Using Statistical Approach to Derive Priority Weights in the Analytic Hierarchy Process with Application to Public Investment Allocation

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Abstract

The Analytic Hierarchy Process (AHP) is one of the most used multi-criteria techniques that can be used for decision-making problems; it depends on judgments of decision maker/expert about different alternatives using some criteria to make the best decision about these alternatives. However, the eigenvector method used for deriving the priorities in the AHP has been criticized due to its deterministic mechanism where the error in judgments is not taking into consideration. The main aims of this paper are first to discuss using of statistical method to obtain priority weights instead of the eigenvector method, and then apply this statistical method to define the priorities of regional allocation of public investment in Egypt. Seven criteria have been used in this study to judge seven alternatives (regions), where the local priority weights as well as their standard errors are calculated for the criteria and the alternatives with regard to the criteria, and then the final (global) priority weights are calculated for the alternatives. Final results of the application case revealed that regions of upper Egypt had the highest priority for investment allocation compared to the other regions.

Keywords: Decision Making; Statistical Method; Analytic Hierarchy Process; Public Investments.

استخدام منهجية إحصائية لحساب أوزان الأولوية ضمن أسلوب التحليل الهرمي مع التطبيق على عملية تخصيص الاستثمارات العامة

ملخص

يعد أسلوب التحليل الهرمي أحد أكثر الأساليب متعددة المعايير استخدامًا في حل المشاكل المتعلقة بعملية اتخاذ القرار؛ حيث يعتمد هذا الأسلوب على تقييمات صانعي القرار/ الخبراء حول مجموعة من

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البدائل المختلفة المراد اتخاذ قرار بشأنها استنادًا إلى عدد من المعايير المختلفة لاتخاذ أفضل قرار بشأن أولويات هذه البدائل. وبالرغم من ذلك، فقد تم تعرض هذا الأسلوب لبعض الانتقادات نتيجة استخدام طريقة رياضية وهي طريقة المتجهات الذاتية لاشتقاق الأوزان ذات الأولوية حيث تتسم هذه الطريقة بأنها حتمية أو يقينية حيث لا يتم أخذ الخطأ في الأحكام في الاعتبار. لذا، تتمثل الأهداف الرئيسية لهذه الورقة أولاً في مناقشة استخدام أسلوب إحصائي للحصول على أوزان الأولوية بدلاً من طريقة المتجهات الذاتية، ثم تطبيق هذه الطريقة الإحصائية لتحديد أولويات التخصيص الإقليمي للاستثمار العام في مصر. لتطبيق هذه المنهجية تم استخدام سبعة معايير في هذه الدراسة لتقييم سبعة بدائل (مناطق جغرافية)، حيث تم حساب أوزان الأولوية المحلية بالإضافة إلى الأخطاء القياسية لكل من المعايير وكذلك البدائل وفقا لكل معيار من المعايير المستخدمة في التقييم، ثم في النهاية تم حساب أوزان الأولوية النهائية لكل بديل من البدائل لتحديد أولوياتها من حيث تخصيص الاستثمارات العامة في مصر. أظهرت النتائج النهائية أن مناطق صعيد مصر لها الأولوية القصوى لتخصيص الاستثمار مقارنة بالمناطق الأخرى.

الكلمات المفتاحية: اتخاذ القرار–أسلوب إحصائي–التحليل الهرمي–الاستثمارات العامة

1. Introduction

The Analytic Hierarchy Process (AHP) is a structured technique for organizing and analyzing complex decision problems. It was developed by Thomas L. Saaty in 1970s as a qualitative technique that processes the subjective and personal preferences in making a decision. AHP is working by decomposing the complex problem into sub-problems and then the solutions of all these sub-problems are aggregated into a conclusion about the alternatives of interest. Through this process, a decision maker/expert makes simple pairwise comparison judgments (organized in a matrix form) for the alternatives themselves and for the criteria which are used to judge the alternatives, and then overall priorities (priority weights) are developed for ranking these alternatives. By AHP, decision maker can find the best decision that fits his objective **(Forman and Gass, 2001)**.

The traditional method to calculate the values of the priority weights from the pairwise comparison judgments, as introduced by Saaty, is the Eigen Vector Method (EVM) by taking the eigenvector corresponding to the largest eigenvalue of the pairwise comparison matrices. Although it is the most popular method to derive priority weight vector, EVM has been criticized regarding its calculation difficulties and the lack of practical statistical theory behind it; it is a deterministic (non-stochastic) method where the errors in judgments are not taken into consideration. Therefore, several researchers attempted to present different stochastic methods to derive priority weights in AHP.

In this paper, a statistical method to derive priority weights in the AHP is discussed, and then this statistical method is applied to derive priorities of public investments allocation to different regions in Egypt. The paper is divided into 6 sections; the next section discusses the AHP and how it works, section 3 discusses the statistical method as stochastic approach to derive local priority weights, section 4 presents a detailed description for the applied model, section 5 presents main results of deriving local and global priority weights for the decision problem of regional allocation of public investments in Egypt, and finally the last section presents conclusion of the study.

2. Analytic Hierarchy Process

The goal of the AHP techniques is to find a unique vector of priority weights $(w = w_1, w_2, \dots, w_{n_A})$ of n_A decision alternatives with respect to a given n_c criteria, and then the calculated weights are used to rank these alternatives to choose the best one.

To obtain final priority weights for the alternatives using AHP, one should follow four primary steps:

a) Structuring the Problem in a Hierarchical Form

Users of the AHP first decompose their decision problem into its constituent parts and then presenting them in a hierarchy form, each of which can be analyzed independently. This step is also called decision modeling as it consists of building a hierarchy to analyze the decision **(Mu and Pereyra-Rojas, 2017).** This structure comprises the main goal at the top level, criteria at the intermediate level, and finally the lowest level contains the options or alternatives. The criteria can be further broken down into sub criteria, sub-sub criteria, and so on, in as many levels as required **(Ramanujam and Saaty, 1981; Saaty and Shih, 2009)**.

b) Pairwise Comparison Judgments Using Ratio Scale

Once the hierarchy is built, the decision maker or the expert is asked to evaluate the various elements of the problem; each element is evaluated with regard to the other one. The set of all evaluations (judgments) are represented in a square matrix called Pairwise Comparison Matrix (PCM), in which the set of elements are put in the rows and columns of the matrix where the set of elements is compared with itself. A number of comparison judgment matrices are constructed; one matrix for the criteria in terms of their importance to achieve the overall goal, and a number of matrices are constructed for the alternatives with regard to each criterion (one matrix for each criterion).

Saaty introduced a ratio scale measurement, which is ranging from one (Equal Importance) to 9 (Extreme Importance) as presented in the following table, to be used by the policy maker or the expert to indicate how more important or dominant one element is over the other one for each pair of elements (criteria or alternatives) **(Saaty, 1990 and 2003)**.

Table (1): The Fundamental Ratio Scale Used in AHP

Intensity of Importance	Definition
1	Equal Importance
2	Weak or slight
3	Moderate Importance
4	Moderate plus
5	Strong Importance
6	Strong plus
7	Very Strong Importance
8	Very, very strong
9	Extreme Importance

Source**:** (Saaty, 1977, 1990 and 1994).

c) Deriving Local Priority Weights (Prioritization Procedures)

In this step, vector of priority weights $\mathbf{w}$ is derived from each pairwise comparison matrix separately. Firstly, priority weights vector is derived for the n_C criteria, and then priority weights vectors are derived for the n_A alternatives with regard to each criterion. These calculated priority weights are called local priority weights.

To derive local priority weights from the pairwise comparison matrices, Saaty proposed using the Eigenvectors Method (EVM) by solving the system of homogeneous linear equations $\mathbf{Aw} = \lambda_{max}\mathbf{w}$, where $\mathbf{A}$ is the pairwise comparison matrix, $\mathbf{w}$ is the vector of local priority weights, and λ_{max} is the maximal eigenvalue of $\mathbf{A}$.

According to Saaty, a necessary and sufficient condition for using the pairwise comparison matrix for deriving local priority weights, is to be acceptably consistent. To measure the consistency level for each matrix, Consistency Ratio (CR) measure was proposed which is a ratio of the Consistency Index (CI) to the Random Index (RI), as follows **(Saaty, 1990 and 1994)**:

$$CR = \frac{CI}{RI} \qquad (1)$$

$$CI = \frac{\lambda_{Max} - n}{n - 1} \qquad (2)$$

where n is the order of the pairwise comparison matrix and RI is calculated as the average value of consistency indices (CIs) of a 500 randomly generated reciprocal matrices from the scale 1 to 9. Based on this, Saaty stated that if the CR is less than or equal to 10%, the matrix can be considered to have an acceptable consistency level, with a perfect consistency level if $\mathrm{CR} = 0$.

d) Synthesizing Through the Structure to Get the Final (Global) Weights

In the final step, the overall priority weights (global priority weights) are obtained. This is done for each alternative by multiplying the weight obtained from the alternatives' comparisons according to a specific criterion by the corresponding weight of this criterion, and then added over all the criteria. The alternative with the highest priority weight value should be taken as the best one.

This synthesizing process can be illustrated as follows **(El-Hefnawy and Mohamed, 2014)**:

$$w^g(A_j) = \sum_{i=1}^{n}\left[w(c_i) \times w(A_j)|_{c_i}\right],\ j = 1, \dots, n_A \qquad (3)$$

Where: $w^g(A_j)$ is the global priority weight value of alternative A_j, $w(c_i)$ is the local priority weight value of the i^{th} criterion (c_i), and $w(A_j)|_{c_i}$ is the local priority weight value of alternative A_j with regard to the i^{th} criterion, $i = 1, \dots, n_C$, $j = 1, \dots, n_A$. This process is repeated to all the alternatives to get the global priority weight vector $\mathbf{w} = (w_1, \dots, w_{n_A})$ for the alternatives.

3. A Statistical Method for Deriving Local Priority Weights

Using statistical method, as an alternative to the EVM, for deriving local priority weights in the AHP decision making problems was first suggested by some researchers. For example: **(De Jong, 1984)** and **(Crawford and Williams 1985)** investigated the use of logarithmic least square method approach to derive the priority weights; **(Alho, et al., 1996)** used the variance component decomposition approach to analysis the uncertainties in the estimated priorities; and the work of **(Laininen and Hamalainen, 2003)** focused on investigating the effect of outliers by elaborating robust regression technique. They showed that regression method turns out to be as good to produce extremely close priorities to Saaty's EVM. However, if the comparison judgments are severely inconsistent, the results may differ considerably (**Alho, et al., 1996).** In this paper, the statistical approach will be discussed in details and then applied to a decision-making problem in Egypt.

To show how the statistical method works to derive priority weights in AHP, let's take a pairwise comparison matrix **A** of size $n \times n$ with the entities a_{ij}, where:

$$a_{ij} = \frac{w_i}{w_j} e_{ij}, \quad i \neq j = 1, \ldots, n \qquad (4)$$

where, a_{ij} is the ijth element of the pairwise comparison matrix **A**, $\frac{w_i}{w_j}$ is the actual ratio of the priority weight, and e_{ij} is a multiplicative error, n is the order of matrix **A**. This error term represents the measurement errors and inconsistencies of the pairwise judgments expressed by decision makers/experts, and:

$$e_{ij} > 0, \text{ if } e_{ij} = 1 \rightarrow a_{ij} = \frac{w_i}{w_j}, \; i \neq j = 1, \ldots, n. \qquad (5)$$

Converting Equ. (4) to a linear regression form by taking the logarithm for two sides, yields the following:

$$\ln a_{ij} = \ln w_i - \ln w_j + \ln e_{ij} \qquad (6)$$

Or equivalently,

$$y_{ij} = \beta_i - \beta_j + \varepsilon_{ij}, \quad i \neq j = 1, \ldots, n \qquad (7)$$

where $y_{ij} = \ln(a_{ij}),\; \beta_i = \ln(w_i), and\; \varepsilon_{ij} = \ln(e_{ij})$

In matrix notation, the linear model takes the following form:

$$\mathbf{y} = \mathbf{X}\beta + \varepsilon \qquad (8)$$

Therefore, linear regression technique can be employed to estimate model parameters using OLS method, by minimizing the sum of square of the errors $\varepsilon_{ij}, i \neq j = 1, \dots, n$.

Where ε is the $(m \times 1)$ vector of the disturbances assumed to have a normal distribution $N(0, \sigma^2)$, **y** is $(m \times 1)$ vector of dependent variable where $y_{ij} = \ln(a_{ij}), i \neq j = 1, \dots, n$, β is $n \times 1$ vector of parameters, and **X** is $m \times n$ matrix of categorical explanatory variables. n is the number of compared elements ($n = n_C$ for criteria and $n = n_A$ for alternatives), and m is the number of observations (pairwise judgments) used for regression analysis where $m = n(n-1)$.

The **X** matrix of explanatory variables take values $(-1,0,1)$ according to the values of i and j. For example, for $i = 1$ and $j = 2$ $(y_{12}) \Rightarrow x_{q1} = 1, x_{q2} = -1, x_{qp} = 0, p \neq 1,2$, and for $i = 3$ and $j = 2$ $(y_{32}) \Rightarrow x_{q3} = 1, x_{q2} = -1, x_{qp} = 0, p \neq 2,3$, and $q = 1, \dots, n(n-1)$. To show how **X** matrix is constructed by a numerical example; let $n = 3$, then the regression equations and matrix **X** take the following form:

$$\begin{aligned} y_{12} &= \beta_1 - \beta_2 + \varepsilon_{12} \\ y_{13} &= \beta_1 - \beta_3 + \varepsilon_{13} \\ y_{23} &= \beta_2 - \beta_3 + \varepsilon_{23} \\ y_{21} &= \beta_2 - \beta_1 + \varepsilon_{21} \\ y_{31} &= \beta_3 - \beta_1 + \varepsilon_{31} \\ y_{32} &= \beta_3 - \beta_2 + \varepsilon_{32} \end{aligned} \qquad \mathbf{X}_{(6\times 3)} = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix}$$

In this case, only the upper and lower diagonal elements of the pairwise comparison matrix **A** are taken into account, while the diagonal elements are excluded. In fact, the solution of model (8) needs an additional constraint for the parameters because the model is over-parameterized. Therefore, a practical constraint of $\beta_n = 0$ $(or\, w_n = 1)$ is needed. Then the last column of the matrix X is deleted **(Laininen and Hamalainen, 2003)**.

It is important to note that using regression approach to derive local priority weights is under certainty, where the pairwise judgments are considered as random variables rather than constants. Moreover, regression analysis here can be considered as a tool of parameter estimation based on the relation between the judgments and actual weights rather than a causal model. The focus here is not on the factor affecting the judgments of the policy maker/expert or the magnitude of the coefficients but rather on developing an estimation tool for the actual priority weights. Thus the parameter estimates should not be interpreted as the effect of the explanatory variables on the judgment values.

According to the linear regression technique, the OLS estimations for the regression parameters are given by $\hat{\beta} = (\acute{X}X)^{-1}\acute{X}Y$. Therefore, the results of the regression model give the estimated parameters $\hat{\beta}_i$ for $i = 1,\dots,n-1$, $\hat{\beta}_n = 0$ and, referring to Equ. (7), the estimated priority weights $\hat{w}_i$ are obtained by the following formula:

$$\hat{w}_i = \exp(\hat{\beta}_i), \quad i = 1, \dots, n . \qquad (9)$$

Then, the estimated weights $\hat{w}_i$ are normalized by dividing by the sum to give the normalized priority weights $\hat{w}_i^*$ for the criteria and alternatives as follows:

$$\hat{w}_i^* = \frac{\hat{w}_i}{\sum_{i=1}^{n} \hat{w}_i} = \frac{\exp(\hat{\beta}_i)}{\sum_{i=1}^{n} \exp(\hat{\beta}_i)}, \ i = 1, \dots, n \qquad (10)$$

Compared to EVM, using regression technique for deriving priority weight in AHP has some advantages, the most important, is that it gives both point and interval estimation for the parameters, so one can calculate point and interval estimation for the un-normalized priority weight value w_i .

Regression approach provides us with some statistical properties that can be very useful for the analysis. From normality assumption of the regression model, $\hat{\beta}_i$ is normally distributed with expected value β_i and variance δ_i^2, So, $\hat{w}_i = \exp(\hat{\beta}_i)$ is log-normally distributed with expected value $E(\hat{w}_i) = \exp[\beta_i + (\delta_i^2 / 2)]$ and variance $\mathrm{Var}(\hat{w}_i) = \left[\exp(2\beta_i + \delta_i^2)\right]\left[\exp(\delta_i^2) - 1\right]$.

By inserting the estimates values $\hat{\beta}_i$ and $\hat{\delta}_i^2$ obtained from regression analysis results into the expected value and variance of $\hat{w}_i$, the variance $\mathrm{Var}(\hat{w}_i)$ and standard error $Se(\hat{w}_i)$ of the estimated un-normalized priority weights can be calculated.

To calculate the standard error for the normalized priority weights, one have to calculate the covariance matrix for these normalized weights **($\mathbf{Cov}(\hat{w}_i^*, \hat{w}_j^*)$). (Laininen and Hamalainen, 2003)** proved that the:

$$\mathbf{Cov}(\hat{w}_i^*, \hat{w}_j^*) = \left[\frac{\partial w_i^*}{\partial w_i}\right]_{n\times(n-1)} \times \left[\mathrm{Cov}(\hat{w}_i, \hat{w}_j)\right]_{(n-1)\times(n-1)} \left[\frac{\partial w_i^*}{\partial w_i}\right]^T_{(n-1)\times n} \qquad (11)$$

Where,

$$\left[\frac{\partial w_i^*}{\partial w_i}\right]_{n\times(n-1)} = -w_n^* \begin{bmatrix} w_1^*-1 & w_1^* & \dots & w_1^* \\ w_2^* & w_2^*-1 & \dots & w_2^* \\ \dots & \dots & \dots & \dots \\ w_{n-1}^* & w_{n-1}^* & \dots & w_n^*-1 \\ w_n^* & w_n^* & \dots & w_n^* \end{bmatrix} \tag{12}$$

$$[\mathbf{Cov}(\widehat{w}_i, \widehat{w}_j]_{(n-1)\times(n-1)} = \left[\exp\left(\beta_i + \beta_j + \frac{(\delta_i^2+\delta_j^2)}{2}\right)\right] \times \left[\exp\left(\mathrm{cov}(\widehat{\beta}_i, \widehat{\beta}_j)\right) - 1\right] \tag{13}$$

for $i \neq j$.

Noting that, as $\hat{\beta}_n = 0$, a constant, then $E[\exp(\widehat{\beta}_n) = 1]$, $Var[\exp(\widehat{\beta}_n) = 0]$, and $Cov[\exp(\widehat{\beta}_i), \exp(\widehat{\beta}_n) = 0]$, for $i = 1, \dots, n-1$.

By inserting the estimates $\widehat{\beta}_i$, $\widehat{\delta}_i^2$ and $\mathrm{cov}(\widehat{\beta}_i, \widehat{\beta}_j)$ into (13), and inserting the estimates of the normalized weights $\widehat{w}_i^*$ into (12), $Cov(\widehat{w}_i^*, \widehat{w}_j^*)$ in (11) can be calculated and then the standard errors $Se(\widehat{w}_i^*)$.

The standard error for the estimated weight can be used for statistical inferences about the priority weights such as hypothesis testing and confidence interval. However, the probability distribution for the normalized priority weights is needed in this case. It is worth mentioning that, even though regression result gives intervals for the model parameters (β_i) and not the un-normalized weights (w_i), the simplest way to derive interval priority weights for (w_i) is to take the anti-log transformation of the two bounds of the confidence intervals of β_i obtained by the regression results **(Olsson, 2005)**.

4. Application for Regional-Level Investment Allocation

Localization of development and reducing the geographic gaps among regions (governorates) is one of the main objectives of the sustainable development agenda for the government of Egypt. Egypt is a large country with high regional diversity in terms of socioeconomic characteristics and developmental gaps; it includes 27 governorates, split across 7 economic regions. The government of Egypt is exerting great efforts to reduce these gabs and develop the under-developed areas by pumping more public investments to improve the infrastructure, creating job opportunities and enhancing the quality of life in the most needed areas.

However, defining geographic priorities for allocating local investments among regions and governorates in a fair and objective manner is a challenge facing the government of Egypt, due to the high disparities among these regions and governorates as well as the different and sometimes contradicted criteria which can be employed to define the priorities of allocating fund.

As a part of the government's efforts to improve the efficiency of public investment management and enhance equality and fairness among governorates as well as promote transparency of public investment allocation; an inter-ministerial committee, chaired by Ministry of Planning and Economic Development (MPED), and representatives from Ministry of Local Development, Ministry of Finance, National Investment Bank and some experts in local finance & development, has adopted a formula-based process for allocating local investment funds among the 27 governorates, which is announced as Governorate-level Investment Allocation Formula (GIAF) **(MPED, 2021).**

According to this formula, four factors have been selected to reflect the state of development of each governorate and capture any development gaps;

1. The past three years average of investment allocated to the governorate,
2. The share of the governorate's population in total population; modeled to make it the formula concave with respect to population share,
3. The incident of poverty in the governorate,
4. Factor capturing whether the governorate is a frontier one.

However, there is no available information about the detailed methodology which is used for the GIAF or even its results. To provide a practical multi-criteria decision-making tool than can help decision maker improve the efficiency of public investment management, this paper is trying to employ the AHP technique for prioritizing the allocation of governorate-level local investment in Egypt using the statistical approach by applying regression model as discussed earlier in this paper.

4.1 Model Description

As mentioned above, the first step in applying AHP is building the hierarchy structure form for the problem. The following figure presents the hierarchy structure form for our AHP model in this study.

Egypt is divided into seven economic regions; each has number of governorates, as follows:

1. **Cairo Region,** which includes the governorates of Cairo, Giza, and Qalyubia.
2. **Alexandria Region,** which includes the governorates of Alexandria, Beheira, and Matrouh.

3. **Delta Region,** which includes the governorates of Damietta, Dakahlia, Kafr El-Sheikh, Gharbia, and Menoufia.
4. **Canal Region,** which includes the governorates of Port Said, Suez, Sharqia, Ismailia, North Sinai, and South Sinai.
5. **North Upper Egypt Region,** which includes the governorates of Beni Suef, Fayoum, and Minya.
6. **South Upper Egypt Region,** which includes the governorates of Sohag, Qena, Luxor, Aswan, and the Red Sea.
7. **Central Upper Egypt Region,** which includes the governorates of Assiut and New Valley.

Due to the large number of governorates (27 governorates) in Egypt, it will be more applicable and informative to use the regions (groups of governorates) instead of individual governorates as alternatives.

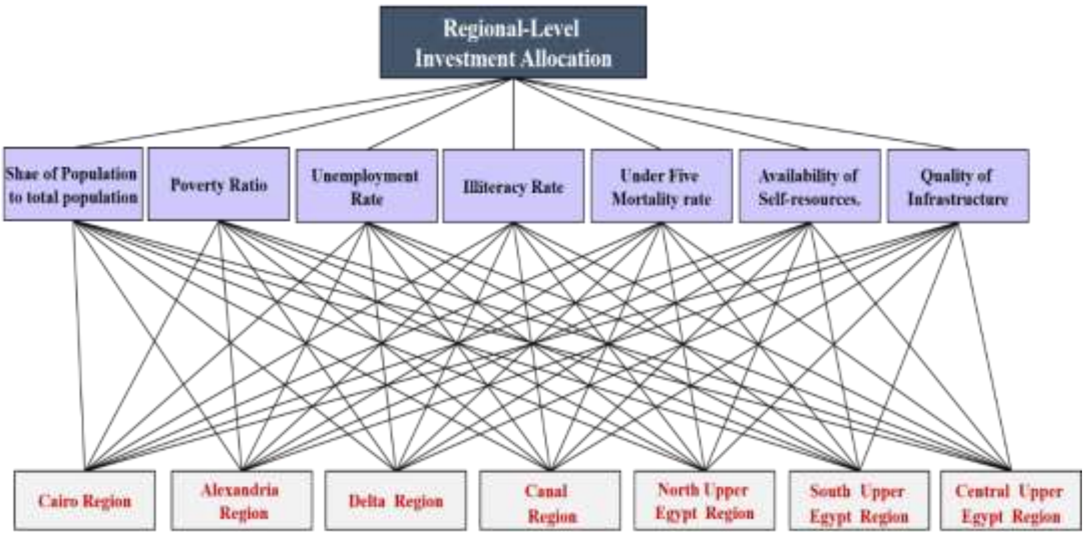

Figure (1): AHP Hierarchy Structure for Regional-Level Investment Allocation.

Source: Prepared by the Authors.

The above hierarchy structure comprises three main levels:

- The top level represents the main goal of the model, which is the prioritization of public investment allocation to the different regions in Egypt.
- The intermediate level represents the criteria identified to be used to judge the alternatives.
- The lowest level contains the alternatives (regions) that need to be prioritized in terms of public investment allocation.

Choosing criteria is the most important step in the application of AHP model; criteria represent the indexes which enable alternatives to be compared from a specific point of view. It has been indicated that the selection of criteria is of prime importance in the resolution of a given problem. Seven criteria have been chosen to be used in the model, these criteria are:

- Share of population to total population,
  The alternative (region) with higher population share has higher priority.
- Poverty ratio,
  The alternative (region) with higher poverty rate has higher priority.
- Unemployment rate,
  The alternative (region) with higher unemployment rate has higher priority.
- Illiteracy rate,
  The alternative (region) with higher illiteracy rate has higher priority.
- Under five mortality rate,
  The alternative (region) with higher under five mortality rate has higher priority.
- Availability of self-resources,
  The alternative (region) with less self-resources has higher priority.
- Quality of infrastructure.
  The alternative (region) with poor infrastructure has higher priority.

These criteria are chosen to measure the need or demand for public investments to improve people's quality of live in different developmental dimensions (demography, income, education, health, infrastructure). On the other hand, they represent very important factors for the government to measure, assess and improve the efficiency of public investment management as well as reduce the developmental gaps among the governorates. It is important to mention that these criteria have been discussed with experts and policy makers in the Egyptian Ministry of Planning and Economic Development.

4.2 Data Sources

According to the above hierarchal form, eight pairwise comparison matrices are constructed; one matrix of order (7×7) for the criteria, and seven matrices of order (7×7) to be used to get the judgments of the 7 alternatives with regard to each criterion. For the first five criteria, data were obtained from the Central Agency for Public Mobilization and Statistics (CAPMAS) which is the national statistical agency in Egypt, then the pairwise comparison judgments for the alternatives with regard to these five criteria were calculated. For the pairwise comparison judgments of criteria and the pairwise comparison judgments for the alternatives with regard to the last two criteria, data were collected by and expert (and former policy maker) at the Egyptian Ministry of Planning and Economic Development.

Table (2): Pairwise Comparison Matrix for the Criteria

↓ (i) (j) →	Share of Population	Poverty	Unemployment Rate	Illiteracy	Under Five Mortality	Self-resources	Quality of Infrastructure
Share of Population	1	1	1	2	2	1	1
Poverty	1	1	1	2	2	1	1
Unemployment Rate	1	1	1	2	2	1	1
Illiteracy	1/2	½	½	1	1	1/2	¼
Under Five Mortality	1/2	½	½	1	1	1	½
Self-resources	1	1	1	2	1	1	½
Quality of Infrastructure	1	1	1	4	2	2	1

Table (3): Pairwise Comparison Matrix for the Alternatives with regard to the Availability of Self-Resources.

↓ (i) (j) →	Share of Population	Poverty	Unemployment Rate	Illiteracy	Under Five Mortality	Self-resources	Quality of Infrastructure
Share of Population	1	1	½	1	1/3	1/2	½
Poverty	1	1	1	1	1/2	1	1/2
Unemployment Rate	2	1	1	1	1/2	1	1
Illiteracy	1	1	1	1	1/3	½	1/2
Under Five Mortality	3	2	2	3	1	2	1
Self-resources	2	1	1	2	1/2	1	1
Quality of Infrastructure	2	2	1	2	1	1	1

Table (4): Pairwise Comparison Matrix for the Alternatives with regard to the Quality of Infrastructure

↓ (i) (j) →	Share of Population	Poverty	Unemployment Rate	Illiteracy	Under Five Mortality	Self-resources	Quality of Infrastructure
Share of Population	1	1	½	1	1/2	1/3	1/3
Poverty	1	1	½	1	1/2	1/3	1/3
Unemployment Rate	2	2	1	2	1	1	1
Illiteracy	1	1	½	1	1/2	½	1/2
Under Five Mortality	2	2	1	2	1	1	1
Self-resources	3	3	1	2	1	1	1
Quality of Infrastructure	3	3	1	2	1	1	1

The following figure (Figures 2 to 6) show the data of the first five criteria for each alternative.

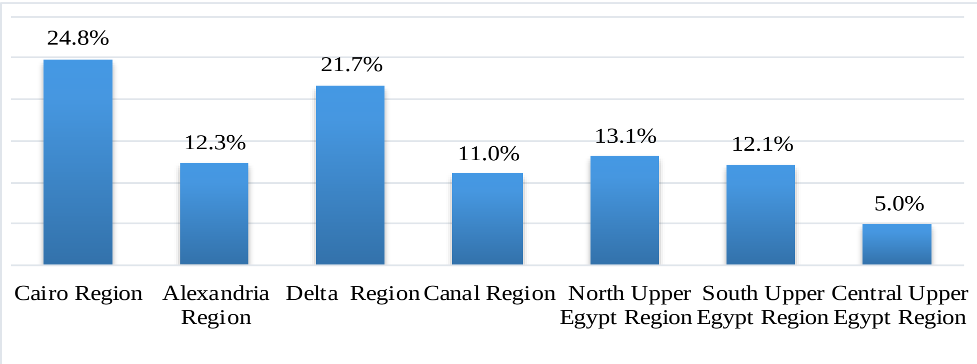

Figure (2): Share of Population by Region

Source: CAPMAS (2021).

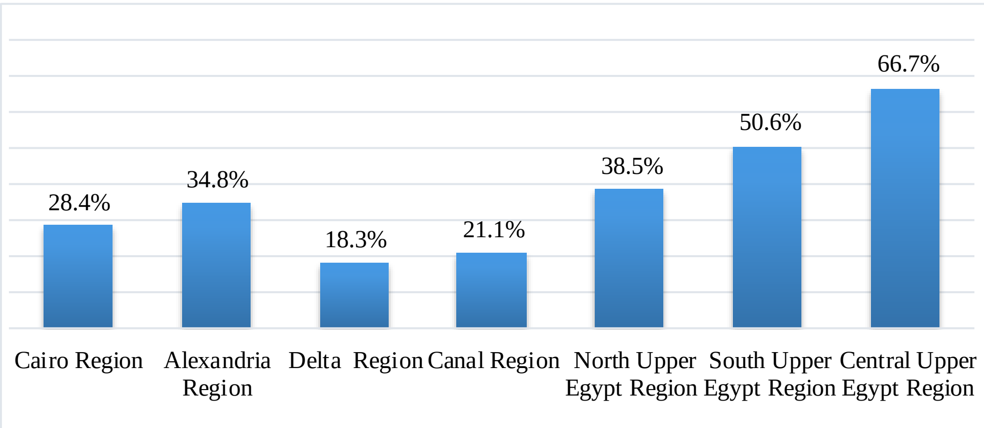

Figure (3): Poverty Ratio by Region

Source: CAPMAS (2019).

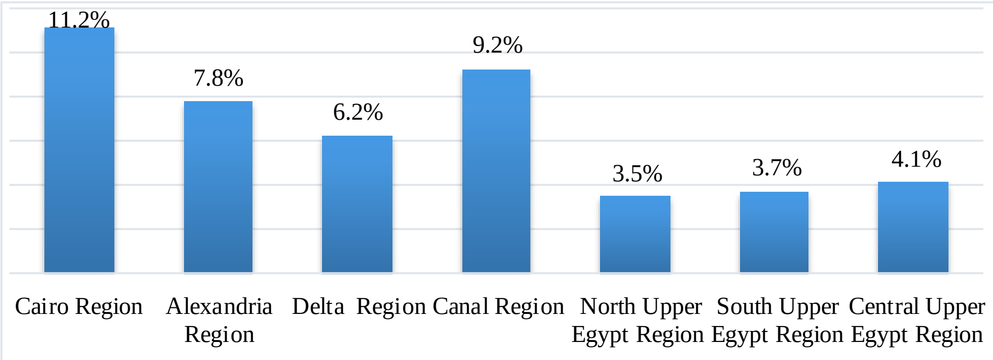

Figure (4): Unemployment Rate by Region

Source: CAPMAS (2022).

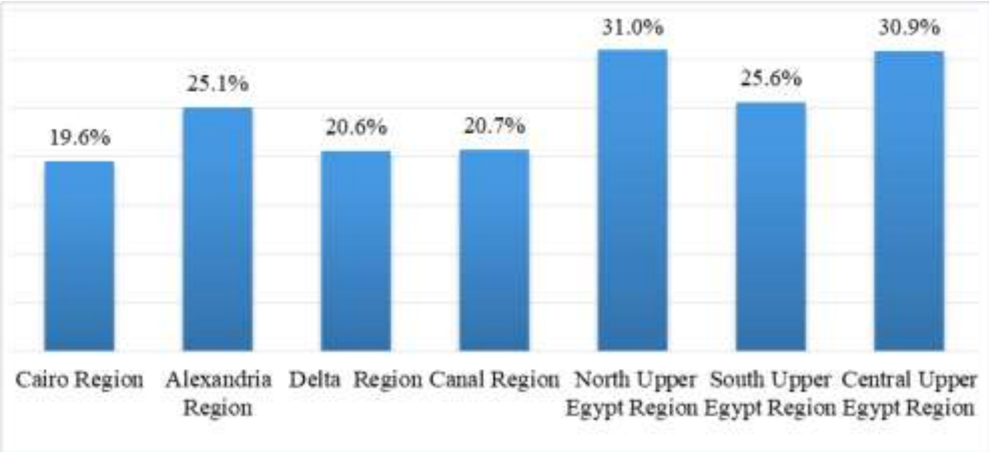

Figure (5): Illiteracy Rate by Region

Source: EAEA (2021).

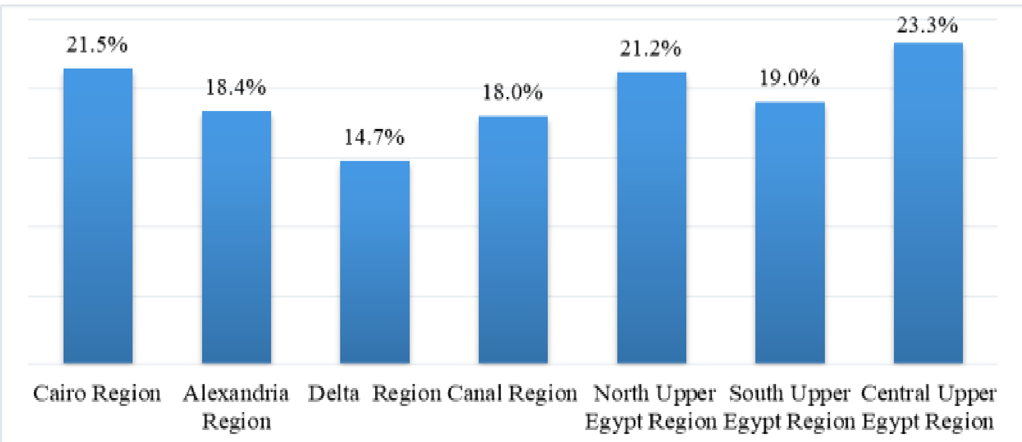

Figure (6): Under Five-Mortality Rate by Region

Source: CAPMAS (2021).

4.3 Check for Consistency for the Comparison Matrices

To measure consistency of the collected judgments, Saaty' CR was used for the eight pairwise comparison matrices. Results showed that all matrices are consistent, where the CR is less than 10%. The following table presents the CR for the all matrices.

Table (5): Consistency Ratio for the Constructed Pairwise Comparison Matrices

	CR
Criteria	1.4%
The Alternatives with regard to	
Share of Population	1.1%
Poverty	1.5%
Unemployment Rate	0.9%
Illiteracy	1.5%
Under Five Mortality	1.1%
Self-resources	1.5%
Quality of Infrastructure	0.5%

Source: Calculated by the Author

5. Results and Discussion

5.1 Local Priority Weights

The priority weight values (for the criteria and alternatives with regard to each criteria) are calculated using the following regression model as described above:

$$\boldsymbol{y} = \boldsymbol{X}\beta + \varepsilon$$

where the priority weight values are calculated using formula (9), and then the local priority weight (normalized weights) for the criteria and alternatives are calculated using formula (10). In addition, the standard errors of the local priority weights are calculated using formula (11). Table 3 presents the local priority weights for the criteria and their standard errors using regression.

Table (6): Priority Weights for the Criteria

Criteria	local priority weights	Standard Errors
Share of Population	0.165	0.042
Poverty	0.165	0.042
Unemployment Rate	0.165	0.042
Illiteracy	0.075	0.021
Under Five Mortality	0.091	0.026
Self-resources	0.136	0.036
Quality of Infrastructure	0.202	0.022

Source: Calculated by the Author

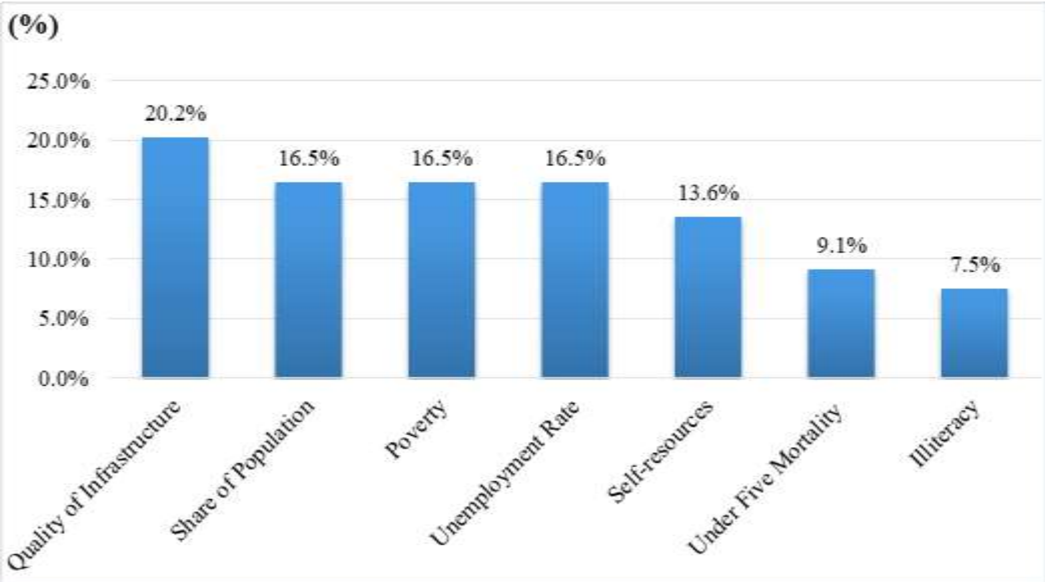

Figure (7): Criteria Ranking According to their Local Priority Weights

Source: Calculated by the Author.

The results show that quality of infrastructure has the most important impact for judging the alternatives (20.2%), followed by share of population, poverty and unemployment with the same weight (16.5%), while self-resources come in the fifth rank by 13.6%. Under five mortality and illiteracy are ranked in the last two positions by 9.1% and 7.5% respectively.

By the same way, the following table shows the derived local priority weights and their standard errors for the alternatives with regard to each criterion. The results show that Cairo and Delta regions have the highest priorities regarding share of population criterion compared to other regions, while Central Upper Egypt region and South Upper Egypt regions have the highest priorities regarding poverty criterion. The unemployment gives highest priorities to Cairo and Canal regions, while North Upper Egypt region and Central Upper Egypt region are prioritized the highest according to illiteracy. For under five mortality rate, results refer to that Central Upper Egypt region and Cairo region have the highest priorities compared to the other regions.

On the other hand, it is clear from the results that upper Egypt regions are the best choices according to both self-resources criterion (North Upper Egypt region and Central Upper Egypt region) and quality of infrastructure (South Upper Egypt region and Central Upper Egypt region) in terms of priorities of public investment allocatio

Table. (7): Priority Weights for Alternatives with regard to Each Criterion

	Share of Population		Poverty		Unemployment Rate		Illiteracy		Under Five Mortality		Self-resources		Quality of Infrastructure	
	Weight	Stand. Error	Weight	Stand. Error	Weight	Stand. Error	Weight	Stand. Error	Weight	Stand. Error	Weight	Stand. Error	Weight	Stand. Error
Cairo Region	0.259	0.032	0.110	0.0003	0.244	0.0007	0.113	0.0003	0.158	0.0005	0.085	0.025	0.079	0.017
Alexandria Region	0.123	0.018	0.135	0.0004	0.170	0.0005	0.145	0.0004	0.135	0.0004	0.110	0.031	0.079	0.017
Delta Region	0.207	0.028	0.071	0.0002	0.136	0.0004	0.119	0.0003	0.108	0.0004	0.134	0.037	0.177	0.035
Canal Region	0.110	0.016	0.082	0.0003	0.202	0.0006	0.120	0.0003	0.132	0.0004	0.094	0.027	0.089	0.019
North Upper Egypt Region	0.131	0.019	0.149	0.0004	0.076	0.0003	0.179	0.0005	0.156	0.0005	0.247	0.059	0.177	0.035
South Upper Egypt Region	0.121	0.018	0.196	0.0005	0.082	0.0003	0.148	0.0004	0.140	0.0004	0.148	0.040	0.199	0.038
Central Upper Egypt Region	0.050	0.003	0.258	0.0003	0.090	0.0001	0.178	0.0002	0.171	0.0002	0.181	0.021	0.199	0.016

Source: Calculated by the Author.

1.1 Global Priority Weights

After calculating local priority weights for criteria and alternatives with regard to each criterion, one comes to the final step in AHP analysis which is to obtain the overall priority weights or the global priority weights for the alternatives. The global weights are calculated for each alternative by multiplying the normalized local priority weight value for the alternative according to a specific criterion by the corresponding normalized local priority weight value of this criterion, and then they are added over all the criteria, as it is shown in formula (3).

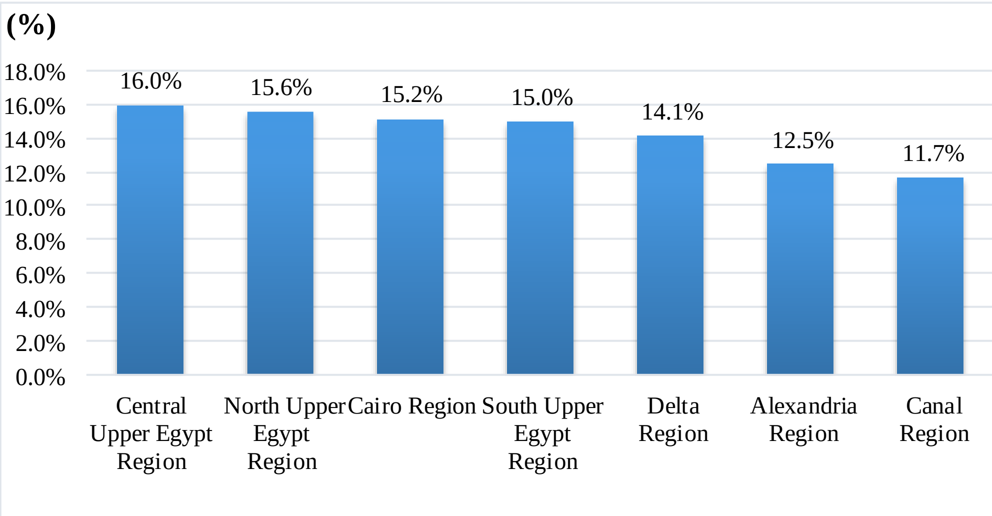

Figure (8): The Final Priority Weights for the Alternatives

Source: Calculated by the Author.

These results show that North and Central Upper Egypt regions have the highest priorities for public investment allocation (16% and 15.6% respectively); they are socially deprived with high poverty rates, high illiteracy rates and high under-five mortality rates. Moreover, they have low self-resources and poor infrastructure compared to the other regions. Cairo region came in the third rank with priority 15.2%, followed by South Upper Egypt region with 15.2%, followed by Delta region with 14.1%, while Alexandria region and Canal regions are ranked last with 12.5% and 11.7% respectively.

2. Main Outcomes and Conclusions

In this paper, a statistical technique using regression model has been discussed to derive priority weights in the analytic hierarchy process instead of the traditional eigenvector method which was suggested by Saaty. Using regression approach for deriving priority weight in AHP has some advantages; it is a stochastic method where the errors in judgments are taken into consideration for deriving priorities. Moreover, one can derive the standard errors for the priorities, so that some statistical inferences can be made about these priorities.

Then, this statistical approach has been applied to derive priorities of public investments allocation to regions in Egypt. Seven regions have been assessed using seven evaluation criteria. The alternatives included Cairo Region, Alexandria Region, Delta Region, Canal Region, North Upper Egypt Region, South Upper Egypt Region, and Central Upper Egypt Region. The selected evaluation criteria included share of population to total population, poverty ratio, unemployment rate, illiteracy rate, under five mortality rate, under five mortality rate, availability of self-resources, and quality of infrastructure. An expert has participated in the study to make the judgments for the criteria and some of the alternatives with regard to the criteria, where the local priority weights as well as their standard errors are calculated for the criteria and the alternatives with regard to the criteria, then the final (global) priority weights are calculated for the alternatives.

Results showed that quality of infrastructure had the most important impact for judging the alternatives (20.2%), while three criteria had the same priorities for judging the alternatives; they are share of population to the total population, poverty and unemployment (16.5%), the availability of self-resources came in the fourth rank by 13.6%, under five mortality and illiteracy was ranked in the last two positions by 9.1% and 7.5% respectively.

Final results revealed that regions of upper Egypt had the highest priority for investment allocation compared to the other regions, where North and Central Upper Egypt regions had the highest priorities (16% and 15.6% respectively). Cairo region, which includes the capital governorate (Cairo), came in the third rank with priority 15.2% due to the high share of population and unemployment rate, followed by South Upper Egypt region with 15.2%, followed by Delta region with 14.1%, while Alexandria region and Canal region are ranked last with 12.5% and 11.7% respectively.

Despite the significant efforts which were exerted by the governments of Egypt over the past years to achieve comprehensive and sustainable development, developmental gaps between different regions still persist. On the contrary, these gaps are widening in light of the crises that the Egyptian economy has faced as a result of local and international conditions, which have clearly reflected on the living conditions of citizens, especially in deprived or under-developed areas. The government of Egypt places great hopes on the localization of sustainable development to reduce the regional developmental gabs.

No doubt that the most important and effective factor to reduce the developmental gabs between regions is the accurate targeting of deprived and needed regions based on quantitative and scientific methods that take into account various criteria and characteristics that cover all aspects of sustainable development. A complementary work may be done in the future to do such multi-criteria decision-making analysis on disaggregated level (governorates) to overcome the disparities among the regions. This, of course, requires the availability of recent and accurate data covering all regions, and more detailed data about governorates, cities and even villages within the governorates may be needed.

On the other hand, reducing the developmental gabs between regions in Egypt requires effective participation of the private sector and civil society has become inevitable in light of the limited public resources, and the government's inability to face all challenges on its own.

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