Physical-based Modeling Identification and Simulation of a Small UAV Geometric Model

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Abstract* – *Nowadays Unmanned Aerial Vehicles (UAVs) have become one of the biggest interests in the whole world either in military applications or in the civilian one such as agricultural observation and traffic monitoring, also a major tendency in UAVs modeling has increased due to the accurate mathematical modeling and open loop simulation are very important to understand the UAV behaviors for various inputs, accuracy of guidance and control design also depends on the accuracy of the open loop model. So, accurate UAV model is a crucial at all. Accuracy of UAV model depends on the accuracy of its subsystems. In this work, a small fixed wing UAV Cessna is taken as a case study and an accurate modeling using experimental measurements of geometric and inertia mass model is performed.

I. Introduction

The moment of inertia is a quantity that expresses a body's tendency to resist angular acceleration from torque about a specified axis. It is also the sum of the mass of each particle in the body with the square of its distance from the axis of rotation. So it s very important as in flight, the control surfaces of an aircraft produce aerodynamic forces. These forces are applied at the center of pressure of the control surfaces which are some distance from the aircraft center of gravity and produce torques (or moments) about the principal axes. The torques cause the aircraft to rotate. The ability to vary the amount of the force and the moment allows the pilot to maneuver or to trim the aircraft Analytically Determining an Object's Moment of inertia.

The moment of inertia of any object having a shape that can be described by a mathematical formula such as a disk or solid rectangle can be easily calculated. It is when an object's shape or the shapes that make an object become irregular in which it is difficult to analytically determine said object's moment of inertia

Experimentally Determining an Objects Moment of Inertia using a pendulum, which employs the relation between for a body with a mathematically indescribable shape, the moment of inertia can be obtained via experiment the period of oscillation and the moment of inertia of the suspended mass.

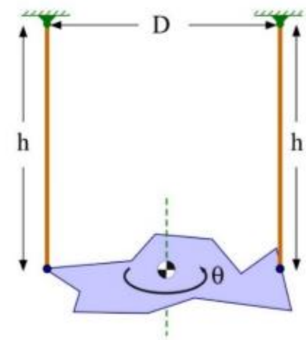


Figure 1: pendulum configuration

To experimentally determine the moment of inertia, the apparatus must first be tested as a simple pendulum to ensure it is setup properly. To do this, gravity is experimentally estimated by swinging the object as a simple pendulum, measuring the period, as shown in fig1 and then using the equations below:

$$w = 2\Pi f = \frac{2\Pi}{T}$$

$$w = \sqrt{\frac{g}{h}} \quad , \text{g}=w^2 h = \frac{4\Pi^2 h}{T^2}$$

If the experimental calculation for gravity is close enough to the known value, then the moment of inertia can be estimated using the equation below. To determine this, the period in the equation below results from rotating the object about the vertical axis rather than in a simple pendulum motion:

II. Project description

The objective of this project is to suggest that the moment of inertia for irregular shaped objects can easily be determined experimental. The object consider here is a model of airplane (cessna316).

To ensure that the method and configuration is sufficient a wooden block with known moments of inertia was first tested, and the results from the experiment were compared to the known and accepted values.

For this particular experiment, the periods were determined by timing how long 10 oscillations lasted for 10 different trials, and then taking the average of those 10 trials to best account for any outliers and human error.

III. EQUIPMENT

Two strings or wires of equal length
Model airplane (cessna316)
Phone for stopwatch and camera
Ruler and measuring tape
Electrical tape
Screwdriver
Screw eyes
Scissors
Binder clips
Block of wood

IV. PROCEDURE

Measure the mass, m, length, a, height, b, and depth, c, of the wooden block. Analytically calculate the moment of inertia of the wooden about the x, y, and z axes using known equations.

Configure pendulum with two strings or wires and suspend the block using the strings ensuring that the strings are parallel and of equal distance as shown in Figure: 2. Note: the block must be suspended about each axis.

Record measurements for D, the distance between wires, and h, the length of the wires. To ensure that the pendulum is set up correctly a known value must be tested.

For the purpose of this experiment, that value null be gravity. To test for gravity, swing the object in a pendulum motion starting from a small angle, as seen in figure 3 and measure the period, T, the time it takes for the object to return to the starting position. One full back and forth, a swing is a period.

To minimize human error 10 oscillations were measured for ten separate trials and the average period was calculated. To calculate the gravity use the equation below:

$$g = w^2 h = \frac{4\Pi^2 h}{T^2}$$

Once gravity is correctly calculated, the moment of inertia can be experimentally determined. Repeat step 3 to measure the period of the oscillations except rotate the object about the vertical axis as shown in figure 4.

To determine the experimental values for moment of inertia use the equation:

$$I = \frac{mgD^2T^2}{16h\pi^2}$$

Compare the results from the experimental values to the accepted, analytical values. Note: If deviation > 10%, something was likely done wrong.

a. Extremely well (less than 0.1% deviation)
b. Very well (less than 1% deviation)
c. Good (less than 10% deviation)
d. Sort of (less than 50% deviation)
e. Poor (less than 100% deviation)

Now that the comparison between the analytical and experimental values for the block's moments of inertia have confirmed that the configuration works, repeat steps replacing the wooden block with the model airplane.

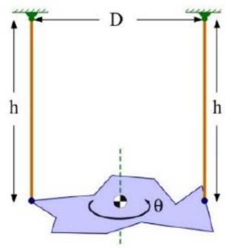


Figure 2: pendulum configuration

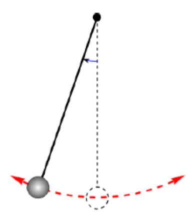


Figure 3: Swinging motion of the pendulum

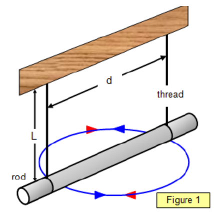


Figure 4: Rotation about the vertical axis

A. DETERMINATION OF THE MOMENT OF INERTIA OF A SOLID WOODEN BLOCK

To calculate the moments of inertia about the solid rectangular block, the known equations below were used:

$$I_x = \frac{m(b^2 + c^2)}{12}$$
$$I_y = \frac{m(a^2 + c^2)}{12}$$
$$I_z = \frac{m(a^2 + b^2)}{12}$$

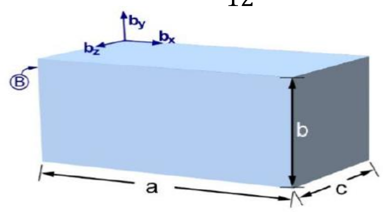


Figure 5: Axes and variables used for block

B. EXPERIMENTAL DETERMINATION

The configuration, measurements, and calculations for the test to determine the block's moment of inertia can be found below.

a (cm)	b (cm)	C (cm)	m (g)	I_x(g*c m^2)	Iy(g* cm^2)	I_z(g $*\, cm^2$)
12.3825	3.81	8.89	178.8	556,387	1363.0402	984.58

Table 1: Physical properties of wooden block

axis	M(g)	h(cm)	D(cm)	T, swinging pendulum (s)	T, rotation about said axis (s)	w (rad)
x	178.8	59.84875	10.795	1.5518	.81	4.05
Y	178.8	67.6275	10.00125	1.65	1.45	3.80
z	178.8	60.6425	14.76375	1.5621	.8	4.02

Table 2: Measurements of pendulum configuration per axis

	Experimental (cm/S^2)	Known (cm/s^2)	Deviation	
x	981.1766	981.456	0.03%	
y	977.9254	981.456	0.03%	
z	981.1258	981.456	0.03%	

Table 3: Experimental vs. known measurements of gravity

	Experimental (g*in^2)	Analytical (g*in^2)	Deviation
I_x	219.72	216.05	1.70%
Iy	534.38	536.63	0.42%
I_z	393.86	387.63	1.61%

Table: 4 Experimental vs. analytical measurements of the block's moments of inertia

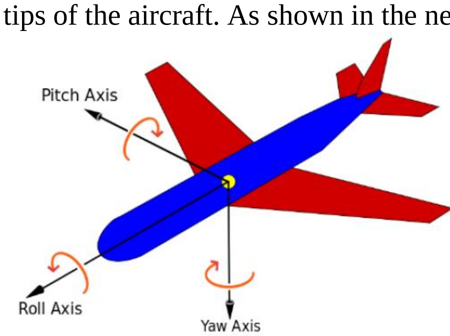
Figure 6: Setup to determine the moment of inertia about the y-axis

C. DETERMINATION OF THE MOMENT OF INERTIA OF A MODEL AIRPLANE

Yaw, Roll, and Pitch Axes

It is necessary to control the attitude or orientation of a flying aircraft in all 3 dimensions. In flight, any aircraft will rotate about its center of gravity, a point that is the average location of the mass of the aircraft. We can define a 3-dimensional coordinate system through the center of gravity with each axis of this coordinate system perpendicular to the other two axes.

The orientation of the aircraft can then be defined by the amount of rotation of the parts of the aircraft along these principal axes. The yaw axis is defined to be perpendicular to the plane of the wings with its origin at the center of gravity and directed towards the bottom of the aircraft. A yaw motion is a movement of the nose of the aircraft from side to side. The pitch axis perpendicular to the yaw axis and is parallel to the plane of the wings with its origin at the center of gravity and directed towards the right wingtip. A pitch motion is an up or down movement of the nose of the aircraft. The roll axis is perpendicular to the other two axes with its origin at the center of gravity, and is directed towards the nose of the aircraft. A rolling motion is an up and down movement of the wing tips of the aircraft. As shown in the next figure.



	Experimental (cm/s2)	Known (cm/s2)	Deviation
yaw	980.7	981.45	.07%
roll	974.75	981.45	.07%
pitch	980.33	981.45	.07%

Table 6:Experimental vs. known measurements of gravity

	Experimental (g*in2)
yaw	103.73
roll	40.25
pitch	103.59

Table 7: Experimental vs. analytical measurements of the airplane's moments

V. DISCUSSION OF RESULTS

Wooden Block Experiment Prior to credibly being able to experimentally determine the moment of inertia of an irregularly shaped object, a model airplane, an object with a known moment of inertia, a wooden block, had to be tested first. To ensure that the pendulum was accurate, gravity had to be tested by measuring the period of the block swinging in a pendulum motion, and then using the equation below:

$$g_y = w^2 h = \frac{4\Pi^2 h}{T^2} = \frac{4\Pi^2 * 26.625in}{(1.65s)^2} = 385.01/s^2 \approx 386.40in/s^2$$

Now that the pendulum's results have been confirmed, the moment of inertia can be determined by measuring the period of the block rotating it about its vertical axis rather than swinging it like a pendulum, and then using the equation below:

$$I_y = \frac{mgD^2T^2}{16h\pi^2} = \frac{178,8g*\frac{386.40in}{s^2}*(3.9375in)^2*(1.45s)^2}{16\pi^2*26.625in} = 534.38g*in^2 \approx 536,63g*in^2$$

Model Airplane Experiment

Now that the experimental method used in this experiment for determining the moments of inertia of an object have been confirmed, the moments of inertia of the model airplane can be determined. Similar to the test for the wooden block, gravity must first be confirmed to ensure a credible configuration:

$$g_{roll} = w^2 h = \frac{4\Pi^2 h}{T^2} = \frac{4\Pi^2 * 25.187in}{(1.66095s)^2} = 383.76\text{in}/s^2 \approx 386.40in/s^2$$

As stated in the previous section, the moment of inertia can then be calculated about the same axis

$$I_{roll} = \frac{mgD^2T^2}{16h\pi^2} = \frac{77.0g*386.40in/s^2*(2.437in)^2*(.9517s)^2}{16\pi^2*25.187in} = 40.25\text{g}*in^2$$

One can see how we determine the moment of inertia about the axis as in figure: 7(a), we calculate the moment of inertia around the x-axis, then in figure: 7(b), we calculate the moment of inertia around the y-axis and in the figure: 7(c) we calculate the moment of inertia around the z-axis, the following two tables show the results of the experiment of the airplane model as Table10 shows the time for the block to complete (10) times oscillations in a pendulum motion to confirm gravity for each axis, finally the time to complete (10) times oscillations rotating about the same axis in Table 11 to determine the moment of inertia.

Figure 7:(a) I_xx measurement

Figure 7(b) I_zz measurement.

Figure 7(c) I_yy measurement.

trial	yaw	roll	pitch
1	17.5	17.5	17.03
2	17.52	17.50	17.02
3	17.53	17.52	17.05
4	17.54	17.59	17.02
5	17.52	17.53	17.04
6	17.53	17.52	17.02
7	17.50	17.5	17.08
8	17.59	17.55	17.05
9	17.56	17.53	17.04
10	17.56	17.52	17.05
average	17.535	17.526	17.04

Table 10: Time for the block to complete 10 oscillations in a pendulum motion to confirm gravity for each axis

trial	yaw	roll	pitch
1	*25.02*	*11.59*	*17.83*
2	*25.00*	*11.13*	*17.23*
3	*25.89*	*11.55*	*17.12*
4	*25.07*	*11.60*	*17.56*
5	*25.10*	*11.48*	*17.8*
6	*25.04*	*11.61*	*17.2*
7	*25.19*	*11.41*	*17.3*
8	*25.05*	*11.23*	*17.2*
9	*25.13*	*11.1*	*17.56*
10	*25.09*	*11.2*	*17.59*
average	*25.158*	*7.39*	*17.43*

Table 11: Time for the block to complete 10 oscillations rotating about said axis to determine the moment of inertia

VI. CONCLUSION

Based on the wooden block part of this experiment the test and results may be considered sufficient. Deviations between the experimentally determined and known values for gravity ranged from 0.03% to 0.68% for both the block and the airplane, and the deviations between the experimentally determined moments of inertia for the block ranged from 0.42% to 1.70%. The results from the airplane experiment make sense because a plane is most likely to rotate about its roll axis versus its yaw and pitch axes based on its weight distribution. The most likely source of error was likely human error while timing the periods of the block. To minimize this as much as possible 10 trials throughout the experiment. These results can be seen in Tables 10 and 11. Overall, this experiment proves that the moment of inertia can be determined experimentally not only of an airplane, but of any irregular shaped object.

VII. REFERENCES

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