution state)
Quantitative	Gradient following in a single pheromone field	Ant cemetery clustering
Qualitative	Decisions based on combinations of pheromones	Wasp nest construction

In general, stigmergy in natural Complex Adaptive Systems (CAS) allows a collection of agents to achieve *global results* for hard task through *local interactions*. Fortunately, stigmergy can be artificially synthesized for the Multi-Agent Systems (MAS) as done in ACO algorithms. Special models for synthesizing stigmergy were presented. For instance, O'Reilly and Ehlers [43] have presented a model called ACEUS that can be used in a business environment to build a software system that imitates a CAS as well as synthesizes stigmergy for interactions between agents and agents and the environment. The design of ACEUS entailed three layers, namely: organic agent layer, insilica agent layer, and stigmergy layer. These layers give some modularity to this model.

However, stigmergic coordination isn't limited to insects. Human stigmergy has been also identified to describe human-human interaction, based on human-level cognition. Fig. 4 summarizes the basic components of a *stigmergic system*: a population of agents sharing an environment.

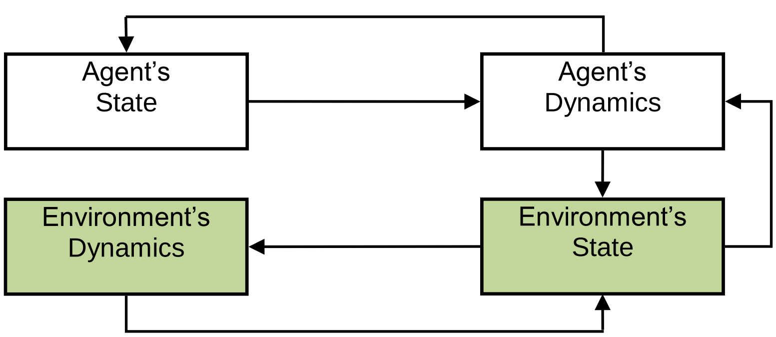

Fig. 4. Basic architecture of stigmergy (Van Dyke Parunak [40]).

As stated in Van Dyke Parunak [40]: the most important distinction between agents and the environment is that the internal state of agents is hidden, while the state of the environment is accessible to an agent with appropriate sensors. In most cases, a second distinction can be observed. Each agent is monolithic, a self-contained computational object with a well-defined boundary. Typically, the environment isn't monolithic, but is structured according to some topology. Ricci et Al. [44] have adopted the term *cognitive stigmergy* to denote stigmergy in the cognitive MAS, i.e. societies of goal/task-oriented/driven agents interacting at the cognitive level. In other words, the term *cognitive stigmergy* supports high-level, knowledge-based social activities. Because of having cognitive capabilities, such agents (*cognitive agents*) aren't necessarily simple and reactive ones, as in the ant case, but can typically be rational, heterogeneous, adaptive, and capable of learning. Cognitive stigmergy uses what is called artifacts (environment abstractions perceived by agents) to model *stigmergic mechanisms* with the shared knowledge and information upon which emergent coordination processes are based. Artifacts are (*i*) the subject of *cognitive agent* activity, (*ii*) the enabler and rulers of agent interaction, and (*iii*) the natural loci for *cognitive stigmergy* processes. (For more details, review Ricci et Al. [44].) Intuitively, pheromone is a non-cognitive.

Sreevalsan-Nair et Al. [45] have described a tool for visualizing and interacting with an ACO algorithm, which is challenging in a 3D environment. That is an interface interacting directly with the operation of this ACO algorithm by allowing the user to deposit some pheromone. That, in turn, influences the coming paths selection to help faster convergence. Thus, the user can be seen here as an external ant/agent having direct interaction, but not a *cognitive agent*. This approach represents a continuation for the "human-guided search" of Mitsubishi research labs. Notice that direct interactions don't denote stigmergy, since they are based on physical senses of agents. Nevertheless, using internal or external direct interactions can be useful for right solution conversion in MAS.

A stigmergic application seems to be recent; that is modeling the artificial emotion. Aqel et Al. [46] have described a simulation model for analyzing artificial emotion supplied to design the interactive game characters. They exploited the *pheromone update* formula of AS to present a formula for emotional state and mood update. They have identified five types of pheromone labels for emotion: acceptance, rejection, frustration, astonishment with fear, and tolerance. Thus, they mimic the behavior of ants to formulate the human emotion. That is human is seen as ants/agents, or in other words ants playing a game. Notice that pheromone decay can mimic human dissatisfaction.

We can conclude that the term stigmergy was originally defined and used in the context of social insects. Stigmergy was used in MAS to describe and realize growing coordination between ant-like nonrational agents. The two main differentiating characteristics of stigmergy are: (*i*) it is an indirect, non-symbolic communication mechanism via agents' environment, and (*ii*) *stigmergic information* is local; see Dorigo et Al. [4]. The term *cognitive stigmergy* (Ricci et Al. [44]) has been defined in the context of social cognitive/rational agents. We can synthesize advanced forms of stigmergy composed of both classes, say *hybrid stigmergy*, to be adopted in MAS. For instance, we can identify an artificial ant having both classes of stigmergy; that can yield more global results. Ricci et Al. [44] proposed their conceptual framework for *cognitive stigmergy* to be a basic reference for engineering practical experimentation in the field of MAS. They identified three main objectives of their framework: (*i*) constructing a model for stigmergic coordination in MAS going beyond ant-like metaphors; (*ii*) providing predictive models for MAS based on *cognitive stigmergy*; and (*iii*) hybridize MAS behavior through both, *cognitive stigmergy* and *non-cognitive stigmergy*.

Thus, we can summarize the terms that can come coherent with the term stigmergy as cognitive, non-cognitive, emotional, non-emotional, quantitative, qualitative, tangible, intangible, explicit, implicit, positive, negative, and user-based (external). In addition, we can use the term *pseudo-pheromone* to express intangible signs of stigmergy. Thus, we can synthesize composite structures of stigmergy and forms for updating their signs. For instance, we can synthesize a pseudo-pheromone, for ACO, to express satisfaction/dissatisfaction of an ant, as if it is human, providing all information, during all solution steps, such as number of accompanying ants, ant's achievement, achievements of other ants, and all problem information. Investigating topics like human needs and motivation (Corning [47]; Wilk [48]) can help to identify paradoxes for a human-like artificial ant or briefly *human ant*. That requires information and knowledge sharing. A helpful concept called *accumulated experience* was introduced for ACO based on ACS and information sharing-weighting system (Montgomery and Randall [49]). The AHP and other *decision systems* can help such aims. Fig. 5 illustrates a basic structure for decision making at transition states by the *human ant*. This structure can be adopted as a base/guide/reference for developing ACO algorithms for *multiple objective* problems. Here, each visibility (heuristic information) is a function of an objective.

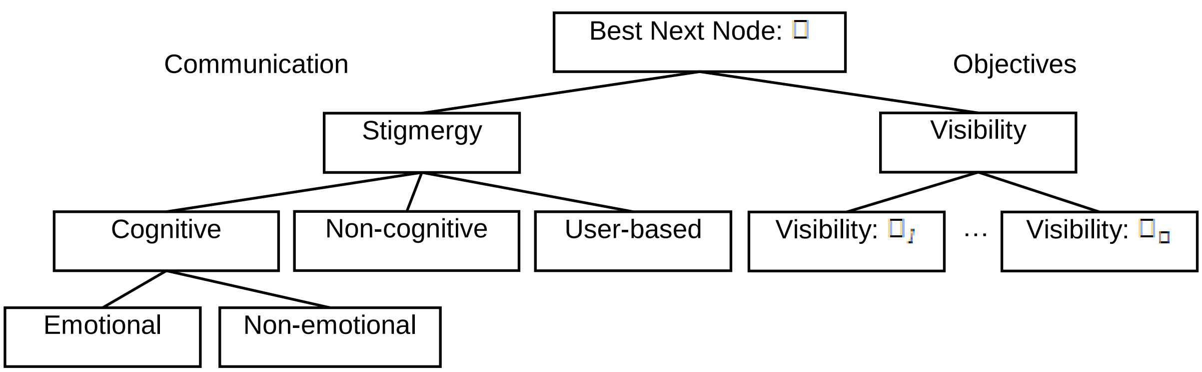

Fig. 5. A basic structure for the human ant's decisions with hybrid stigmergy.

7. CONCLUSIONS AND FUTURE INSIGHTS

The ACO algorithms are construction algorithms based on assigning ants as simple agents, which can iteratively solve NP-hard problems guided by artificial pheromone trails (*stigmergic information*) and *heuristic information*. Mainly, an ACO algorithm is configured by investigating these two kinds of information. The main determinants of an ACO algorithm become the *transition probability* and the way for *pheromone update*, which control the ants' travel. Other characteristics can be also considered. The ant simulates the decision maker, when it has to choose a path to follow on the construction graph. Therefore, this paper proposes that integrating a *decision system* like AHP into ACO will derive and improve a variety of ACO algorithms suiting the problems of *multiple objective*. Stigmergy is discussed in the context of insects and human societies and some *stigmergic paradoxes* have been viewed. The term *human ant* is viewed for the artificial ants if they are actuated with some *cognitive stigmergy*. The term *emotional stigmergy* is viewed to formulate positive and negative feedbacks; that can be also viewed for all types of *cognitive stigmergy*.

The AHP can be used in ACO for several purposes: (1) constructing the *transition probability* function; (2) switching *pheromone update* way during iterations; and (3) rank the performance of the ants during iterations to make presumable decisions of removing some ants from the system or reinforcing others. Here, the focus has been on the first one. The developed function (Formula (16)) has the advantages of simpler operations and availability to integrate unlimited number of guiding information (criteria). This function can primarily replace that in AS (Formula (5)) to provide the system called, in this paper, the AHP Ant System (AHPAS).

Integrating AHP into ACO can be strengthened by the possibility of describing the foraging behavior of ants using fuzzy modeling (Rozin and Margaliot [50]), since AHP itself is highly amenable to fuzzy modeling (see Fuzzy AHP in Chang [51], Lien and Chan [52], and Jaganathan et Al. [53]). Analytic Network Process (ANP) developed by Saaty in 1996 (Saaty [54]) is a direct extension for AHP; therefore, it can be also used to extend this work. Fortunately, ANP is also highly amenable to fuzzy modeling (see Fuzzy ANP in Mikhailov and Madan [55]). Other *decision systems* can be used such as ELECTRE and PROMETHEE (see Figueira et Al. [32]).

However, this paper is a trial to open new topics of research in ACO based on *decision systems* and *cognitive stigmergy*. These topics can be called Decision Support ACO and Cognitive ACO, respectively. This will be promising for MOACO. Therefore, the current work still needs to a series of numerical experiments for the purposes of verification, validation, modification, and extension.

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APPENDICES

Appendix A

Suppose that the pairwise comparison matrices are constructed at each level of the hierarchy and the weight vectors are computed. Weight vector of criteria C relative to the goal, G, is represented as

$$W_C(G) = [w_{C_1} \quad w_{C_2} \quad \cdots \quad w_{C_n}]. \tag{A.1}$$

Weight vector of the edges E, relative to the criterion C_i is represented as

$$W_E(C_i) = [w_{E_1}(C_i) \quad w_{E_2}(C_i) \quad \cdots \quad w_{E_m}(C_i)] . \qquad (A.2)$$

Thus, weight matrix of the edges E relative to the criteria C becomes

$$W_E(C) = \begin{bmatrix} w_{E_1}(C_1) & w_{E_2}(C_1) & \cdots & w_{E_m}(C_1) \\ w_{E_1}(C_2) & w_{E_2}(C_2) & \cdots & w_{E_m}(C_2) \\ \vdots & \vdots & \ddots & \vdots \\ w_{E_1}(C_n) & w_{E_2}(C_n) & \cdots & w_{E_m}(C_n) \end{bmatrix} , \qquad (A.3)$$

and score vector of the edges E is then computed as

$$W_E(G) = W_C(G) W_E(C) . \qquad (A.4)$$

Each entry in the vector $W_E(G)$ represents the weight, $w_{E_x}(G)$, of edge x, relative to G. This entry is computed as

$$w_{E_x}(G) = \sum_{i=1}^{n} w_{C_i} w_{E_x}(C_i), \qquad (A.5)$$

which can be normalized to yield the *transition probability* as Formula (11), where rs replaces E_x and $\mathcal{N}_k(r)$ replaces E.

Appendix B

Suppose that the pairwise comparison matrices are constructed at each level and the weight vectors are computed. Let the weight vector of the pheromone and visibility, relative to the goal, G, be

$$W_{Ant} = [w_F \quad w_V], \qquad (B.1)$$

where w_F and w_V are pheromone and visibility weights, respectively, relative to G. Weight vector of sub-visibilities v, relative to the visibility V can be represented as

$$W_v(V) = [w_{v_1}(V) \quad w_{v_2}(V) \quad \cdots \quad w_{v_n}(V)], \qquad (B.2)$$

then, the corresponding composite weight vector of sub-visibilities v relative to G becomes

$$W_v(G) = [w_V w_{v_1}(V) \quad w_V w_{v_2}(V) \quad \cdots \quad w_V w_{v_n}(V)], \qquad (B.3)$$

and combining the pheromone weight w_F with $W_v(G)$ yields

$$W_{v;F}(G) = [w_V w_{v_1}(V) \quad w_V w_{v_2}(V) \quad \cdots \quad w_V w_{v_n}(V) \quad w_F]. \qquad (B.4)$$

Weight vector of the edges E, relative to the sub-visibility v_i can be represented as

$$W_E(v_i) = [w_{E_1}(v_i) \quad w_{E_2}(v_i) \quad \cdots \quad w_{E_m}(v_i)] \,. \tag{B.5}$$

Thus, weight matrix of the edges E relative to the sub-visibilities v becomes

$$W_E(v) = \begin{bmatrix} w_{E_1}(v_1) & w_{E_2}(v_1) & \cdots & w_{E_m}(v_1) \\ w_{E_1}(v_2) & w_{E_2}(v_2) & \cdots & w_{E_m}(v_2) \\ \vdots & \vdots & \ddots & \vdots \\ w_{E_1}(v_n) & w_{E_2}(v_n) & \cdots & w_{E_m}(v_n) \end{bmatrix} . \tag{B.6}$$

Weight vector of the edges E, relative to the pheromone F can be represented as

$$W_E(F) = [w_{E_1}(F) \quad w_{E_2}(F) \quad \cdots \quad w_{E_m}(F)] \,. \tag{B.7}$$

Combining $W_E(v)$ and $W_E(F)$ yields

$$W_E(v;F) = \begin{bmatrix} w_{E_1}(v_1) & w_{E_2}(v_1) & \cdots & w_{E_m}(v_1) \\ w_{E_1}(v_2) & w_{E_2}(v_2) & \cdots & w_{E_m}(v_2) \\ \vdots & \vdots & \ddots & \vdots \\ w_{E_1}(v_n) & w_{E_2}(v_n) & \cdots & w_{E_m}(v_n) \\ w_{E_1}(F) & w_{E_2}(F) & \cdots & w_{E_m}(F) \end{bmatrix} . \tag{B.8}$$

Score vector of the edges E is then computed as

$$W_E(G) = W_{v;F}(G) W_E(v;F) \,. \tag{B.9}$$

Each entry in the vector $W_E(G)$ represents the weight, $w_{E_x}(G)$, of edge x, relative to G,

$$w_{E_x}(G) = \left(\sum_{i=1}^{n} w_V w_{v_i}(V) w_{E_x}(v_i)\right) + w_F w_{E_x}(F), \tag{B.10}$$

which can be normalized to yield the *transition probability* as Formula (14), where rs replaces E_x and $\mathcal{N}_k(r)$ replaces E.

Appendix C

Suppose that the pairwise comparison matrices are constructed at each level and the weight vectors are computed. Let the weight vector of the pheromone and visibility, relative to the goal, G, be

$$W_{Ant} = [w_F \quad w_V], \tag{C.1}$$

where w_F and w_V are pheromone and visibility weights, respectively, relative to G. Weight vector of sub-visibilities v, relative to the visibility V can be represented as

$$W_v(V) = [w_{v_1}(V) \quad w_{v_2}(V) \quad \cdots \quad w_{v_n}(V)], \qquad (C.2)$$

then, the corresponding composite weight vector of sub-visibilities v relative to G becomes

$$W_v(G) = [w_V w_{v_1}(V) \quad w_V w_{v_2}(V) \quad \cdots \quad w_V w_{v_n}(V)]. \qquad (C.3)$$

Weight vector of sub-pheromones f, relative to the pheromone F can be represented as

$$W_f(F) = [w_{f_1}(F) \quad w_{f_2}(F) \quad \cdots \quad w_{fl}(F)], \qquad (C.4)$$

then, the corresponding composite weight vector of sub-pheromones f relative to G becomes

$$W_f(G) = [w_F w_{f_1}(F) \quad w_F w_{f_2}(F) \quad \cdots \quad w_F w_{fl}(F)]. \qquad (C.5)$$

Combining $W_f(G)$ with $W_v(G)$ yields

$$W_{v;f}(G) = [w_V w_{v_1}(V) \quad w_V w_{v_2}(V) \quad \cdots \quad w_V w_{v_n}(V) \quad w_F w_{f_1}(F) \quad w_F w_{f_2}(F) \quad \cdots \quad w_F w_{fl}(F)]. \qquad (C.6)$$

Weight vector of the edges E, relative to the sub-visibility v_i can be represented as

$$W_E(v_i) = [w_{E_1}(v_i) \quad w_{E_2}(v_i) \quad \cdots \quad w_{E_m}(v_i)]. \qquad (C.7)$$

Weight vector of the edges E, relative to the sub-pheromone f_j can be represented as

$$W_E(f_j) = [w_{E_1}(f_j) \quad w_{E_2}(f_j) \quad \cdots \quad w_{E_m}(f_j)]. \qquad (C.8)$$

Thus, weight matrix of the edges E relative to the sub-visibilities v becomes

$$W_E(v) = \begin{bmatrix} w_{E_1}(v_1) & w_{E_2}(v_1) & \cdots & w_{E_m}(v_1) \\ w_{E_1}(v_2) & w_{E_2}(v_2) & \cdots & w_{E_m}(v_2) \\ \vdots & \vdots & \ddots & \vdots \\ w_{E_1}(v_n) & w_{E_2}(v_n) & \cdots & w_{E_m}(v_n) \end{bmatrix}. \qquad (C.9)$$

Weight matrix of the edges E, relative to the sub-pheromones f can be represented as

$$W_E(f) = \begin{bmatrix} w_{E_1}(f_1) & w_{E_2}(f_1) & \cdots & w_{E_m}(f_1) \\ w_{E_1}(f_2) & w_{E_2}(f_2) & \cdots & w_{E_m}(f_2) \\ \vdots & \vdots & \ddots & \vdots \\ w_{E_1}(f_l) & w_{E_2}(f_l) & \cdots & w_{E_m}(f_l) \end{bmatrix}. \qquad (C.10)$$

Combining $W_E(v)$ and $W_E(f)$ yields

$$W_E(v;f) = \begin{bmatrix} w_{E_1}(v_1) & w_{E_2}(v_1) & \cdots & w_{E_m}(v_1) \\ w_{E_1}(v_2) & w_{E_2}(v_2) & \cdots & w_{E_m}(v_2) \\ \vdots & \vdots & \ddots & \vdots \\ w_{E_1}(v_n) & w_{E_2}(v_n) & \cdots & w_{E_m}(v_n) \\ w_{E_1}(f_1) & w_{E_2}(f_1) & \cdots & w_{E_m}(f_1) \\ w_{E_1}(f_2) & w_{E_2}(f_2) & \cdots & w_{E_m}(f_2) \\ \vdots & \vdots & \ddots & \vdots \\ w_{E_1}(f_l) & w_{E_2}(f_l) & \cdots & w_{E_m}(f_l) \end{bmatrix}. \qquad (C.11)$$

Score vector of the edges E is then computed as

$$W_E(G) = W_{v;f}(G) W_E(v;f). \qquad (C.12)$$

Each entry in the vector $W_E(G)$ represents the weight, $w_{E_x}(G)$, of edge x, relative to G,

$$w_{E_x}(G) = \left(\sum_{i=1}^{n} w_V w_{v_i}(V) w_{E_x}(v_i)\right) + \left(\sum_{j=1}^{l} w_F w_{f_j}(F) w_{E_x}(f_j)\right), \qquad (C.13)$$

which can be normalized to yield the *transition probability* as Formula (15), where rs replaces E_x and $\mathcal{N}_k(r)$ replaces E.

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