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The behavior of the vibration for nonlinear system with mix active controller

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ABSTRACT **: Vibration control can be divided into two main categories : passive control and active control. Active control is provided by the negative linear velocity and acceleration feedback controller. The vibration of a nonlinear dynamical system is reduced by the Negative linear velocity and acceleration feedback controller in the worst resonance case ($\Omega = \omega$).This system has one degree of freedom, which contains the third order of nonlinear terms, as well as an external force. The response of the nonlinear system is determined using the multiple scale perturbation technique. Frequency response equations are used to test the stability of the numerical solution. The impacts of different parameters on the vibrating system is studied and reported on. Numerically, we investigated the time histories of the system before and after using Negative linear velocity acceleration feedback controller using Runge – Kutta of the fourth order and studied the response curve with detuning parameter σ . Finally, the approximate and numerical solutions accord well.**

KEYWORDS: Vibration control; Negative linear velocity feedback; Resonance case; Stability.

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I. INTRODUCTION

Mechanical and civil constructions' dynamic responses to high-amplitude vibrations are frequently destructive and unwanted. The oscillator is the most harmful of these motions. Vibration can occur in any mechanical system due to a variety of variables. Material fatigue, structure damage, failure, decrease of system performance, and higher noise levels are all common consequences of vibration. These effects are usually most noticeable around the system's natural frequencies**.** In many technological and physical applications, the Duffing oscillator is one of the most essential models [1] and [2] it is useful in Electric circuits, plasma oscillations, optical stability, and buckling beams all benefit from it. [3] investigated the usefulness of the feedback gain on the bifurcation point using a nonlinear time delayed feedback controller to control the vibrations of a Van der Pol oscillator. [4] explained the behavior of the ring of coupled Van der Pol oscillators' stable and unstable responses numerically. If the stability conditions are not satisfied, the amplitude of the Van der Pol oscillator increases, according to[5-7]applying the method of multiple time scale MTSPT of hybrid Van der Pol–Duffing–Rayleigh oscillator for modelling the lateral walking force on a rigid floor and Investigations on the bifurcation of a noisy, quantitative analysis of the nonlinear behavior of the forced and self-excited beam was studied. [8]. modulated and displayed the Van der Pol – Duffing – Rayleigh oscillator bifurcation analysis. Suppression of vibrations of a forced and self-excited nonlinear beam by using positive position feedback controller PPF.The vibrations of the Van der Pol – Duffing – Rayleigh oscillator were restrained using nonlinear integral positive position feedback controller NIPPF, Nonlinear integral positive position feedback (NIPPF) was introduced as a novel method that combines the advantages of both integral resonant controllers (IRC) and positive position feedback controllers (PPF) to control nonlinear systems. Moreover, one of its main advantages is to reduce vibrations in a short time as shown in [9].

The stability of the system is investigated using both frequency response curves and phase-plane trajectories. All possible resonance cases are extracted. The effects of different parameters of the system are studied numerically as shown in Sayed M. [10]. The vibration of nonlinear coupled Van der Pol oscillators has been researched and solved under external and parametric excitations. examined at the frequency response equations for this system's primary resonance case. The study is focused on the stability of the periodic solution and the nonlinear behavior of this system as shown in [11]. Modeled as a self-sustained oscillator capable of generating: I self-sustained motion; (ii) a lateral periodic force signal; and (iii) a stable limit cycle. The suggested oscillator is a modification of the hybrid Van der Pol Duffing Rayleigh oscillator, as presented in [12] by including a nonlinear hardening term. To produce both the odd and even harmonics observed in experimentally observed force measurements, certain additional nonlinear components and parameters are introduced, as indicated in [13-14].

The procedure of multiple time scale disruption is used to overcome nonlinear differential equations and achieve approximate solutions. Multiple scale approaches are one of the most extensively utilized perturbation techniques. In contrast to some other strategies, which only produce the steady state solution, this method produces both transient and steady state solutions by [15]. Many types of controllers are used for suppressing the vibrations of different non-linear dynamical systems. the vibrations of many vibrating systems [16-18] and [19] has been studied. Because of the time delayed and active controls springiness in controlling many vibrating systems, many papers used time delay for suppressing the vibrations of non-linear systems. Studied vibrations analysis and dynamic responses of a Hybrid Rayleigh-Van der Pol- Duffing oscillator with proportional derivative controller (PD) that used to control the vibration of the main system. The goal of using the PD controller is to increase the system stability by improving control since it has an ability to predict the future error of the framework response. This control method makes stability increased, maximum peak overshoot decreased, settling time decreases. This controller is applied to recover the framework transient response [20-21].

Finally, The Negative linear velocity and acceleration feedback controller used to suppress the vibration of the main system excited by an external force. To examine the system's time histories before and after adopting The Negative linear velocity and acceleration feedback controller, we used Runge – Kutta of the fourth order. The approximate solution obtained applying the method of multiple scales up to second approximation. The stability of the system investigated at the primary resonance case. The behavior of the system without and with The Negative linear velocity and acceleration feedback controller is simulated numerically. The influence of some chosen coefficient is illustrated. The settlement between numeric and approximate solution is offered. The efficacy of different parameters and behavior of the device was demonstrated using the MATLAB program.

Table (1): list of symbols

$x, \dot{x}, \ddot{x}$	Displacement, velocity and acceleration of main system respectively.
μ	The damping coefficients of main system.
ω	The natural frequency of main system.
F	External excitation force amplitude frequency of external force.
$\alpha_i \, (i = 1, 2, 3)$	Nonlinear coefficients of the main system.
λ	The gain of control signal
ε	Small perturbation parameter

II. MATERIALS AND METHODS

1. Closed loop model

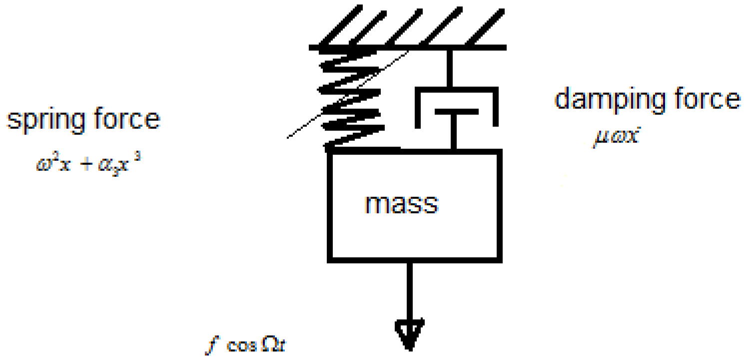

Fig. 1. The model of duffing system oscillator

Kumar (2018), presented the equation of motion of the nonlinear dynamical system by:

$$\ddot{x} + 2\mu\omega\dot{x} + \alpha_1\dot{x}x^2 + \alpha_2\dot{x}^3 + \alpha_3 x^3 + \omega^2 x = f\cos\Omega t - \lambda R_C(t) - GR_f(t) \tag{1}$$

Take scale $\alpha_i = \dfrac{\hat{\alpha}_i}{\varepsilon}, f = \varepsilon^2\hat{f}, \lambda = \varepsilon\hat{\lambda}, G = \varepsilon\hat{G}$ .we insert the negative linear velocity and acceleration feedback controller which is linked to the main system through a control low. We cheek the controls signal as $R_C = \dot{x}, R_f(t) = \ddot{x}$ so the closed loop of controller system equation is

$$\ddot{x} + 2\varepsilon\mu\omega\dot{x} + \frac{\hat{\alpha}_1}{\varepsilon}\dot{x}x^2 + \frac{\hat{\alpha}_2}{\varepsilon}\dot{x}^3 + \frac{\hat{\alpha}_3}{\varepsilon}x^3 + \omega^2 x = \varepsilon^2\hat{f}\cos\Omega t - \varepsilon\hat{\lambda}\dot{x} - \varepsilon\hat{G}\ddot{x} \tag{2}$$

Perturbation techniques

apply multiple scale method Nayfeh. A. H (1979). we get first order approximate solution for equation (2) in the form:

$$x(t;\varepsilon) = \varepsilon x_1(T_0,T_1) + \varepsilon^2 x_2(T_0,T_1) + O(\varepsilon^3) \tag{3}$$

where, the fast scale is T_0 and the slow scale is $T_1 = \varepsilon t$. The derivatives are converted as:

$$\frac{d}{dt} = D_0 + \varepsilon D_1 + \ldots, \frac{d^2}{dt^2} = D_0^2 + 2\varepsilon D_0 D_1 + \ldots, D_j = \frac{\partial}{\partial T_j} \quad (\mathrm{j} = 0,1) \tag{4}$$

Subset equations [3]and [4] into equation [2], then equating the coefficient of same power of ε we get the following differential equations:

$O(\varepsilon)$

$$\left[D_0^{\;2} + \omega^2\right]x_1 = 0 \tag{5}$$

$O(\varepsilon^2)$

$$\begin{aligned}\left(D_0^{\;2} + \omega^2\right)x_2 &= -2D_0D_1x_1 - 2\mu\omega D_0 x_1 - \hat{\alpha}_1(D_0x_1)x_1^{\;2} - \hat{\alpha}_2(D_0x_1)^3 \\ &- \hat{\alpha}_3 x_1^{\;3} + \hat{f}\cos(\Omega\,\mathrm{t}) - \hat{\lambda}D_0x_1 - \hat{G}D_0^{\;2}x_1\end{aligned} \tag{6}$$

The general solution of homogenous differential equation (5) takes the following forms:

$$x_1(\mathrm{T}_0,\mathrm{T}_1) = A(\mathrm{T}_1)e^{i\omega T_0} + \bar{A}(\mathrm{T}_1)e^{-i\omega T_0} \tag{7}$$

From equation (7) into equation (6), we get:

$$\left(D_0^{\,2}+\omega^2\right)x_2=\frac{\hat{f}}{2}e^{i\Omega t_0}-\hat{\lambda}i\,\omega Ae^{i\omega T_0}+\hat{G}\,\omega^2Ae^{i\omega T_0}-2i\,\omega DAe^{i\omega T_0}-2i\,\mu\omega^2Ae^{i\omega T_0}$$
$$-\hat{\alpha}_1 i\,\omega A^2\bar{A}e^{i\omega T_0}-3\hat{\alpha}_2 i\,\omega^3A^2\bar{A}e^{i\omega_1 T_0}-3\hat{\alpha}_3A^2\bar{A}e^{i\omega_1 T_0} \quad (8)$$
$$+(-\hat{\alpha}_1 i\,\omega A^3+\hat{\alpha}_2 i\,\omega^3A^3-\hat{\alpha}_3A^3)e^{3i\omega T_0}+CC$$

The particular solutions of equations (8), after eliminating the secular terms take the followings forms:

$$x_2=\frac{-\hat{\alpha}_1 i\,\omega A^3+\hat{\alpha}_2 i\,\omega^3A^3-\hat{\alpha}_3A^3}{-8\omega^2}e^{3i\omega_1 T_0}+\frac{\hat{f}}{2(\omega^2-\Omega^2)}e^{i\Omega T_0} \quad (9)$$

2. Periodic Solution

From the second approximation at the worst resonance case $\Omega\cong\omega$ we introduced the detuning parameter σ to examine the stability of the system at the primary resonance case, as the following

$$\Omega=\omega+\sigma$$
(10)

Appending equation (10) into the secular terms in eq. (8) then the solvability condition takes the form:

$$-2i\,\omega DA-2i\,\mu\omega^2A-\hat{\alpha}_1 i\,\omega A^2\bar{A}-3\hat{\alpha}_2 i\,\omega^3A^2\bar{A}-3\hat{\alpha}_3A^2\bar{A}-\hat{\lambda}i\,\omega A+\hat{G}\,\omega^2A+\frac{\hat{f}}{2}e^{i\sigma T_1}=0$$
(11)

To elucidate the solution of (11) we take A the polar form as:

$$A=\frac{1}{2}\mathrm{a}\,\mathrm{e}^{i\theta}\quad ,DA=\frac{1}{2}\left(\mathrm{a}'+ia\theta'\right)\mathrm{e}^{i\theta}$$
(12)

where a the steady state amplitudes of the motion of the system ,and ϕ are the phases of the motion. Which ($\phi=\sigma T_1-\theta$) elucidate (12) into (11) we get the following amplitude – phase modulating equations:

$$a'=-\mu\omega a-\frac{1}{8}\hat{\alpha}_1\mathrm{a}^3-\frac{3}{8}\hat{\alpha}_2\omega^2\mathrm{a}^3-\frac{1}{2}\hat{\lambda}a+\frac{\hat{f}}{2\omega}\sin\phi$$
(13)

$$a\theta'=\frac{3}{8\omega}\hat{\alpha}_3\mathrm{a}^3-\frac{1}{2}\hat{G}a\omega-\frac{\hat{f}}{2\omega}\cos\phi$$
(14)

Back to the main system parameters, we have the following system of differential equations:

$$\dot{a}=-\mu\omega a-\frac{1}{8}\alpha_1\mathrm{a}^3-\frac{3}{8}\alpha_2\omega^2\mathrm{a}^3-\frac{1}{2}\lambda a+\frac{f}{2\omega}\sin\phi$$
(15)

$$\dot{\phi}=\sigma-\frac{3}{8\omega}\alpha_3\mathrm{a}^2+\frac{1}{2}G\,\omega+\frac{f}{2\omega a}\cos\phi$$
(16)

where $a'=\frac{\dot{a}}{\varepsilon}$ the performance of the control law will be evaluated by calculating the equilibrium solutions of (15) and (16) examining their stability as a function in the parameters $\mu,\hat{\alpha}_1,\hat{\alpha}_2,\hat{\alpha}_3,\hat{\lambda},\sigma$ and f.

4. Fixed point solution

The steady-state solution happens when, $\frac{da}{dt}=\frac{d\phi}{dt}=0$ so, the steady state solution is given by,

$$\frac{f}{2\omega}\sin\phi=\mu\omega a+\frac{1}{8}\alpha_1\mathrm{a}^3+\frac{3}{8}\alpha_2\omega^2\mathrm{a}^3+\frac{1}{2}\lambda a$$

(17) $$\frac{f}{2\omega}\cos\phi = \frac{3}{8\omega}\alpha_3 a^3 - a\sigma - \frac{1}{2}G\,\omega a$$

(18) Squaring then adding both sides of equations (17) and (18) to obtain the following equation:

$$\left(\mu\omega a + \frac{1}{8}\alpha_1 a^3 + \frac{3}{8}\alpha_2\omega^2 a^3 + \frac{1}{2}\lambda a\right)^2 + \left(\frac{3}{8\omega}\alpha_3 a^3 - a\sigma - \frac{1}{2}G\,\omega a\right)^2 = \frac{f^2}{4\omega^2}$$

(19) Equations (19) is the frequency-response equations that used to describes the steady state solutions of the behavior of the system for the practical case i.e.($a \neq 0$).

5. Equilibrium solution of a fixed point

To determine the stability of the equilibrium solution, the eigenvalues of the Jacobian matrix on the right-hand side of equations (15) and (16) were analyzed. If the real part of each eigenvalue is negative, the corresponding equilibrium solution is asymptotically stable. If the real component of any eigenvalue is positive, the corresponding equilibrium solution is unstable. While moving to improve the stability of the steady state solution. We must investigate the behavior of tiny deviations from steady-state solutions in order to determine the stability requirements. As a result, we assume

$$a = a_1 + a_0, \quad \phi = \phi_1 + \phi_0$$
$$\dot{a} = \dot{a}_1, \quad \dot{\phi} = \dot{\phi}_1$$

(20)

where a_0, ϕ_0 satisfy (15) and (16) and a_1, ϕ_1 are perturbations which are assumed to be small compared to a_0, ϕ_0. Substituting equation (20) into equations (15)–(16), expanding for small a_1, ϕ_1, and keeping linear terms in a_1, ϕ_1, we get

$$\dot{a}_1 = \left(-\mu\omega - \frac{3}{8}\alpha_1 a_0^2 - \frac{9}{8}\alpha_2\omega^2 a_0^2 - \frac{1}{2}\lambda\right)a_1 + \left(\frac{f}{2\omega}\cos\phi_0\right)\phi_1$$

(21)

$$\dot{\phi}_1 = \left(\frac{\sigma}{a_0} - \frac{9\alpha_3}{8\omega}a_0 + \frac{1}{2a_0}G\,\omega\right)a_1 - \left(\frac{f}{2\omega a_0}\sin(\phi_0)\right)\phi_1$$

(22)

For the above system's solution be stable, the real parts of its Eigen-values must be negative.

III. RESULTS AND DISCUSSION

Numerical Consequence

To examine the results of the system numerically, we used "Ode 45" package in MATLAB program. Also, we investigate the stability of the main system using the multiple scale method and the influence of different parameters on the behavior of controlled system was illustrated. We introduced a comparison between the approximate solution which obtained from the multiple scale method and the numerical one. We used the following parameters values

$$\omega{=}1;\mu{=}0.1;\alpha_1{=}0.5;\alpha_2{=}0.5;\alpha_3{=}0.05;e{=}1;f{=}0.08;\lambda{=}0.5;G{=}1.2;$$

Time History

We showed the time history of uncontrolled system at primary resonance in Figure 2. We can see from this figure that the system's response is approximately 0.36. The vibrations were decreased by 94.7. This means that the efficiency of the negative linear velocity and acceleration feedback controller E_a is about **8** (E_a= the amplitude of uncontrolled system/the amplitude of controlled system). after utilizing the negative linear velocity and acceleration feedback controller, and the amplitude of the vibrating system was reduced from about 0.36 to about 0.05. Figure 3 illustrates the influence of the main system parameters (damping coefficient μ and nonlinearities coefficient $\alpha_1, \alpha_2, \alpha_3$). The amplitude of the main system is monotonic decreasing in the damping coefficient μ and nonlinearities coefficient $\alpha_1, \alpha_2, \alpha_3$, as shown in this figure3. More increasing of the damping coefficient leads to saturation phenomena. the system might be needing a control. After using negative linear velocity and acceleration feedback controller, the main system amplitude reduced to reach 0.05 as represented on Fig4.

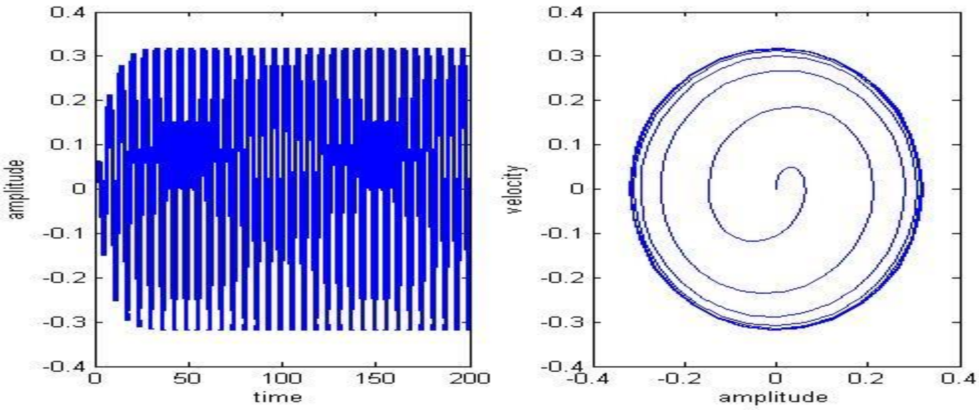

Figure 2: The time history and phase plane of uncontrolled system at primary resonance case.

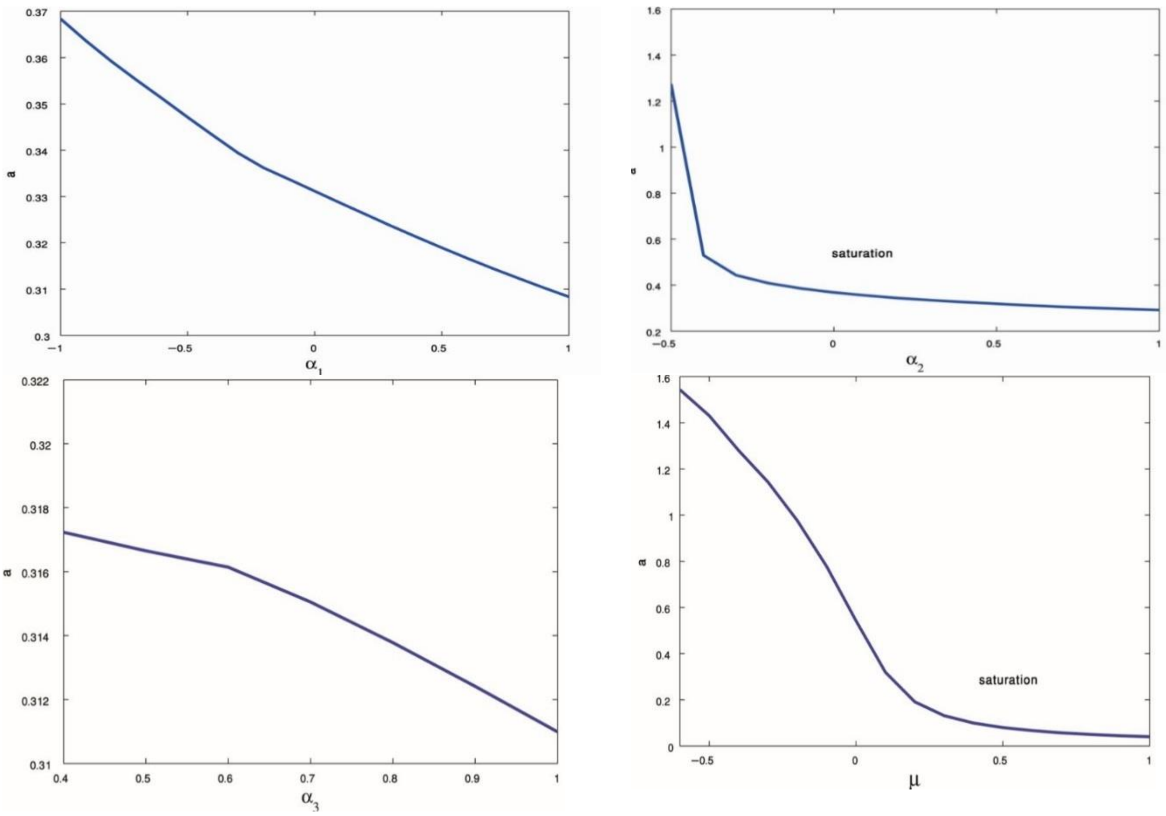

Figure 3: The influence of the parameters of the main system without control.

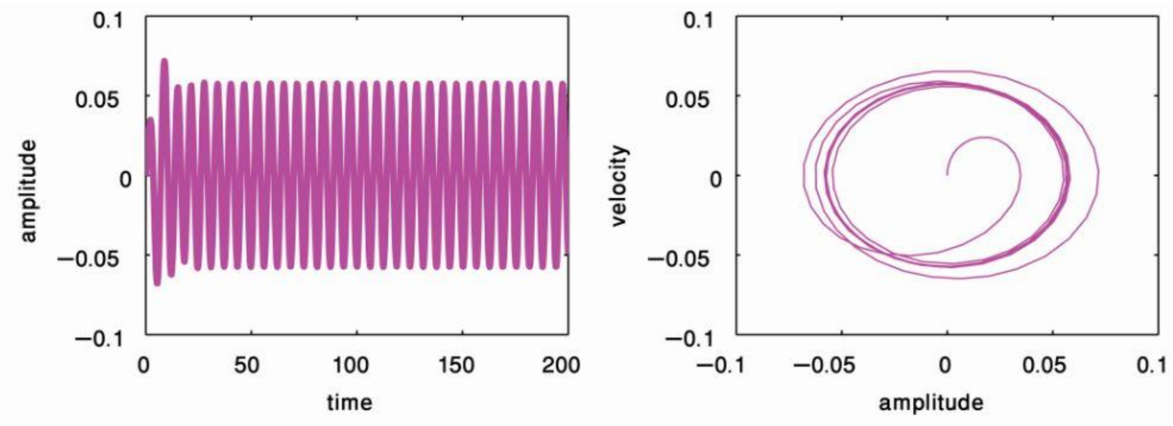

Figure 4: The time history and phase plane of controlled system at primary resonance case.

Response Curves

We used the frequency response curves to examine the influence of different parameters on the controlled system. The relation between the amplitude of the controlled system a and the detuning parameter σ is given by equation (19). This equation was simulated as shown in Figure 5. We may determine from this figure that the frequency response curve is represented by a red curve at the primary resonance case. such that the solid line expresses the stable solution of Eq. (19) while, the dash one expresses the unstable solution of the same equation. The behavior of the controlled system increased as the external excitation force f was increased as shown in figure 6. the natural frequency ω and damping coefficient μ increased, the main system's response decreased as illustrated in figures 7 and 8. The response of the main system decreased with increasing the negative linear velocity coefficient λ ,which is advantageous in the performance of the negative linear velocity feedback controller in figure 9. The amplitude shift to right for small values of the negative linear acceleration feedback coefficient G shown figure10. The Eqns. (21) and (22) solved analytically and presented graphically by (———) lines which be in agreement with the numerical solution of Eq. 11as shown in Fig 12. Fig. 13, there is a good agreement between the frequency response curves (FRC) which given by the sold line and the numerical solution of Eq.1using (RK-4) that marked by green circles.

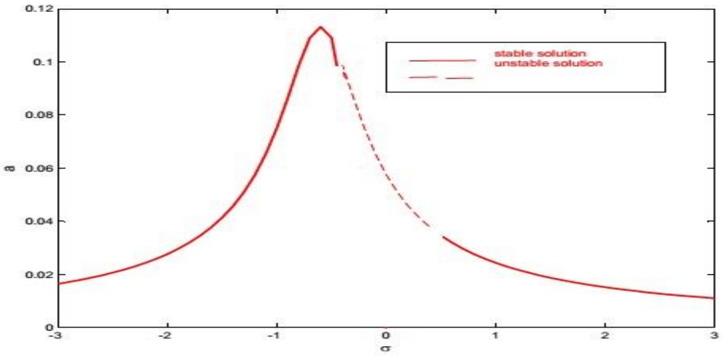

Figure 5: The FRC of the controlled system.

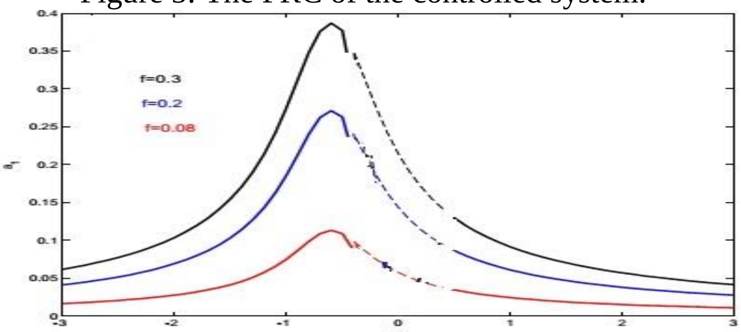

Figure 6: The external force action of the controlled system.

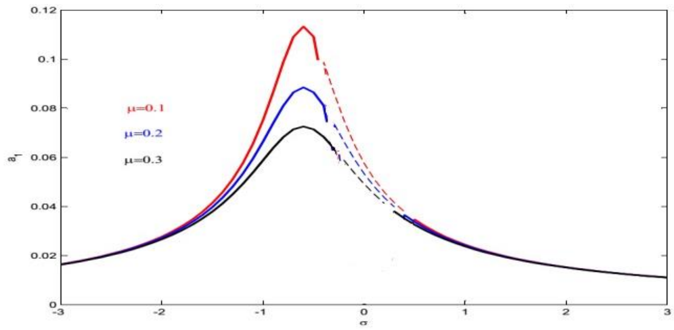

Figure 7: The effectiveness of damping parameter μ .

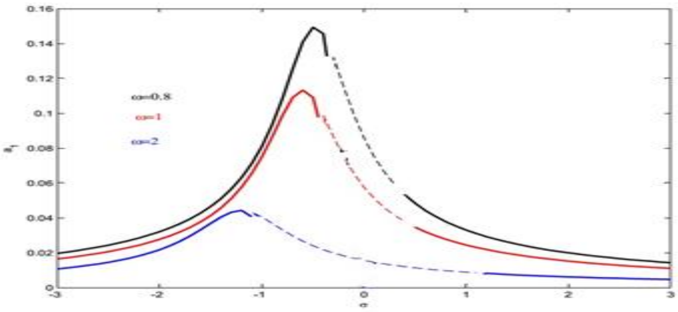

Figure 8: The influence of the internal frequency ω .

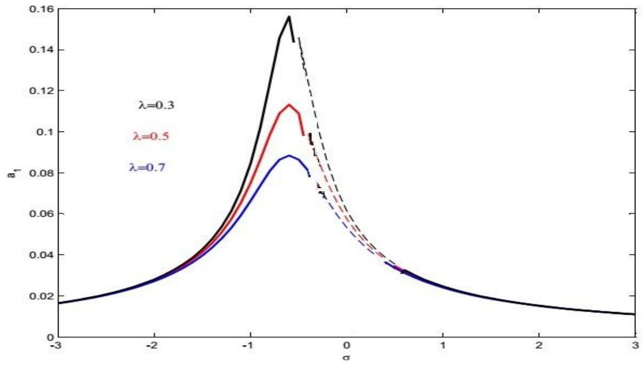

Fig. 9: The influence of the negative linear velocity coefficient λ .

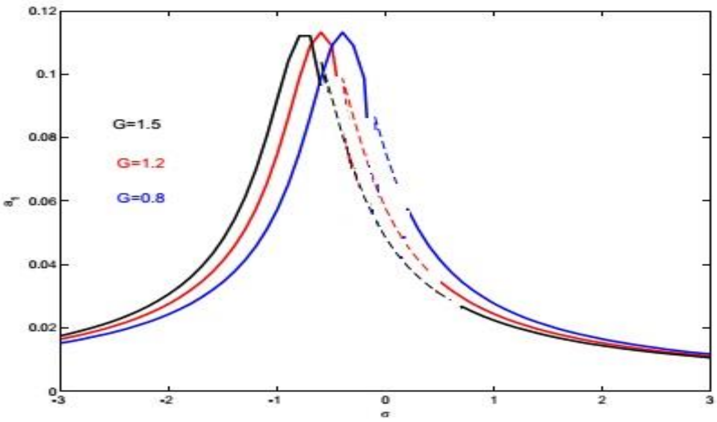

Fig. 10: The influence of the negative linear acceleration coefficient G.

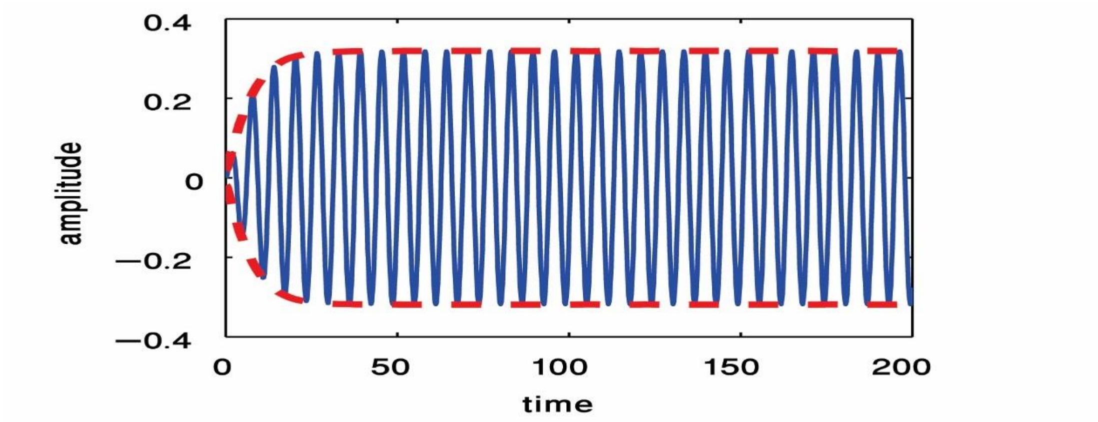

Figure 11: Comparison between the numerical solution (———) and the perturbation analysis (——————) for the uncontrolled system.

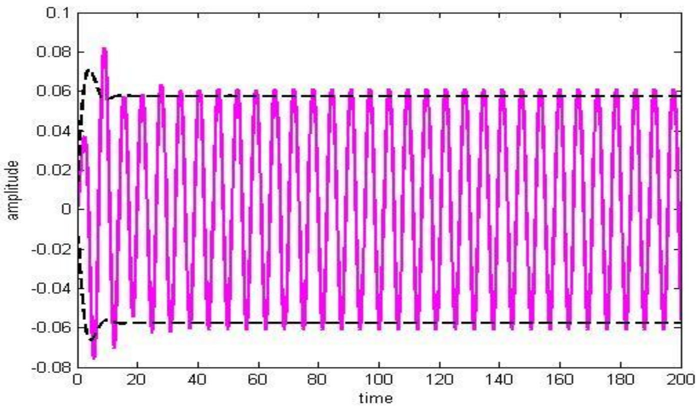

Figure 12: Comparison between the numerical solution (———) and the perturbation analysis (——————) for the controlled system.

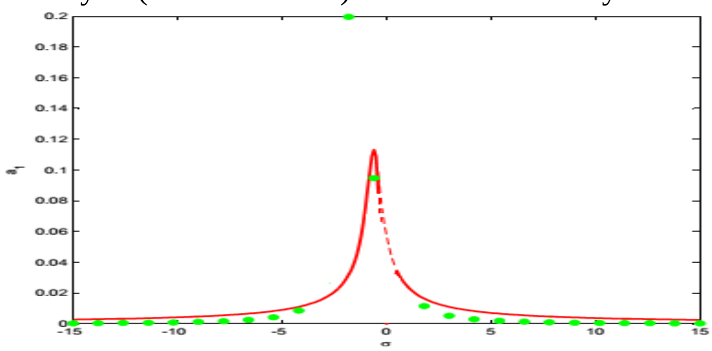

Figure 13: Comparison between the FRC Solution and RK-4 Solution

V –CONCLUION

In many engineering and physical applications, the system is one of the most important models. The vibrations analysis and dynamic responses of system subjected to external excitation force f were investigated. For obtaining the approximate solution of the vibrating system, we applied the multiple scales method. We used negative linear velocity feedback controller to suppress the vibrations of system, the amplitude of the vibrating system was repressed from about 1.2 to about 0.15 and the vibrations were reduced by about 94.7% from its value without control and the effectiveness of the negative linear velocity feedback controller Ea is nearly about 12. From this study, we can note some important results for the influence of the vibrating system's parameter such that:

- The behavior of the controlled system increased with increasing the external excitation force f.

- The response of the main system decreased with increasing the natural frequency ω.
- The response of the main system decreased with increasing the negative linear velocity coefficient λ.
- The amplitude shift to right of the negative linear acceleration feedback coefficient G.
- There is a conformity of FRC solutions with RK-4 solutions.

REFERENCES

[1]. Siewe MS, Kakmeni FM, Bowong S, Tchawoua. C. Non-linear response of a self-sustained electromechanical seismographs to fifth resonance excitations and chaos control. Chaos, Solitons & Fractals 29(2): 431-445. (2006).

[2]. Kakmeni. C .et al (2004) Siewe MS, Kakmeni FM, Tchawoua Resonant oscillation and homoclinic bifurcation in a Φ^6-van der pol oscillator. Chaos, Solitons & Fractals 21(4): 841-853 (2004).

[3]. Huang. C. Multiple scales scheme for bifurcation in a delayed extended van der Pol oscillator. Physica A (Statistical Mechanics and its Applications) 490: 643-652 (2018).

[4]. Barron MA Stability of a ring of coupled van der Pol oscillators with non-uniform distribution of the coupling parameter. Journal of applied research and technology 14(1): 62-66. (2016)

[5]. Kumar P, Narayanan S, Gupta S Investigations on the bifurcation of a noisy Duffing–van der Pol oscillator. Probabilistic Engineering Mechanics 45: 70-86. (2016)

[6]. Kumar P, Kumar A, Erlicher S A modified hybrid Van der Pol–Duffing–Rayleigh oscillator for modelling the lateral walking force on a rigid floor. Physica D (Nonlinear Phenomena) 358: 1-14. (2017)

[7]. Kumar P, Kumar A, Racic V, Erlicher S Modelling vertical human walking forces using self-sustained oscillator. Mechanical Systems and Signal Processing 99: 345-363. (2018).

[8]. Abdel Hafez HM, & Nassar ME. Suppression of vibrations of a forced and self-excited nonlinear beam by using positive position feedback controller PPF. *Journal of Advances in Mathematics and Computer Science*, 1-19(2016).

[9]. Amer YA, El-Sayed AT, Abd EL-Salam MN Outcomes of the NIPPF Controller Linked to a Hybrid Rayleigh–Van der Pol-Duffing Oscillator. Journal of Control Engineering and Applied Informatics 22(3): 33-41. (2020).

[10]. Sayed M. Hamed Y S, & Amer Y A. (2011). Vibration reduction and stability of non-linear system subjected to external and parametric excitation forces under a non-linear absorber. International Journal of Contemporary Mathematical Sciences, 6(22), 1051-1070.

[11]. Kamel MM Nonlinear behavior of Van der Pol oscillators under parametric and harmonic excitations. Physica Scripta 79(2): 025004. (2009).

[12]. Kumar P, Narayanan S, Gupta S Investigations on the bifurcation of a noisy Duffing–van der Pol oscillator. Probabilistic Engineering Mechanics 45: 70-86. (2016).

[13]. Kumar P, Kumar A, Erlicher S A modified hybrid Van der Pol–Duffing–Rayleigh oscillator for modelling the lateral walking force on a rigid floor. Physica D (Nonlinear Phenomena) 358: 1-14. (2017).

[14]. Kumar P, Kumar A, Racic V, Erlicher S Modelling vertical human walking forces using self-sustained oscillator. Mechanical Systems and Signal Processing 99: 345-363. (2018)

[15]. Nayfeh A H, & Younis M I.: Dynamics of MEMS resonators under super harmonic and subharmonic excitations. J. Micromech. Microeng. **15(10)**, 1840-1847.

[16]. Amer Y A, EL-Sayed, A T, & EL-Salam, M. A.: Non-linear Saturation Controller to Reduce the Vibrations of Vertical Conveyor Subjected to External Excitation. Asian Research Journal of Mathematics, **11(2)**, 1-26. (2018).

[17]. Amer Y A, El-Sayed AT, & El-Bahrawy F T.: Torsional vibration reduction for rolling mill's main drive system via negative velocity feedback under parametric excitation. J. Mech. Sci. Technol. **29(4)**, 1581-1589. (2015).

[18]. Amer YA, El-Sayed AT, & Kotb AA.: Nonlinear vibration and of the Duffing oscillator to parametric excitation with time delay feedback. Nonlinear Dynamics, **85(4)**, 2497-2505. (2016).

[19]. Eftekhari, M., Ziaei-Rad, S., & Mahzoon, M: Vibration suppression of a symmetrically cantilever composite beam using internal resonance under chordwise base excitation. Int. J. Non-Linear Mech. 48, 86-100. (2013).

[20]. Abd EL-Salam, M. N. Behavior of a Hybrid Rayleigh-Van der Pol-Duffing Oscillator with a PD Controller. *Journal of Applied Research and Technology*, *20*(1), 58-67. (2022).

[21]. Abualnaja, K. M., Amer, Y. A., El-Sayed, A. T., Ahmed, E. E. E., & Hamed, Y. S. Response Analysis and Controlling the Nonlinear Vibration of Van Der-Pol Duffing Oscillator Connected to the NIPPF Controller. *IEEE Access*, *9*, 91836-91849. (2021).