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## **MIMO Communication System Optimization Using RNS**

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### **ABSTRACT:**

This paper evaluates the Bit Error Rate (BER) performance of MIMO communication system using Residue Number System (RNS). The basic idea behind the proposed scheme is to make use of the independent parallel feature of RNS and reduced signal amplitude to convert the input signal into parallel smaller residue signals that have a smaller dynamic range. This approach would reduce the channel distortions and thus improves the received signal-to-noise ratio. The Peak-to-Average Power Ratio (PAPR) is measured with and without using RNS in the multi-transmission system.

The simulation is implemented using MATLAB tool and the results had shown the performance of transmission/reception model with and without RNS implementation.

### **KEY WORDS**

MIMO communication, OFDM system, RNS, BER, PAPR.

### **NOMENCLATURE**

- |    |       |   |
|----|-------|---|
| 1) | S     | The transmitted symbol  |
| 2) | N     | Channel Noise   |
| 3) | H     | The MIMO channel matrix can be represented by a $N_t \times N_r$ matrix |
| 4) | l     | Varies from 0 to N-1  |
| 5) | E     | The expectation operator  |
| 6) | k     | 0, 1, 2 . . . , N - 1   |
| 7) | i     | 1 to v  |
| 8) | $M_r$ | Dynamic range   |
| 9) | $T_i$ | Multiplicative Inverse  |

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## 1.0 Introduction:

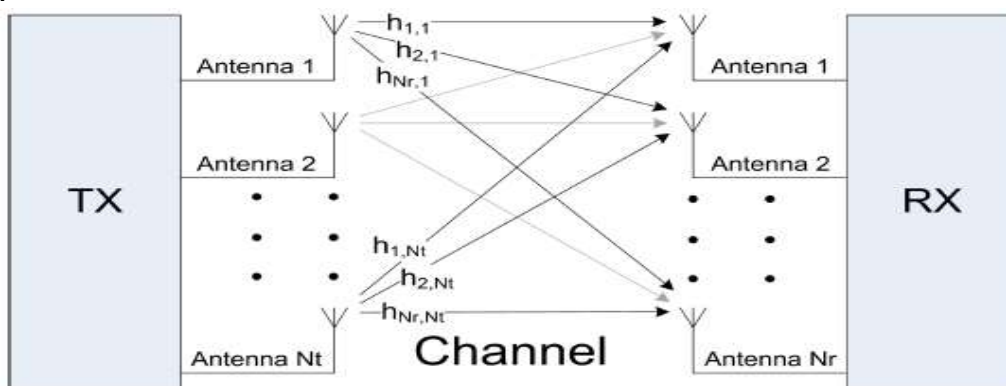
Multiple-Input-Multiple-Output (MIMO) technology stands out in wireless communication through its ability to provide higher data rate and high quality of service (QoS), by utilizing the spatial multiplexing gain and improving the link reliability by antenna diversity [1], [2]. Combining this technology with orthogonal frequency-division multiplexing (OFDM); that divides a radio channel into a large number of closely spaced sub-channels, would provide higher reliable communications at high speeds [3]. Unfortunately, the drawback for this topology is that the variation of OFDM signal amplitudes is very wide with high Peak-to-Average Power Ratio (PAPR), which introduces signal distortion when the dynamic range of transmitted signals is larger than the amplifier accommodation and thus, degrades the system performance due to high PAPR. [4]

Introducing Residue number system (RNS), which is a parallel number system based on Chinese remainder theory (CRT), to make use of its ability for independent parallel communication and studying its performance specially the change in the system PAPR. The paper introduces in section 2 a basic background on the technologies utilized in the system, section 3 provide MIMO-OFDM system model, section 4 illustrates a review for current power reduction approaches, section 5 describe the MIMO-RNS communication system, section 6 defines the way in which the system is evaluated, section 7 the simulation results are presented to measure the system performance and finally in section 8, a conclusion will be provided.

## 2.0 Historical Background

### 2.1 MIMO system

As shown in Fig. 1, the MIMO system is an advanced antenna configuration that uses multiple antennas at both transmit and receive sides in the communication system [5], [6].



**Fig.1.** MIMO communication Channel

The standard received signal vector (R) can be calculated as:

$$R = S H + N \tag{1}$$

The multiple antenna configurations offer a mean for increasing system coverage and/or capacity and the improvement comes from diversity and/or spatial multiplexing gain, which generally have to be traded off against each other, depending on the particular operating conditions and their different channel knowledge requirements.

## 2.2 Space-time block coding

Space-Time Block Coding (STBC) exploits the transmit diversity of the MIMO system by transmitting the information simultaneously on different antennas and thus, pushes the system performance close to the theoretical limits [6]. Alamouti algorithm which is a STBC, presents an orthogonal block code using (N) transmitters and (M) receivers. The information data bits are first modulated and mapped into their corresponding constellation points. So, if  $x_1$  and  $x_2$  are two modulated symbols that enter the space-time encoder, these two symbols are transmitted at two consecutive time instances  $t_1$  and  $t_2$  from two antennas, separated by a constant time duration  $T$ .

The space-time encoding mapping of Alamouti's two-branches transmit diversity technique can be represented by the following coding matrix:

$$X = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} \quad (2)$$

And, the output would be shown as;

$$Y = XH + N \quad (3)$$

## 2.3 RNS properties

A residue number system (RNS) is a technique in which large integers are presented using a set of smaller integers that are mutually independent due to the absence of carry information. Some of these small integers can be discarded without affecting the results, making the arithmetic computation faster and more efficient. This is based on the Chinese remainder theorem that was introduced in the 4<sup>th</sup> century AD. [7 - 9]

This technique is defined by the choice of  $v$  positive integers  $m_i$  ( $i = 1, 2, \dots, v$ ) referred to as moduli, which are pair-wise relative primes. So, any integer  $N$ , describing a non-binary message, can be uniquely and unambiguously represented by the so-called residue sequence  $(r_1, r_2 \dots r_v)$  in the range  $0 < N < M_i$ , where;

$$r_i = N \pmod{m_i}; \text{ represents the residue digit of } N \text{ upon division by } m_i \quad (4)$$

$$M_i = \prod m_i; \text{ is the information symbols' dynamic range.} \quad (5)$$

To recover the symbols, CRT is implemented such that for any given  $v$ -tuple  $(r_1, r_2 \dots r_v)$  where  $0 \leq r_i < m_i$ ; the numerical value of  $N$  can be computed according to the equation:

$$T_i M_i = 1 \pmod{m_i}; \quad (6)$$

$$N = \sum r_i T_i M_i \pmod{M_r}; \text{ such that; } M_i = M_r/m_i \quad (7)$$

In this paper the RNS will be introduced as a "coding" scheme that is used to transfer very big numbers to multiple of short numbers.

### 2.4 Direct and Indirect mapping schemes

The mapping “modulation” of non-binary information symbols to complex constellations for transmission over wireless channels is performed in two methods; direct and indirect mapping schemes [10].

In direct mapping, the symbols are mapped directly to constellation points using a one to one mapping. However, the Indirect mapping scheme, residues are converted back to binary bits before mapping them to the M-ary constellation using binary bit assignment strategy.

Fig. 2.a and Fig. 2.b presented hereunder, show the system block for direct and in-direct mapping schemes;

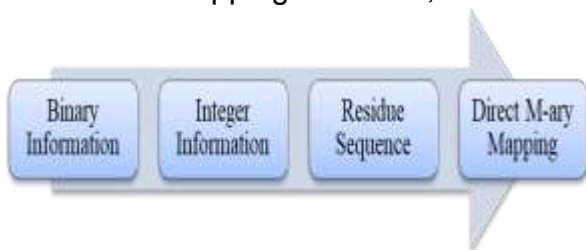


Fig. 2.a. System for Direct Mapping



Fig. 2.b. System for Indirect Mapping

### 2.5 OFDM technique

It is a method of encoding digital data on multiple carrier frequencies to be able to transport it over a wideband digital channel [11]. This transportation technique; as shown in Fig.3 is used to support higher data rates, especially when the multipath effect causes inter-symbol interference that cannot be ignored anymore.

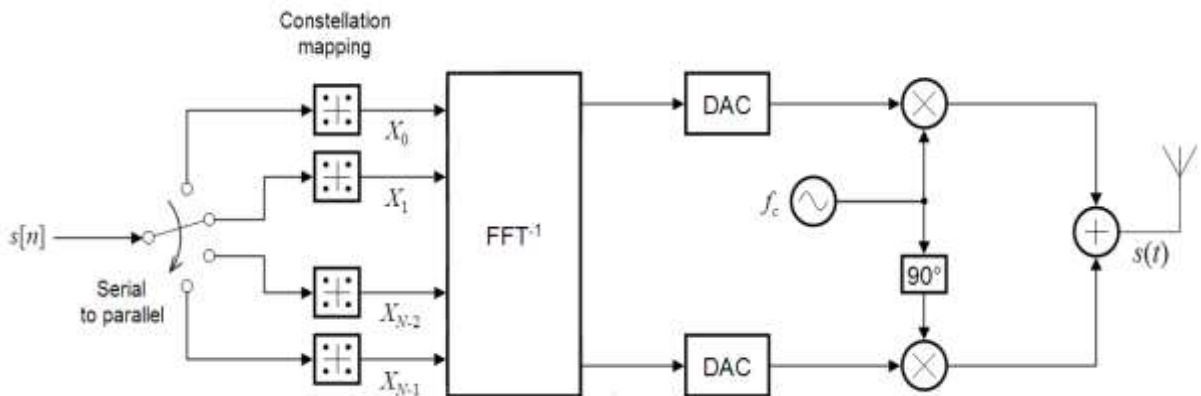


Fig.3.a. OFDM Transmitter

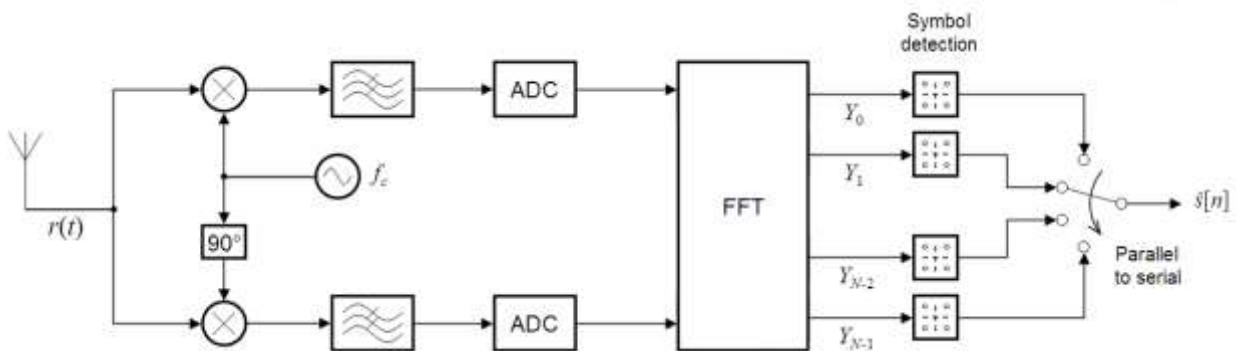
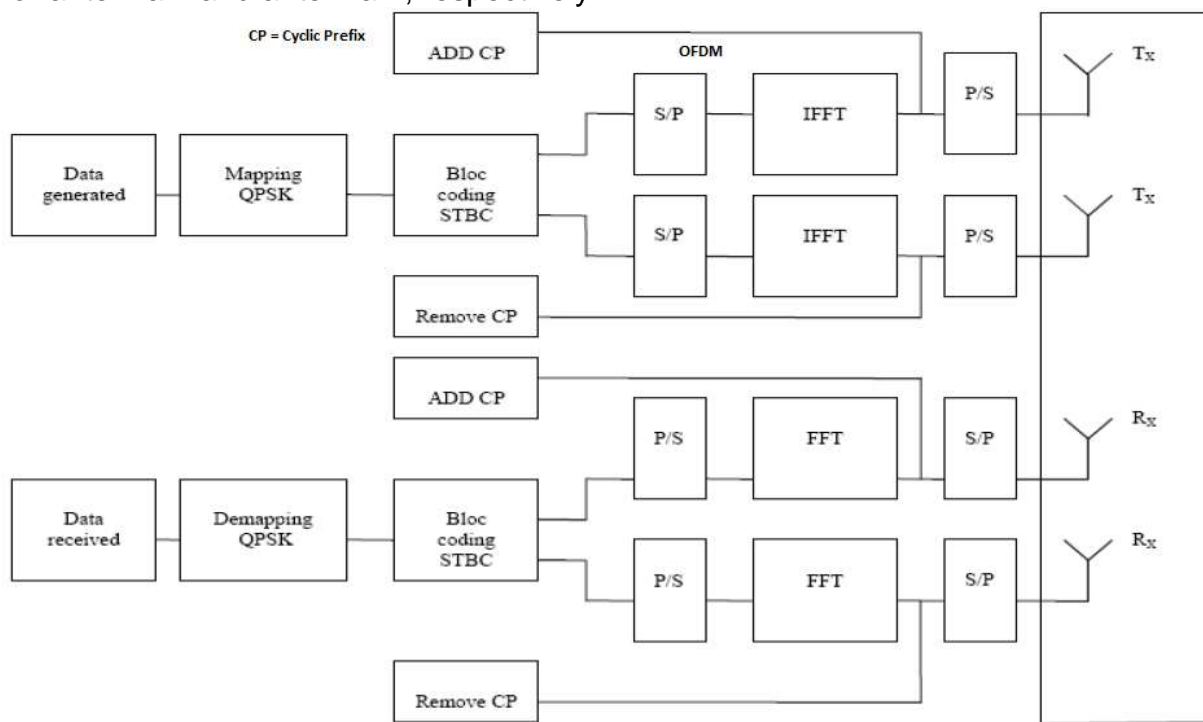


Fig.3.b. OFDM Receiver

The basic principle is that the larger the number of used carriers (tones), the lower the data rates has to carry. So, by using an appropriate number of carriers; the symbol duration can be made to be significantly longer than the multipath delay differences, and this reduces the level of inter-symbol interference and also creates channels that have flat fading characteristics. Furthermore, the inter-symbol interference can be completely eliminated by introducing a cyclic prefix with duration is longer than the channel delay spread. However, this is at the expense of increased channel overhead.

### 3. MIMO-OFDM System Model

As shown from the transmit/receive block diagram provided in Fig.4, the signal is modulated, coded through the STBC encoder, and passed to a serial-to-parallel (S/P) converter that generates a complex vector, which is passed through each IFFT block for antenna 1 and antenna 2, respectively.



**Fig.4.** MIMO-OFDM Structure

The main advantage of OFDM-based systems lies in its robustness to frequency selective fading, high spectral efficiency, low-complexity equalization ...etc. However, since the transmitted signal of multicarrier transmission is the sum of data on different subcarriers, the variation of OFDM signal amplitudes is very wide with high PAPR [4], [12]. The system performance could be degraded due to high PAPR, which introduces signal distortion when the dynamic range of transmitted signals is larger than the amplifier accommodation.

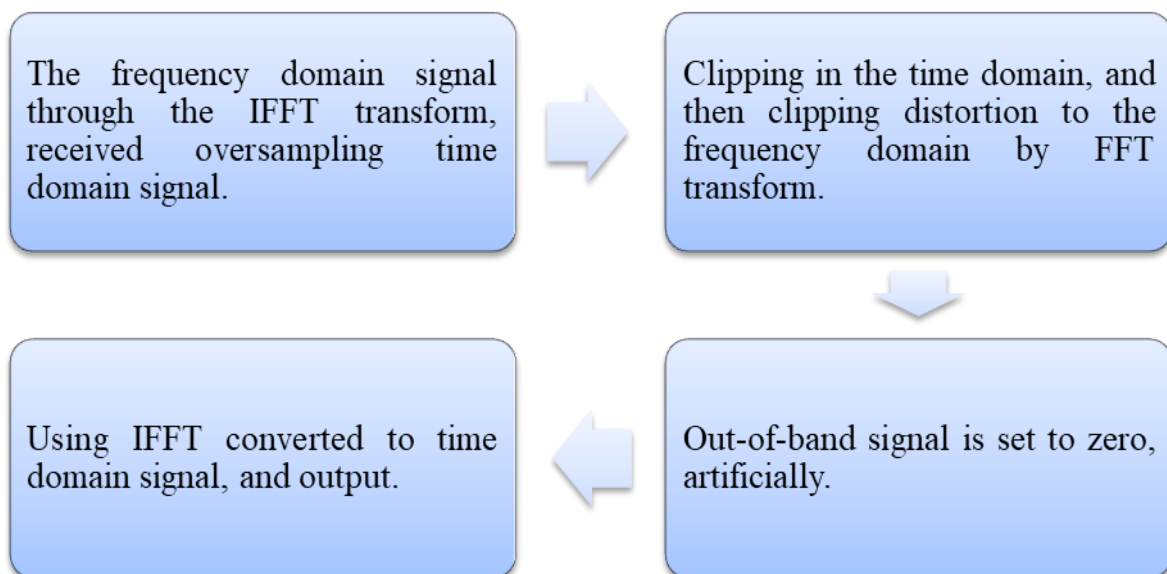
As a consequence, PAPR becomes one of the bottlenecks for OFDM-based systems in practical applications. In next section we will try to provide a background on the current existing power reduction approaches.

#### 4.0 Power Reduction Approaches:

We could categorize current power reduction techniques into two categories [12, 13]: lossy and lossless approaches, which depends whether the transmitted signals is distorted or not.

##### 4.1 Lossy Approaches

This scheme includes techniques as clipping, peak windowing, etc. Clipping scheme limits parts of the signals over the allowed region; this approach is considered as the simplest and most widely used, where the OFDM signal is intentionally clipped at a defined threshold value before amplification. Also, the presence of large peaks of OFDM signal has very low probability; the operation could be an effective technique for the reduction of the PAPR. The algorithm is shown in Fig.5.

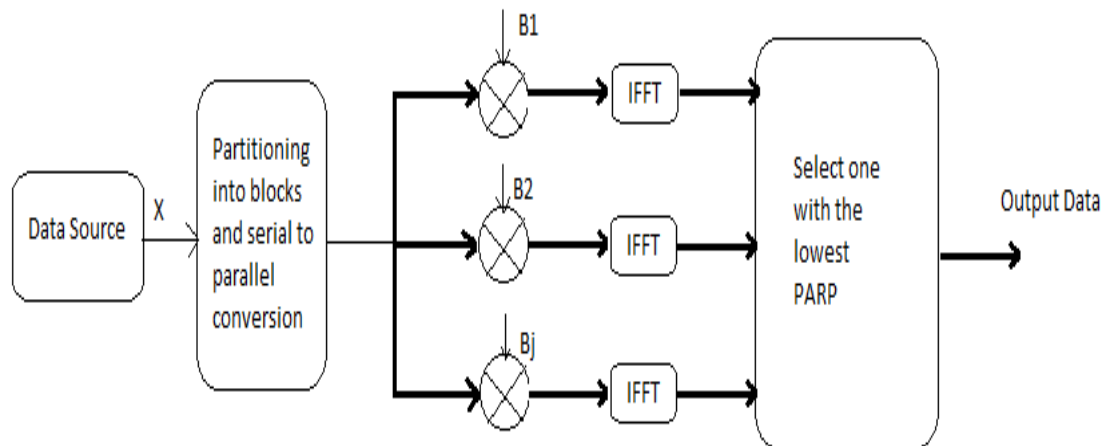


**Fig.5.** Clipping and Filtering Process Steps

It should be noted that this approach causes important in-band distortion and out-of-band noise, which will indirectly degrades the bit error rate performance and the spectral efficiency.

##### 4.2 Lossless Approaches

This approach includes coding, selective mapping and probabilistic scheme. Coding scheme as Selective Mapping (SM) selects the codeword that reduces the PAPR for transmission and may address the problem of error control, but it is hard to be adapted to OFDM with a larger number of subcarriers. In SM the data stream is divided into different blocks of N symbols, were every block is multiplied with B different phase factors to generate U modified blocks before the IFFT block. Different IFFT blocks are used to generate OFDM symbols. PAPR is calculated for each modified block and select the block which is having minimum PAPR ratio, as seen in Fig.6.

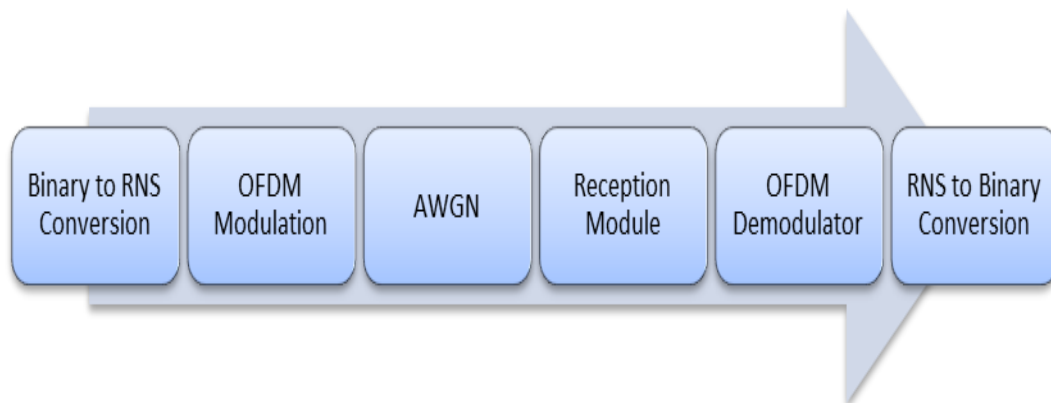


**Fig.6.** OFDM Transmitter with the SM method

Although it is able to reduce PAPR effectively, the drawback is the increased circuit complexity as it contains several IFFT blocks.

### 5.0 MIMO-OFDM-RNS System Model

In this section, the system model is introduced, analyzed, and studied. As shown in Fig.7, a basic block diagram for RNS-OFDM system is presented, where the initial frequency band is divided into  $v$  sub-bands and the input signal is converted into  $v$  residuals. The frequency-domain symbols in residue form for each residue sub-channel are modulated based on the so-called “Direct Mapping” scheme by IFFT to the RNS-based OFDM symbols through the OFDM modulator.



**Fig.7.** RNS-OFDM Tx/Rx Block diagram

In this scheme, parallel transmitting signals of  $v$  residue sub-channels are simultaneously sent to the channel in  $v$  frequency band portions. The reception module of the receiver is dedicated to the received signals on the corresponding residue sub-channel. FFT is used to demodulate the signals for each residue sub-channel. Then, the signals are recovered after R/B (residue to binary conversion).

### 6.0 PAPR Evaluation Performance Method

The performance is evaluated by measuring the Peak-to-Average Power Ratio (PAPR) of the signal,  $x(t)$  [14], as seen in equation (8);

$$\text{PAPR} = 10 \log ( \max|x_k|^2/ E[|x_k|^2] ), \text{ in dB} \tag{8}$$

It could also be illustrated through “Complementary Cumulative Distribution Function” (CCDF), which is the probability of PAPR exceeding a threshold and its mathematical expression as seen in equation(9);

$$\begin{aligned} \text{CCDF}(\text{PAPR}(x(n))) &= P ( \text{PAPR}(x(n)) > \text{PAPR}_0 \\ &= (1 - e^{-\text{PAPR}_0})^N, \text{ N mutual independent signals} \end{aligned} \tag{9}$$

By taking into account M transmitted antennas; equation (9) would yield to equation (10) for MIMO-OFDM system;

$$\begin{aligned} \text{CCDF}(\text{PAPR}(x(n))) &= P ( \text{PAPR}(x(n)) > \text{PAPR}_0 \\ &= (1 - e^{-\text{PAPR}_0})^{M \cdot N} \end{aligned} \tag{10}$$

### 7.0 Simulation Results

Using MATLAB simulations, the system performance will be examined where the proposed MIMO-OFDM communication system that uses RNS algorithms, is evaluated for optimizing the system performance.

#### 7.1 SISO OFDM vs. MIMO-OFDM Communication Systems

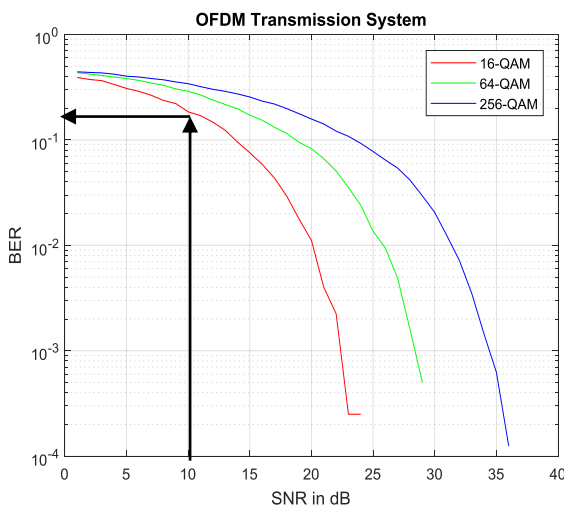


Fig.8.a. OFDM-System BER vs. SNR

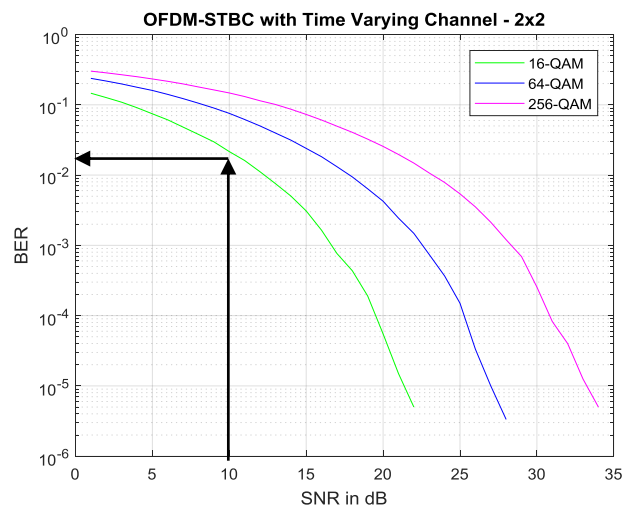


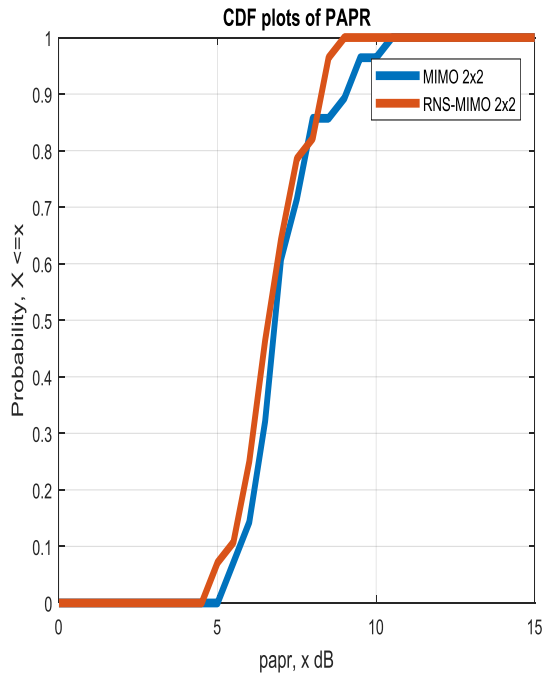
Fig.8.b. MIMO-OFDM-System BER vs. SNR

From Fig.8.a and Fig.8.b, it is seen that the achieved enhancement is due to the usage of MIMO technique, where for SNR = 10 dB and using 16-QAM, the BER of 2\*2 MIMO-OFDM is 0.02, while it is 0.2 for OFDM only.

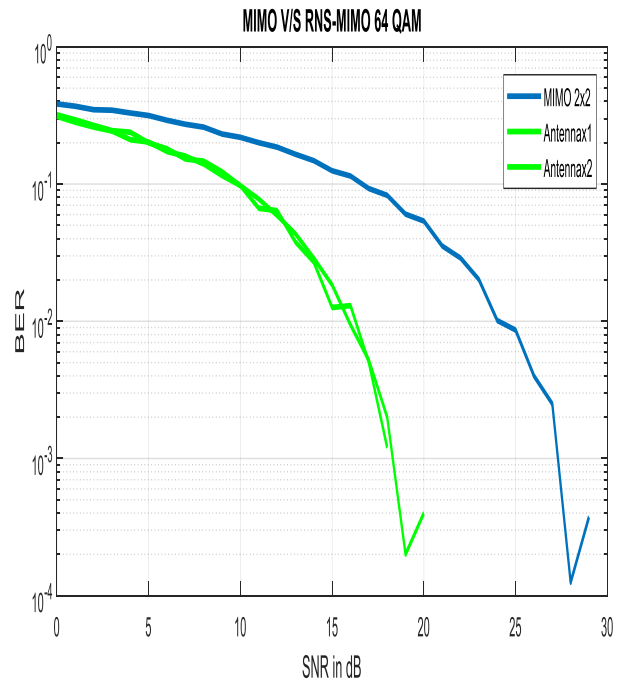


### 7.2 RNS parallel transmission scheme

The effect of RNS coding scheme for parallel transmission of each RNS moduli over the MIMO channels is shown in Fig.9.a and Fig.9.b.



**Fig.9.a.** PAPR with and without RNS



**Fig.9.b.** BER Vs. SNR with and without RNS

It could be seen from Fig.9.a that PAPR reduction from 11db to 9db was achieved. Thus, the RNS would provide around 20% reduction compared to conventional MIMO system without RNS. Also, the received SNR seen in Fig.9.b has been improved compared to that without using the RNS scheme.

### 7.3 RNS sequential transmission scheme

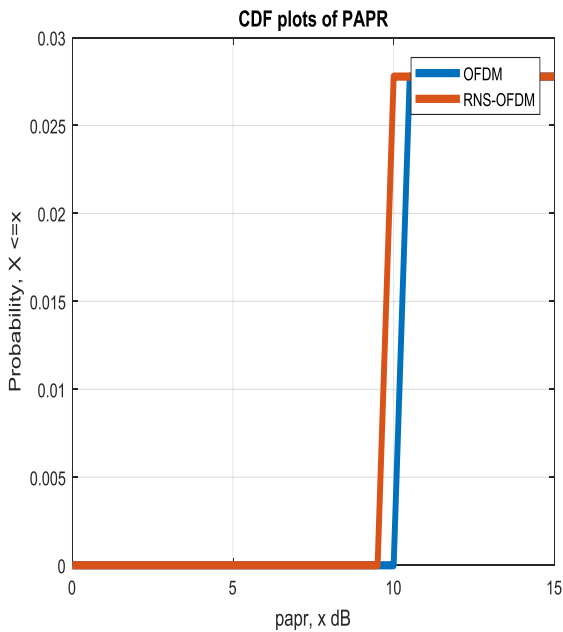
The RNS coding scheme can be modified by using a sequential transmission for the RNS moduli's over each MIMO channel instead of using a parallel transmission for each RNS moduli. In this way the following advantages can be obtained:

- Regardless of any number of moduli sets - we can have fixed hardware configuration say (2x2).
- Improving the SNR.

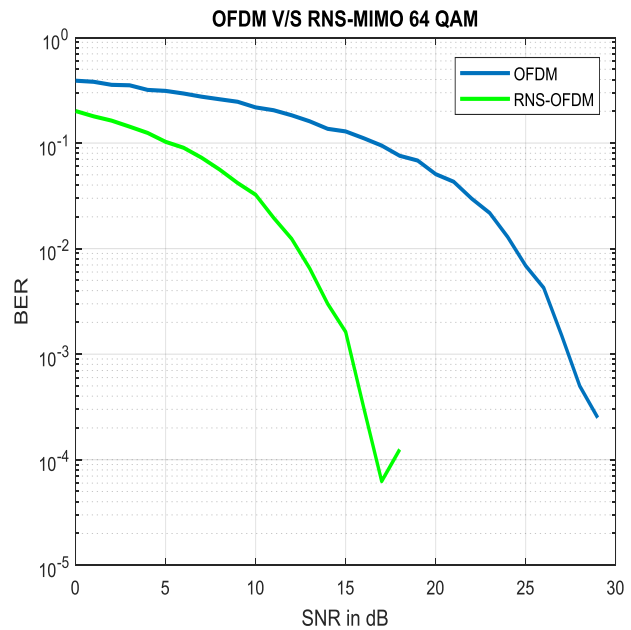
Also, both of the moduli range and the number of Moduli would have an effect not only on the system performance over SNR, but as well on the transmitted Peak-Average-Power Ratio. Through the selection of the RNS sets, to present a wider range of data representation, a PAPR gain and better SNR are obtained.

Focusing on this scheme, several simulation scenarios can be conducted as seen in the following subsections.

a) The selective RNS set of [11, 7, 5, 3]; would produce the enhanced results seen in Fig.10.a and Fig. 10.b;



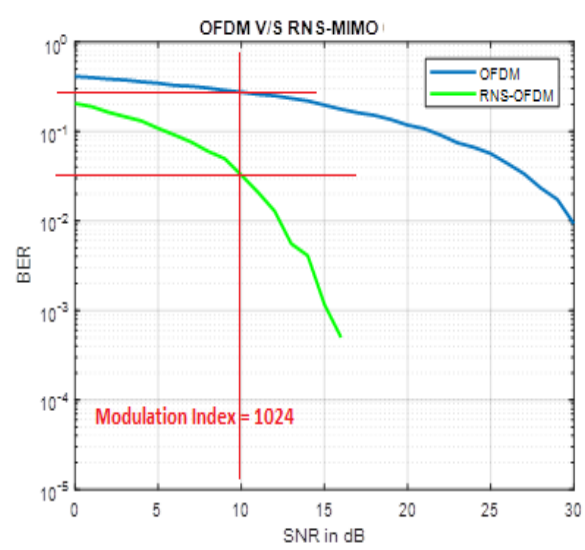
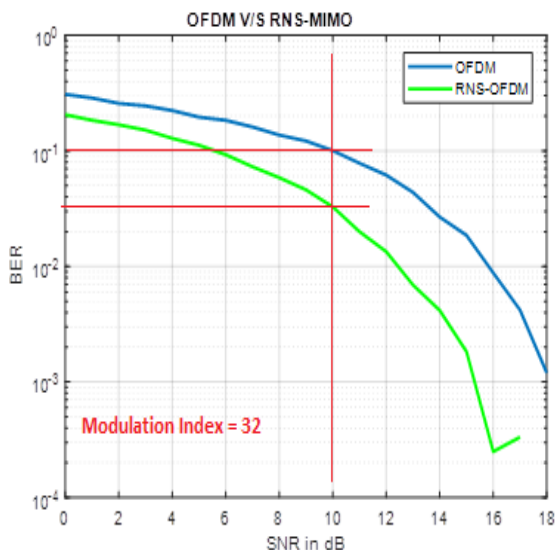
**Fig.10.a.** PAPR with and without RNS



**Fig.10.b.** BER Vs. SNR with and without RNS

Where, it could be seen the reduction provided in the PAPR when using RNS scheme in Fig.10.a, and the received SNR seen in Fig.10.b has been improved compared to that without using the RNS scheme

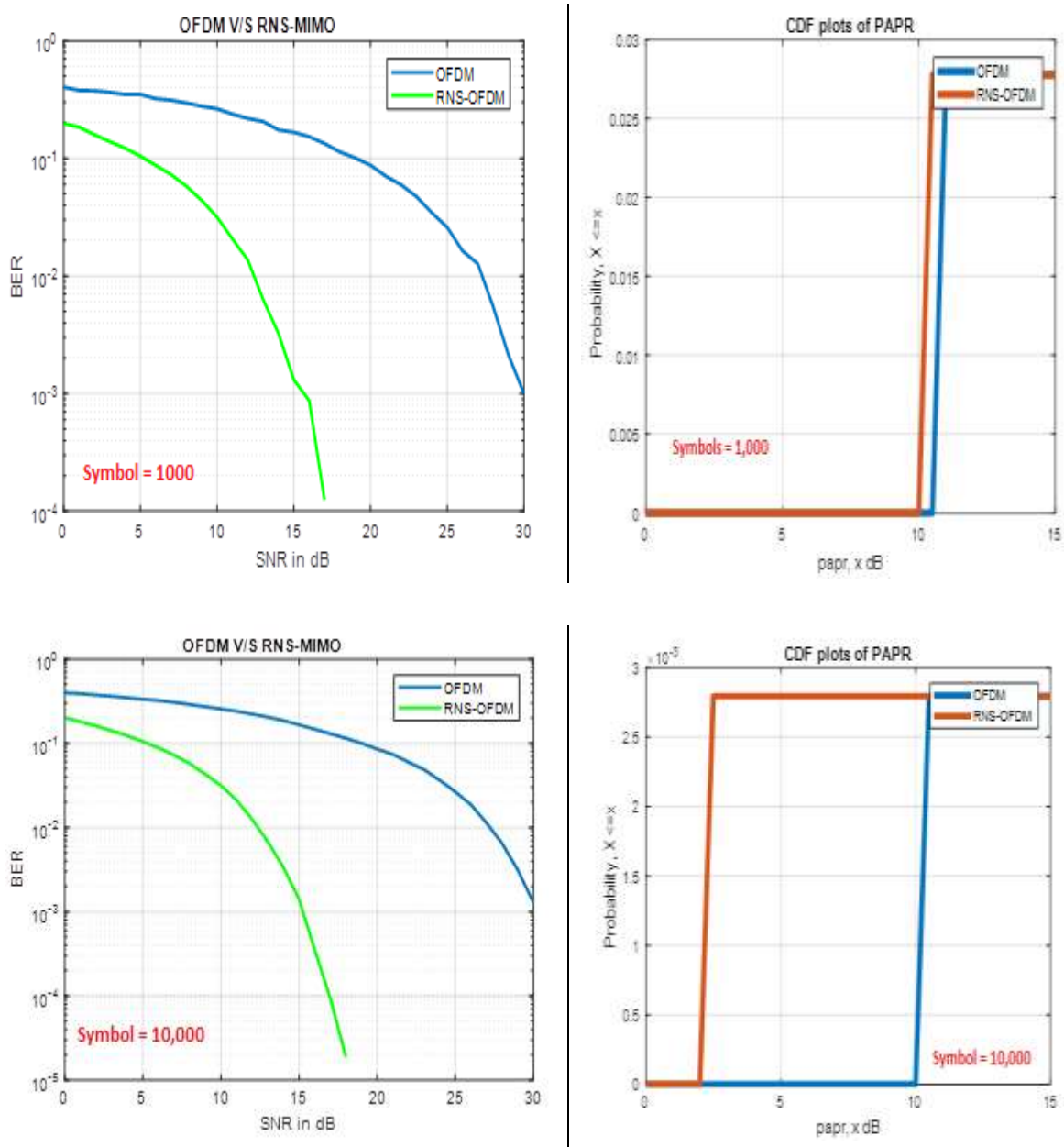
b) The effect of modulation index on a fixed RNS scheme [11, 7, 5]; as shown in Fig.11;



**Fig.11.** BER vs. SNR for different modulation index

From Fig.11, it is clear that the SNR is degraded at higher modulation index for OFDM coding scheme without RNS. However using RNS, the SNR it is not affected by the increase of the modulation index.

c) The effect of increasing the number of transmitted symbols on the communication performance is illustrated in Fig 12, by fixing the RNS moduli set = [11, 7, 5, 3] and using a modulation index set to 512, and changing the Tx symbols. The following results will be obtained:



**Fig.12.** BER vs. SNR & PAPR for different Tx symbols

The same advantage seen in the previous section in Fig.11; could also be noticed from Fig.12, where as the number of transmitted symbols increases, the improvement in the PAPR is more for RNS based scheme. Thus, the RNS would provide better performance with higher order modulation index and thus it is capable of providing higher throughput system.

- d) For a fixed Modulation scheme ( $M = 256$ ), we change the RNS moduli and see the effect on the system performance, as seen in the table 1;

**Table 1.** System performance vs. RNS moduli

RNS moduli	BER for RNS-OFDM vs. OFDM	$\Delta$ PAPR RNS-OFDM vs. OFDM
[128, 127, 63]	@ SNR = 10; $\Delta = 0.041$ db (improved)	-0.5 db (worst)
[17, 15, 13]	@ SNR = 10; $\Delta = 0.1328$ db (improved)	-0.5 db (worst)
[15, 13]	@ SNR = 10; $\Delta = 0.1896$ db (improved)	-1.5 db (worst)
[11, 7, 5, 3]	@ SNR = 10; $\Delta = 0.1863$ db (improved)	+1.0 db (improved)
[13, 11, 7, 5, 3]	@ SNR = 10; $\Delta = 0.1884$ db (improved)	+1.0 db (improved)

It can be deduced that when the difference RNS moduli and modulation index is increased (which mean utilizing lower RNS set), the PAPR is improved. Also, in all cases; the sequential RNS improves significantly the SNR, compared to the system without RNS coding scheme.

## 8. Conclusion

Utilization of RNS coding scheme in MIMO-OFDM communication system, enable to utilize the advantage of OFDM in its ability to offer higher data rates.

In this paper, RNS coding scheme is used to exploit the OFDM transmission technique that encode digital data over multiple carrier frequencies, and through the implementation of parallel and sequential RNS moduli transmission. It was able to reduce the transmitted signal dynamic range such that the obtained PAPR was around 20% less compared to the conventional MIMO-OFDM system, and at the same time enhance the SNR of the system.

The sequential RNS moduli transmission scheme can be used for any transmission technique of MIMO - regardless of any modulation format and also with low computational complexity without side information.

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