



Free convective heat for nanofluids stagnation-point flow past a shrinking sheet in a saturated porous medium

Emad M. Abo-Eldahab^a, Rasha Adel^{a*}, Yasmin M. Saad^b

^a Department of Mathematics, Faculty of Science, Helwan University, Cairo, Egypt

^b Basic Science department, School of Engineering, Canadian International College, Giza, Egypt

ARTICLE INFO

Article history:

Received 7 February 2023

Received in revised form 8 April 2023

Accepted 20 May 2023

Available online 3 June 2023

doi:10.21608/ABAS.2023.192448.1004

Keywords: Free convection, stagnation point, saturated porous medium

ABSTRACT

The effects of porous medium saturated with nano-fluid and the free convective heat in a steady two-dimensional stagnation-point flow over a stretching or (shrinking) sheet are to be studied. The study of flow over stretching or shrinking surface very importance in industry like glass fiber and paper production. The governing partial differential equations are transformed into the ordinary differential equations by using a similarity transformation. The ordinary differential equations which are obtained are solved numerically by using the Rung-Kutta Fehlberg method (shooting technique). The numerical results which are obtained for different values of the governing parameters and their influence on the velocity and temperature profiles have been presented graphically and discussed in detail. The velocity increases with increase of Grashof number but decreases with increase of permeability parameter and shrinking/stretching parameter. The temperature within the boundary layer decreases with increase of Grashof number, permeability parameter. by assuming that the nano-particle is *Cu* with nanometer-size, its diameter less than 50 nm and its shape is the sphere and the based fluid is water.

1. Introduction

The free convection heat in laminar boundary layer flow over shrinking or stretching sheet has very important application in industries, during a lot of mechanical forming processes, such as extrusion, melt-spinning, cooling of large metallic plate in a bath, manufacture of plastic and rubber sheets, glass blowing. The previous research studied the free convection over stretching or shrinking surface, in this research produce a new addition, the presence of a porous

medium in stretching or shrinking surface. Nano-fluid is the new kind of fluid which is coined by choi [1] describes a liquid containing a suspension of submicron solid particle (diameter less than 50 nm), it called “nanoparticles”, the base of fluids is usually water, ethylene glycol and oil. In modern days, due to the requirement of the compact systems the heat dissipation rate is point of more concern. Air cooling systems and water-cooling systems are not that much effective in small systems because of their size. So, it is preferable to use nanofluids in industries for heating and cooling purposes and

Nomenclatures		Greek Letters	
u, v	Velocity components along $x-y$ axes	σ	Electrical conductivity
K	Permeability of the porous medium	μ_{nf}	Nanofluid dynamic viscosity
K_1	Permeability parameter	ρ_{nf}	Density of the nanofluid
Gr	Grashof number	α_{nf}	Thermal diffusion of the nanofluid
E	Electric field	φ	Nano particle volume fraction
g	The gravitational acceleration	ρ_f	Density of base fluid
p_r	Prandtl number	μ_f	Dynamic viscosity of water
u_w	Surface velocity	α_f	Thermal diffusion of water
u_e	Velocity of inviscid flow	ρ_s	Solid fraction
T_w	temperature of the shrinking sheet	ν_f	Kinematic viscosity
a, c	constants	c_p	Specific heat of fluid
x, y	Dimensionless coordinates	ν_{nf}	Kinematic viscosity of nanofluid
k_f	Thermal conductivity of the base fluid	η	Similarity variable
T_∞	The ambient temperature	ψ	Stream function
k_{nf}	Thermal conductivity of nanofluid	β	volumetric volume expansion coefficient of the fluid
nf	Nanofluid		
f	fluid		
s	solid		
w	condition on the sheet		
∞	condition far away from the plate		

at the same time cost can also be optimized by reducing the size of system. Nanofluid helps in improving the performance of the thermal systems by enhancing heat transfer rate. Nanofluids are widely used in various applications like as fuel, as coolant in automobiles, in medical and electronic equipment to reduce the thermal resistance. The stagnation-point flow is a flow that explains the behavior of the fluid motion near the stagnation region. This type of flow happens when the flow hits the solid surface and the fluid velocity at the stagnation-point equals zero. Recently the study on the flow past a stretching/ shrinking sheet has received more attention. In the book by Das et al [2] it is worth mentioning that while modeling the boundary layer flow and heat transfer of stretching /shrinking surface. Khan and Pop [3] studied the boundary layer flow of a nanofluid past a stretching sheet. El bashbeshy and Bazid[4] studied flow and heat transfer in a porous medium over a stretching surface with internal heat generation and suction/blowing when the surface is held at a constant temperature. Loket et al. [5] analyzed MHD stagnation-point flow from a shrinking sheet, the dual solutions existed for small values of the magnetic parameter has been found by them. The stagnation-point flow over a stretching or shrinking sheet in a nanofluid was investigated by Bachok et al. [6], showing that adding nanoparticles to a base fluid increased the skin friction and heat transfer coefficients. Recently, Narayana and Sibanda [7] investigated the laminar

flow of a nano liquid film over an unsteady stretching sheet, noticing that the effect of an increase in the nanoparticle volume fraction was to reduce the axial velocity and free stream velocity in the case of a Cu -water nano-fluid. However, the opposite appeared to be true in the case of an Al_2O_3 –water nanofluid. Heat transfer can be enhanced by employing various techniques and methodologies, such as increasing either the heat transfer surface or the heat transfer coefficient between the fluid and the surface, that allow high heat transfer rates in a small volume. Cooling is one of the most important technical challenges facing many diverse industries, including microelectronics, transportation, solid state lighting and manufacturing. There is an urgent need for new and innovative coolants with improved performance. The addition of micrometer- or millimeter-sized solid metal or metal oxide particles to the base fluids shows an increment in the thermal conductivity of resultant fluids. But the presence of milli- or micro sized particles in a fluid poses several problems, which is that micro-particles do not form a stable solution and tend to settle down, A part from the application in the field of heat transfer, nanofluids (nanometer particles in a fluid) can also be synthesized for unique magnetic, electrical, chemical, and biological applications, in addition causing erosion and clogging of the heat transfer channels. A very perfect collection of articles which, investigate the heat transfer characteristics in nanofluids [8-12] found out that in the presence of the

nanoparticles in the fluids, the effective thermal conductivity of the fluid increases appreciably and consequently enhances the heat transfer characteristics. However, in natural convection (free convection) the fluid motion occurs by natural means such as buoyancy. Since the fluid velocity associated with natural convection is relatively low, the heat transfer coefficient encountered in natural convection is also low. Convective heat transfer in nanofluids is a topic of major contemporary interest both in science and engineering. Khanafer et al. [13] analyzed the two-dimensional natural convective flow of a nanofluid in an enclosure and found that for any given Grashof number, the heat transfer rate increased as the volume fraction of nanoparticles increased. Kim et al. [14] introduced a new friction factor f to describe the effect of nanoparticles on the convective instability and heat transfer characteristics of the base fluid. Convective flow has been widely studied in the recent years due to its wide application in engineering as solar collectors, drying processes, heat exchangers, etc. (Nield and Bejan[15], Ingham and Pop[16], Vafai[17], Vadasz[18], etc.). It is well known that conventional heat transfer fluids including oil, water, and ethylene glycol mixture are poor heat transfer fluids, since the thermal conductivity of these fluids plays an important role on the heat transfer coefficient between the heat transfer medium and heat transfer surface, so that used the nanoparticles for improving heat transfer in the fluid. The convective heat transfer and thermal conductivity studies on nanofluids by numerous studies and in the review papers Buongiorno [19]; Daungthongsuk and Wong Wises [20]; Ahmad and Pop [21] and Bachok et al. [11]. Hady and Ibrahim [22] studied the effect of the presence of an isotropic solid matrix on the forced convection heat transfer rate from a flat plate to power-law non-Newtonian fluid-saturated porous medium. Cheng and Minkowycz [23] studied the problem of natural convection from a vertical flat plate whose temperature varied with power of the distance from the leading edge and embedded in a fluid-saturate porous medium. The present of this paper is to extend the work by Roslinda Nazeret al. [24] which the steady stagnation-point flow towards a shrinking sheet in a nanofluid with free convective in porous medium is studied. The three different types of nanoparticles Cu, Al_2O_3 and TiO_2 . The models in this paper involves only five parameters, namely the Prandtl number Pr , the nanoparticle volume fraction ϕ , the stretching/shrinking parameter λ , the permeability parameter K_1 and finally Grashof number Gr .

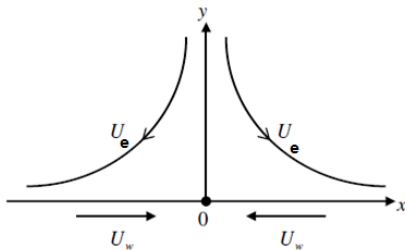


Fig. 1: Physical model and coordinate system.

2. Formulation of the problem

Consider the nano-fluid is steady, two dimensions and free convection stagnation-point flow over a stretching/shrinking sheet in a saturated porous medium. The stretching (or shrinking) velocity $u_w(x)$ and the velocity of ambient fluid is $u_e(x)$ are assumed to vary linearly from the stagnation-point, i.e., $u_w(x) = cx$ and $u_e(x) = ax$, where c, a are constants and x is the coordinate measured along the stretching surface. Note that if $c > 0$ and $c < 0$ correspond to stretching and shrinking sheets. The flow takes place at $y \geq 0$, where y is the coordinate measured normal to the stretching surface. A steady uniform stress leading to equal and opposite force is applied along the x -axis so that the sheet is stretched keeping to fixed origin (Fig. 1). The basic steady conservation of mass, momentum and energy equations for a nanofluid in Cartesian coordinates x and y are (see Tiwari and Das [25]).

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{-1}{\rho_{nf}} \frac{\partial p}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \frac{\mu_{nf}}{\rho_{nf} K} u + (1 - \phi_{\infty}) g \beta (T - T_{\infty}) \tag{2}$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \frac{-1}{\rho_{nf}} \frac{\partial p}{\partial y} + \frac{\mu_{nf}}{\rho_{nf}} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{\mu_{nf}}{\rho_{nf} K} v \tag{3}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \tag{4}$$

Subject to the boundary conditions

$$v = 0, \quad u = u_w(x) = cx, \quad T = T_w \text{ at } y = 0 \tag{5}$$

$$u \rightarrow u_e(x) = ax, \quad v = 0, \quad T \rightarrow T_{\infty} \text{ at } y \rightarrow \infty$$

where u and v are the velocity components along the x and y axes, T is the temperature of the nanofluid, it is assumed that the constant temperature of the shrinking sheet is T_w and that of the ambient nano-fluid is T_{∞} , where $T_w > T_{\infty}$ (heated stretching sheet). p is the pressure, K is the permeability of the porous medium, ρ_f, μ, β are the density, viscosity, and volumetric volume expansion coefficient of the fluid, the gravitational acceleration is denoted by g , α_{nf} is the thermal diffusion of the nanofluid, ρ_{nf} is the effective density of the nanofluid, and μ_{nf} is the effective viscosity of the nanofluid, which are given by (Oztop and Abu-Nada [9])

$$\mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}},$$

$$\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_s,$$

$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}$$

$$(\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s,$$

$$\frac{k_{nf}}{k_f} = \frac{(k_s + 2k_f) - 2\varphi(k_f - k_s)}{(k_s + 2k_f) + \varphi(k_f - k_s)}$$

where φ is nanoparticle volume fraction, $(\rho C_p)_{nf}$ is the heat capacity of the nanofluid, k_{nf} is the thermal conductivity of the nanofluid, k_f and k_s are the thermal conductivities of the fluid and the solid fractions, respectively, ρ_f and ρ_s are the densities of the fluid and solid fraction.

$$\eta = \sqrt{\frac{a}{\nu_f}} y, \quad \psi = \sqrt{a\nu_f} x f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad (7)$$

Using Eqn. (7) obtained and applying similarity solution to Eqns. (1)-(4) with the boundary conditions (5).

where η is the similarity variable and θ is the dimensionless temperature, and ν_f is the kinematic viscosity of the base, $\psi(x, y)$ is the physical stream function and is defined in the usually way as $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$, which identically satisfy Eq. (1). On substituting (6) and (7) into Eqns. (2)-(4), then obtain following nonlinear ordinary differential equations:

$$\frac{1}{(1 - \varphi)^{2.5} \left(1 - \varphi + \varphi \frac{\rho_s}{\rho_f}\right)} f''' + f f'' + 1 - f'^2 - \frac{K_1}{(1 - \varphi)^{2.5} \left(1 - \varphi + \varphi \frac{\rho_s}{\rho_f}\right)} f' + Gr\theta = 0, \quad (8)$$

$$\frac{1}{Pr} \left(\frac{\left(\frac{k_{nf}}{k_f}\right)}{1 - \varphi + \varphi \frac{(\rho C_p)_s}{(\rho C_p)_f}} \right) \theta'' + f\theta' - f'\theta = 0 \quad (9)$$

Subject to the boundary conditions:

$$f(0) = 0, f'(0) = \lambda, \theta(0) = 1, f'(\infty) = 1, \theta(\infty) = 0 \quad (10)$$

Where primes denote differentiation with respect to η , $Pr = \frac{\nu_f}{\alpha_f}$ is the prandtl number, $K_1 = \frac{\nu_f}{ka}$ is the permeability parameter, $Gr = \frac{(1-\varphi_\infty)\beta g(T_w - T_\infty)}{a^2 x}$ is the Grashof number, and $\lambda = \frac{c}{a}$ is the stretching parameter when $\lambda > 0$ and a shrinking parameter when $\lambda < 0$. The pressure p can be determined from Eq. (3) as

$$p = p_0 - \frac{\rho_{nf} a^2 x^2}{2} - \frac{\rho_{nf} \nu^2}{2} + \frac{\mu_{nf} \partial v}{\partial y}, \quad (11)$$

Where p_0 is the stagnation pressure, it is worth mentioning Eqns. (8) and (9) reduce to those first derived by Wang [26] when the nanoparticles volume fraction parameter $\varphi = 0$ (regular Newtonian fluid), and $Pr = 1$.

3. Result and discussion

The system of ordinary differential equations (8) and (9) with boundary condition (10) was solved numerically using Rung-Kutta Fehlberg method (shooting technique). Consider Cu –water nanofluid. The thermophysical properties of the nanofluids used in this paper are given in Table 1 [24].

The results of this research are excellent agreement with Roslinda Nazer et al. [24]. The Prandtl number considered in this paper is $(Pr) = 1$, the influence of (Gr) Grashof number, (K_1) permeability parameter, (φ) nanoparticles volume fraction and (λ) shrinking/stretching parameter on the velocity $f'(\eta)$ and temperature $\theta(\eta)$.

In Figs. (2.1,2.2) show that, the influence of Prandtl number (Pr) on the velocity $f'(\eta)$ and temperature $\theta(\eta)$. When $Pr = 1, 3, 5, 6.2$, with $Gr = 4, K_1 = 0.1, \lambda = -1.2, \varphi = 0.1$. It is observed that with increasing in the Prandtl number (Pr) decreases both the velocity $f'(\eta)$ and temperature $\theta(\eta)$.

Table 1. Thermophysical properties of fluid and nanoparticles

Properties	Fluid (water) <i>cu</i>	<i>TiO₂</i>	<i>Al₂O₃</i>
c_p (J/kg K)	4,179	385	686.2 765
ρ (kg/m ³)	997.1	8,933	4250 3,970
K (W/mK)	0.613	400	4040

Figs. (3.1,3.2) shows that the variation of $f'(\eta)$ with different value of Grashof number (Gr) which $Gr = 1, 2, 4, 6$ with $Pr = 1, K_1 = 0.1, \lambda = -1.2, \varphi = 0.1$. The effect of Grashof number Gr on the velocity $f'(\eta)$ observed that the increasing in Grashof number leads to increases in velocity $f'(\eta)$, but there is no obvious effect in temperature profiles.

Figs. (4.1,4.2) shows that, the effect of (K_1) permeability parameter on the profiles of velocity $f'(\eta)$, by taking various values of $K_1 = 0.1, 0.2, 0.3, 1$ with $Pr = 1, Gr = 4, \lambda = -1.2, \varphi = 0.1$. It is observed that, increases the permeability parameter K_1 decreases the velocity $f'(\eta)$, and there is no obvious effect in temperature profiles $\theta(\eta)$.

Figs. (5.1,5.2) illustrates that the influence of nanoparticles volume fraction (φ), when $\varphi = 0, 0.1, 0.2, 0.3$ with $Pr = 1, Gr = 4, \lambda = -1.2, K_1 = 0.1$. The nanoparticles volume fraction (φ) increases, resulting in increases the velocity $f'(\eta)$, and there is no obvious effect on the temperature profiles.

Figs. (6.1,6.2) shows that, the effect of the shrinking/stretching parameter(λ) on the velocity $f'(\eta)$. The shrinking parameter takes various values $\lambda = 0, -0.25, -0.5, -0.75, -1.2$ and $\lambda = 1, Gr = 4, \varphi = 0.1, K_1 = 0.1$. It is observed that increase of the shrinking/stretching parameter(λ) decreases the velocity $f'(\eta)$ and there is no obvious effect on temperature profiles.

Conclusions

This study focused on the presence of porous medium in stretching or shrinking surface and its effects on velocity and temperature. The nano-fluid of steady two-dimensional laminar stagnation-point flow has been studied theoretically and numerically with the influence of free convection. The governing partial differential equations are transformed to ordinary differential equations by using similarity transformation, the obtained equations are solved numerically by using the Rung-Kutta Fehlberg method (shooting technique). Numerical results are obtained for of Prandtl number (Pr), (Gr) Grashof number, (K_1) permeability parameter, (φ) nanoparticles volume fraction and (λ) shrinking/stretching parameter and their effect on the velocity $f'(\eta)$ and temperature $\theta(\eta)$ as well as the velocity and temperature profiles for some values of the governing parameters, namely

1) The velocity within the boundary layer increases with increase of (Gr) Grashof number and (φ) nanoparticles volume fraction but decreases with increase of (K_1) permeability parameter, and (λ) shrinking/stretching parameter and Prandtl number (Pr).

2) The temperature within the boundary layer decreases with increase of Prandtl number (Pr), and there is no obvious effect on temperature profiles with (Gr) Grashof number, (φ) nanoparticles volume fraction, (K_1) permeability parameter, and (λ) shrinking/stretching parameter.

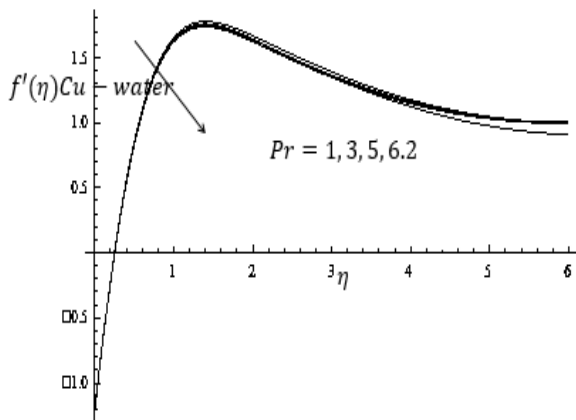


Fig. 2.1: velocity profile $f'(\eta)$ of Prandtl number Pr when $Gr = 4, K_1 = 0.1, \lambda = -1.2, \varphi = 0.1$.

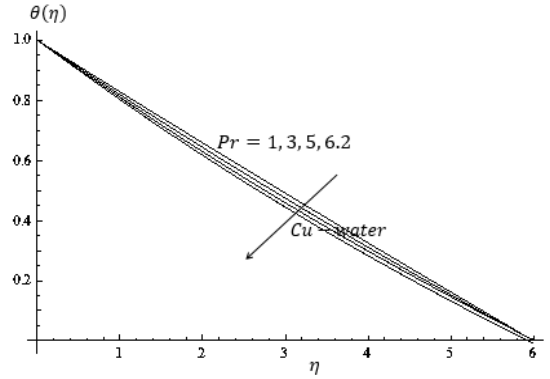


Fig. 2.2: Temperature profiles $\theta(\eta)$ of Prandtl number Pr when $Gr = 4, K_1 = 0.1, \lambda = -1.2, \varphi = 0.1$.

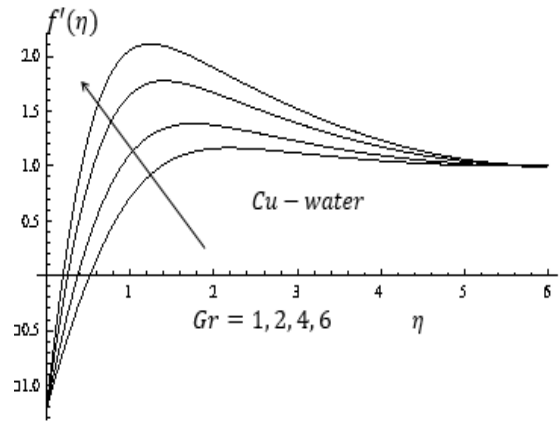


Fig. 3.1: Velocity profiles $f'(\eta)$ of Grashof number Gr when $Pr = 1, K_1 = 0.1, \lambda = -1.2, \varphi = 0.1$.

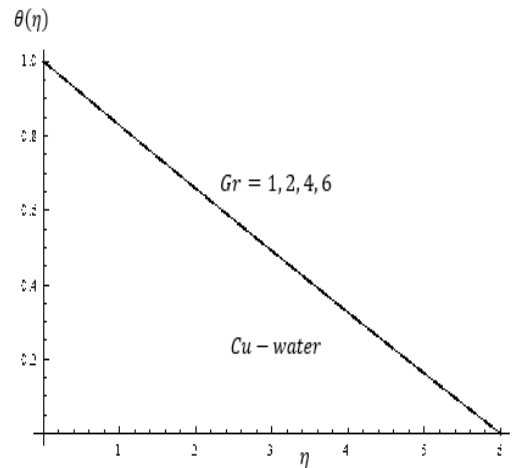


Fig. 3.2: Temperature profiles $\theta(\eta)$ of of Grashof number Gr when $Pr = 1, K_1 = 0.1, \lambda = -1.2, \varphi = 0.1$.

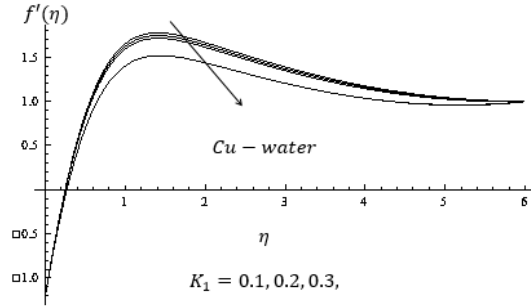


Fig. 4.1 Velocity profiles $f'(\eta)$ of permeability parameter K_1 when $Pr = 1$, $Gr = 4$, $\lambda = -1.2$, $\omega = 0.1$.

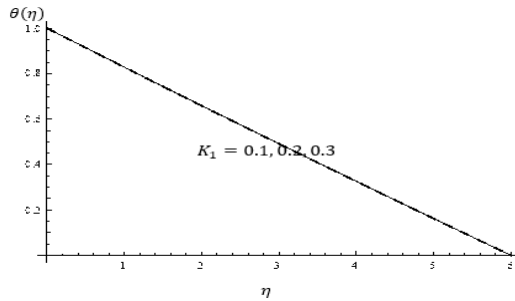


Fig. 4.2: The temperature profiles $\theta(\eta)$ of permeability parameter K_1 when $Pr = 1$, $Gr = 4$, $\lambda = -1.2$, $\omega = 0.1$.

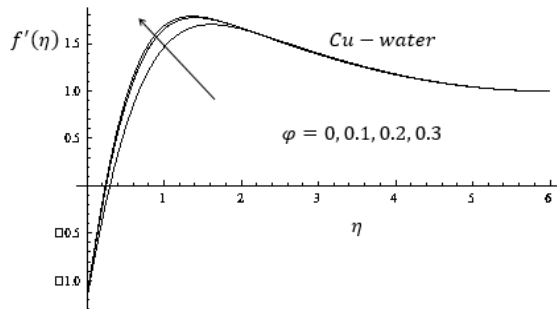


Fig. 5.1: Velocity profiles $f'(\eta)$ of nanoparticles volume fraction ϕ when $Pr = 1$, $Gr = 4$, $\lambda = -1.2$, $K_1 = 0.1$.

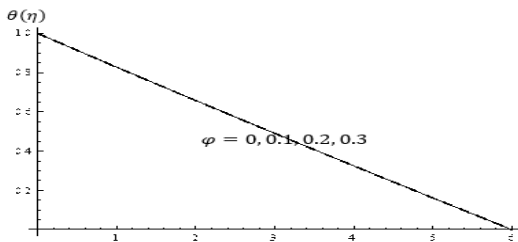


Fig. 5.2: The temperature profiles $\theta(\eta)$ of nanoparticles volume fraction ϕ when $Pr = 1$, $Gr = 4$, $\lambda = -1.2$, $K_1 = 0.1$

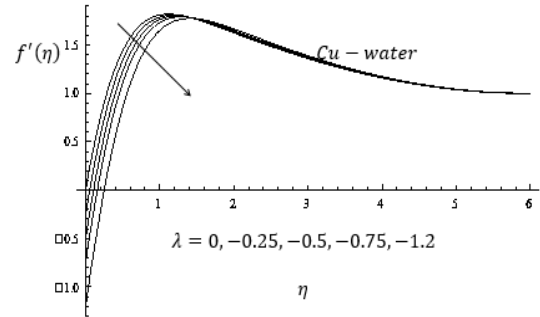


Fig. 6.1: Velocity profiles $f'(\eta)$ of the shrinking parameter λ when $Pr = 1$, $Gr = 4$, $\omega = 0.1$, $K_1 = 0$.

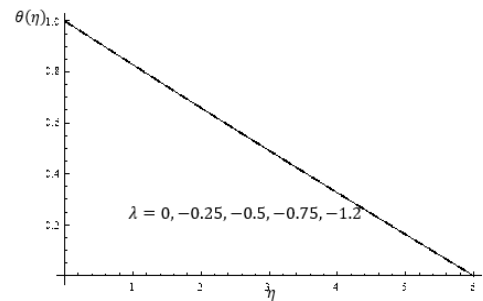


Fig. 6.2: Temperature profiles $f'(\eta)$ of the shrinking parameter λ when $Pr = 1$, $Gr = 4$, $\omega = 0.1$, $K_1 = 0$.

References

1. Choi, S.: Enhancing Thermal Conductivity of Fluids with Nanoparticles. In: Siginer, D.A., Wang, H.P., (Eds.), Developments and Applications of Non-Newtonian Flows. ASME FED-231, MD-66, 99- 105(1995).
2. [2] Das SK, Choi SUS, Yu W, Pradeep T. Nanofluids: Science and Technology N. J.:Wiley; 2007.
3. [3] Das SK, Choi SUS, Yu W, Pradeep T: Nanofluids: Science and Technology N.J.. Wiley; 2007.
4. [4] Khan W A., I. Pop: Boundary-layer Flow of a Nanofluid Past a Stretching Sheet. International Journal of Heat and Mass Transfer (2010);53:2477-2483.
5. [5] Elbashaeshy E M A, Bazid M A A . Heat Transfer in a Porous Medium Over a Stretching Surface with Internal Heat Generation and Suction or Injection. Appl. Math. Comp. (2004);158: 799–807.
6. [6] Lok, Y Y, Ishak A, Pop, I: MHD Stagnation-Point Flow Towards a Shrinking Sheet. Int. J. Numer. Methods Heat Fluid Flow. (2011); 21: 61-72
7. Bachok, N, Ishak, A, Pop, I: Stagnation Point Flow Over a Stretching/Shrinking Sheet in a Nanofluid. Nanoscale Res. Lett. (2011); 6(1), 623
8. Narayana, M, Sibanda, P: Laminar Flow of a Nanoliquid Film Over an Unsteady Stretching Sheet. Int. J. Heat Mass Transf. (2012);55: 7552-7560

9. Abu-Nada E: Application of Nanofluids for Heat Transfer Enhancement of Separated Flows Encountered in a Backward Facing Step. *Int. J. Heat Fluid Flow*. 2008; 29:242-249.
10. Oztop HF, Abu-Nada E: Numerical Study of Natural Convection in Partially Heated Rectangular Enclosures Filled with Nanofluids. *Int. J. Heat Fluid Flow*. 2008; 29:1326-1336.
11. Nield DA, Kuznetsov AV: The Cheng-Minkowycz Problem for Natural Convective Boundary-Layer Flow in a Porous Medium Saturated by a Nanofluid. *Int. J. Heat Mass Transfer*. 2009; 52:5792-5795.
12. Bachok N, Ishak A, Pop I: Boundary-Layer Flow of Nanofluids Over a Moving Surface in a Flowing Fluid. *Int. J. ThermSci*. 2010; 49:1663-1668.
13. Buongiorno J.: Convective Transport in Nanofluids. *ASME. J. Heat Transfer*. 2006; 128:240-250.
14. K.Khanafer, K. Vafai, M. Lightstone: Buoyancy-Driven Heat Transfer Enhancement Utilizing Nanofluids, *Int. J. Heat Mass Transfer* (2003); 46:3639-3653.
15. J.Kim, Y.T.Kang, C.K.Choi: Analysis of Convective Instability and Heat Transfer Characteristics of Nanofluids, *Phys. Fluids*. (2004) ;16:2395-2401.
16. D.A.Nield, A.Bejan: *Convection in Porous Media*, Third edition. Springer, New York. 2006.
17. D.B. Ingham, I. Pop (Eds.): *Transport Phenomena in Porous Media*, Vol.III, Elsevier, Oxford. 2005.
18. K. Vafai (Ed), *Handbook of Porous Media*, second ed., Taylor&Francis, New York, 2005.
19. P. Vadasz, *Emerging Topics in Heat and Mass Transfer in Porous Media*, Springer, New York, 2008.
20. J. Buongiorno, W. Hu: Nanofluid Coolants for Advanced Nuclear Power Plants, Paper no. 5705, Proceedings of ICAPP '05, Seoul. 2005 May 15-19,.
21. W. Daungthongsuk, S. Wongwises: A Critical Review of Convective Heat Transfer Nanofluids, *Renewable and Sustainable Energy Review* (2007);11: 797-817.
22. S. Ahmad, I. Pop: Mixed Convection Boundary Layer Flow From a Vertical Flat Plate Embedded in a Porous Medium Filled with Nanofluids, *International Communications in Heat and Mass Transfer*. (2010);37:987-991.
23. Hady, F.M. and Ibrahim, F.S: Forced Convection Heat Transfer on a Flat Plate Embedded in Porous Media for power-law fluids. *Trans. Porous Media*. (1997);28:125-134.
24. Cheng, P. and Minkowycz, W.J: Free Convection about a Vertical Flat Plate Embedded in a Porous Medium with Application to Heat Transfer From a Dike. *Journal of Geophysical Research*. (1977); 82:2040- 2044
25. Roslinda Nazar, Mihaela Jaradat, Norihan M.A., I. Pop: Stagnation-Point Flow Past a Shrinking Sheet in a Nanofluid. *Central European Journal of Physics*. 2011;9(5)..1195-1202.
26. R.J. Tiwar, M.K.Das, *Int. J..Heat Mass Tran*.50, 2002 (2007).
27. C.Y.Wang, *Int . J. Nonlin. Mech*. 43,377(2008).