Military Technical College
Kobry Elkobbah,
Cairo, Egypt
May 27-29,2008

4th International Conference on
Mathematics and Engineering
Physics (ICMEP-4)

EM-9

# APPLICATION OF CLIFFORD ALGEBRA TO KINEMATICS OF GUIDED MISSILES

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## ABSTRACT

One of the main issues within aerospace and robotics applications is the Kinematics equations of guided missiles. Investigation and analysis of these equations in both homing and command guidance systems consider the missile as an immaterial/massless point moving with a known velocity. In addition, the nature of applications necessitates this process to be fast in computations and easy or concise in manipulation. Therefore, the objective is to reformulate these equations using the properties and the geometrical significance of Clifford algebra. This formulation may be easier and more obvious than the other traditional methods as it unites vectors of different planes into a single mathematical system with a comprehensive geometric significance.

## *KEY WORDS*

*Kinematics equations, Ideal bonds, Clifford algebra and Projection.*

## *NOMENCLATURE*

| | |
|---|---|
| $\frac{dr}{dt}\mathbf{E}_o$ | the rate of change of the magnitude of vector **R** (transitional motion). |
| $\frac{dr_M}{dt}\mathbf{E}_M$ | the rate of change of the missile range in radial direction. |
| $\mathbf{E}_0$ | the unit vector in the direction of **R**. |
| $\mathbf{E}_M$ | the unit vector in the direction of $\mathbf{R}_M$. |
| $\mathbf{F}_1$ and $\mathbf{F}_2$ | are functions selected to fulfill the condition of missile-target hit. |
| $\hat{\mathbf{n}}_1, \hat{\mathbf{n}}_2, \hat{\mathbf{n}}_3$ | are unit vectors. |
| $\mathbf{R}$ | the vector of distance between target and missile. |
| $\mathbf{R}_M$ | the vector of distance between the control point and the missile. |
| $\mathbf{t}$ | time. |
| $\mathbf{V}_K$ | the vector of the control point velocity. |
| $\mathbf{V}_M$ | the missile velocity vector. |
| $\mathbf{V}_T$ | the target velocity vector. |
| $\mathbf{W}_R$ | the angular rate of change of vector **R**. |
| $\mathbf{W}_{R_M}$ | The angular rate of change of vector $\mathbf{R}_M$. |
| $\mathbf{W}_{R_M} \otimes \mathbf{R}_M$ | the rate of change of the missile range in lateral directions. |
| $\mathbf{X}_1$ | the unit vector coincident with the missile-longitudinal axis. |
| $\boldsymbol{\varepsilon}_p$ and $\boldsymbol{\sigma}_p$ | are the lead angles. |

## INTRODUCTION

The missile motion can be categorized into translatory and rotary motion, each of which is divided into Kinematics and Dynamics. That is, the missile motion consists of its c.g. translation and the body rotation around its c.g. The whole set of equations representing the guided missile motion describes the missile modeling and can be used for further investigation. To close the set of guidance equations, the law of ideal bonds is used, which is a geometrical condition, imposed upon the missile motion during its flight. That is, this condition has to be kept through the missile flight till the interception point. The ideal bond equation can be expressed by measuring the relative coordinates of missile, target and the control point or guidance station. This ideal bond might be named law of control or law of guidance according to its utilization in design and analysis.

In the second section, the Kinematics equations of homing guided missile will be discussed and then it will be manipulated using the Clifford algebra in the third section.

Similarly; in the fourth section, the Kinematics equations of the command guided missile will be discussed and then it will be reformulated using Clifford algebra.

## KINEMATIC EQUATIONS OF HOMING GUIDED MISSILES

The vector of the missile position w.r.t. target is determined in polar coordinate system by radius vector R according to Fig. (1).The principle law of motion of a homing-guided missile is given by the following relationship:

$$\frac{\mathbf{dR}}{\mathbf{dt}} = \mathbf{V_T} - \mathbf{V_M} \tag{1}$$

Both magnitude and direction of the vector **R** are functions of time. Thus, the rate of change of the range vector **R** has the following form [1]:

$$\begin{aligned}\frac{\mathbf{dR}}{\mathbf{dt}} &= \frac{\mathbf{d\{rE_o\}}}{\mathbf{dt}} = \frac{\mathbf{dr}}{\mathbf{dt}}\mathbf{E_o} + \frac{\mathbf{dE_o}}{\mathbf{dt}}\mathbf{r} \\ &= \frac{\mathbf{dr}}{\mathbf{dt}}\mathbf{E_0} + \mathbf{W_R} \otimes \mathbf{R}\end{aligned} \tag{2}$$

**Substituting equation (1) into equation (2) yields the following form:**

$$V_T - V_M = \frac{dr}{dt}E_o + W_R \otimes R \tag{3}$$

It is clear from equation (3) that the position of the missile to target is represented by vector R and can be limited in the radial direction and in two normal directions. However, the radial component of the change in range is influenced only by the change of the engine thrust. Therefore, the motion is controlled only in normal (lateral) directions. That is, there are only two ideal bond equations and instead of guiding the missile to the point of interception (hit) it will be guided on a trajectory at the end of which it must intercept the target. The ideal bond equations can be determined according to the methods of housing the coordinator on the missile board. The coordinator measures the mutual position of the missile and the target. There exist three possible methods of housing the coordinator in a missile so that it is possible to determine continuously the mutual position of the coordinator and the specified reference frame. These frames include the body coordinate system, the velocity coordinate system, and the ground coordinate system.

The ideal bond equations of homing-guided missiles have the following general form:

$$\begin{aligned} F_1(R, V_M, X_1, t) &= 0 \\ F_2(R, V_M, X_1, t) &= 0 \end{aligned} \tag{4}$$

To determine the scalar Kinematics equations, a reference frame should be specified along its axes the vector form (3) can be projected. Since the objective is to minimize the distance between missile and target to zero for interception, the directions of increasing/decreasing (radial and lateral) distances are used to establish the required frame of reference. That is; the axes of this frame are $\hat{n}_1, \hat{n}_2$ and $\hat{n}_3$ as shown in Fig. 1. where;

(a) $\hat{n}_1$ is a unit vector in the direction of the vector **R**.

(b) $\hat{n}_2$ is a unit vector perpendicular to $\hat{n}_1$ in the direction of $\hat{\dot{\varepsilon}}$ (which is a vector perpendicular to the plane of angle $\varepsilon$ according to the right hand rule).

(c) $\hat{n}_3$ is a unit vector perpendicular to the plane of $R - \hat{\dot{\varepsilon}}$ such that an orthogonal frame is obtained.

The components of the vector $\frac{dR}{dt}$ along these directions are ($\dot{r}, r\dot{\varepsilon}, r\cos\varepsilon\,\dot{\sigma}$), where $\dot{\sigma}$ is in the direction perpendicular to the plane of the angle $\sigma$. These components can be obtained through the vectorial derivative [1] as follows:

$$\begin{aligned} W_R &= \hat{\dot{\varepsilon}} + \hat{\dot{\sigma}} \\ &= \dot{\varepsilon}\hat{n}_3 - \dot{\sigma}\sin\varepsilon\,\hat{n}_1 - \dot{\sigma}\cos\varepsilon\,\hat{n}_2 \end{aligned} \tag{5}$$

Therefore, the normal component of (3) can be obtained as follows:

$$W_R \otimes R = \begin{vmatrix} \hat{n}_1 & \hat{n}_2 & \hat{n}_3 \\ -\dot{\sigma}\sin\varepsilon & -\dot{\sigma}\cos\varepsilon & \dot{\varepsilon} \\ r & 0 & 0 \end{vmatrix} = r\dot{\varepsilon}\hat{n}_2 + r\dot{\sigma}\cos\varepsilon\,\hat{n}_3 \tag{6}$$

From (1), (3) and (6), the rate of change of the line of sight (LOS) can be obtained as follows:

$$\frac{dR}{dt} = \dot{r}\,\hat{n}_1 + r\dot{\varepsilon}\,\hat{n}_2 + r\dot{\sigma}\cos\varepsilon\,\hat{n}_3 \tag{7}$$

In the same way the other side of (3) can be projected along the directions of the reference axes to yield the scalar Kinematics equations of the homing guided missile c.g. motion as follows:

$$\begin{aligned} \dot{r} &= v_T\cos(\sigma-\varphi_T)\cos(\varepsilon-\theta_T) - v_M\cos(\sigma-\varphi_M)\cos(\varepsilon-\theta_M) \\ r\dot{\varepsilon} &= v_M\cos(\sigma-\varphi_M)\sin(\varepsilon-\theta_M) - v_T\cos(\sigma-\varphi_T)\sin(\varepsilon-\theta_T) \\ r\cos\varepsilon\,\dot{\sigma} &= v_M\sin(\sigma-\varphi_M)\cos\theta_M - v_T\sin(\sigma-\varphi_T)\cos\theta_T \end{aligned} \tag{8}$$

The ideal bond equations (or equations of the law of guidance) depend upon the guidance method used to guide the missile for intercepting the target and can have one of the following forms:

(a) Pure pursuit guidance method where the ideal bond equations have the form:

$$\begin{aligned} \varepsilon - \theta_M &= 0 \\ \sigma - \varphi_M &= 0 \end{aligned} \tag{9}$$

(b) Deviated pursuit guidance method where the ideal bond equations have the form:

$$\begin{aligned} \varepsilon - \theta_M &= \varepsilon_p \\ \sigma - \varphi_M &= \sigma_p \end{aligned} \tag{10}$$

where $\varepsilon_p$ and $\sigma_p$ are constants.

(c) Constant bearing guidance method where the ideal bond equations have the form:

$$\begin{aligned} \varepsilon &= \varepsilon_p \\ \sigma &= \sigma_p \end{aligned} \tag{11}$$

where $\varepsilon_p$ and $\sigma_p$ are constants.

(d) Proportional navigation guidance method which utilizes ideal bond equations of the form

$$\begin{aligned} \dot{\theta}_M &= K_1\dot{\varepsilon} \\ \dot{\varphi}_M &= K_2\dot{\sigma} \end{aligned} \tag{12}$$

where $K_1$ and $K_2$ are the navigation constants.

Obviously, there are 5-equations {three scalar kinematics equations and two ideal bonds} and 6-unknowns which are {$r$, $\varepsilon$, $\sigma$, $v_M$, $\theta_M$, $\varphi_M$}. Thus, the guided missile motion is not only defined by the Kinematics coupling but also, by the forces acting upon it. Considering the influence of these forces, the velocity of the missile is considered to be known as a function of time and so the system of equations becomes closed (6-equations in 6-unknowns). The solution of this set of equations yields the missile trajectory in addition to the time of flight, the demanded turning rate or normal acceleration and the miss distance [2].

## KINEMATICS EQUATIONS OF HOMING GUIDED MISSILES USING CLIFFORD ALGEBRA

This section is devoted to the formulation of the Kinematics of homing guided missile motion using Clifford algebra. It is clear from the derivation of the Kinematics equations of homing guided missile motion that it mainly depends on the projection of the vector form (3) along the axes of the reference frame as shown in Fig. 1.Though, by using the properties of projection in Clifford algebra [3] this formulation can be feasible and easily derived.

It is known in the projection using Clifford algebra, for any vector A there are projections in the direction parallel and orthogonal to the unit vector E, respectively, as follows:

$$A_p = \tfrac{1}{2}(A + EAE) \tag{13}$$

$$A_0 = \tfrac{1}{2}(A - EAE) \tag{14}$$

where $A_p$ is the projection of vector A in the direction parallel to E and $A_o$ is the projection of vector A in the direction orthogonal to E as shown in Fig. 2. The vector forms of the time rate of change of the range vector R and the reference frame $(\hat{n}_1, \hat{n}_2, \hat{n}_3)$ are shown in Fig. 3. And from the equations (13) and (14) the projections of vector $V_M$ along the directions of $\hat{n}_1$, $\hat{n}_2$ and $\hat{n}_3$ have the form:

$$V_{Mp1} = \tfrac{1}{2}(V_M + \hat{n}_1 V_M \hat{n}_1) \tag{15a}$$

$$V_{Mp2} = \tfrac{1}{2}(V_M + \hat{n}_2 V_M \hat{n}_2) \tag{15b}$$

$$V_{Mp3} = \tfrac{1}{2}(V_M + \hat{n}_3 V_M \hat{n}_3) \tag{15c}$$

Similarly, the projections of $V_T$ in the directions of $\hat{n}_1, \hat{n}_2$ and $\hat{n}_3$ have the form

$$V_{Tp1} = \tfrac{1}{2}(V_T + \hat{n}_1 V_T \hat{n}_1) \tag{16a}$$

$$V_{Tp2} = \tfrac{1}{2}(V_T + \hat{n}_2 V_T \hat{n}_2) \tag{16b}$$

$$V_{Tp3} = \tfrac{1}{2}(V_T + \hat{n}_3 V_T \hat{n}_3) \tag{16c}$$

From equations (1), (7), (15) and (16) the Kinematics equations have the following form:

$$\begin{aligned} \dot{r}\hat{n}_1 &= \tfrac{1}{2}(V_T + \hat{n}_1 V_T \hat{n}_1) - \tfrac{1}{2}(V_M + \hat{n}_1 V_M \hat{n}_1) \\ r\dot{\varepsilon}\hat{n}_2 &= \tfrac{1}{2}(V_T + \hat{n}_2 V_T \hat{n}_2) - \tfrac{1}{2}(V_M + \hat{n}_2 V_M \hat{n}_2) \\ r\cos\varepsilon\,\dot{\sigma}\hat{n}_3 &= \tfrac{1}{2}(V_T + \hat{n}_3 V_T \hat{n}_3) - \tfrac{1}{2}(V_M + \hat{n}_3 V_M \hat{n}_3) \end{aligned} \tag{17}$$

Thus, equations (17) are the Kinematics equations of the homing missile motion, formulated in the Clifford form. For the meanwhile, its simplicity comes from its form in which all of its elements multiplied together are Clifford products. In addition, the simple form has been seen in the multiplication of its velocity vectors with the unit vectors $(\hat{n}_1, \hat{n}_2, \hat{n}_3)$ of the reference frame.

Kinematics equations using the pure pursuit method

The ideal bond equations (or equations of the guidance law) are dependent upon the method of guidance used to guide the missile for intercepting the target. For example, in the pure pursuit guidance method the ideal bond equations have the form:

$$\varepsilon - \theta_M = 0$$
$$\sigma - \varphi_M = 0$$

In this case, the vector $V_M$ is in the direction of $\hat{n}_1$ or its angle with $\hat{n}_1$ is zero and can be written as follows:

$$V_M = v_M \hat{n}_1 \tag{18}$$

Substituting (18) into (17) yields the Kinematics equations as follows:

$$\begin{aligned}
\dot{r}\hat{n}_1 &= \tfrac{1}{2}(V_T + \hat{n}_1 V_T \hat{n}_1) - \tfrac{1}{2}(v_M \hat{n}_1 + \hat{n}_1 v_M \hat{n}_1 \hat{n}_1) \\
r\dot{\varepsilon}\hat{n}_2 &= \tfrac{1}{2}(V_T + \hat{n}_2 V_T \hat{n}_2) - \tfrac{1}{2}(v_M \hat{n}_1 + \hat{n}_2 v_M \hat{n}_1 \hat{n}_2) \\
r\cos\varepsilon\,\dot{\sigma}\hat{n}_3 &= \tfrac{1}{2}(V_T + \hat{n}_3 V_T \hat{n}_3) - \tfrac{1}{2}(v_M \hat{n}_1 + \hat{n}_3 v_M \hat{n}_1 \hat{n}_3)
\end{aligned} \tag{19}$$

From the properties of the Clifford Algebra the following relations can be considered [3],

$$E_1 E_1 = \hat{n}_1\hat{n}_1 = \hat{n}_2\hat{n}_2 = \hat{n}_3\hat{n}_3 = I$$
$$\hat{n}_1\hat{n}_2 = -\hat{n}_2\hat{n}_1,\ \hat{n}_1\hat{n}_3 = -\hat{n}_3\hat{n}_1,\ \hat{n}_2\hat{n}_3 = -\hat{n}_3\hat{n}_2$$

*and consequently, the Kinematics equations have the following form:*

$$\begin{aligned}
\dot{r}\hat{n}_1 &= \tfrac{1}{2}(V_T + \hat{n}_1 V_T \hat{n}_1) - \tfrac{1}{2}(v_M \hat{n}_1 + \hat{n}_1 v_M) \\
r\dot{\varepsilon}\hat{n}_2 &= \tfrac{1}{2}(V_T + \hat{n}_2 V_T \hat{n}_2) - \tfrac{1}{2}(v_M \hat{n}_1 + v_M \hat{n}_2 \hat{n}_1 \hat{n}_2) \\
r\cos\varepsilon\,\dot{\sigma}\hat{n}_3 &= \tfrac{1}{2}(V_T + \hat{n}_3 V_T \hat{n}_3) - \tfrac{1}{2}(v_M \hat{n}_1 + v_M \hat{n}_3 \hat{n}_1 \hat{n}_3)
\end{aligned} \tag{20}$$

*That is, it can be simplified to the following form:*

$$\begin{aligned}
\dot{r}\hat{n}_1 &= \tfrac{1}{2}(V_T + \hat{n}_1 V_T \hat{n}_1) - v_M \hat{n}_1 \\
r\dot{\varepsilon}\hat{n}_2 &= \tfrac{1}{2}(V_T + \hat{n}_2 V_T \hat{n}_2) - \tfrac{1}{2}(v_M \hat{n}_1 - v_M \hat{n}_1) \\
r\cos\varepsilon\,\dot{\sigma}\hat{n}_3 &= \tfrac{1}{2}(V_T + \hat{n}_3 V_T \hat{n}_3) - \tfrac{1}{2}(v_M \hat{n}_1 - v_M \hat{n}_1)
\end{aligned} \tag{21}$$

*Finally, these equations can be put in the following form:*

$$\begin{aligned} \dot{r}\hat{n}_1 &= \tfrac{1}{2}(V_T + \hat{n}_1 V_T \hat{n}_1) - v_M \hat{n}_1 \\ r\dot{\varepsilon}\hat{n}_2 &= \tfrac{1}{2}(V_T + \hat{n}_2 V_T \hat{n}_2) \\ r\cos\varepsilon\,\dot{\sigma}\hat{n}_3 &= \tfrac{1}{2}(V_T + \hat{n}_3 V_T \hat{n}_3) \end{aligned} \tag{22}$$

## KINEMATICS EQUATIONS OF COMMAND GUIDED MISSILE

In command-controlled guided missiles, the position of the missile and the target are both related to the point of control as shown in Fig. 4. In this figure, K represents the control point position, and $R_M$ is the range vector between the control point and the missile. The following relation gives the basic law of motion of a command guided missile in the vector form [2]:

$$\frac{dR_M}{dt} = V_M - V_K \tag{23}$$

$$\begin{aligned} \frac{dR_M}{dt} &= \frac{d\{r_M E_M\}}{dt} = \frac{dr_M}{dt}E_M + \frac{dE_M}{dt}r_M \\ &= \frac{dr_M}{dt}E_M + W_{R_M} \otimes R_M \end{aligned} \tag{24}$$

It is clear that in command guided missiles, Eqn. (24) is similar to Eqn. (3) in homing guided missiles where there exist three components, one in the radial direction and two in the lateral directions. Moreover, the ideal bond is determined due to the methods of guidance used to limit the motion of the guided missile. That is, the law of guidance can be expressed by two equations as follows:

$$\begin{aligned} F_1(R_M, R_T, t) &= 0 \\ F_2(R_M, R_T, t) &= 0 \end{aligned} \tag{25}$$

Considering the velocity vector $V_M$ is a known function of time, this couple of equations can be solved. Similarly, to determine the scalar Kinematics equations, a reference frame should be specified along its axes the vector form (24) is projected. Since the objective in the command guidance method is to control the distance between the missile and the control point until interception, the directions of increasing/decreasing (radial and lateral) this distance are used to establish the required frame of reference. That is; the axes of this frame are also $\hat{n}_1, \hat{n}_2$, and $\hat{n}_3$ where,

(a) $\hat{n}_1$ is a unit vector in the direction of the vector $R_M$.

(b) $\hat{n}_2$ is a unit vector perpendicular to $\hat{n}_1$ in the direction of $\hat{\dot{\varepsilon}}_M$ (in the direction perpendicular to the plane of angle $\varepsilon_M$ according to the right hand rule).

(c) $\hat{n}_3$ is a unit vector perpendicular to the plane of $R_M$-$\hat{\dot{\varepsilon}}_M$.

And, the components of the vector $\frac{dR_M}{dt}$ along the directions of the unit vectors are $(\dot{R}_M, R_M\dot{\varepsilon}_M, R_M\cos\varepsilon\,\dot{\sigma}_M)$. In addition, the other side of equation (24) can be projected along these directions to yield the scalar Kinematics equations of the command guided missile c.g. motion as follows:

$$\begin{aligned} \dot{r}_M &= v_M\cos(\sigma_M-\varphi_M)\cos(\varepsilon_M-\theta_M) - v_K\cos(\sigma_M-\varphi_K)\cos(\varepsilon_M-\theta_K) \\ r_M\dot{\varepsilon}_M &= v_K\cos(\sigma_M-\varphi_K)\sin(\varepsilon_M-\theta_K) - v_M\cos(\sigma_M-\varphi_M)\sin(\varepsilon_M-\theta_M) \\ r_M\cos\varepsilon_M\,\dot{\sigma}_M &= v_K\sin(\sigma_M-\varphi_K)\cos\theta_K - v_M\sin(\sigma_M-\varphi_M)\cos\theta_M \end{aligned} \tag{26}$$

The ideal bond equations (or equations of the law of guidance) depend upon the guidance method used to guide the missile for intercepting the target and these ideal bonds can be summarized as follows [2]:

(1) 3-Point guidance method in which the control point, the missile and the target lie on a straight line throughout the missile flight. That is, the ideal bond equation has the form:

$$\begin{aligned} \varepsilon_M - \varepsilon_T &= 0 \\ \sigma_M - \sigma_T &= 0 \end{aligned} \tag{27}$$

(2) Lead angle guidance method in which the missile line of sight (LOS) is deviated from the target LOS by a certain lead angle. Consequently, the ideal bond equations have the form:

$$\begin{aligned} \varepsilon_M - \varepsilon_T &= \varepsilon_p \\ \sigma_M - \sigma_T &= \sigma_p \end{aligned} \tag{28}$$

And according to the values of $\varepsilon_p$ and $\sigma_p$ there are three guidance methods:

- Half lead angle
- Full lead angle
- K- or C- method

## KINEMATICS EQUATIONS OF COMMAND GUIDED MISSILES USING CLIFFORD ALGEBRA

Similar to the formulation done in the previous section for the Kinematics equations describing the homing guided missile motion, the formulation for the command guided missile motion can be carried out and investigated on the same principal of projection in Clifford algebra.

From the equations (13) and (14), the projections of vector $V_M$ along the directions of $\hat{n}_1, \hat{n}_2$, and $\hat{n}_3$ have the following form:

$$V_{Mp1} = \tfrac{1}{2}(V_M + \hat{n}_1 V_M \hat{n}_1) \tag{29a}$$

$$V_{Mp2} = \tfrac{1}{2}(V_M + \hat{n}_2 V_M \hat{n}_2) \tag{29b}$$

$$V_{Mp3} = \tfrac{1}{2}(V_M + \hat{n}_3 V_M \hat{n}_3) \tag{29c}$$

Similarly, the projections of $V_K$ along the directions of $\hat{n}_1, \hat{n}_2,$ and $\hat{n}_3$ have the form:

$$V_{Kp1} = \tfrac{1}{2}(V_K + \hat{n}_1 V_K \hat{n}_1) \tag{30a}$$

$$V_{Kp2} = \tfrac{1}{2}(V_K + \hat{n}_2 V_K \hat{n}_2) \tag{30b}$$

$$V_{Kp3} = \tfrac{1}{2}(V_K + \hat{n}_3 V_K \hat{n}_3) \tag{30c}$$

From equations (26), (29) and (30) the Kinematics equations have the following form:

$$\begin{aligned} \dot{r}_M \hat{n}_1 &= \tfrac{1}{2}(V_K + \hat{n}_1 V_K \hat{n}_1) - \tfrac{1}{2}(V_M + \hat{n}_1 V_M \hat{n}_1) \\ r_M \dot{\varepsilon}_M \hat{n}_2 &= \tfrac{1}{2}(V_K + \hat{n}_2 V_K \hat{n}_2) - \tfrac{1}{2}(V_M + \hat{n}_2 V_M \hat{n}_2) \\ r_M \cos \varepsilon_M \dot{\beta}_M \hat{n}_3 &= \tfrac{1}{2}(V_K + \hat{n}_3 V_K \hat{n}_3) - \tfrac{1}{2}(V_M + \hat{n}_3 V_M \hat{n}_3) \end{aligned} \tag{31}$$

where, equations (31) are the Kinematics equations of the command guided missile motion as shown in fig. 5. which is formulated in Clifford form. It is clear that each term of (31) is a Clifford number and its elements multiplied with each other are Clifford products.

## KINEMATICS EQUATIONS FOR PLANNER MOTION OF MISSILE AND TARGET

The motion of the target and missile can be considered as planer [either in vertical or horizontal]. Such situation is simpler in the mathematical description of the problem than that in the spatial motion.

The kinematics equations of a homing guided missile in the Clifford form are simplified to the form:

$$\begin{aligned} \dot{r} \hat{n}_1 &= \tfrac{1}{2}(V_T + \hat{n}_1 V_T \hat{n}_1) - \tfrac{1}{2}(V_M + \hat{n}_1 V_M \hat{n}_1) \\ r \dot{\varepsilon} \hat{n}_2 &= \tfrac{1}{2}(V_T + \hat{n}_2 V_T \hat{n}_2) - \tfrac{1}{2}(V_M + \hat{n}_2 V_M \hat{n}_2) \end{aligned} \tag{32}$$

While, the kinematics equations of a command guided missile in the Clifford form are simplified as follows:

$$\begin{aligned} \dot{r}_M \hat{n}_1 &= \tfrac{1}{2}(V_K + \hat{n}_1 V_K \hat{n}_1) - \tfrac{1}{2}(V_M + \hat{n}_1 V_M \hat{n}_1) \\ r_M \dot{\varepsilon}_M \hat{n}_2 &= \tfrac{1}{2}(V_K + \hat{n}_2 V_K \hat{n}_2) - \tfrac{1}{2}(V_M + \hat{n}_2 V_M \hat{n}_2) \end{aligned} \tag{33}$$

## SUMMARY AND CONCLUSIONS

This chapter presented the Kinematics equations of the guided missiles for both homing and command guidance systems. Then, a formulation for these equations had investigated using the Clifford algebra. This approach is based on the projection of the vector form relation w.r.t. a reference frame. The formulation was investigated for the Kinematics equations of homing, and command guided missiles in details, followed by the case of planner motion. The new formulation of these equations can be considered as a first trial to search for simpler, more significant and easier formula than that of the traditional one, which may posses' great benefits in the design of autopilot in further works. The point that needs justification, the future is the numerical implementation of this approach with real applications.

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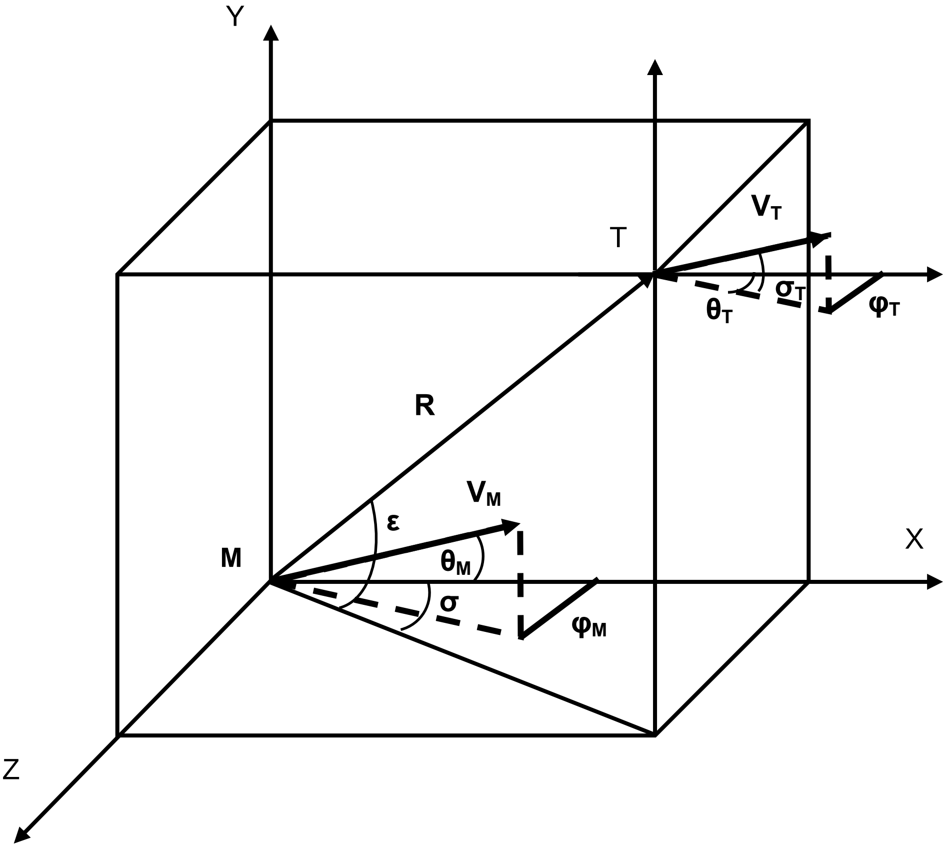


Fig. 1. Missile-Target engagement geometry

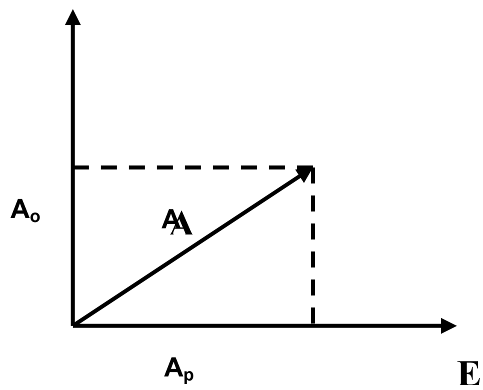


Fig. 2. Projection of vector A

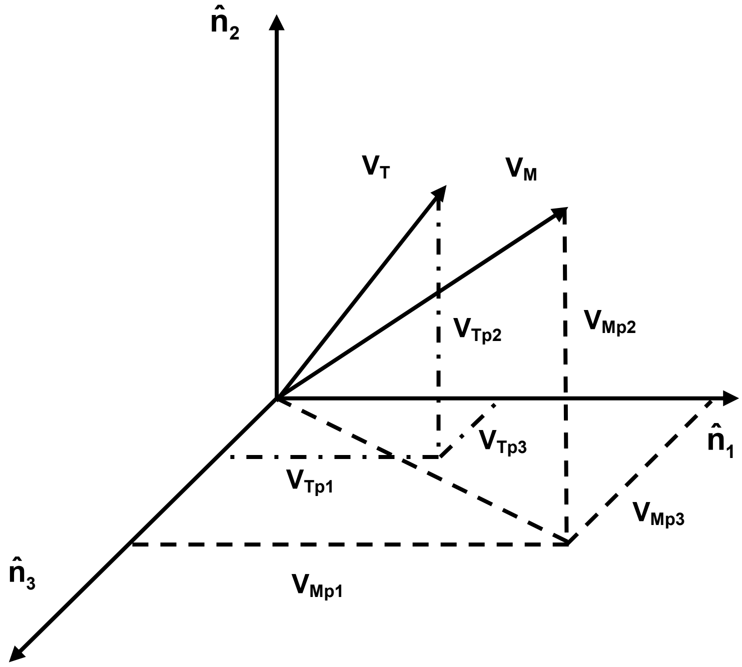


Fig. 3. Velocity vectors along reference axes

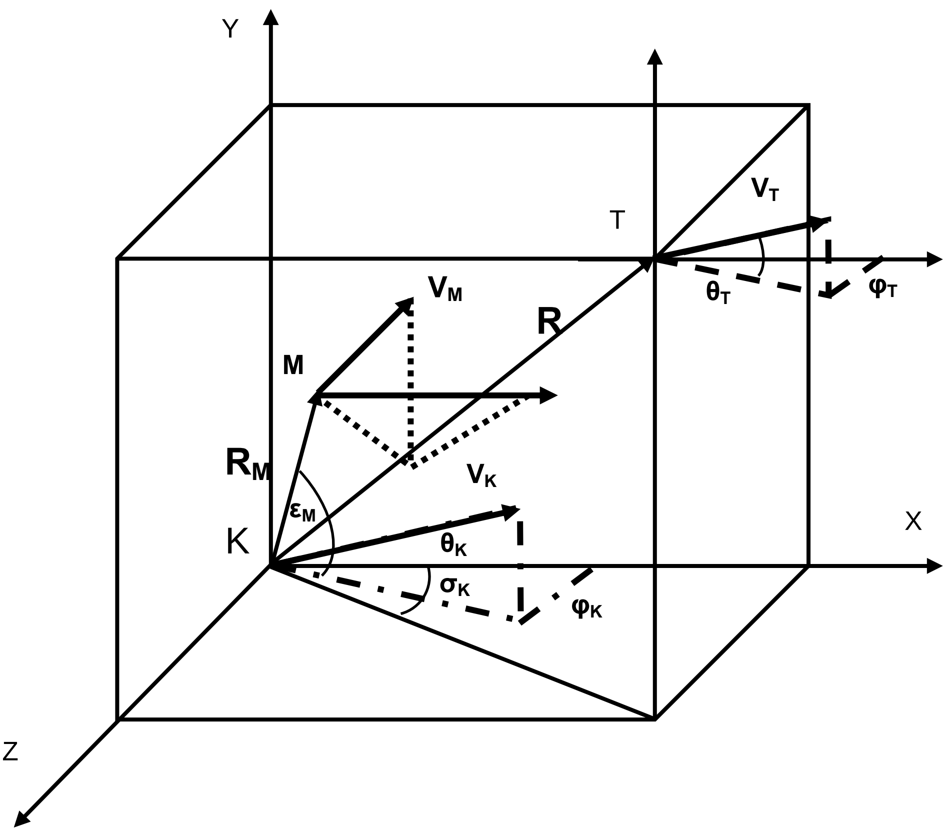


Fig. 4. Geometry of the command guidance system