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Performance investigation and control parameters choice for sliding mode control of coupled tanks system

Performance investigation and control parameters choice for sliding mode control of coupled tanks system

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Abstract. In several food processing and chemical industries, liquid is pumped and kept in interrelating coupled tanks. However, automatic regulation of the liquid level and flow control between these tanks is a challenging problem because of the complexity and high non linearity of such system. This paper deals with the liquid level control of two horizontal coupled tanks system. A comprehensive comparative study is made for most popular sliding mode control (SMC) algorithms found in literature, namely Proportional-Derivative Sliding Mode Control (PD-SMC), Proportional-Integral-Derivative SMC (PID-SMC), Fractional Order SMC and finally dynamic SMC. Special emphasis is put on the effect of the sensor noise on the controller performance. Simulated experiments including robustness to variation in plant parameters and step input disturbances are made. Control algorithms parameters are selected to optimize designed performance indices by using MATLAB optimization toolbox. Simulation results reveal that dynamic SMC is superior to other control algorithms in the presence of sensor noise and has a significant reduction in the actuator chattering phenomenon.

1. Introduction
The liquid level and flowing control between two or more tanks are essential applications in several industrial areas such as food processing, water distribution and paper industry. However, a coupled tank system is an interesting control problem due to complexity and high non linearity.An exact model besides a proper control strategy are very important so as to maintain the required tank level under uncertainties and disturbances. Numerous control tuning approaches of PID controller were used because of its structure simplicity and parameters adjustment easiness. [1], [2]. Yet, whenever requiring a good tracking with high precision of liquid level, traditional tuning methods of PID controllers failed providing an appropriate performance to level control. Moreover, most of the PID controllers were designed using lower order linearized process models [3], [4], [5] and [6]. Because of the model order reduction, this leads to additional parametric uncertainty.Nevertheless, not many PID controllers are designed on the basis of higher order model[7], [8]. However, the robustness problem still unsolved, if not considering the robustness measure in designing process.
Conventional PID was compared with fuzzy control[9]. For the nonlinear quadruple tank system, nonlinear Artificial Neural Network (ANN) control was designed [10]. Also, for coupled tank system, backstepping controllers and Adaptive controllers are implemented [11], [12] and [13]. Observer-based back stepping controller waspresented[14]. For regulating the coupled tank systemlevel, a robust decentralized PI controller was introduced[15]. A fractional order PI (FOPI) controller for coupled tank system was introduced[16] and [17]. A comparison between cascaded FOPD, FOPI and integer order PD (IOPD), integer order PI (IOPI)controllers for coupled tank system was presented [18].For

nonlinear processes control, an evaluation for multi-model FO controller is made, and was compared with multi-model IO controller [19]. Fuzzy FOPID controller for controlling thelevel of liquid of spherical coupled tanks system was introduced[20].FOID controller for theproblem of level control in two input two output liquid level system was proposed [21].

SMC has several attractive features as better disturbance rejection capabilities, good transient performance, and faster response. Mainly, in face of the uncertainties, SMC laws are inherently more robust[22], [23]. Variable structure systems (VSS) with sliding modes design and systems analysiswerestudied[24], [25]. A fuzzy SMCwith nonlinear sliding surface waspresentedfor coupled tanks system [26], such thata fuzzy logic controller was usedfor improving the chattering. An adaptive fuzzy SMCwas proposed for coupled tanks system [27]. Aneuro-fuzzy-SMCusing a nonlinear sliding surface was developed for coupled tanks system in order to smooth the switching signal[28]. A PI-SMC and a backstepping PI-SMC for a quadruple tank system are presented [29].

A static SMC design for a coupled tank system was introduced[30]. To reduce the chattering problem,two different dynamic SMC algorithms werealso proposed [30]. In a quadruple tank system,feedback linearization combinedwith SMC algorithmwas applied [31]. A second order SMC algorithm was presented[32], [33].For four coupled tank system using higher order SMC, a robust observer based controller was proposed [34].A Feed-forward adaptive second order SMC is presented to reduce the disturbance effect on the coupled tank system [35]. A chattering free SMCwas proposed to realize level position control of a coupled tank system[36]. A nonlinear SMC with varying boundary layer was presented to improve the tracking performance of a coupled tank system against various uncertainties [37]. Fuzzy FOSMC is proposed to find a chattering-free robust method for a coupled tank system [38]. SMC for quadrupletank withmulti input multi output process with time delay compensation was introduced [39].

In many industrial circumstances, sensor signal is interfered with noise. This may be caused by many reasons such as, long wires connections, close proximity to other electrical equipment, etc... In such cases the ability of the closed-loop controller to manage these noises is crucial. In many researches that are found in literature, SMC is reported as a robust faster response control that can deal with nonlinear uncertain systems. However, sensor noise effect on the SMC chattering phenomenon has not been investigated.

In this paper, a comprehensive comparative study for four control algorithms; namely PD-SMC, PID-SMC, fractional-SMC and dynamic-SMC are conducted. The comparison in the performances of the four algorithms was done by using Simulink-MATLAB. The novelty in this paper is the investigation done about the effect of the sensor noise on the performance of these famous SMC algorithms, Specially on the chattering phenomenon. The paper is organized as follows. The coupled tank system model presented in Section 2. The Controller design is proposed in Section 3. The simulation results are discussed and shown in Section 4. Finally, the conclusion in section 5.

2. The coupled tank system model

Considering the horizontal coupled tanks shown in figure 1, the system governing equations can be expressed as,

$$C\frac{dh_1}{dt} = q - q_1,\ C\frac{dh_2}{dt} = q_1 - q_2 \tag{1}$$

where, q(t) is the inlet flow rate in (m^3/sec), $q_1(t)$ is the flow rate from tank 1 to tank 2 in (m^3/sec), $h_1(t)$ is the level in the first tank in (m), $h_2(t)$ is the level in the second tank in (m), $q_2(t)$ is the flow rate out of tank 2 in (m^3/sec) and C is the cross-section area of tank 1 and tank 2 in (m^2).

The intermediate and outlet flow rates q_1 and q_2 can be expressed as,

$$q_1 = c_{12}\sqrt{2g(h_1 - h_2)} \quad \text{for } h_1 > h_2,\ q_2 = c_2\sqrt{2gh_2} \quad \text{for } h_2 > 0 \tag{2}$$

where c_{12} is the coupling orifice area in (m^2), c_2 is the outlet orifice area in (m^2), and g is the gravitational constant in (m^2/sec). In the coupled tanks system, the fluid flow rate (q) into tank 1 will be positive because the water pumped only into the tank by the pump. Therefore, the inflow rate will be

$$q \geq 0 \tag{3}$$

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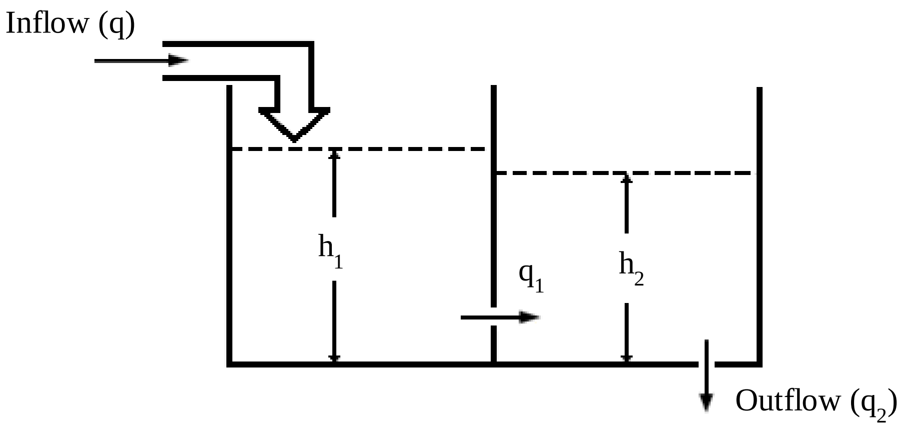


Figure 1. Horizontal coupled tanks problem.

$$\dot{h}_1 = -\frac{c_{12}}{A}\sqrt{2g|h_1-h_2|}sgn(h_1-h_2)+\frac{q}{A}$$
$$\dot{h}_2 = \frac{c_{12}}{A}\sqrt{2g|h_1-h_2|}sgn(h_1-h_2)-\frac{c_2}{A}\sqrt{2gh_2} \quad (4)$$

At equilibrium, for constant liquid level desired point, the derivatives must be zero, i.e.,
$\dot{h}_1 = \dot{h}_2 = 0,$ (5)
therefore,
$$-\frac{c_{12}}{C}\sqrt{2g\,|h_1-h_2|}\,sgn(h_1-h_2)+\frac{Q}{C}=0, \frac{c_{12}}{C}\sqrt{2g\,|h_1-h_2|}\,sgn(h_1-h_2)-\frac{c_2}{C}\sqrt{2gh_2}=0 \quad (6)$$
where Q is the equilibrium inlet flow rate. From equation (6) and to justify the restriction in equation (3) on the inlet flow rate,$sgn(h_1-h_2)$ should be positive.
Considering $z_1 = h_2 > 0,\ z_2 = h_1 - h_2 > 0,\ \mathbf{Z} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$, u = q(t)
Also let $a_1 = \frac{c_2\sqrt{2g}}{C}$ &$a_2 = \frac{c_{12}\sqrt{2g}}{C}$
thus, the dynamic model can be written as,
$\dot{z_1} = -a_1\sqrt{z_1} + a_2\sqrt{z_2}\ , \dot{z_2} = a_1\sqrt{z_1} - 2a_2\sqrt{z_2} + \frac{u}{C}, y = z_1$ (7)
where$z_1 = h_2$ is considered the system output.
The control algorithm objective is regulating the output $y(t) = z_1(t) = h_2(t)$ to a required value h_{2d}. As can be seen, the coupled tanks system dynamic model is complex and highly nonlinear. Therefore, a transformation will be defined so that the dynamic model in equation (7) can be converted to a form that will facilitates the control design.
Considering the state vector $\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and defining the transformation $\boldsymbol{x} = T(\boldsymbol{z})$
such that,
$x_1 = z_1, x_2 = -a_1\sqrt{z_1} + a_2\sqrt{z_2}$ (8)
The inverse transformation $\boldsymbol{z} = T^{-1}(\boldsymbol{x})$ is such
$z_1 = x_1, z_2 = \left(\frac{a_1\sqrt{x_1}+x_2}{a_2}\right)^2$ (9)
The dynamic model in equation (7) then can be written as,
$\dot{x_1} = x_2, \dot{x_2} = \frac{a_1a_2}{2}\left(\frac{\sqrt{z_1}}{\sqrt{z_2}} - \frac{\sqrt{z_2}}{\sqrt{z_1}}\right) + \frac{a_1^2}{2} - a_2^2 + \frac{a_2}{2c}\frac{1}{\sqrt{z_2}}u$ (10)
where the values of $z_1$and$z_2$are function of x_1 and x_2 as given by equation (9).
Thus, the system dynamic model can be written as,

$\dot{x}_1 = x_2,\ \dot{x}_2 = f + \phi u, y = x_1$ (11)

where,

$f = \frac{a_1 a_2}{2}\left(\frac{\sqrt{z_1}}{\sqrt{z_2}} - \frac{\sqrt{z_2}}{\sqrt{z_1}}\right) + \frac{a_1^2}{2} - a_2^2,\ \phi = \frac{a_2}{2c}\frac{1}{\sqrt{z_2}}$ (12)

3. Controller design

3.1. Classical sliding mode controller (SMC)

SMC is a robust nonlinear control algorithm which is basing on Lyapunov method, where an n^{th} order uncertain and nonlinear system is converted to a 1^{st} order system. SMC has many features such as: the relative simplicity in the design, robustness to process dynamic characteristics and external disturbances. It is known that robust solution is provided by SMC to the control problem; hence it allows coping with the changes in plant without noteworthy deterioration of the performance. Even that SMC is fundamentally a discontinuous control result, however, it is clear that the control to be designed must drive the trajectory to the switching surface and once it has been reached is preserved on this surface. The problem of the control is making the system respond to track a desired and specified trajectory using a SMC. The procedure will be introduced by means of the next Single input single output system [22]:

$x^{(n)} = f(x) + \phi(x)u,$ (13)

such that x is a state vector, u is a scalar input, n is the order of the system and f and ϕ are nonlinear functions of the states. The objective of the control is that the state must follow a required vector state trajectory $x_d(t)$. Considering the surface S(t) in the state-variable form:

$$S(x,t) = \left(\frac{d}{dt} + \lambda\right)^{n-1} \tilde{x} \quad (14)$$

such that $\tilde{x} = x - x_d$ represents the error, while λ is a positive constant. Considering Lyapunov function as $V = \frac{1}{2}S^2$,selection of the control law should be in done in a way that reduces the distance to this surfacein (14) along with all state trajectories of the system (sliding condition). In other words,

$$\frac{1}{2}\frac{d}{dt}S^2 \le -\eta|S|, \quad (15)$$

such that η is a positive constant. The whole state trajectories were obviously enhanced and are getting nearer to the sliding surface in finite time, and for all future times, staying on the surface. As soon as settling the system behavior on the surface, it's said that the sliding mode ($\dot{S} = 0$) took place. When the surface is touched by the initial state for the first time, that time will be as follows

$$t_{reach} \le \frac{S(t=0)}{\eta} \quad (16)$$

Therefore, under SMC, an ideal motion consists of two phases; reaching and sliding, such that the motion is limited to the sliding surface during these phases [22]. The designing method of the sliding mode consists of two steps. The first step involving the design of a switching function $S = 0$, such that the design specifications have been satisfied by the sliding motion. The second step was the concerning of the description of the sliding mode by the control law which will be selected, so that its existence and reaching conditions are satisfied[40]. The next subsections explain the designing of switching surfaces for a coupled tank using four alternative methods. Firstly, presenting sliding surfaces based on linear compensation networks PD and PID. Then, the fractional form of one of these networks $PI^{\lambda}D^{\mu}$ is used so as to obtain the sliding surfaces. Finally, a dynamic sliding mode controller is presented.

3.1.1. Sliding surface through a PD controller. Assume H is the required system output constant value; $h_2(t)$, then defining the error

$e = (h_2 - H) = (z_1 - H)$ (17)

Therefore, obtaining PD-SMC as

$S = K_d\dot{e} + K_p e = K_d\dot{z}_1 + K_p(z_1 - H)$(18)
Differentiate both sides of equation (18) with respect to time, yields
$\dot{S} = K_d\ddot{z}_1 + K_p\dot{z}_1$(19)
$\dot{S} = K_d\left((-a_1\dot{z}_1/2\sqrt{z_1}) + (a_2\dot{z}_2/2\sqrt{z_2})\right) + K_p\dot{z}_1$(20)
Substituting from (7) into (20) leads to
$\dot{S} = K_d\left[\frac{a_1^2-2a_2^2}{2} + \left(\frac{a_1a_2}{2}\right)\left(\frac{\sqrt{z_1}}{\sqrt{z_2}} - \frac{\sqrt{z_2}}{\sqrt{z_1}}\right) + \left(\frac{a_2}{2C\sqrt{z_2}}\right)u\right] + K_p(-a_1\sqrt{z_1} + a_2\sqrt{z_2})$(21)
To satisfy Lyapunov stability criterion, we introduce
$\dot{S} = -K\,\mathrm{sgn}(S)$(22)

$$\text{where}\,\mathrm{sgn}(S) = \begin{cases} +1, & \text{if } S > 0, \\ 0, & \text{if } S = 0, \\ -1, & \text{if } S < 0, \end{cases}$$

substituting in (21)we get
$-K\,\mathrm{sgn}(S) = K_d\left[\frac{a_1^2-2a_2^2}{2} + \left(\frac{a_1a_2}{2}\right)\left(\frac{\sqrt{z_1}}{\sqrt{z_2}} - \frac{\sqrt{z_2}}{\sqrt{z_1}}\right) + \left(\frac{a_2}{2C\sqrt{z_2}}\right)u\right] + K_p(-a_1\sqrt{z_1} + a_2\sqrt{z_2})$(23)
solving for u we get
$u = \left(\frac{2C\sqrt{z_2}}{a_2}\right)\left[-\frac{a_1^2}{2} + a_2^2 - \left(\frac{a_1a_2}{2}\right)\left(\frac{\sqrt{z_1}}{\sqrt{z_2}} - \frac{\sqrt{z_2}}{\sqrt{z_1}}\right)\right] - (K_pK_d)(-a_1\sqrt{z_1} + a_2\sqrt{z_2}) - (K/K_d)\,\mathrm{sgn}(S)$(24)
Using the control law in the above equation, the system states are now reaching to the hyperplane. The error vectors are forced to tend to zero asymptotically. At the same time, the required value H will be asymptotically converged by the height $h_2(t)$. Therefore, the SMC assures the output asymptotic convergence to the desired value.
The switching function control scheme suffers from the chattering problem. In the control signal, the sign function is included causing the chattering. This means that the control could switch its value at any instant and with nearly zero-time delay. The chattering could be reduced with the usage of a saturation function [22], equation (25).

$$K\,\mathrm{sat}(S/\Delta) = \begin{cases} +1, & \text{if } (S/\Delta) \geq 1, \\ S/\Delta, & \text{if} -1 < S < 1, \\ -1, & \text{if } (S/\Delta) \leq 1, \end{cases} \quad (25)$$

such that K is a switch gain with positive value, Δ is the boundary layer width and rewriting equation (24)using the saturation function we get
$u = \left(\frac{2C\sqrt{z_2}}{a_2}\right)\left[-\frac{a_1^2}{2} + a_2^2 - \left(\frac{a_1a_2}{2}\right)\left(\frac{\sqrt{z_1}}{\sqrt{z_2}} - \frac{\sqrt{z_2}}{\sqrt{z_1}}\right) - \left(\frac{K_p}{K_d}\right)(-a_1\sqrt{z_1} + a_2\sqrt{z_2}) - \left(\frac{K}{K_d}\right)\mathrm{sat}\left(\frac{S}{\Delta}\right)\right.$(26)

3.1.2 Sliding surface through a PID controller. The PID controller helps getting system output to the desired value, in a short time, with minimal overshoot, and with little error [41]. It is also the most adopted controller in the industry due to the its low cost and simplicity.
A PID-SMC is designed as follows:
$S = K_p e + K_I\int e\,dt + K_d\dot{e} = K_p(z_1 - H) + K_I\int(z_1 - H)dt + K_d\dot{z}_1$(27)
Differentiate equation (27) with respect to time, results in
$\dot{S} = K_p\dot{z}_1 + K_I(z_1 - H) + K_d\ddot{z}_1$(28)
Substituting (7)in (28),yields
$\dot{S} = K_p(-a_1\sqrt{z_1} + a_2\sqrt{z_2}) + K_I(z_1 - H) + K_d\left[\frac{a_1^2-2a_2^2}{2} + \left(\frac{a_1a_2}{2}\right)\left(\frac{\sqrt{z_1}}{\sqrt{z_2}} - \frac{\sqrt{z_2}}{\sqrt{z_1}}\right) + \left(\frac{a_2}{2C\sqrt{z_2}}\right)u\right]$ (29)
Same as the procedure in (24), when $\dot{S}$is forced to zero, the control signal yields

$$u = \left(\frac{2C\sqrt{z_2}}{a_2}\right)\left[-\left(\frac{K_P}{K_d}\right)(-a_1\sqrt{z_1} + a_2\sqrt{z_2}) - \left(\frac{K_I}{K_d}\right)(z_1 - H) - \frac{a_1^2}{2} + a_2^2 - \left(\frac{a_1a_2}{2}\right)\left(\frac{\sqrt{z_1}}{\sqrt{z_2}} - \frac{\sqrt{z_2}}{\sqrt{z_1}}\right)\right] - \left(\frac{K}{K_d}\right)\mathrm{sat}\left(\frac{S}{\Delta}\right) \quad (30)$$

3.1.3 Sliding surfaces through fractional $PI^{\lambda}D^{\mu}$ controller. As many systems in real life are described using fractional order differential equations, the notion of fractional order controllers or processes were the subject of considerable researches [42], [43], [44], [45] and [46]. A popularization of the PID

controller is presented, namely; the $PI^{\lambda}D^{\mu}$ controller, such that λ and μ are the indices [46]. Obviously, the PID controller is a special case of FOPID. The expansion of fractional order of derivative and integral terms could provide much more flexibility in PID controller design.
A $PI^{\lambda}D^{\mu}$-SMC is as follow:

$$\begin{aligned} S &= K_p e + K_I D^{-\lambda} e + K_d D^{\mu} e \\ &= K_p(z_1 - H) + K_I D^{-\lambda}(z_1 - H) + K_d D^{\mu}(z_1 - H) \end{aligned} \quad (31)$$

such that D is a general fundamental operator which denotes the fractional order differentiator and K_p, K_I, K_d, λ and μ are the design parameters which have to be determined. Taking the time derivative for both side of equation (31), results in

$$\begin{aligned} \dot{S} &= K_p \dot{z}_1 + K_I D^{1-\lambda}(z_1 - H) + K_d D^{1+\mu}(z_1 - H) \\ &= K_p \dot{z}_1 + K_I D^{1-\lambda}(z_1 - H) + K_d D^{\mu-1} \ddot{z}_1 \end{aligned} \quad (32)$$

Using the defined variable in (7) resulting in

$$\dot{S} = K_p \dot{z}_1 + K_I D^{1-\lambda}(z_1 - H) + K_d D^{\mu-1}\left(\left(-\frac{c_1 \dot{z_1}}{2\sqrt{z_1}}\right) + \left(\frac{c_2 \dot{z_2}}{2\sqrt{z_2}}\right)\right) \quad (33)$$

$$\dot{S} = K_p(-a_1\sqrt{z_1} + a_2\sqrt{z_2}) + K_I D^{1-\lambda}(z_1 - H) + K_d D^{\mu-1}\left[\frac{a_1^2 - 2a_2^2}{2} + \left(\frac{a_1 a_2}{2}\right)\left(\frac{\sqrt{z_1}}{\sqrt{z_2}} - \frac{\sqrt{z_2}}{\sqrt{z_1}}\right) + \left(\frac{a_2}{2C\sqrt{z_2}}\right) u\right] \quad (34)$$

Forcing $\dot{S} = 0$, provides:

$$K_d D^{\mu-1}\left(\left(\frac{a_2}{2C\sqrt{z_2}}\right) u\right) = -K_p(-a_1\sqrt{z_1} + a_2\sqrt{z_2}) - K_I D^{1-\lambda}(z_1 - H) - K_d D^{\mu-1}\left[\frac{a_1^2 - 2a_2^2}{2} + \left(\frac{a_1 a_2}{2}\right)\left(\frac{\sqrt{z_1}}{\sqrt{z_2}} - \frac{\sqrt{z_2}}{\sqrt{z_1}}\right)\right] \quad (35)$$

Then obtaining an equivalent control signal as

$$u = \left(\frac{2C\sqrt{z_2}}{a_2}\right)\left[\left(\frac{-K_p}{K_d}\right) D^{1-\mu}(-a_1\sqrt{z_1} + a_2\sqrt{z_2}) + \left(-\frac{K_I}{K_d}\right) D^{2-\lambda-\mu}(z_1 - H) - \frac{a_1^2 - 2a_2^2}{2} - \left(\frac{a_1 a_2}{2}\right)\left(\frac{\sqrt{z_1}}{\sqrt{z_2}} - \frac{\sqrt{z_2}}{\sqrt{z_1}}\right)\right] - \left(\frac{D^{1-\mu} K sat\left(\frac{S}{\Delta}\right)}{K_d}\right) \quad (36)$$

3.1.4 Dynamic sliding mode controller. To decrease the chattering because of the static-SMC (PD-SMC), a dynamic SMC is introduced [30]. Let α_1, α_2 be positive scalers, an input-dependent sliding surface S is defined as,

$$S = \ddot{x}_1 + \alpha_1 \dot{x}_1 + \alpha_2(z_1 - H) \quad (37)$$

where x_1 is defined by the alternate system model defined by equations (11) and (12). Substituting (11) and (12) into (37) we get

$$S = \left(\frac{a_1 a_2}{2}\right)\left(\frac{\sqrt{z_1}}{\sqrt{z_2}} - \frac{\sqrt{z_2}}{\sqrt{z_1}}\right) + \frac{a_1^2}{2} - a_2^2 + \frac{a_2}{2C}\frac{1}{\sqrt{z_2}} u + \alpha_1(-a_1\sqrt{z_1} + a_2\sqrt{z_2}) + \alpha_2(z_1 - H) \quad (38)$$

Taking the derivative of equation (37) with respect to time using equations (8), (10) we obtain

$$\begin{aligned} \dot{S} &= \dddot{x}_1 + \alpha_1 \ddot{x}_1 + \alpha_2 \dot{x}_1 \\ &= f_1 + \alpha_1\left(\frac{a_1 a_2}{2}\left(\frac{\sqrt{z_1}}{\sqrt{z_2}} - \frac{\sqrt{z_2}}{\sqrt{z_1}}\right) + \frac{a_1^2}{2} - a_2^2 + \frac{a_2}{2C}\frac{1}{\sqrt{z_2}} u\right) + \alpha_2(-a_1\sqrt{z_1} + a_2\sqrt{z_2}) + \frac{a_2}{2C}\frac{1}{\sqrt{z_2}}\dot{u} - \frac{a_2}{4C}\frac{1}{\sqrt{z_2^3}}\left(a_1\sqrt{z_1} - 2a_2\sqrt{z_2} + \frac{1}{C} u\right) u \end{aligned} \quad (39)$$

Satisfying Lyapunov stability criterion by equating (39) to, $\dot{S} = -K sat(S/\Delta)$, from which we get,

$$\dot{u} = \frac{-2C\sqrt{z_2}}{a_2}\left[f_1 + \alpha_1\left(\frac{a_1a_2}{2}\left(\frac{\sqrt{z_1}}{\sqrt{z_2}} - \frac{\sqrt{z_2}}{\sqrt{z_1}}\right) + \frac{a_1^2}{2} - a_2^2 + \frac{a_2}{2C}\frac{1}{\sqrt{z_2}}u\right) + \alpha_2(-a_1\sqrt{z_1} + a_2\sqrt{z_2}) + K\mathrm{sat}\left(\frac{S}{\Delta}\right)\right] + \frac{1}{2z_2}\left(a_1\sqrt{z_1} - 2a_2\sqrt{z_2} + \frac{1}{C}u\right)u \quad (40)$$

where

$$f_1 = \frac{-a_1a_2(z_1+z_2)}{4\sqrt{(z_1z_2)^3}} * \left(a_1{z_1}^{\frac{3}{2}} - 2a_2z_1\sqrt{z_2} + \frac{1}{C}z_1u + a_1z_2\sqrt{z_1} - a_2{z_2}^{\frac{3}{2}}\right) \quad (41)$$

The state trajectories accompanying the unforced discontinuous dynamics equation (22) show a finite time reachability to zero from any given initial condition providing that the constant K is positive. Because S is driven to zero in finite time, the output $y = z_1 = h_2$ is governed after such a finite time, by the second-order dynamics $\ddot{y} + \alpha_1\dot{y} + \alpha_2(y - H) = 0$. Hence, the required value H will be asymptotically converged by the output y(t), because α_1 and α_2 are positive scalars. Then, the dynamic SMC given by equations (40) and (41) assures the asymptotic convergence of the output y(t) = $z_1(t) = h_2(t)$ to its required value H.

4. Simulation Results and Discussion

Each control algorithm has between four to seven tuning parameters. Selecting controller parameters by using the trial and error method that was used in many past researches to achieve certain response requirements is not practical. In this research, MATLAB optimization toolbox is used in order to find the optimal values of the controller parameters to achieve the specified response. The optimization toolbox provides functions for finding parameters that minimize or maximize objectives while satisfying certain constraints. The toolbox can perform design optimization tasks, including parameter estimation and parameter tuning. The optimization method used in order to find the optimal parameters values is the gradient descent method. Figure 2 describes the constraints used for the optimization in all proposed algorithms. The rise time (90%) is 35 seconds, the settling time (5%) is 100 seconds, overshoot is 8% and undershoot is 5%.

To ensure finding the best controller parameters, two objective functions are defined. The first is the integral of square of the error between the liquid level, h_2,and the desired height, h_{2d}. This function will be called error index.

$$Q_1 = \int (h_2 - h_{2d})^2 . dt \quad (42)$$

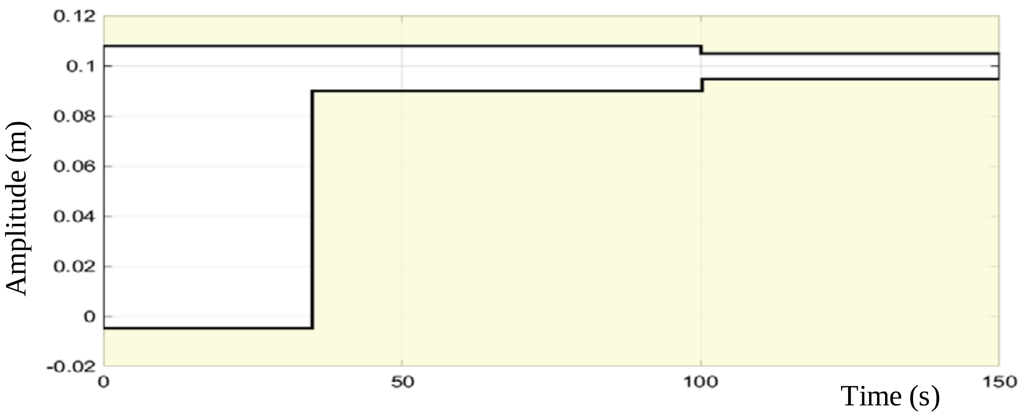


Figure 2. Constraints used for the optimization.

The second objective function is the integral of the square of time derivative of the control signal, u. This function will be called chattering index. It is described as

$$Q_2 = \int \left(\frac{du}{dt}\right)^2 . dt \quad (43)$$

This function decreases as the chattering rate of the control signal decreases. Minimizing both functions while keeping the signal inside the constraints described above ensure the best performance of the controller. The optimization algorithm searches for the best combinations of the controller parameters to satisfy this target. The range for searching for each controller parameter is described in table 1:

Table 1. Searching range for each controller parameter.

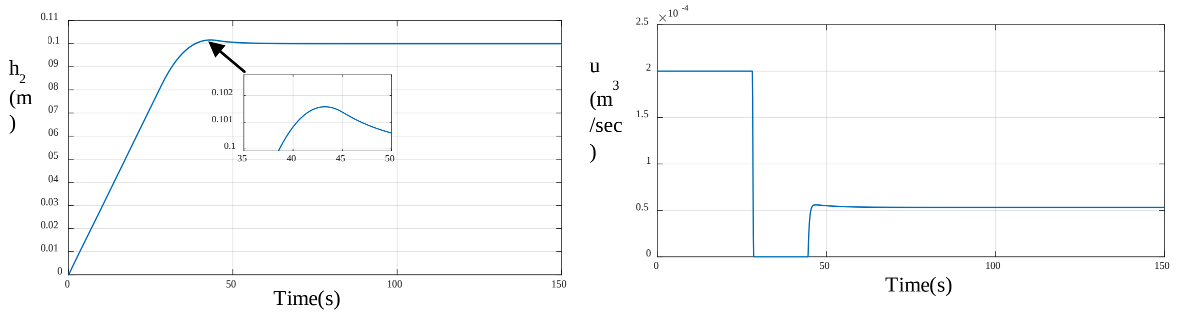
Coefficient	Range	Coefficient	Range
K	$0\rightarrow\infty$	μ	$0\rightarrow 1$
Δ	$0\rightarrow\infty$	Λ	$0\rightarrow 1$
K_p	$0\rightarrow\infty$	α_1	$0\rightarrow\infty$
K_I	$0\rightarrow\infty$	α_2	$0\rightarrow\infty$
K_d	$0\rightarrow\infty$		

4.1 Step response

The coupled tank system is subjected to a step input of 0.1 meters. Table 2 shows the tuning parameters found by the optimization algorithm for each controller to fit the response into the constraints shown in figure 4 and to satisfy both objective functions given by (42) and (43).Figures 3 to 6 show the response of the four control algorithms to this disturbance. As can be seen from the figures, PD-SMC, PID-SMC and Dynamic-SMC reached the required level with zero steady state error, while the fractional PID-SMC has a steady state error of 0.0002 m. PD-SMC has the minimum overshoot of 1.6% while the PID-SMC has the maximum overshoot of 4.9%. The four algorithms almost do not have any chattering as can be seen from figure 3-b to figure 6-b. A comparison between the rise time, percentage overshoot, steady state error, error index, and the settling time (5% criterion) for the four controllers is shown in table 3. As can be seen from the table that PD-SMC has the minimum error index.

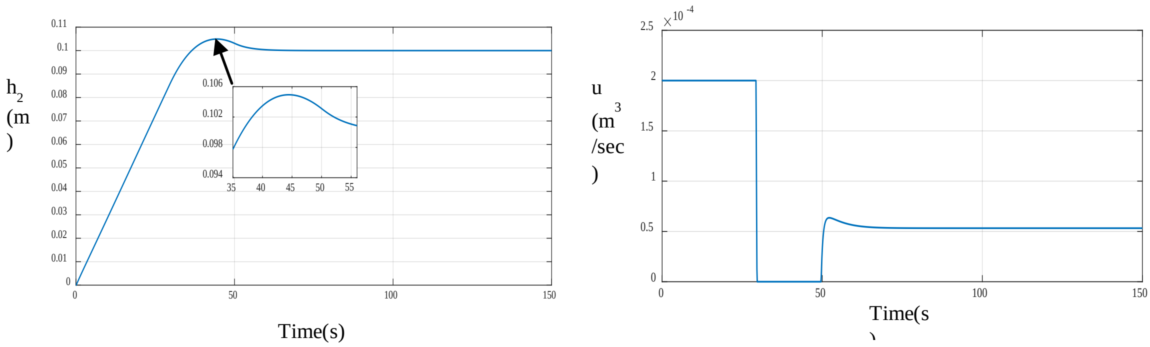
Table 2. Tuning parameters for each controller.

Parameter	PD-SMC	PID-SMC	Fractional-PID	Dynamic-SMC
K	1.935	10.17	2.59	1.036
Δ	0.832	47.213	0.509	0.406
K_p	8.55	1.828	14.765	-
K_I	-	$1.0*10^{-5}$	$1.416*10^{-6}$	-
K_d	50.865	1.099	78.653	-
λ	-	-	0.999	-
μ	-	-	0.965	-
α_1	-	-	-	0.233
α_2	-	-	-	0.012



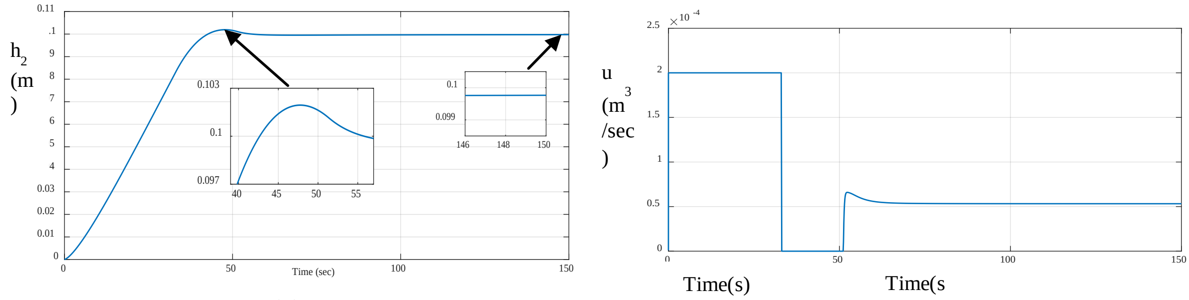
(a) (b)

Figure 3. PD-SMC (a) Step response (b) Control signal.



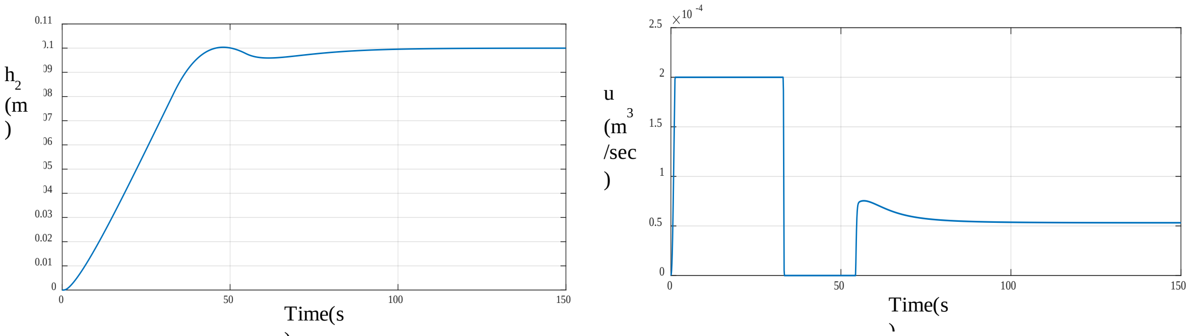
(a) (b)

Figure 4. PID-SMC (a) Step response (b) Control signal.



(a) (b)

Figure 5. FractionalPID-SMC (a) Step response (b) Control signal.



(a) (b)

Figure 6. Dynamic-SMC (a) Step response (b) Control signal.

Table 3. Comparison between the step response of the four controllers.

Controller	Rise time (s)	Overshoot (%)	Steady state error (m)	Error index (m^2.s)	Settling time (s)
PD-SMC	28	1.6	Zero	0.1159	34.2
PID-SMC	27.8	4.9	Zero	0.1162	33.5
Fractional-PID	29.8	1.9	0.0002	0.1488	38.5
Dynamic-SMC	30	-	Zero	0.1573	39.8

4.2 Input disturbance

An input disturbance step signal of -0.0001 m^3/s (50% of the full scale of the pump) is added to the control signal (u) when the running time in the simulation is 100 seconds. Figures 7 to 10 describe the response of the four controllers to that disturbance. As can be seen from the figures, the four controllers almost reach the desired value but all have steady state error. The dynamic-SMC has the

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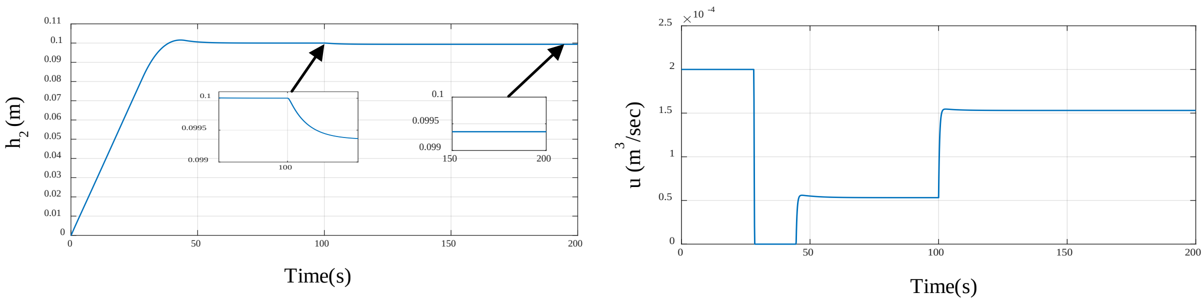
lowest steady state error of 0.0001 m while the PD-SMC and PID-SMC have the largest steady state error of about 0.0007 m.Also, the four algorithms almost do not cause any chattering in the control signal,figures 7-b to 10-b. A comparison in the values of the steady state error, the error index and the settling time (5% criterion) is shown in table 4.

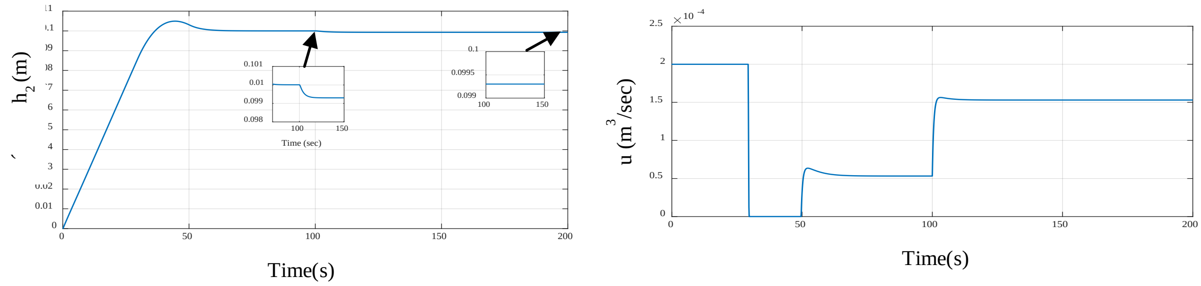
Table 4. Comparison between the four controllers after a disturbance signal is applied.

Controller	Steady state error (m)	Error index (m^2.s)	Settling time (s)
PD-SMC	0.0007	0.11596	34.2
PID-SMC	0.0007	0.11626	33.5
Fractional-PID	0.0005	0.1488	38.5
Dynamic-SMC	0.0001	0.1573	39.8



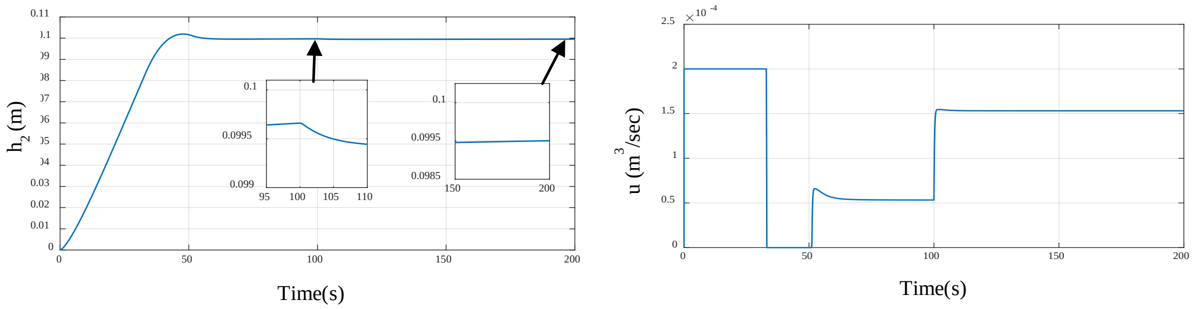
(a) (b)

Figure 7. PD-SMC with a step disturbance in the control signal (a) Response (b) Control signal.



(a) (b)

Figure 8. PID-SMC with a step disturbance in the control signal (a) Response (b) Control signal.



(a) (b)

Figure 9. FractionalPID-SMC with a step disturbance in the control signal (a) Response (b) Control signal.

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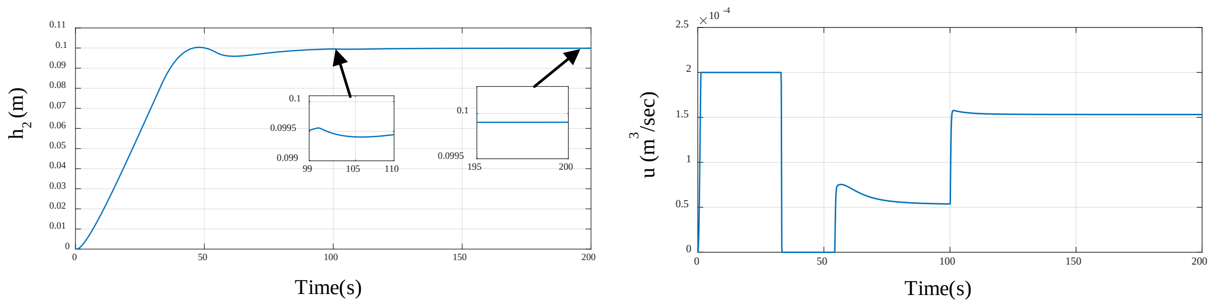
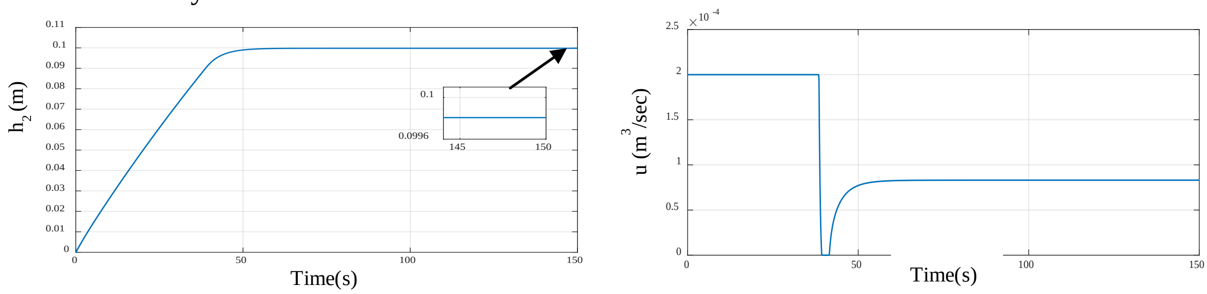


(a) (b)

Figure 10. Dynamic-SMC with a step disturbance in the control signal (a) Response (b) Control signal.

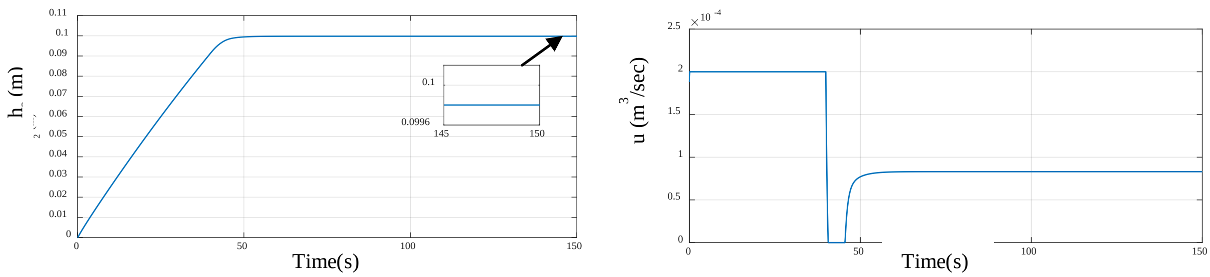
4.3 Robustness test: 25% variation in plant parameters

To investigate the robustness of the proposed control algorithms, a 25% variation in the plant parameters, a_1, a_2 and c is applied. As can be seen from figures 11 to 14, all of the four algorithms reached the required value and still the fractional order PID has the highest steady state error.Also, the four algorithms almost do not have any chattering,figures 11-b to 14-b. However, as can be noticed, the dynamic-SMC response has an overshoot of 24.2%. A comparison between the rise time, percentage overshoot, steady state error, the error index, and the settling time (5% criterion) for the four controllers is given in table 5. As can be seen from the table, PD-SMC has the minimum error index of 0.1342 m^2.s and the minimum rise time of 34.7 s while both PD-SMC and PID-SMC have the minimum steady state error of 0.0002 m.



(a) (b)

Figure 11.PD-SMC with 25% variation in the plant parameters (a) Response (b) Control Signal.



(a) (b)

Figure 12.PID-SMC with 25% variation in the plant parameters (a) Response (b) Control Signal.

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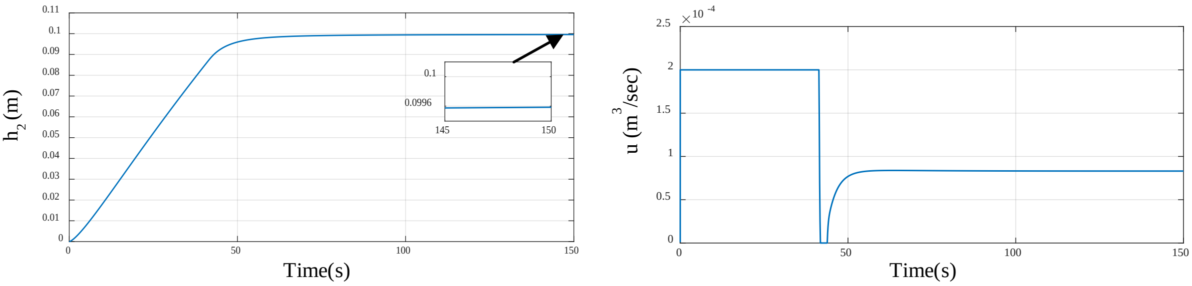
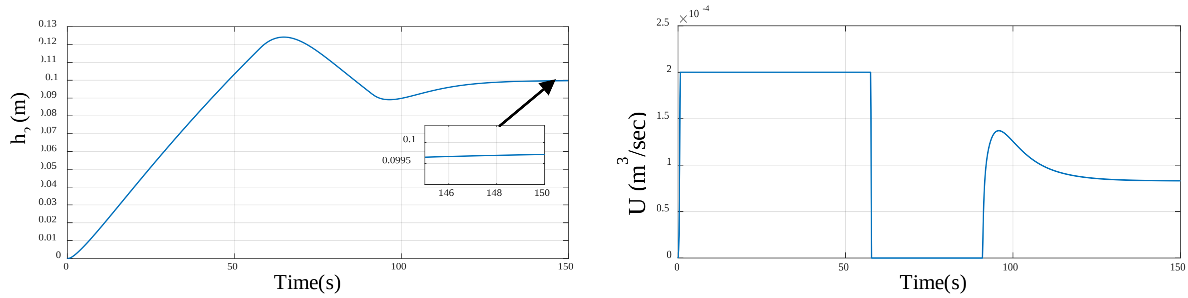


(a) (b)

Figure 13. Fractional PID-SMC with 25% variation in the plant parameters (a) Response (b) Control Signal.



(a) (b)

Figure 14. Dynamic-SMC with 25% variation in the plant parameters (a) Response (b) Control Signal.

Table 5. Comparison between the four controllers in the presence of 25% variation in the plant parameters.

Controller	Rise time (s)	Overshoot (%)	Steady state error (m)	Error index (m^2.s)	Settling time (s)
PD-SMC	34.7	-	0.0002	0.1342	42.1
PID-SMC	35.5	-	0.0002	0.1374	42
Fractional-PID	36.9	-	0.0004	0.166	58
Dynamic-SMC	36.7	24.2%	0.0003	0.1829	122.3

4.4 Sensor noise

In order to test the controllers' sensitivity to sensor noise, a gaussian white noise with signal to noise ratio (SNR) ranges from 19 dB to 23 dB is added to the level sensor, figure 15. The time response to this test showed a high chattering level for all control algorithms. It worth nothing that, the controller parameters in all algorithms have been optimized to reduce the error signal and chattering in the absence of sensor noise. Therefore, a new run of the optimization algorithm is done in order to get the optimized controller parameters in the presence of sensor noise, see table 6.

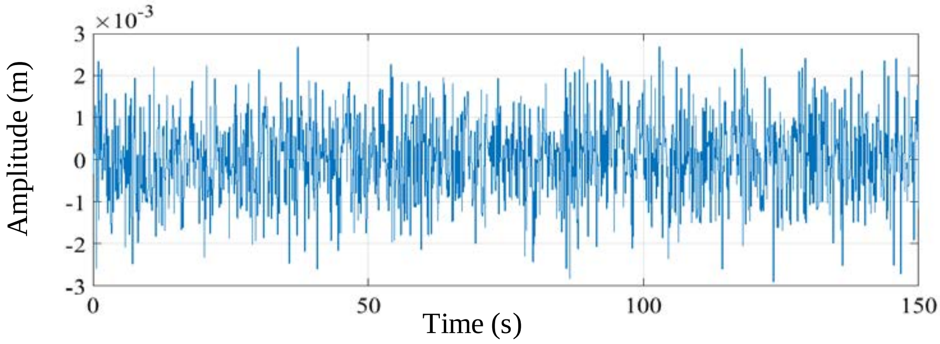


Figure 15. Gaussian white noise added to the level sensor 2.

Table 6 New tuning parameters in the presence of sensor noise.

Parameter	PD-SMC	PID-SMC	Fractional-PID	Dynamic-SMC
K	$7.281*10^{6}$	$2.148*10^{12}$	$7.901*10^{3}$	0.146
Δ	$4.366*10^{7}$	$9.600*10^{14}$	12.0	2.179
K_p	$3.259*10^{6}$	$6.494*10^{7}$	$1.194*10^{10}$	-
K_I	-	$6.182*10^{6}$	$5.786*10^{9}$	-
K_d	$2.042*10^{7}$	$3.890*10^{8}$	$4.215*10^{10}$	-
λ	-	-	0.9999	-
μ	-	-	0.9179	-
α_1	-	-	-	0.3398
α_2	-	-	-	0.0493

Using the new tuning parameters, the chattering in the control signal is decreased significantly in comparison with that using the old ones. However, the fractional-SMC still suffers from heavy chattering in its control signal, while the dynamic-SMC has the least chattering, figures 16-b to 19-b. Also, as can be seen from figures 16 to 19, the largest steady state error was found in PID-SMC, while the fractional-PID has the maximum overshoot percentage

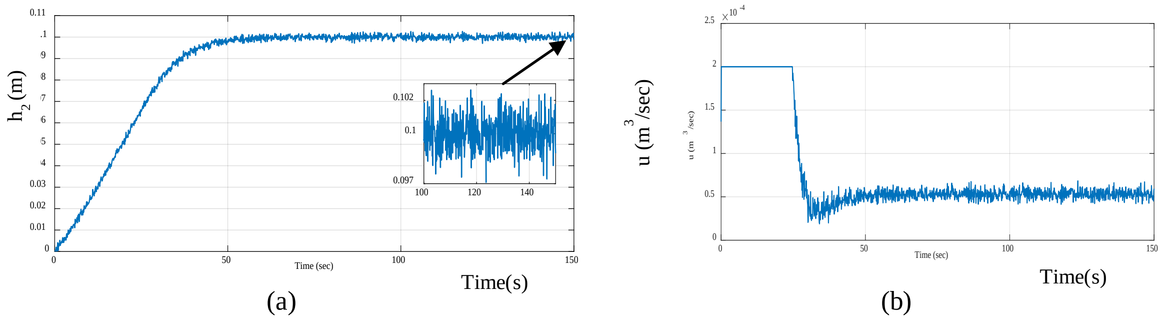


Figure 16.PD-SMC with sensor noise (parameters retuned) (a) Response (b) Control Signal.

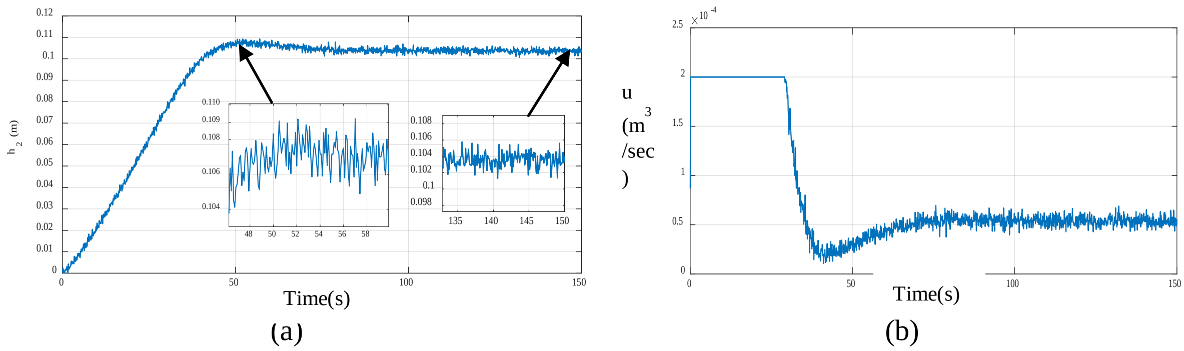
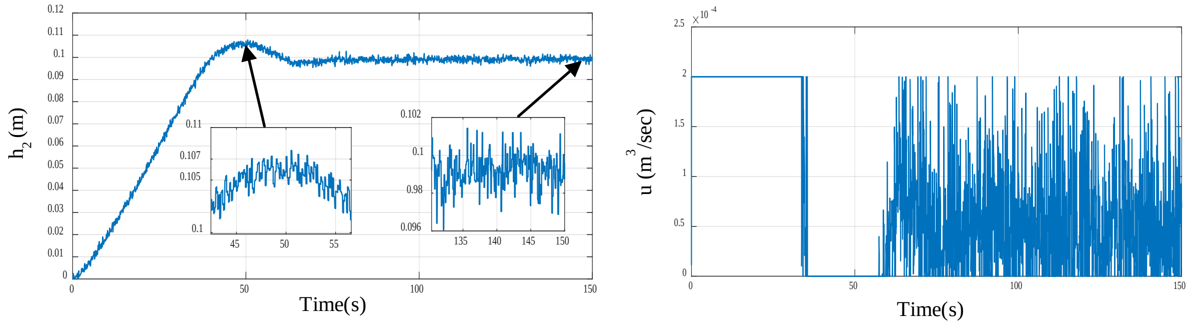
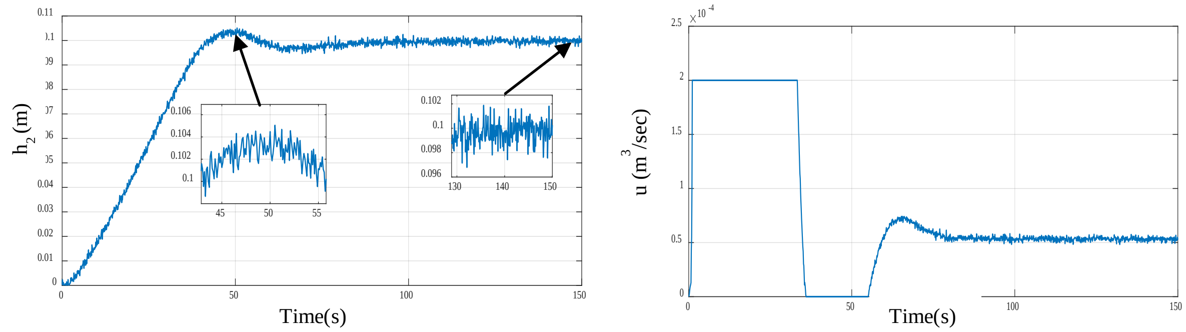


Figure 17.PID-SMC with sensor noise (parameters retuned) (a) Response (b) Control Signal.



(a) (b)

Figure 18. FractionalPID-SMC with sensor noise (parameters retuned) (a) Response (b) Control Signal.



(a) (b)

Figure 19. Dynamic-SMC with sensor noise (parameters retuned) (a) Response (b) Control Signal.

A comparison between the rise time, percentage overshoot, steady state error, the chattering index, and the settling time (5% criterion) for the four controllers is shown in table 7. As can be seen from the table, PD-SMC has in general a better performance in comparison to the other algorithms irrespective of the chattering level. However, in accordance to the application, if the chattering level of the actuator is considered, dynamic-SMC is superior to the other controllers.

Table 7. Comparison between the four controllers in the presence of sensor noise.

Controller	Rise time (s)	Overshoot (%)	Steady state error (m)	Chattering Index	Settling time (s)
PD-SMC	31.4	-	0.0015	$9.999*10^{-7}$	45
PID-SMC	29.8	9.2	0.0035	$8.2296*10^{-7}$	>150
Fractional-PID	29.8	7.8	0.002	0.0038	54.8
Dynamic-SMC	30.7	5.0	0.0016	$1.999*10^{-7}$	75.8

5. Conclusion

In this paper, the performance of four SMC algorithms are investigated for the control of the liquid level in horizontal coupled tank system. The performances of the four controllers are studied when they are exposed to step input, a step input disturbance, a 25% variation in the plant parameters and finally in the presence of sensor noise. The simulation results indicate that the behaviors of the four proposed control algorithms have no significant differences when the step input and input disturbance are applied. In case of plant parameters variation (25%), the PD-SMC, PID-SMC and fractional-SMC give better response than dynamic-SMC. However, the dynamic-SMC has a significant reduction in the chattering level compared to the other three controllers when sensor noise is added. Based on the results presented in this study, the dynamic-SMC is highly recommended in case of sensor used in the

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feedback control system has a significant noise level. This can significantly protect the actuator from damage and increase its life span.

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