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Solving System of Fuzzy Linear Equations

By

Amgad M. F. Gorgi(1) and Hassan M. El Hamouly(2)

Abstract

System of fuzzy linear equations is very important in many applications, especially in control systems, financial data analysis. Ordinary methods, Cramer's Rule, Simple Row operations, failed to solve system. We have tested both methods on triangular fuzzy numbers; and their corresponding α–cut. Here we give reason of the failure of each method and a new proposed method to solve such system. Finally we wrote a *Mathematica*® program to solve system of fuzzy linear system. We have considered the case of triangular fuzzy numbers T.F.N. Similar reasoning may be used for any other form of membership function.

(1)…Teaching Assistant - Ain Shams University – Faculty of Engineering-Engineering Physics and Mathematics Dept.

(2)… Egyptian Armed Forces.

1. Introduction

The system of fuzzy linear equations we have studied is:

$$AX = B \qquad \dots (1)$$

where A is a matrix $n \times m$, $n > m$ whose elements are T.F.N.s $A = (a_{ij})_{n\times m}$, $a_{ij} = (l_{ij}, c_{ij}, h_{ij})$, B is a vector $n \times 1$ whose elements are T.F.N.s $B = (b_i)_{n\times 1}$, $b_i = (l_i, c_i, h_i)$ and X is a unknowns-vector $n \times 1$ whose elements were assumed to be T.F.N.s $X = (x_i)_{n\times 1}$, $x_i = (\alpha_i, \beta_i, \gamma_i)$.

There exists a need to solve simultaneous fuzzy equations in a fuzzy sense. In our proposed method, we will approximate the multiplication of two T.F.N.s to be T.F.N.s , we also use $\alpha - cut$ of T.F.N.

2. Failure of Ordinary methods to Solve Simultaneous fuzzy Linear Equations

Consider the following system of fuzzy linear equations:

$$\begin{aligned} (1,2,4)X_1 &+ (0,1,3)X_2 &= (-19,-4,12) \\ (-1,2,3)X_1 &+ (-1,2,5)X_2 &= (-28,-8,14) \end{aligned} \qquad \dots (2)$$

whose solutions are:

$X_1 = (-1,0,3), X_2 = (-5,-4,0)$

and $X_1 = (-1,0,\frac{18}{11}), X_2 = (-5,-4,\frac{20}{11})$

The system may be written **(**$0 \le \alpha \le 1$**)** [4] :

$$\begin{aligned} &[1+\alpha, 4-2\alpha]X_1 + [\alpha, 3-2\alpha]X_2 = [-19+15\alpha, 12-16\alpha] \\ &[-1+3\alpha, 3-\alpha]X_1 + [-1+3\alpha, 5-3\alpha]X_2 = [-28+20\alpha, 14-22\alpha] \end{aligned} \qquad \dots (3)$$

whose solutions are:

$X_1^{\alpha=1} = [0,0], X_2^{\alpha=1} = [-4,-4]$, $X_1^{\alpha=0} = [-1,3], X_2^{\alpha=0} = [-5,0]$.

and $X_1^{\alpha=1} = [0,0], X_2^{\alpha=1} = [-4,-4]$, $X_1^{\alpha=0} = [-1,\frac{18}{11}], X_2^{\alpha=0} = [-5,\frac{20}{11}]$.

2.1. Failure of Crammer's rule to solve Simultaneous Fuzzy Equations

2.1.1. Using Crammer's rule and fuzzy mathematics

We have:

$$\Delta = \begin{vmatrix} (1,2,4) & (0,1,3) \\ (-1,2,3) & (-1,2,5) \end{vmatrix} = (1,2,4)(\cdot)(-1,2,5) - (0,1,3)(\cdot)(-1,2,3) = (-13,2,23)$$

$$\Delta_1 = \begin{vmatrix} (-19,-4,12) & (0,1,3) \\ (-28,-8,14) & (-1,2,5) \end{vmatrix} = (-19,-4,12)(\cdot)(-1,2,5) - (0,1,3)(\cdot)(-28,-8,14) = (-137,0,144)$$

$$Since: \Delta X_1 = \Delta_1$$

$$(-13,2,23)X_1 = (-137,0,144)$$

$$X_1 = \left(-\frac{137}{23}, 0, \frac{144}{23}\right) \cong (-6,0,6)$$

, this is not one of possible solutions.

2.1.2. Using Crammer's rule and $\alpha - cut$ of T.F.N.s

We have:

$$\Delta = \begin{vmatrix} [1+\alpha, 4-2\alpha] & [\alpha, 3-2\alpha] \\ [-1+3\alpha, 3-\alpha] & [-1+3\alpha, 5-3\alpha] \end{vmatrix}$$
$$= [1+\alpha, 4-2\alpha](\cdot)[-1+3\alpha, 5-3\alpha] - [\alpha, 3-2\alpha](\cdot)[-1+3\alpha, 3-\alpha]$$

$$\Delta_1 = \begin{vmatrix} [-19+15\alpha, 12-16\alpha] & [\alpha, 3-2\alpha] \\ [-28+20\alpha, 14-22\alpha] & [-1+3\alpha, 5-3\alpha] \end{vmatrix}$$
$$= [-19+15\alpha, 12-16\alpha](\cdot)[-1+3\alpha, 5-3\alpha] - [\alpha, 3-2\alpha](\cdot)[-28+20\alpha, 14-22\alpha]$$

$$\alpha = 0:$$

$$\Delta = [1,4](\cdot)[-1,5] - [0,3](\cdot)[-1,3] = [-4,20] - [-3,9] = [-13,23]$$

$$\Delta_1 = [-19,12](\cdot)[-1,5] - [0,3](\cdot)[-28,14] = [-95,60] - [-84,42] = [-137,144]$$

$$Since: \Delta X_1 = \Delta_1$$

$$[-13,23]X_1 = [-137,144]$$

$$X_1^{\alpha=0} = [-6,-6]$$

$$\alpha = 1:$$
$$\Delta = [2,2](\cdot)[2,2] - [1,1](\cdot)[2,2] = [4,4] - [2,2] = [2,2]$$
$$\Delta_1 = [-4,-4](\cdot)[2,2] - [1,1](\cdot)[-8,-8] = [-8,-8] - [-8,-8] = [0,0]$$
$$Since: \Delta X_1 = \Delta_1$$
$$[2,2]X_1 = [0,0]$$
$$X_1^{\alpha=1} = [0,0]$$

, this is not one of possible solutions.

2.2. Failure of Simple Row Operations to Solve Simultaneous Fuzzy Linear Equations

2.2.1. Using Simple Row Operations and fuzzy mathematics

We have:

$$[A|B] = \left[\begin{array}{cc|c} (1,24) & (0,1,3) & (-19,-4,12) \\ (-1,2,3) & (-1,2,5) & (-28,-8,14) \end{array}\right]$$

$$\underset{\approx}{\overset{(-1,2,3)(\cdot)r_1}{\scriptstyle (1,2,4)(\cdot)r_2}} \left[\begin{array}{cc|c} (-1,2,3)(\cdot)(1,24) & (-1,2,3)(\cdot)(0,1,3) & (-1,2,3)(\cdot)(-19,-4,12) \\ (1,2,4)(\cdot)(-1,2,3) & (1,2,4)(\cdot)(-1,2,5) & (1,2,4)(\cdot)(-28,-8,14) \end{array}\right]$$

$$\underset{\approx}{\overset{(-1,2,3)(\cdot)r_1}{\scriptstyle (1,2,4)(\cdot)r_2}} \left[\begin{array}{cc|c} (-4,4,12) & (-3,2,9) & (-57,-8,36) \\ (-4,4,12) & (-4,4,20) & (-112,-16,56) \end{array}\right]$$

$$\overset{r_2--r_1}{\approx} \left[\begin{array}{cc|c} (-4,4,12) & (-3,2,9) & (-57,-8,36) \\ (-16,0,16) & (-13,2,11) & (-148,-8,113) \end{array}\right]$$

Hence, we can not eliminate coefficient of X1 or X2.

2.2.2. Using Simple Row Operations and $\alpha - cut$ of T.F.N.s

We have:

$$[A|B] = \left[\begin{array}{cc|c} [1+\alpha, 4-2\alpha] & [\alpha, 3-2\alpha] & [-19+15\alpha, 12-16\alpha] \\ [-1+3\alpha, 3-\alpha] & [-1+3\alpha, 5-3\alpha] & [-28+20\alpha, 14-22\alpha] \end{array}\right]$$

$\alpha = 0$:

$$\left[\begin{array}{cc|c} [1,4] & [0,3] & [-19,12] \\ [-1,3] & [-1,5] & [-28,14] \end{array}\right]$$

$$\overset{\substack{[-1,3]r_1 \\ [1,4]r_2}}{\approx} \left[\begin{array}{cc|c} [-4,12] & [-3,9] & [-57,36] \\ [-4,12] & [-4,20] & [-112,56] \end{array}\right]$$

$$\overset{r_1 - r_2}{\approx} \left[\begin{array}{cc|c} [-4,12] & [-3,9] & [-57,36] \\ [-16,16] & [-13,11] & [-148,113] \end{array}\right]$$

Hence, we can not eliminate coefficient of X1 or X2. Similarly for $\alpha = 1$.

2.3 Reason of fail of Ordinary Methods

As we have noticed above ordinary methods, Cramer's Rule, Simple Row operations and Gauss Elimination, can not be applied in fuzzy equations because:

IF $A = (a,b,c)$ is T.F.N. and is not singleton $A \neq (a,a,a)$

Then $A - A = (a,b,c) - (a,b,c) = (a-c,0,c-a) \neq (0,0,0)$

Hence we can not eliminate the coefficients of unknowns x_i in equations, and all ordinary methods use coefficient elimination (either directly like Simple Row Operations or indirectly like Cramer's rule).

3. How to solve system of fuzzy linear equations

3.1. Analysis of interval multiplication

Assume $X = (a_x, b_x, c_x)$ is T.F.N. where b_x is known, we have $a_x \leq b_x \leq c_x$. The possible outcomes of interval multiplication $[a_1, c_1](\cdot)[a_x, c_x]$, where a_1, c_1 are known constants:

a_1	c_1	a_x	c_x	$[a_1,c_1](\cdot)[a_x,c_x]$	Case numbering
+	+	+	+	$[a_1 a_x, c_1 c_x]$	I
+	+	-	+	$[c_1 a_x, c_1 c_x]$	ii
+	+	-	-	$[c_1 a_x, a_1 c_x]$	iii
-	+	+	+	$[a_1 c_x, c_1 c_x]$	iv

-	+	-	+	$[a_1c_x, a_1a_x]$	v
				$[a_1c_x, c_1c_x]$	iv
				$[c_1a_x, a_1a_x]$	vi
				$[c_1a_x, c_1c_x]$	ii
-	+	-	-	$[c_1a_x, a_1a_x]$	vi
-	-	+	+	$[a_1c_x, c_1a_x]$	vii
-	-	-	+	$[a_1c_x, a_1a_x]$	v
-	-	-	-	$[c_1c_x, a_1a_x]$	viii

Then there exit 8-possible outcomes i, ii, iii, iv, v, vi, vii and viii. Those results may be summarized in the following table:

Case	i	ii	iii	iv
$[a_1,c_1](\cdot)[a_x,c_x]$	$[a_1a_x, c_1c_x]$	$[c_1a_x, c_1c_x]$	$[c_1a_x, a_1c_x]$	$[a_1c_x, c_1c_x]$
a_1	+	±	+	-
c_1	+	+	+	+
b_x	+	±	-	±
Condition	$a_1 > 0 \,\&\, b_x > 0$	$c_1 > 0$	$a_1 > 0 \,\&\, b_x < 0$	$a_1 < 0 \,\&\, c_1 > 0$

Case	v	vi	vii	viii
$[a_1,c_1](\cdot)[a_x,c_x]$	$[a_1c_x, a_1a_x]$	$[c_1a_x, a_1a_x]$	$[a_1c_x, c_1a_x]$	$[c_1c_x, a_1a_x]$
a_1	-	-	-	-
c_1	±	+	-	-
b_x	±	±	+	-
Condition	$a_1 < 0$	$a_1 < 0 \,\&\, c_1 > 0$	$c_1 < 0 \,\&\, b_x > 0$	$c_1 < 0 \,\&\, b_x < 0$

3.2. Proposal to Solve System of fuzzy linear equations

We have $\alpha-cut$ representation of system of fuzzy linear equations:

$$\begin{aligned}&[1+\alpha, 4-2\alpha]X_1 + [\alpha, 3-2\alpha]X_2 = [-19+15\alpha, 12-16\alpha]\\&[-1+3\alpha, 3-\alpha]X_1 + [-1+3\alpha, 5-3\alpha]X_2 = [-28+20\alpha, 14-22\alpha]\end{aligned} \quad \ldots (3)$$

Assume $X_1 = (a_1, b_1, c_1), X_2 = (a_2, b_2, c_2)$ are T.F.N.s

then $X_1 = [a_1 + (b_1 - a_1)\alpha, c_1 - (c_1 - b_1)\alpha]$,

$X_2 = [a_2 + (b_2 - a_2)\alpha, c_2 - (c_2 - b_2)\alpha]$

Step (1):

Obtain solution for $\alpha = 1$:

$$[2,2][b_1, b_1] + [1,1][b_2, b_2] = [-4,-4]$$
$$[2,2][b_1, b_1] + [2,2][b_2, b_2] = [-8,-8]$$

$$\begin{aligned} 2b_1 + b_2 &= -4 \\ 2b_1 + 2b_2 &= -8 \end{aligned} \qquad \ldots \text{(i)}$$

$$b_1 = 0, b_2 = -4$$

$$\boxed{X_1 = (a_1, 0, c_1) \,\&\, X_2 = (a_2, -4, c_2)}$$

Step (2):

Obtain solution for $\alpha = 0$:

$$[1,4][a_1, c_1] + [0,3][a_2, c_2] = [-19,12]$$
$$[-1,3][a_1, c_1] + [-1,5][a_2, c_2] = [-28,14]$$

$$[\min(a_1, 4a_1), 4c_1] + [3a_2, \max(0, 3c_2)] = [-19,12]$$
$$[\min(3a_1, -c_1), \max(-a_1, 3c_1)] + [\min(5a_2, -c_2), \max(-a_2, 5c_2)] = [-28,14]$$

$$[\min(a_1, 4a_1) + 3a_2, 4c_1 + \max(0, 3c_2)] = [-19,12]$$
$$[\min(3a_1, -c_1) + \min(5a_2, -c_2), \max(-a_1, 3c_1) + \max(-a_2, 5c_2)] = [-28,14]$$

$$\begin{aligned} \min(a_1, 4a_1) + 3a_2 &= -19 \\ 4c_1 + \max(0, 3c_2) &= 12 \\ \min(3a_1, -c_1) + \min(5a_2, -c_2) &= -28 \\ \max(-a_1, 3c_1) + \max(-a_2, 5c_2) &= 14 \end{aligned} \qquad \ldots \text{(ii)}$$

Equations (ii) are ordinary simultaneous linear equations that can be solved using Crammer's Rule or Simple Row Operations. However, the result of such $\max(0, 3c_2)$ is not known because they depend on unknowns, therefore we need to test existence of solution twice, first we assume $\max(0,3c_2)=0$ then we assume $\max(0,3c_2)=3c_2$. Applying the same idea on other min () and max (), we find that we need to test at least 64 assumptions which can not be done without computer aid.

3.3. *Mathematica*® Program to Solve System of Fuzzy Linear Equations:

```
(* This program solves system of n×m, n>m, linear equations whose coefficient and unknowns are
Triangular Fuzzy Number. It has been developed by T.A. Amgad Gorgi under supervision of Prof.Dr.
Hassan El Hamouly, You may copy program and paste in mathematica 5.1 or higher to use it, republish
is prohibited without written notice from authors.*)
S="N";
While[S!="Y",
(* Enter No. of Equations (n) and Variables(m) *)
n=Input["Please Enter No. of Equations (n):"]; m=Input["Please Enter No. of Variables (m):"];
(* Print Equations Form *)
Print["The Simultaneous Fuzzy linear equations Form is:"];
For[i=1,i<n+1,i++,F="";For[j=1,j<m+1,j++,F=StringForm["``(a`2``3`,b`2``3`,c`2``3`)X`3`",F,i,j];
If[j<m,F=StringForm["``+",F],F=StringForm["``=",F]];];F=StringForm["``(a`2``3`,b`2``3`,c`2``3`)",F,i,j]
; Print[F];]; Print[""];
(*Enter Traingular Fuzzy Coefficients A and Constants B*)
Array[a,{n,m+1}];Array[b,{n,m+1}];Array[c,{n,m+1}];For[i=1,i<n+1,i++,For[j=1,j<m+2,j++,{a[i,j],b[i,
j],c[i,j]}= Input[StringForm["Please Enter {a`1``2`,b`1``2`,c`1``2`}",i,j]];];];
(* Print The Given Equations and wait confirmation*)
Print["Your Simultaneous Fuzzy linear equations are:"];
For[i=1,i<n+1,i++,F="";For[j=1,j<m+1,j++,F=StringForm["``(``,``,``)X``",F,a[i,j],b[i,j],c[i,j],j];
If[j<m,F=StringForm["``+",F],F=StringForm["``=",F]]]; F=StringForm["``(``,``,``)",F,a[i,j],b[i,j],c[i,j]];
Print[F];]; Print[""];S=Input["Are your equations right?[Y/N]"]; If[n<m,S="N";Print[" No. of equations
< No. of Variables"];];];
(*End of Inputs*)
(*Step (1) to solve fuzzy linear equations, finding (b)'s*)
bX=RowReduce[Array[b,{n,m+1}]];
(* Calculating Number of Possible Solutions "t" and Possible Combinations "PS2"*)
Array[k,{n,m}];Array[ca,{n*m,4}];l=1;t=1; For[i=1,i<n+1,i++,For[j=1,j<m+1,j++,k[i,j]=0;
If[a[i,j]>=0&&bX[[j,m+1]]>=0,k[i,j]=k[i,j]+1;ca[m(i-1)+j,l]=1;l++];If[c[i,j]>=0,k[i,j]=k[i,j]+1;ca[m(i-
1)+j,l]=2;l++];If[a[i,j]>=0&&bX[[j,m+1]]<0,k[i,j]=k[i,j]+1;ca[m(i-
1)+j,l]=3;l++];If[a[i,j]<0&&c[i,j]>=0,k[i,j]=k[i,j]+1;ca[m(i-
```

```
1)+j,l]=4;l++];If[a[i,j]<0,k[i,j]=k[i,j]+1;ca[m(i-
1)+j,l]=5;l++];If[a[i,j]<0&&c[i,j]>=0,k[i,j]=k[i,j]+1;ca[m(i-
1)+j,l]=6;l++];If[c[i,j]<0&&bX[[j,m+1]]>=0,k[i,j]=k[i,j]+1;ca[m(i-
1)+j,l]=7;l++];If[c[i,j]<0&&bX[[j,m+1]]<0,k[i,j]=k[i,j]+1;ca[m(i-1)+j,l]=8;l++];If[l<4,ca[m(i-
1)+j,l]=0;ca[m(i-1)+j,l+1]=0]; l=1; t=t*k[i,j];];];Print["The program will test ",t," possible case. Please
Wait....\n"];PS=Tuples[Array[ca,{n*m,4}]];
(*Possible Combinations*)
q=1; l=1;For[i=1,i<(4^(n*m))+1,i++,For[j=1,j<n*m+1,j++,q=Min[q,PS[[i,j]]]];
If[q>0,For[j=1,j<n*m+1,j++,PS2[l,j]=PS[[i,j]]];l++];q=1;];
(* Step (2): Testing Possible Solutions to (a)'s and (c)'s)
Array[e,{2n,2m+1}];For[i=1,i<n+1,i++,e[2i-
1,2m+1]=a[i,m+1];e[2i,2m+1]=c[i,m+1];];q=1;d=0;XH=0;XL=0;For[l=1,l<t+1,l++,For[i=1,i<n+1,i++,F
or[j=1,j<m+1,j++,If[PS2[l,m(i-1)+j]==1,{e[2i-1,2j-1],e[2i-1,2j],e[2i,2j-1],e[2i,2j]}={a[i,j],0,0,c[i,j]}];
If[PS2[l,m(i-1)+j]==2, {e[2i-1,2j-1],e[2i-1,2j],e[2i,2j-1],e[2i,2j]}={c[i,j],0,0,c[i,j]}]; If[PS2[l,m(i-
1)+j]==3, {e[2i-1,2j-1],e[2i-1,2j],e[2i,2j-1],e[2i,2j]}={c[i,j],0,0,a[i,j]}]; If[PS2[l,m(i-1)+j]==4, {e[2i-
1,2j-1],e[2i-1,2j],e[2i,2j-1],e[2i,2j]}={0,a[i,j],0,c[i,j]}]; If[PS2[l,m(i-1)+j]==5, {e[2i-1,2j-1],e[2i-
1,2j],e[2i,2j-1],e[2i,2j]}={0,a[i,j],a[i,j],0}];If[PS2[l,m(i-1)+j]==6, {e[2i-1,2j-1],e[2i-1,2j],e[2i,2j-
1],e[2i,2j]}={c[i,j],0,a[i,j],0}];If[PS2[l,m(i-1)+j]==7, {e[2i-1,2j-1],e[2i-1,2j],e[2i,2j-
1],e[2i,2j]}={0,a[i,j],c[i,j],0}];If[PS2[l,m(i-1)+j]==8, {e[2i-1,2j-1],e[2i-1,2j],e[2i,2j-
1],e[2i,2j]}={0,c[i,j],a[i,j],0}];];];If[MatrixRank[Array[e,{2n,2m}]]==MatrixRank[Array[e,{2n,2m+1}]],
acX=RowReduce[Array[e,{2n,2m+1}]];Array[X,{m,3}];For[j=1,j<m+1,j++,X[j,1]=acX[[2j-1,2m+1]];
X[j,2]=bX[[j,m+1]]; X[j,3]=acX[[2j,2m+1]];];
(*Checking a<b<c*)
If[X[j,1]>X[j,2]||X[j,2]>X[j,3],q=0];
(*Checking that obtained values of (a)s and (c)s are satisfying equations*)
For[i=1,i<n+1,i++,For[j=1,j<m+1,j++,XL=XL+Min[a[i,j]X[j,1],a[i,j]X[j,3],c[i,j]X[j,1],c[i,j]X[j,3]];
XH=XH+Max[a[i,j]X[j,1],a[i,j]X[j,3],c[i,j]X[j,1],c[i,j]X[j,3]];]; If[a[i,m+1]!=XL||c[i,m+1]!=XH,q=0];
XL=0;XH=0;];
(*Remove repeated values of (X)s*)
For[i=1,i<d+1,i++,For[j=1,j<m+1,j++,If[{Y[i,j,1],Y[i,j,2],Y[i,j,3]}=={X[j,1],X[j,2],X[j,3]},q=0];];];
(*Record Correct Unrepeated Solution*)
If[q==1, d++;For[j=1,j<m+1,j++,{Y[d,j,1],Y[d,j,2],Y[d,j,3]}={X[j,1],X[j,2],X[j,3]};];];q=1;];];
Print["There is at least ",Length[Array[Y,{d,m,3}]]," Possible Solutions\n"];
(* Printing Correct Unrepeated Solutions *)
For [i=1,i<d+1,i++,Print[StringForm["Solution ``
:",i]];For[j=1,j<m+1,j++,Print[StringForm["X``=(``,``,``)",j,Y[i,j,1],Y[i,j,2],Y[i,j,3]]];];Print["\n"];];
```

3.4. Examples

We have used our *Mathematica*® program to solve two simple examples of systems of fuzzy linear equations:

Example 1:-

$$\begin{aligned}(1,2,4)X_1 &+ (0,1,3)X_2 &= (-19,-4,12)\\ (-1,2,3)X_1 &+ (-1,2,5)X_2 &= (-28,-8,14)\end{aligned}$$

The program tests 64 possible cases, from which it found the two possible solutions:

$X_1 = (-1,0,3), X_2 = (-5,-4,0)$ **and** $X_1 = (-1,0,\frac{18}{11}), X_2 = (-5,-4,\frac{20}{11})$

Example 2:-

We were also able to obtain the solution of linearly dependent system using our program:

$$\begin{aligned}(1,2,4)X_1 &+ (0,1,3)X_2 &= (-19,-4,12)\\ (-1,2,3)X_1 &+ (-1,2,5)X_2 &= (-28,-8,14)\\ (0,4,7)X_1 &+ (-1,3,8)X_2 &= (-47,-12,26)\end{aligned}$$

The program tests 512 possible cases, from which results the following two possible solutions

$X_1 = (-1,0,3), X_2 = (-5,-4,0)$ **and** $X_1 = (-1,0,\frac{18}{11}), X_2 = (-5,-4,\frac{20}{11})$

4. Error Analysis

We have approximated the result of multiplication of two T.F.N.s to T.F.N. during our analysis therefore we only investigate solution for $\alpha = 0$ and $\alpha = 1$, and the resulting error would be that of approximating the function X(x) by linear spline function , that is the error E(x) will be given by the formula:

$$E(x) = \frac{(x - x_1)(x - x_2)}{2} X''(x),$$

where x_i , i = 1 , 2 represents the value of x corresponds to $\alpha = 0, or\, \alpha = 1$ according to whether we are approximating the increasing or decreasing edge of the triangular fuzzy number.

5. Conclusion and Future Work

From results, we could see that our proposal can be used to solve any Simultaneous Fuzzy Linear system of equations n×m, n>m. However, more studies should be done to deduce a theory of existence of solution.

6. References

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