Design, Modelling, and Control of an Inertial Stabilized Platform (ISP)

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Abstract-A gun stabilizer is a device that facilitates aiming an artillery piece by compensating for the motion of the platform on which it is mounted. We aim to learn and try to design and control our own stabilizer. A dual axis inertial stabilized platform (ISP) that has 2 degrees of freedom (2-DOF) about both Y and Z axis as a turret was developed using CAD software then followed by designing a model for a tracked vehicle, to make the model similar to a real tank. Kinematic modelling and dynamic modelling are discussed and explained to make sure that the ISP can withstand maximum loading conditions and to select needed equipment and devices. A simulation is made on the model using Matlab Simscape Multibody to see how it reacts to different input signals. PID controller is then used to enhance performance of the model, in our case to reduce overshoots and rise time.

Keywords- Tank, Stabilizer, Line of Sight, LOS, ISP, 2-DOF, PID Controller, Simulation, Simscape Multibody.

I. Introduction

The LOS or line of sight is an imaginary line drawn between an observer and an object (or between two objects) [1]. ISPs or Inertial stabilized platforms are used to stabilize sensors, cameras, weapon systems. Their main goal is to hold or control the line of sight of a one object relative to another one. ISPs have many applications **Fig. *1*** including surveillance cameras, telescopes, drones, missile guidance, target tracking, weapon systems and military applications (as in our application which is gun-turret control system) [1] [2].

Most Tanks in service at the Egyptian army have stabilizers which enables Tank gun to be fixed on the target, no matter what disturbances are subjected to the tank. We aim to design, manufacture, and control of our own stabilizer to learn more about stabilizers and increase our experience in that field.

Fig. 1: ISP Applications

II. 3D-CAD Model

The 3D-CAD model design of a 2-DOF ISP used to stabilize the LOS of a tank turret about two perpendicular axes, (elevation and azimuth) about the Y and Z axes, respectively. It was conducted using Inventor software aiming to simulate the shape of a real tank turret **Fig. *2*** and **Fig. *3***, taking in account some considerations such as easy assembly and disassembly, total mass and size, symmetrical mass distribution of the rotating masses about the rotation axes to decrease their principal moments of inertia.

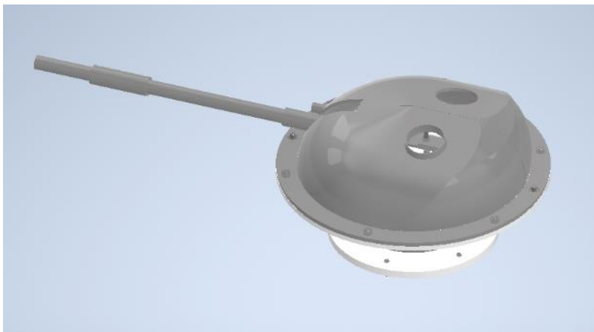

Fig. 2:3D-CAD model (turret only)

Fig. 3: 3D-CAD model (whole vehicle)

2.1 Construction:

The elevating parts are a barrel assembly (made of two parts), a counter weight, and an IMU MPU-9250 which are all driven by a Servo Motor Towerpro (360) 2.2 kg.cm Metal Gears "MG90S".

The rotating parts consists of a two-part plate which carries the elevating parts, 9V-battery, an Arduino MEGA, and covered with a turret like cover which are all driven by a High Torque Servo Motor (15 kg.cm - Metal Gear).

2.2 Design Constraints:

The design limitations for the rotational motion is 360 degrees, and -5 to 20 degrees in elevation (as in real tank turrets).

2.3 Stress Analysis:

a stress analysis was conducted using Autodesk Inventor software. It was conducted for both the turret and vehicle chassis to check stresses and deflection as indicated in **Fig. *4***,**Fig. *5***,**Fig. *6***, and **Fig. *7***.

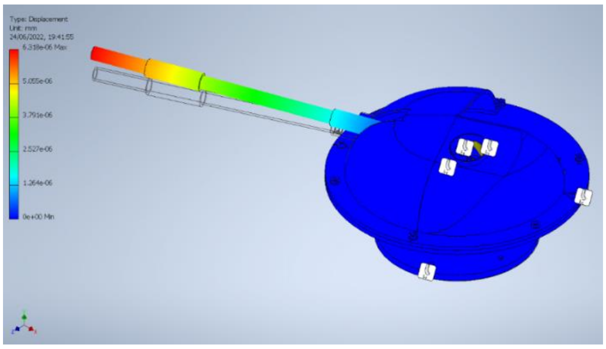

Fig. 4:Turret stress analysis against deflection

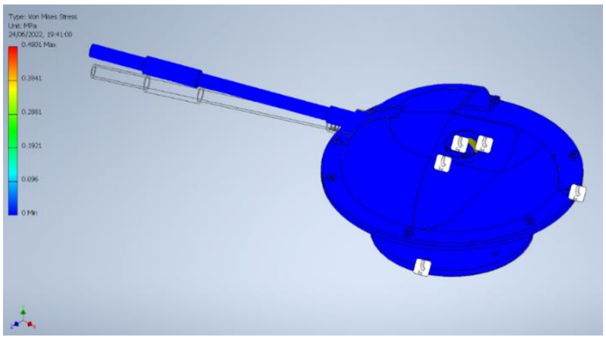

Fig. 5: Turret stress analysis against stresses

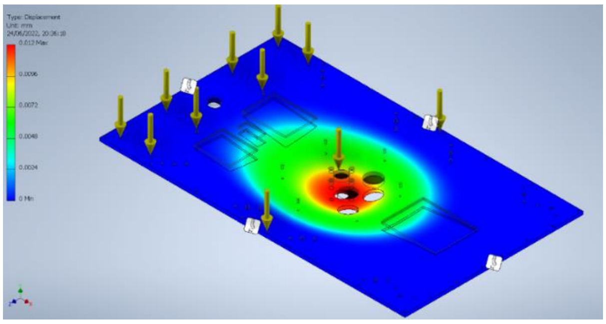

Fig. 6: Vehicle base stress analysis against deflection

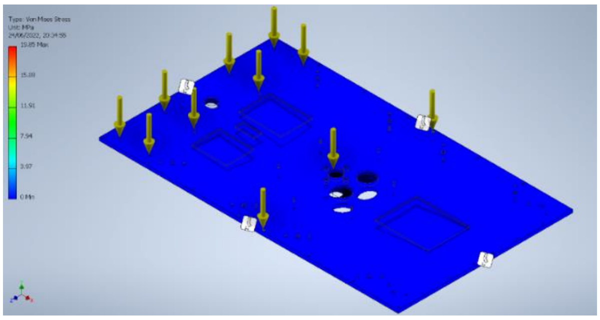

Fig. 7: Vehicle base stress analysis against stresses

As shown in the previous stress analysis it was shown that the model is safe and can withstand maximum possible loading conditions (torques and forces).

III. Mathematical Modelling

For the designed Turret Rotation system, four reference frames as shown in **Fig. *8***, they must be considered namely:

1. A fixed reference frame with respect to the earth OF.

2. The carrier or platform reference frame OP.

3. The outer gimbal (rotating parts) reference frame O_O.

4. The inner gimbal (elevating parts) reference frames that carrying Barrel O_I.

The relation between any two frames is expressed by the sum of a rotation and translation matrices. Our interest is only on the angles, angular velocities and angular accelerations, the translation matrix has no influence on how the angular velocity of one coordinate frame relates to another frame and it can be neglected from the calculations [3].

We will discuss relations between different frames to be able to study the model kinematics and dynamics later, this is achieved by using Euler coordinate system (Euler matrices).

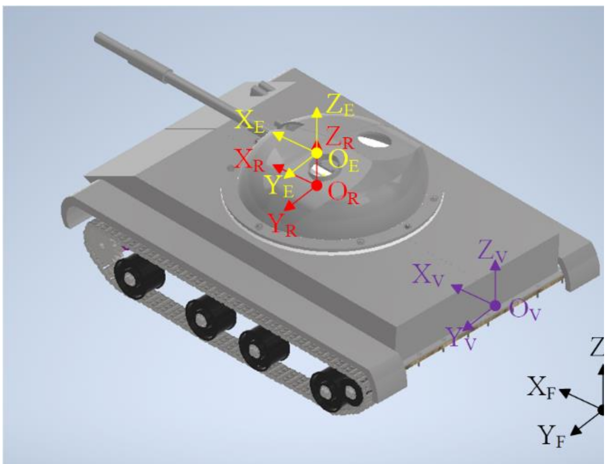

Fig. 8: Model coordinate frames

IV. Kinematic modelling

4.1 Euler Matrices:

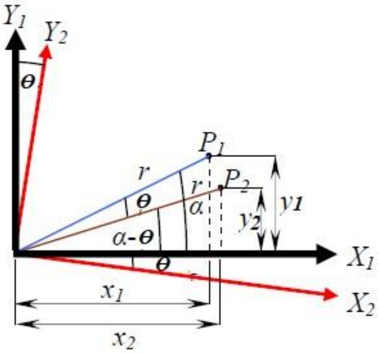

Fig. 9: Rotation about z-axis *[3]*

Fig. *9* represents a coordinate system X1-Y1-Z1, having a Point P1 of the position vector $[x1\ y1\ 0]^T$ and making an angle α with the X1-axis. If the coordinate system rotates about the Z-axis in a clockwise direction (from Y to X) with an angle ϴ, the new coordinate system will be X2-Y2-Z1, and the point P1 will turn to be P2 with the new position vector $[x2\ y2\ 0]^T$ and angle (α- ϴ) with the X1-axis. The equations describe this system are:

$$\cos\alpha = \frac{x_1}{r}$$

$$\sin\alpha = \frac{y_1}{r}$$

$$\cos(\alpha - \Theta) = \frac{x_2}{r}$$

$$\sin(\alpha - \Theta) = \frac{y_2}{r}$$

$$\cos(\alpha - \Theta) = \cos\alpha\,\cos\Theta + \sin\alpha\,\sin\Theta$$

$$\frac{x_2}{r} = \frac{x_1}{r}\cos\Theta + \frac{y_1}{r}\,\sin\Theta$$

$$x_2 = x_1\,\cos\Theta + \,y_1\,\sin\Theta$$

$$\sin(\alpha - \Theta) = \sin\alpha\,\cos\Theta - \cos\alpha\,\sin\Theta$$

$$\frac{y_2}{r} = \frac{y_1}{r}\cos\Theta - \frac{x_1}{r}\,\sin\Theta$$

$$y_2 = y_1\,\cos\Theta - \,x_1\,\sin\Theta$$

$$z_2 = \,z_1$$

In matrix form:

$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} C\theta & S\theta & 0 \\ -S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$$

$$E_z = \begin{bmatrix} C\theta & S\theta & 0 \\ -S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

4.2 Angular velocities of different frames:

The inertia reference frame O_F, has a fixed position on the earth with respect to the fixed stars. So, its angular rate ω_F is:

$$\omega_F = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

During the motion, the carrier platform coordinate frame OP, rotates about XF, YF, and ZF of the fixed inertia frame with angular rates of P, Q, and R respectively. So, the relative angular velocity of the platform frame about the inertia reference frame is ω_{PF}:

$$\omega_{PF} = \begin{bmatrix} P \\ Q \\ R \end{bmatrix}$$

the angular velocity of the carrier platform ω_P as observed from the platform reference frame OP, is:

$$\omega_P = E.\ \omega_F + \omega_{PF} = \begin{bmatrix} P \\ Q \\ R \end{bmatrix}$$

Then, the outer frame rotates about its Z-axis with an angle θ, results in relative angular rate between the outer and the platform frames ω_{OP}:

$$\omega_{OP} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta} \end{bmatrix}$$

from eqns 1,4 the angular velocity of the outer gimbal platform ω_O as observed from its reference frame O_O, is:

$$\omega_O = E_z.\ \omega_P + \omega_{OP} = \begin{bmatrix} C\theta & S\theta & 0 \\ -S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} P \\ Q \\ R \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} PC\theta + QS\theta \\ -PS\theta + QC\theta \\ R + \dot{\theta} \end{bmatrix}$$

Finally, the Inner frame rotates about the Y-axis with an angle ϕ and angular rate $\dot{\phi}$. Results in relative angular rate between the inner and the outer frames ω_{IO}:

$$\omega_{IO} = \begin{bmatrix} 0 \\ \dot{\phi} \\ 0 \end{bmatrix}$$

from eqns 2,4,5The angular velocity of the inner gimbal ω_I as observed from its reference frame OI, is:

$$\omega_I = E_y.\ \omega_O + \omega_{IO}$$

$$= \begin{bmatrix} (PC\theta + QS\theta)C\phi + (R + \dot{\theta})S\phi \\ -PS\theta + QC\theta + \dot{\phi} \\ -(PC\theta + QS\theta)S\phi + (R + \dot{\theta})C\phi \end{bmatrix}$$

4.3 Angular accelerations of different frames:

The angular acceleration of the platform, outer, and Inner gimbals (α_P, α_O, α_I) are calculated by differentiating the angular velocities ω_P, ω_O, and ω_I:

$$\alpha_P = \dot{\omega_P} = \begin{bmatrix} \dot{P} \\ \dot{Q} \\ \dot{R} \end{bmatrix}$$

$$\alpha_O = \dot{\omega_O} = E_z \dot{\omega_P} + \dot{E_z} \omega_P + \dot{\omega_{OP}}$$

$$\dot{E_z} = \begin{bmatrix} -S\theta & C\theta & 0 \\ -C\theta & -S\theta & 0 \\ 0 & 0 & 0 \end{bmatrix} \dot{\theta}$$

$$\dot{\omega_{OP}} = \ddot{\theta} = \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta} \end{bmatrix}$$

$$\alpha_O = \dot{\omega_O} = \begin{bmatrix} (\dot{Q} - P\dot{\theta})S\theta + (\dot{P} + Q\dot{\theta})C\theta \\ -(\dot{P} + Q\dot{\theta})S\theta + (\dot{Q} - P\dot{\theta})C\theta \\ \dot{R} + \ddot{\theta} \end{bmatrix}$$

$$\alpha_I = \dot{\omega_I} = E_y \dot{\omega_O} + \dot{E_y} \omega_O + \dot{\omega_{IO}}$$

$$\dot{E_y}=\begin{bmatrix}-S\phi & 0 & C\phi\\ 0 & 0 & 0\\ -C\phi & 0 & -S\phi\end{bmatrix}\dot{\phi}$$

$$\dot{\omega_{IO}}=\ddot{\phi}=\begin{bmatrix}0\\ \ddot{\phi}\\ 0\end{bmatrix}$$

$$\alpha_I=\dot{\omega}_I=$$

$$\begin{bmatrix}-\dot{\phi}S\phi(PC\theta+QS\theta)+C\phi(\dot{P}C\theta+Q\dot{\theta}C\theta+\dot{Q}S\theta-P\dot{\theta}S\theta)+\dot{\phi}C\phi(R+\dot{\theta})-S\phi(\dot{R}+\ddot{\theta})\\ -PS\theta-Q\dot{\theta}S\theta+\dot{Q}C\theta-P\dot{\theta}C\theta+\ddot{\phi}\\ -\dot{\phi}C\phi(PC\theta+QS\theta)+S\phi(-\dot{P}C\theta+P\dot{\theta}S\theta-\dot{Q}S\theta-Q\dot{\theta}C\theta)-\dot{\phi}S\phi(R+\dot{\theta})+C\phi(\dot{R}+\ddot{\theta})\end{bmatrix}$$

It is clear from the above equations that the angular velocity of the outer gimbal can't be analysed explicitly without considering the effect of the relative motion between the outer frame and the platform frame. Also, the relative rotation between the outer and inner frames must be considered when dealing with the inner gimbal rotation.

V. Dynamic Model Derivations

From the rigid body dynamics considering a fixed reference frame X Y Z with basis unit vectors i, j, and k, and another rotating reference frame X_r Y_r Z_r with basis unit vectors i_r, j_r, and k_r. If the rotation is with angular velocity ω about the Z-axis of the fixed frame, the new position of the rotating frame is:

$$\begin{bmatrix}x_r\\ y_r\\ z_r\end{bmatrix}=\begin{bmatrix}C\omega t & S\omega t & 0\\ -S\omega t & C\omega t & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}xC\omega t+yS\omega t\\ -xS\omega t+yC\omega t\\ z\end{bmatrix}$$

Where:

$i_r=(C\omega t, S\omega t, 0)$, $j_r=(-S\omega t, C\omega t, 0)$, $k_r=(0,0,1)$

$$\frac{d}{dt}i_r=\omega(-S\omega t, C\omega t, 0)=\omega\, j_r$$

$$\frac{d}{dt}j_r=\omega(-C\omega t, -S\omega t, 0)=-\omega\, i_r$$

Since: k x i = j , k x j = i , $\hat{\omega}=[0\ 0\ \omega]=\omega k$

$$\omega\, j_r=\omega\, k_r \times i_r=\hat{\omega}\times i_r=\frac{d}{dt}i_r$$

$$-\omega\, i_r=\omega\, k_r \times j_r=\hat{\omega}\times j_r=\frac{d}{dt}j_r$$

Using the concept of conservation of the angular momentum, where I the mass moment of inertia: L=Iω , $L=l_x i+l_y j+l_z k$

$$\frac{d}{dt}L=\frac{dl_x}{dt}i+\frac{di}{dt}l_x+\frac{dl_y}{dt}j+\frac{dj}{dt}l_y+\frac{dl_z}{dt}k+\frac{dk}{dt}l_z$$

$$\frac{d}{dt}L=\frac{dl_x}{dt}i+\frac{dl_y}{dt}j+\frac{dl_z}{dt}k+[\omega\times(l_x i+l_y j+l_z k)]$$

$$\frac{d}{dt}L=\left(\frac{dL}{dt}\right)_r+\omega\times L$$

where $(dL/dt)_r$, the rate of change of the angular momentum L as observed in the rotating coordinate system. Hence, the derivative of the angular momentum L in a rotating reference frame has two components, one from the explicit time dependence due to motion of the rotating frame itself, and another from the frame's carrier rotation. During the rotation of the outer gimbal, a torque is applied to rotate the Z-axis of the gimbal. Since the inner gimbal is also rotating about its Y-axis, the outer gimbal will affect the inner by a torque around Z and Y-axes of the inner gimbal. Applying the above relations to the relative rotation between the outer and inner gimbals frames, the moments M exerted on the inner gimbal frame by the outer frame can be calculated from the rate of change of the angular momentum L of the outer frame as observed from the inner frame. Since L and ω are changing with time during the rotation, M can't be solved with them. So, we change to a coordinate frame fixed with the rotating body using L_r and ω.

$$M=\dot{L}=\dot{L_r}+\omega\times L_r=I\dot{\omega}+\omega\times I\omega$$

$$\omega\times I\omega=\begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix}\times\begin{bmatrix}I_x\omega_x\\ I_y\omega_y\\ I_z\omega_z\end{bmatrix}=\begin{vmatrix}i & j & k\\ \omega_x & \omega_y & \omega_z\\ I_x\omega_x & I_y\omega_y & I_z\omega_z\end{vmatrix}$$

$\omega \times I\omega = (I_z\omega_y\omega_z - I_y\omega_y\omega_z)i - (I_z\omega_x\omega_z - I_x\omega_x\omega_z)j + (I_y\omega_x\omega_y - I_x\omega_x\omega_y)k$

$M_x = I_x\dot{\omega}_x + (I_z - I_y)\ \omega_y\omega_z$

$M_y = I_y\dot{\omega}_y + (I_x - I_z)\ \omega_x\omega_z$

$M_z = I_z\dot{\omega}_z + (I_y - I_x)\ \omega_x\omega_y$

where M_x, M_y, and M_z are the moments applied on the inner frame due to the effect of both the rotation of the inner frame itself, and the coupling between the inner and outer frames. All these equations are assuming that the rotating body has its axes parallel to the body's principle axes of inertia. Generally, the dynamics of rotating 3-axes gimbals are given by Euler's equations:

1- For nonsymmetrical, inhomogeneous mass:

$T_x = \alpha_x I_x + \omega_y\omega_z(I_z - I_y) - (\omega_y^2 - \omega_z^2)\ I_{yz} - (\omega_x\omega_y + \dot{\omega_z})I_{xz} + (\omega_x\omega_z - \dot{\omega_y})I_{xy}$

$T_y = \alpha_y I_y + \omega_x\omega_z(I_x - I_z) - (\omega_z^2 - \omega_x^2)\ I_{xz} - (\omega_z\omega_y + \dot{\omega_x})I_{xy} + (\omega_x\omega_y - \dot{\omega_z})I_{yz}$

$T_z = \alpha_z I_z + \omega_x\omega_y(I_y - I_x) - (\omega_x^2 - \omega_y^2)\ I_{xy} - (\omega_x\omega_z + \dot{\omega_y})I_{yz} + (\omega_x\omega_z - \dot{\omega_x})I_{xz}$

2- If the rotating object's coordinate frame is aligned with its center of mass, it is called principle axes of inertia. Then, I_{xx}, I_{yy}, and I_{zz} are called principle moments of inertia. Where, $I_{xy} = I_{yx} = I_{xz} = I_{zx} = I_{yz} = I_{zy} = 0$

$T_x = \alpha_x I_x + \omega_y\omega_z(I_z - I_y)$

$T_y = \alpha_y I_y + \omega_x\omega_z(I_x - I_z)$

$T_z = \alpha_z I_z + \omega_x\omega_y(I_y - I_x)$

3- if the rotating part is symmetric about:

X-axis : $T_x = \alpha_x I_x$ where $I_y = I_z$

Y-axis : $T_y = \alpha_y I_y$ where $I_x = I_z$

Z-axis : $T_z = \alpha_z I_z$ where $I_y = I_x$

In our model it is the second case where the center of rotation is at the assembly center of gravity $I_{xy} = I_{yx} = I_{xz} = I_{zx} = I_{yz} = I_{zy} = 0$ (for both elevating and rotating parts) so we will be using the following equations in our dynamic modeling and torque calculations:

$T_x = \alpha_x I_x + \omega_y\omega_z(I_z - I_y)$

(7) $T_y = \alpha_y I_y + \omega_x\omega_z(I_x - I_z)$

(8) $T_z = \alpha_z I_z + \omega_x\omega_y(I_y - I_x)$

VI. Torque Calculations of the Inner Gimbal

the moments (M_I) applied on the inner gimbal frame due to the effect of rotation are given by:

$$M_I = I_I\dot{\omega}_I + \omega_I \times I_I\omega_I$$

The sum of the torques about the inner gimbal (T_I)is:

$$T_I = T_E - M_I - T_g - T_f$$

T_E ……. Motor elevation torque about the Y-axis

T_g ……Gravity torques about each axis of the inner gimbal

T_f ……. Friction and cable restraint torques expressed as:

$$T_{Ify} = k_{If}\dot{\phi} + T_{Ifn} + k_{Ic}\phi + T_{Icn}$$

k_{If} ……..Viscous friction coefficient

T_{Ifn}……..Nonlinear friction torques

k_{Ic}……...Cable restraint coefficient

T_{Icn}…..…Nonlinear cable restraint torques.

$$T_I = \begin{bmatrix} 0 \\ T_E \\ 0 \end{bmatrix} - \begin{bmatrix} M_{Ix} \\ M_{Iy} \\ M_{Iz} \end{bmatrix} - \begin{bmatrix} T_{Igx} \\ T_{Igy} \\ T_{Igz} \end{bmatrix} - \begin{bmatrix} 0 \\ T_{fy} \\ 0 \end{bmatrix}$$

$$T_{Iy} = T_E - I_{Iy}\dot{\omega}_{Iy} - (I_{Ix} - I_{Iz})\,\omega_{Ix}\omega_{Iz} - T_{Igy} - T_{Ify}$$

Since the inner gimbal motor only controls the rotation about the Y-axis, so, to stabilize the LOS in the elevation direction, it is required to null the torques about the Y-axis:

$$\sum T_{Iy} = 0$$

$$T_E = I_{Iy}\dot{\omega}_{Iy} + (I_{Ix} - I_{Iz})\,\omega_{Ix}\omega_{Iz} + T_{Igy} + T_{Ify}$$

the last two terms (T_{Igy} + T_{Ify}) are too small compared to the first two, for simplicity they will be neglected and their effect can be dealt with as disturbances' noise that can be diminished by the feedback servo control system. [3] Hence:

$$T_E = I_{Iy}\dot{\omega}_{Iy} + (I_{Ix} - I_{Iz})\,\omega_{Ix}\omega_{Iz}$$

Assuming the carrier base frame is fixed then $P=Q=R=\dot{P} = \dot{Q} = \dot{R}=0$

$$\theta = 0{:}360°$$

$$\phi = -5{:}20°$$

$$\dot{\omega}_{Oz} = \ddot{\theta} = 15°/s^2$$

$$\dot{\omega}_{Iy} = \ddot{\phi} = 15°/s^2$$

$$\omega_{Ix} = -\dot{\theta} S\phi$$

$$\omega_{Iz} = \dot{\theta} C\phi$$

$$\dot{\theta}^2 = \dot{\theta}_o{}^2 + 2\ddot{\theta}\theta = 2\ddot{\theta}\theta$$

$$\omega_{Ix}\omega_{Iz} = -\dot{\theta}^2 S\phi C\phi = -0.5\dot{\theta}^2 S2\phi = -\ddot{\theta}\theta\, S2\phi$$

$$\dot{\theta}_o = 0 \text{ initial angular rotation velocity}$$

$$T_E = I_{Iy}\ddot{\phi} + (I_{Iz} - I_{Ix})\,\ddot{\theta}\theta\, S2\phi$$

By substituting in the above equation at different elevation and rotation angles we get the following curves:

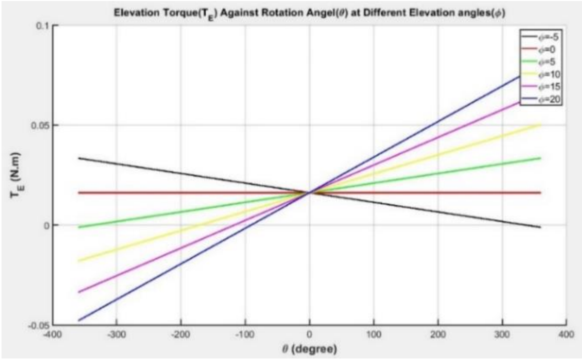

Fig. 10: Required motor elevation torque at different elevation angles against rotation angle

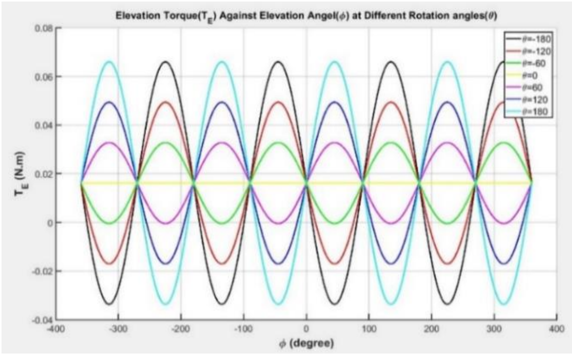

Fig. 11: Required motor elevation torque at different rotation angles against elevation angle

From **Fig. *10*** and **Fig. 11** we choose the motor that will drive the inner gimbal (elevating parts) to be Servo Motor Micro (180) 2.2 kg.cm Metal Gears (FS90MG) which has a maximum torque of 0.21582 N.m

Torque Calculations for The Outer Gimbal

Due to the rotation of the inner gimbal with respect to the outer one about the Y-axis, the inner gimbal moments M_I induce a torque $(M_I)_O$ from the inner gimbal on the outer one. To transform the torque done by the inner gimbal from the inner gimbal reference coordinate frame to the outer gimbal reference coordinate frame, we use the Euler's angular transformation about the Y-axis. [3]

$$M_I = E_y(M_I)_O$$

$$(M_I)_O = Ey^{-1} M_I$$

$(M_I)_O$ The inner gimbals' torque as observed from the outer gimbals' reference frame.

Since E_y is an orthogonal matrix: $Ey^T = Ey^{-1}$

$$(M_I)_O = Ey^T M_I$$

$$= \begin{bmatrix} C\phi & 0 & S\phi \\ 0 & 1 & 0 \\ -S\phi & 0 & C\phi \end{bmatrix} \begin{bmatrix} M_{Ix} \\ M_{Iy} \\ M_{Iz} \end{bmatrix} = \begin{bmatrix} M_{Ix}C\phi + M_{Iz}S\phi \\ M_{Iy} \\ -M_{Ix}S\phi + M_{Iz}C\phi \end{bmatrix}$$

$$(M_I)_{Oz} = -(I_{Ix}\dot{\omega}_{Ix} + (I_{Iz} - I_{Iy})\,\omega_{Iy}\omega_{Iz})\,S\phi + (I_{Iz}\dot{\omega}_{Iz} + (I_{Iy} - I_{Ix})\,\omega_{Ix}\omega_{Iy})\,C\phi$$

The same as the inner gimbal:

$$M_{Oz} = I_{Oz}\dot{\omega}_{Oz} + (I_{Oy} - I_{Ox})\,\omega_{Ox}\omega_{Oy} + (M_I)_{Oz}$$

where M_o: moments applied on the outer gimbal frame due to the effect of rotation. The sum of the torques about the outer gimbal T_O and the motor rotation torque T_R are:

$$T_O = \begin{bmatrix} 0 \\ 0 \\ T_R \end{bmatrix} - \begin{bmatrix} M_{Ox} \\ M_{Oy} \\ M_{Oz} \end{bmatrix} - \begin{bmatrix} T_{Ogx} \\ T_{Ogy} \\ T_{Ogz} \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ T_{fz} \end{bmatrix}$$

Since the outer gimbal motor only controls the rotation about the Z-axis, the only used equation will be:

$$T_{Oz} = T_R - I_{Oz}\dot{\omega}_{Oz} - (I_{Oy} - I_{Ox})\,\omega_{Ox}\omega_{Oy} - (M_I)_{Oz} - T_{Ogz} - T_{fz}$$

For eliminating the torques about the Z-axis of the LOS:

$$\sum T_{Oz} = 0$$

$$T_R = I_{Oz}\dot{\omega}_{Oz} + (I_{Oy} - I_{Ox})\,\omega_{Ox}\omega_{Oy} - (I_{Ix}\dot{\omega}_{Ix} + (I_{Iz} - I_{Iy})\,\omega_{Iy}\omega_{Iz})\,S\phi + (I_{Iz}\dot{\omega}_{Iz} + (I_{Iy} - I_{Ix})\,\omega_{Ix}\omega_{Iy})\,C\phi + T_{Ogz} + T_{fz}$$

As for the inner gimbal, the last two terms of this equation will be neglected. [3] To determine the required torque for the outer gimbal, we assume again the carrier base frame is fixed then $P = Q = R = \dot{P} = \dot{Q} = \dot{R} = 0$

$$\theta = 0{:}\,360°$$

$$\phi = -5{:}\,20°$$

$$\dot{\omega}_{Oz} = \ddot{\theta} = 15°/s^2$$

$$\omega_{Iy} = \dot{\phi}$$

$$\dot{\phi}^2 = \dot{\phi}_o^{\,2} + 2\ddot{\phi}\phi$$

$$\omega_{Iy} = \dot{\phi} = \sqrt{2\ddot{\phi}\phi}$$

$$\omega_{Ix} = -\dot{\theta}S\phi$$

$$\dot{\theta}^2 = \dot{\theta}_o^{\,2} + 2\ddot{\theta}\theta$$

$$\dot{\theta}_o = 0$$

$$\omega_{Ix} = -\dot{\theta}S\phi = \sqrt{2\ddot{\theta}\theta}\,S\phi$$

$$\omega_{Iz} = \dot{\theta}C\phi = \sqrt{2\ddot{\theta}\theta}\,C\phi$$

$$\dot{\omega}_{Ix} = -\dot{\theta}\dot{\phi}C\phi - \ddot{\theta}S\phi = -\sqrt{2\ddot{\phi}\phi}\sqrt{2\ddot{\theta}\theta}\,C\phi - \ddot{\theta}S\phi$$

$$\dot{\omega}_{Iz} = -\dot{\theta}\dot{\phi}S\phi + \ddot{\theta}C\phi = -\sqrt{2\ddot{\phi}\phi}\sqrt{2\ddot{\theta}\theta}S\phi + \ddot{\theta}C\phi$$

$$T_R = 15I_{Oz} - ((-30C\phi\sqrt{\theta\phi} - 15S\phi)I_{Ix} + (I_{Iz} - I_{Iy})\,30C\phi\sqrt{\theta\phi})\,S\phi + (I_{Iz}(-30S\phi\sqrt{\theta\phi} + 15C\phi) - (I_{Iy} - I_{Ix})\,30S\phi\sqrt{\theta\phi})\,C\phi$$

$$T_R = 15I_{Oz} + 15S2\phi\sqrt{\theta\phi}(2I_{Ix} - 2I_{Iz}) + 15(I_{Ix}S^2\phi + I_{Iz}C^2\phi)$$

By substituting in the above equation at different elevation and rotation angles we get the following curves:

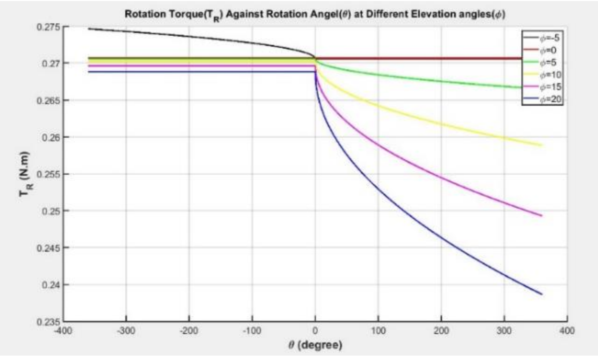

Fig. 12: Required motor rotation torque at different elevation angles against rotation angle

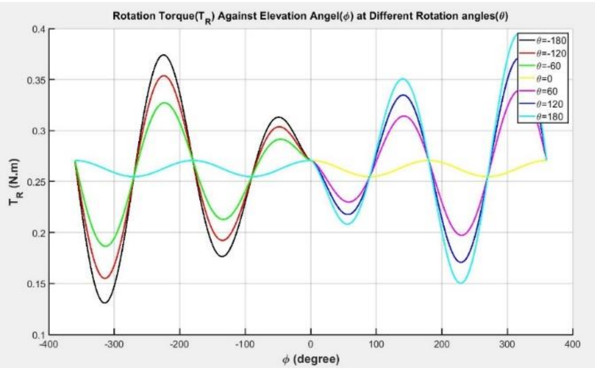

Fig. 13: Required motor rotation torque at different rotation angles against elevation angle

From **Fig.** ***12*** and **Fig.** ***13*** we choose the motor that will drive the outer gimbal (rotating parts) to be Servo Motor Continuous (360) 14 kg.cm Metal Gears (FS5113R) which has a maximum torque of 1.3734 N.m

Control System Design

Using SimscapeMultibody the CAD model was linked to Matlab to see how the model reacts with inputs of rotation and elevation angles and how much error the model will produce.

We try different configurations to control our model according to relations between input and output and different control methods used.

8.1 Closed-Loop Control System:

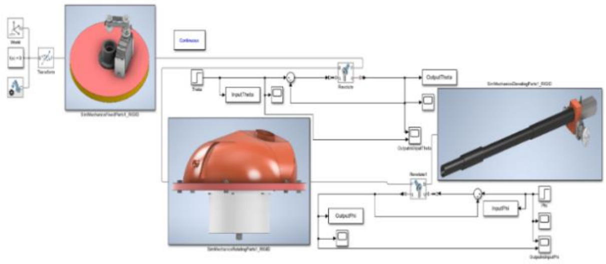

Fig. 14: Closed-loop control system

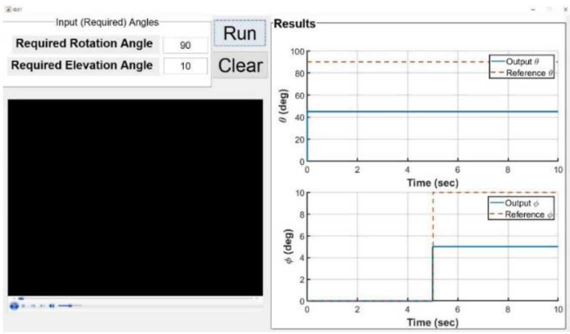

Fig. 15: Closed-loop control system output vs input

Fig. ***15*** shows there is a considerable error (difference between the reference input and output) which must be reduced, which can be done using different control methods.

8.2 Closed-Loop PID Control System:

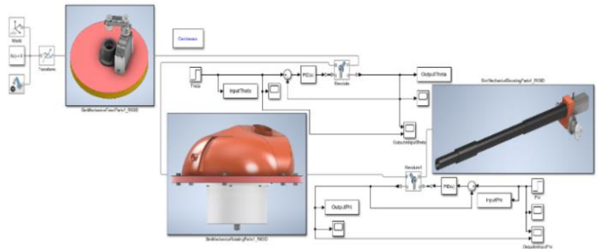

Fig. 16: Closed-loop PID control system

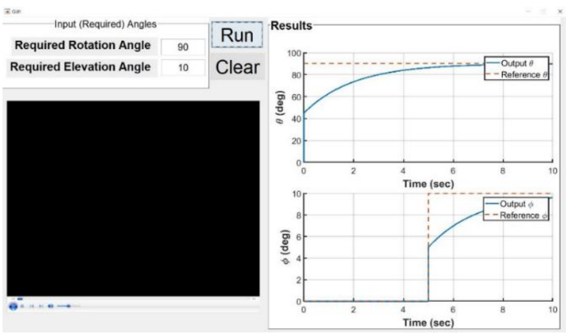

Fig. 17: Closed-loop PID control system output vs input

Fig. *17* shows that PID controller will not get the best result instantly instead it should be tuned (the values of proportional, integral, and derivative constants must be changed according to the desired response of the application to obtain the best results).

By using Matlab auto-tuner, we adjust response time and transient behavior sliders, then Matlab changes proportional, integral, and derivative constants automatically to obtain the required results as shown in **Fig. *18*** and **Fig. *19***.

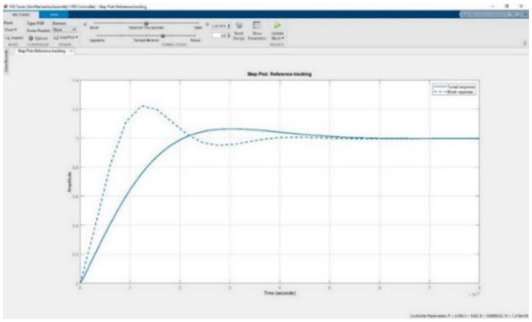

Fig. 18: Results before using PID auto-tuner

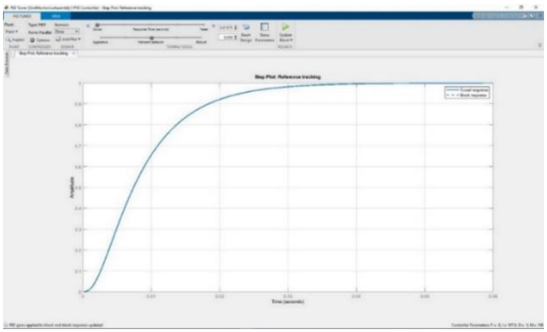

Fig. 19: Results after using PID auto-tuner

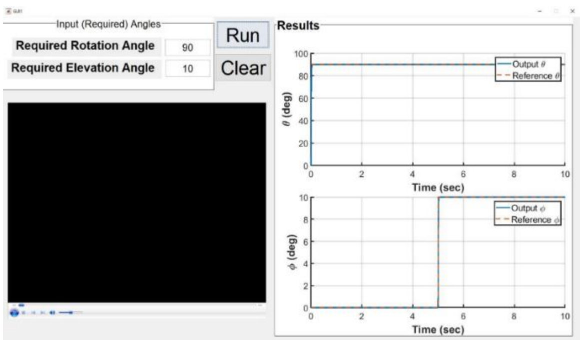

Fig. 20: Control system results after using PID auto-tuner

VII. Conclusion

In this study, a 2-DOF model is designed. The mathematical model is presented with its kinematics and dynamics equations. A detailed analysis is performed to show the relationship between the dynamics of inner and outer gimbals in addition to a detailed stress analysis. We ended up using a servo motors of torques 0.21582 and 1.3734 N.m for moving the elevating and rotating parts respectively. Then after applying different control methods we concluded that the best proposed control method is to use PID controller and adjust its values via Matlab PID auto-tuner as it showed in **Fig. *20*** that the steady state error is greatly reduced (small value that can be neglected), and the response is enhanced heavily. Finally, this study will be further used and developed in real-time implementation of our model.

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