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ELECTROMAGNETIC SCATTERING COMPUTATION METHODS FOR VERY SMALL PARTICLES

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ABSTRACT

An electromagnetic wave propagating through a cloud of very small particles (say dust/sand) experiences extinction (absorption and scattering) .The size of these particles is normally less than 0.2 mm (in radius), much smaller than the microwave wavelength,therefore it is possible to avoid laborious computation by using approximations . Not only are these considerably simpler than existing solutions (e.g. for the sphere) but they can often provide useful physical insight into the processes occurring . In this paper we have checked the Rayleigh theory (Rayleigh scatterer) against a full solution of Maxwell's time-varying fields equations through a point matching technique (PMT) and the agreement between PMT and Rayleigh approximations for small particles is found to be good .

KEY WORD
Scattering computations , Maxwell's equations full solution ,Rayleigh-scatterer approximation .

1. SCATTERING COMPUTATION METHODS

1.1 Rayleigh-Method Ellipsoids

The simplest approximation is the Rayleigh scatterer approximation . This applies in the following limits (simplifications) :

$$\frac{2\pi a}{\lambda} << 1$$

and

$$\frac{2\pi a}{\lambda}\left|\varepsilon - 1\right| << 1$$

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where a=dust/sand particle's radius
λ =transmitting wavelength
ε =permittivity of the particle

The first simplification introduced by the small size is that the particle may be considered to be placed in a homogeneous (uniform) electric field , and the second simplification ensures that the phase of the internal field is not very different to the external field . Then, the scattering and absorption cross sections are given by :

$$\sigma_{sca} = \frac{8}{3}\pi^5\left(\frac{2}{\lambda}\right)^4 a^6\left|\frac{\varepsilon-1}{\varepsilon+1}\right|^2 \qquad (1)$$

and

$$\sigma_{abs} = \frac{8\pi^2}{\lambda}a^3 \operatorname{Im}\left(\frac{\varepsilon-1}{\varepsilon+1}\right) \qquad (2)$$

Since $2\ \pi a/\lambda << 1$ it follows that for most particle in the Rayleigh approximation

$$\sigma_{abs} >> \sigma_{sca}$$

Provided $\operatorname{Im}\varepsilon$ is not very small for a given frequency. The total extinction cross section in this approximation is :

$$\sigma_{ext} = \sigma_{sca} + \sigma_{abs}$$

When $\operatorname{Im}\varepsilon = 0, \sigma_{abs} = 0$ and σ_{sca} dominates no matter how small particle is compared with the wavelength. The lowest value of $\operatorname{Im}\varepsilon$ measured so far is 0.0091 , with the corresponding Re ε equal to 2.5384 , it is found that $\operatorname{Im}\left(\frac{\varepsilon-1}{\varepsilon+1}\right) = 1.23254x10^{-3}$ while $\left|\frac{\varepsilon-1}{\varepsilon+1}\right|^2 = 0.1491$. Taking $a = 0.1$ mm as the largest particle at 10 GHz we have, even for this permittivity:

$$\frac{\sigma_{sca}}{\sigma_{abs}} = 6.889x10^{-4}\ .$$

This justifies neglecting the scatter contribution to extinction at least up to 30 GHz .

Considering a dielectric ellipsoid with principal axes labeled 1 , 2 , and , 3 and the corresponding semi-axes a , b, and c , the particle forward ($\theta = 0$) scattering coefficient S(0) for a plane wave incident along a principal axis and linearly polarized parallel to the i'th principal axis is given in the small particle Rayleigh approximation by :

$$S_i(0) = j\beta^3\alpha_i \qquad (\beta = \frac{2\pi}{\lambda}) \tag{3}$$

$$\alpha_i = \frac{V_p}{4\pi\left(L_i + \frac{1}{\varepsilon_r - 1}\right)} \quad , \text{i=1, 2, 3}$$

Where

V_p = is the particle's volume
L_i = factors which depend on the shape only;
they satisfy: $L_1 + L_2 + L_3 = 1$ (Ph.D, 1984) [1]
$\alpha_i =$ the polarizability [3]
$\varepsilon_r =$ relative permittivity of the particle

L_i are approximated by McEwen and Bashir, 1983 [4] as follows :

$L_1 = 0.2, L_2 = 0.3, L_3 = 0.5$

1.2 Point-Matching-Technique

The Point-Matching-Technique (PMT) is a solution to Maxwell's two main field equations:

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \tag{4}$$

$$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t} \tag{5}$$

where they are exactly satisfied in the boundaries , exterior and interior to the dust/sand particle and the incident plane wave is expanded in a complex Fourier series similar to the treatment used by Morrison et al 1974 [6] , this is simply because of the symmetry of the semi-axis of the ellipsoid at the origin , and this made it easier for solving the unknown coefficients , as opposed to expanding the incident wave in spherical harmonics Oguchi 1974 [5] . The forward scattered amplitudes ($\theta = 0$) for the dual polarizations are given by the following expressions:

$$S_v(0) = \frac{1}{E_v} \sum_{m=-\infty}^{\infty} \sum_{n \supseteq |m|}^{\infty} (-j)^{n-1} x[a_{mn}^v \frac{m}{\sin\alpha} P_n^{|m|}(\cos\alpha) + b_{mn}^v \frac{dP_n^{|m|}(\cos\alpha)}{d\alpha}] \tag{6}$$

and

$$S_h(0) = \frac{1}{E_h} \sum_{m=-\infty}^{\infty} \sum_{n \supseteq |m|}^{\infty} (-j)^{n+2} x[a_{mn}^h \frac{dP_n^{|m|}(\cos\alpha)}{d\alpha} + b_{mn}^h n \frac{m}{\sin\alpha} P_n^{|m|}(\cos\alpha)] \tag{7}$$

where $P_n^{|m|}$ are associated Legendre functions of the first kind, and α is the angle of incidence between the direction of propagation and the axis of symmetry. For more details of equations (6) and (7) see Bashir's Ph.D thesis 1984 [1] and also Bashir 1980 (ITU Journal) [2].

In terrestrial line-of sight radio-rely links, this angle is 90 degrees, with smaller values (70, 50, etc) for satellite links, deep space ships, and so on.

CONCLUSION

A computer programme is available and is used to check the Rayleigh theory results against the fields full solution through the PMT . The results for vertical polarization are depicted in table 1 . The agreement between the two methods is very good ; the relative error is less than 1% for the real part and less than 0.4% for the imaginary part .

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Table 1: A comparison (check) between Rayleigh theory for ellipsoids and the point-matching-technique

Description of method	$S_v(0)$ scattering amplitudes for vertical polarization	(c) radius of miner axis (mm)	c/a ratio of miner to major axis
Rayleigh theory of ellipsoids	$3.251\text{x}10^{-8}$-j$1.119\text{x}10^{-6}$ $2.485\text{x}10^{-7}$-j$8.549\text{x}10^{-6}$ $1.900\text{x}10^{-6}$-j$6.529\text{x}10^{-5}$	0.05 0.10 0.20	0.8 0.8 0.8
P.M.T	$3.282\text{x}10^{-8}$-j$1.123\text{x}10^{-6}$ $2.510\text{x}10^{-7}$-j$8.588\text{x}10^{-6}$ $1.924\text{x}10^{-6}$-j$6.565\text{x}10^{-5}$	0.05 0.10 0.20	0.8 0.8 0.8