Maximum Product Spacings for the Kumaraswamy Distributions Based on Progressive Type-II Censoring Schemes

prepared by

Dr. Osama El Sayed Eraki
Assistant Professor of Mathematical Statistics
Department of Statistics
Faculty of Commerce
Zagazig University
osamaelsayederaky@gmail.com

Dr. Ahmed El Saeed Abdel Hameed
Lecturer of Statistics
Department of Basic Sciences
Obour HighInstitute for Management & Informatics
ahmedramces@oi.edu.eg

Amira Ibrahim El Sayed Ibrahim
Teaching assistant of Statistics
Obour High Institute for Management & Informatics
amirae@oi.edu.eg

Journal of Business Research
Faculty of Commerce -Zagazig University
Volume 4 5- Issue 4 October 2023
link: https://zcom.journals.ekb.eg/

Abstract

The Type-II progressive censoring schemes of maximum product spacing will be discussed. In this paper, we have studied the problem of point estimation of the two parameters for the Kumaraswamy distribution based on progressive Type-II censoring. The maximum product spacing is used to estimate the model's parameters. To evaluate the performance of the point estimator, the simulation study is carried out. To illustrate the usefulness of the study in practice, a real data is used to study the performance of the estimation process under this progressive Type-II censoring scheme. In this paper, we have considered the problem of estimation of the unknown parameters for Kumaraswamy distribution under progressive Type-II censoring. We derived maximum product spacings estimates for the unknown parameters of a Kumaraswamy distribution. To illustrate the usefulness of the study in practice, a real data is used to study the performance of the estimation process under this progressive Type-II censoring scheme. In this paper, although we have mainly considered Type-II progressive censoring cases, the same method can be extended to other censoring schemes also.

Keywords

Kumaraswamy distribution, progressive Type-II censoring, Maximum product spacing estimation.

1. Introduction

The Kumaraswamy distribution is similar to the Beta distribution, but it has a notable advantage of having an invertible cumulative distribution function that can be expressed in a closed-form. Kumaraswamy (1976, 1978) showed that commonly used probability distribution functions like the normal, log-normal, and beta distributions do not adequately model hydrological data such as daily rainfall and stream flow. As a result, Kum. introduced a new probability density function known as the sine power probability density function.

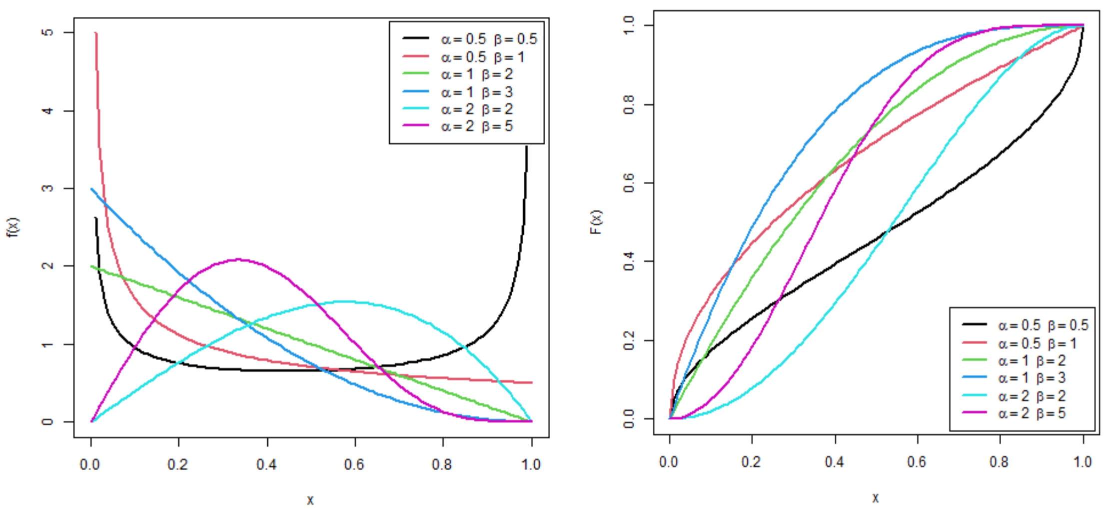
Kumaraswamy (1980) introduced the Kumaraswamy distribution as a versatile probability density function for double-bounded random processes. This distribution is suitable for modeling various natural phenomena with lower and upper bounds, such as individual heights, test scores, atmospheric temperatures, hydrological data, and more. Additionally, the Kumaraswamy distribution can be used when scientists need to model data with finite bounds, even if they are using probability distributions with infinite bounds in their analysis. The Kumaraswamy distribution's probability density function (pdf) is described by

$$f(x) = \alpha\beta x^{\alpha-1}(1 - x^{\alpha})^{\beta-1} \qquad (\mathbf{1})$$

where $0 < x < 1$ and α, β are two positive shape parameters. When $\alpha = 1$ and $\beta = 1$, then one can obtain a Uniform distribution $U(0, 1)$ as special case of the Kumaraswamy distribution. The cumulative distribution function (cdf) of the Kumaraswamy distribution is given by

$$F(x) = 1 - (1 - x^{\alpha})^{\beta}, 0 < x < 1 \qquad \textbf{(2)}$$

Figure 1 showed the behavior of pdf and cdf of the Kumaraswamy distribution at different values of the parameters α and β



(a) pdf of the Kumaraswamy distribution(b) cdf of the Kumaraswamy distribution

Figure 1: Behavior of Kumaraswamy distribution

In industrial life testing and medical survival analysis, it is common for the object of interest to be lost or withdrawn before failure, or for the object's lifetime to be only known within an interval. This results in a sample that is incomplete, often referred to as a censored sample. There are various reasons for removal of experimental units, such as saving them for future use, reducing total test time, or lowering associated costs. Right censoring is a technique used in life-testing experiments to handle censored samples. The conventional Type-I and Type-II censoring schemes are the most common methods of right censoring, but they do not allow for removal of units at points other than the terminal point of the experiment, limiting their flexibility. To address this limitation, a more general censoring scheme called the progressive Type-II censoring scheme has been proposed. as follows:

- Suppose that $n$unite are placed on a test at time zero with m failures to be observed.
- At the first failure, say $x_{(1)}, R_1$of the remaining units are randomly selected and removed.
- At the time of the second failure, $x_{(2)}, R_2$of the remaining units are selected and removed.
- Finally, at the time of the m^{th} failure the rest of the units are all removed, $R_m = n - R_1 - R_2 - \cdots - R_{m-1} - m$.
- Thus, it is possible to witness the data $\{(x_{(1)}, R_1), \dots, (x_{(m)}, R_m)\}$ during a gradual censorship plan. Even though $R_1, R_2, \dots, R_m$ are encompassed as a section of the data, their values are previously known.

The joint probability density function of all $m$progressive Type-II censoring schemestatistics is (Balakrishnan and Aggarwala (2000))

$$L(\alpha,\beta) = \text{C}\prod_{i=1}^{m}\left(f(x_{(i)};\alpha,\beta)\right)\left(1 - F(x_{(i)};\alpha,\beta)\right)^{R_i} \qquad (\mathbf{3})$$

where

$$C = n(n - R_1 - 1)\dots(n - R_1 - R_2 - \cdots - R_m - m + 1)$$

If$R_1 = R_2 = \cdots = R_{m-1} = 0$, then $R_i = n - m$ which corresponds to the Type-II censoring. and If $R_1 = R_2 = \cdots = R_m = 0$, then $n = m$ which corresponds to the complete sample (Wu (2002)).

Sindhu *et al*. (2013) studied the Bayesian and non-Bayesian estimation for the shape parameter of the Kumaraswamy distribution under Type-II censored samples. They obtained maximum likelihood estimation and Bayes estimation using asymmetric loss functions: Degroot loss function, LINEX loss function and General Entropy loss function. They derived Posterior predictive distributions along with posterior predictive intervals under simple and mixture priors. Reyad and Ahmed (2016) introduced the Bayesian and E-Bayesian estimation for the shape parameter of the Kumaraswamy distribution based on Type-II censored schemes. They derived estimates under symmetric loss function [squared error loss] and three asymmetric loss functions: LINEX loss function, Degroot loss function and Quadratic loss function. Ghosh and Nadarajah (2017) discussed Bayesian estimation of Kumaraswamy distributions based on three different types of censored samples: left censoring, singly Type-II censoring and doubly Type-II censoring. They obtained Bayes estimates using two different types of loss functions: LINEX and Quadratic. Pak and Rastogi (2018) considered non-Bayesian and Bayesian estimation of Kumaraswamy parameters when the data are Type-II hybrid censored. The maximum likelihood estimates and its asymptotic variance-covariance matrix are obtained. They used the asymptotic variances and covariance's of the MLEs to construct approximate confidence intervals. In addition, by using the parametric bootstrap method, discussed the construction of confidence intervals for the unknown parameter. Sultana *et al*. (2018) considered estimation of unknown parameters of a two-parameter Kumaraswamy distribution with hybrid censored samples. They obtained maximum likelihood estimates using an EM algorithm. Bayes estimates were derived under the squared error loss function using different approximation methods and an importance sampling technique is also discussed. El-Deen *et al*. (2014) studied non-Bayesian and Bayesian approaches to obtain point and interval estimation of the shape parameters, the reliability and the hazard rate functions of the Kumaraswamy distribution. The estimators are obtained based on generalized order statistics. The symmetric and asymmetric loss functions, the squared error loss function (as a symmetric loss function), LINEX, precautionary and general entropy loss functions (as asymmetric loss functions) are considered for Bayesian estimation. Also, maximum likelihood and Bayesian prediction for a new observation are found. The results have been specialized to Type-II censored data and the upper record values. Kohansal and Bakouch (2019) described the point and interval estimation of the unknown parameters of Kumaraswamy distribution under the adaptive Type-II hybrid progressive censored samples. They obtained the maximum likelihood estimation of the parameters. In addition, the Bayesian estimation of the parameters is approximated by using the MCMC algorithm and Lindley's method due to the lack of explicit forms. Sultana *et al*. (2020) investigated the estimation problems of unknown parameters of the Kumaraswamy distribution under Type-I progressive hybrid censoring. They derived the maximum likelihood estimates of parameters using an EM algorithm. Bayes estimates were obtained under different loss functions using the Lindley method and importance sampling procedure. Tu and Gui (2020) discussed and considered the estimation of unknown parameters featured by the Kumaraswamy distribution on the condition of a generalized progressive hybrid censoring scheme. They derived the maximum likelihood estimators and Bayesian estimators under symmetric loss functions and asymmetric loss functions, like general entropy, squared error as well as Linex loss function. Since the Bayesian estimates fail to be of explicit computation, Lindley approximation, as well as the Tierney and Kadane method, is employed to obtain the Bayesian estimates. Ghafouri and Rastogi (2021) considered the estimation of the parameters and reliability characteristics of Kumaraswamy distribution using progressive first failure censored samples. They derived the maximum likelihood estimates using an EM algorithm and compute the observed information of the parameters that can be used for constructing asymptotic confidence intervals. Also, they computed the Bayes estimates of the parameters using Lindley approximation as well as the Metropolis-Hastings algorithm. Gholizadeh *et al*. (2011) studied the Bayesian and non-Bayesian estimators for the shape parameter, reliability and failure rate functions of the Kumaraswamy distribution in the cases of progressively Type-II censored samples. Maximum likelihood estimation and Bayes estimation, reliability and failure rate functions are obtained using symmetric and asymmetric loss functions, like squared error loss, Precautionary and LINEX loss functions. Feroze and Elbatal (2013) considered the estimation of two parameters of the Kumaraswamy distribution under progressive Type-II censoring with random removals, where the number of units removed at each failure time has a binomial distribution. They obtained the maximum likelihood estimation of the unknown parameters, and the asymptotic variance-covariance matrix was also obtained. Also, they derived the formula to compute the expected test time. Eldin *et al*. (2014) studied the Estimation for parameters of the Kumaraswamy distribution based on progressive Type-II censoring. They derived estimates using the maximum likelihood and Bayesian approaches. In the Bayesian approach, the two parameters are assumed to be random variables and estimators for the parameters are obtained using squared error loss function. Erick *et al*. (2016) considered the parameter estimation problem of test units from Kumaraswamy distribution based on progressive Type-II censoring scheme. The maximum likelihood Estimates of the parameters are derived using EM algorithm. Also the expected Fisher information matrix based on the missing value principle is computed. EL-Sagheer (2019) used the maximum likelihood, Bayes, and parametric bootstrap methods for estimating the unknown parameters, as well as some lifetime parameters reliability and hazard functions, based on progressively Type-II right-censored samples from a two-parameter Kumaraswamy distribution. The classical Bayes estimates proposed by applying the Markov chain Monte Carlo (MCMC) technique.

2. Maximum Product Spacings

Ng *et al.* (2012) and El-Sherpieny*et al.* (2020) introduced maximum product spacing (MPS) method based on progressive Type-II censoring scheme sample method, MPS method chooses the parameter values that make the observed data as uniform as possible, according to a specific quantitative measure of uniformity.

$$G(\alpha,\beta) = \prod_{i=1}^{m+1}\left(F(x_{(i)};\alpha,\beta) - F(x_{(i-1)};\alpha,\beta)\right)\prod_{i=1}^{m}\left(1 - F(x_{(i)};\alpha,\beta)\right)^{R_i}$$

from (2), one can get:

$$G(\alpha,\beta) = \prod_{i=1}^{m+1}\left\{\left(1 - x_{(i-1)}{}^{\alpha}\right)^{\beta} - \left(1 - x_{(i)}{}^{\alpha}\right)^{\beta}\right\}\prod_{i=1}^{m}\left(1 - x_{(i)}{}^{\alpha}\right)^{\beta R_i}$$

The natural logarithm of the product spacings function is

$$S(\alpha,\beta) = \sum_{i=1}^{m+1}\log\left\{\left(1 - x_{(i-1)}{}^{\alpha}\right)^{\beta} - \left(1 - x_{(i)}{}^{\alpha}\right)^{\beta}\right\} + \sum_{i=1}^{m}\beta R_i \log(1 - x_{(i)}{}^{\alpha})$$

where$S(\alpha,\beta) = \log G(\alpha,\beta)$.

The MPS estimators of α and β, denoted by $\hat{\alpha}_{MPS}$ and $\hat{\beta}_{MPS}$, respectively, are obtained by solving the following normal equations simultaneously

$$\frac{\partial S(\alpha,\beta)}{\partial\alpha} = \sum_{i=1}^{m}\beta R_i \frac{-x_{(i)}{}^{\alpha}\log(x_{(i)})}{(1 - x_{(i)}{}^{\alpha})}$$

$$+\sum_{i=1}^{m+1}\left[\frac{\beta(1 - x_{(i)}{}^{\alpha})^{\beta-1}x_{(i)}{}^{\alpha}\log(x_{(i)}) - \beta(1 - x_{(i-1)}{}^{\alpha})^{\beta-1}x_{(i-1)}{}^{\alpha}\log(x_{(i-1)})}{\left(1 - x_{(i-1)}{}^{\alpha}\right)^{\beta} - \left(1 - x_{(i)}{}^{\alpha}\right)^{\beta}}\right]$$

$$= 0,$$

and

$$\frac{\partial S(\alpha,\beta)}{\partial\beta} = \sum_{i=1}^{m} R_i \log(1 - x_{(i)}{}^{\alpha})$$

$$+\sum_{i=1}^{m+1}\left[\frac{\left(1 - x_{(i-1)}{}^{\alpha}\right)^{\beta}\log\left(1 - x_{(i-1)}{}^{\alpha}\right) - \left(1 - x_{(i)}{}^{\alpha}\right)^{\beta}\log\left(1 - x_{(i)}{}^{\alpha}\right)}{\left(1 - x_{(i-1)}{}^{\alpha}\right)^{\beta} - \left(1 - x_{(i)}{}^{\alpha}\right)^{\beta}}\right]$$

$$= 0$$

The MPS, $\hat{\alpha}_{MPS}$and $\hat{\beta}_{MPS}$ are the solution of the two nonlinear equations that the system needs to be solved numerically to obtain parameters estimation values.

3.Simulation Study

In this simulation, the average and mean square error using the MPC Method for Parameters estimation of Kumaraswamy distribution based on a progressive Type-II censoring scheme are now computed using a number of replications 1000using R-Statistical Programming Language for Statistical Computing, based on the following assumptions:

1. values of$(\alpha, \beta) = (0.5, 0.5), (0.5, 1), (1, 2),$ and $(1, 1)$.
2. Sample sizes of n = 40, n = 80 and n = 100.
3. In this simulation, the algorithm proposed by Balakrishnan and Sandhu (1995)can be used to generate a progressively Type-II censored sample, removed items R_i are assumed at different sample sizes n and the number of stages m as shown in Table (1).

Table (1): Numerous patterns for removing items from life test at different number of stages

n	m	censoring schemes			
		S_1	S_2	S_3	S_4
40	20	$(20, 0^{*19})$	$(10, 0^{*18}, 10)$	$(0^{*9}, 10,10, 0^{*9})$	$(0^{*19}, 20)$
	30	$(10, 0^{*29})$	$(5, 0^{*28}, 5)$	$(0^{*14}, 5,5, 0^{*14})$	$(0^{*29}, 10)$
80	40	$(40, 0^{*39})$	$(20, 0^{*38}, 20)$	$(0^{*19}, 20,20, 0^{*19})$	$(0^{*39}, 40)$
	60	$(20, 0^{*59})$	$(10, 0^{*58}, 10)$	$(0^{*29}, 10,10, 0^{*29})$	$(0^{*59}, 20)$
100	60	$(40, 0^{*59})$	$(20, 0^{*58}, 20)$	$(0^{*29}, 20,20, 0^{*29})$	$(0^{*59}, 40)$
	80	$(20, 0^{*79})$	$(10, 0^{*78}, 10)$	$(0^{*39}, 10,10, 0^{*39})$	$(0^{*79}, 20)$

Here, $(2^{*3}, 0)$, for example, means that the censoring scheme employed is (2,2,2,0).

Based on the generated data, we compute the MPSs. In tables (2,3), we display the estimates obtained by using MPS at different values of n and m, respectively. Further, the first column represents the average estimates (Avg.) and the second column represents the mean square error (MSE).

From Tables (2,3), we observed that the Avg. and MSE of the estimates are close together. As a general result, we see that when n increases, for all cases, the MSE decrease.

Table (2): Average, MSE for MPS of the Kumaraswamy distribution for different progressive Type-II censoring scheme at different values of α, β, n, and m.

(n, m)	Sch.	Parm.	$\alpha = 0.05, \beta = 0.05$		$\alpha = 0.05, \beta = 1$	
			Avg.	MSE	Avg.	MSE
(40, 20)	S_1	α	0.6925	0.0919	0.6467	0.0524
		β	0.6665	0.0812	1.4239	0.5218
	S_2	α	0.6379	0.0621	0.6057	0.0390
		β	0.6164	0.0746	1.3238	0.5548
	S_3	α	0.6697	0.0693	0.6355	0.0438
		β	0.7133	0.1368	1.5564	0.9635
	S_4	α	0.6335	0.0602	0.6063	0.0405
		β	0.6382	0.1082	1.4027	0.9064
(40, 30)	S_1	α	0.6593	0.0645	0.6202	0.0367
		β	0.6137	0.0388	1.2881	0.2362
	S_2	α	0.6270	0.0507	0.5929	0.0292
		β	0.5780	0.0299	1.2043	0.1872
	S_3	α	0.6511	0.0584	0.6162	0.0344
		β	0.6259	0.0468	1.3223	0.2971
	S_4	α	0.6209	0.0483	0.5900	0.0289

		β	0.5780	0.0325	1.2111	0.2156
(80, 40)	S_1	α	0.6050	0.0299	0.5819	0.0184
		β	0.5803	0.0213	1.1992	0.1207
	S_2	α	0.5749	0.0214	0.5581	0.0143
		β	0.5539	0.0194	1.1452	0.1210
	S_3	α	0.5899	0.0224	0.5726	0.0148
		β	0.5987	0.0306	1.2460	0.1820
	S_4	α	0.5713	0.0206	0.5573	0.0145
		β	0.5625	0.0252	1.1732	0.1701
(80, 60)	S_1	α	0.5835	0.0219	0.5634	0.0130
		β	0.5573	0.0123	1.1412	0.0687
	S_2	α	0.5654	0.0177	0.5477	0.0107
		β	0.5380	0.0104	1.0968	0.0590
	S_3	α	0.5773	0.0189	0.5600	0.0116
		β	0.5624	0.0142	1.1545	0.0807
	S_4	α	0.5616	0.0167	0.5459	0.0105
		β	0.5379	0.0112	1.0993	0.0662
(100, 60)	S_1	α	0.5805	0.0203	0.5618	0.0123
		β	0.5569	0.0122	1.1402	0.0676
	S_2	α	0.5594	0.0154	0.5446	0.0099
		β	0.5375	0.0110	1.0982	0.0648
	S_3	α	0.5700	0.0155	0.5554	0.0099
		β	0.5657	0.0155	1.1620	0.0879
	S_4	α	0.5557	0.0144	0.5433	0.0098
		β	0.5407	0.0130	1.1097	0.0811
(100, 80)	S_1	α	0.5642	0.0152	0.5488	0.0092
		β	0.5438	0.0080	1.1075	0.0447
	S_2	α	0.5508	0.0127	0.5368	0.0078
		β	0.5291	0.0068	1.0730	0.0386
	S_3	α	0.5601	0.0136	0.5466	0.0084
		β	0.5467	0.0090	1.1152	0.0507
	S_4	α	0.5479	0.0121	0.5352	0.0076
		β	0.5282	0.0072	1.0727	0.0420

Notes: Sch. – scheme, Parm. – Parameter.

Table (3): Average,MSE for MPS of the Kum. distribution for different progressive Type-II censoring schemeat different values of α, β, n, and m.

(n, m)	Sch.	Parm.	$\alpha = 1, \beta = 2$		$\alpha = 1, \beta = 1$	
			Avg.	MSE	Avg.	MSE
(40, 20)	S_1	α	1.2454	0.1456	1.2935	0.2098
		β	3.1189	3.7435	1.4239	0.5218
	S_2	α	1.1776	0.1162	1.2114	0.1559
		β	2.9212	4.6127	1.3238	0.5547
	S_3	α	1.2339	0.1298	1.2710	0.1753
		β	3.4893	7.5024	1.5564	0.9634
	S_4	α	1.1838	0.1269	1.2126	0.1621
		β	3.1945	8.5153	1.4027	0.9066
(40, 30)	S_1	α	1.1989	0.1006	1.2405	0.1471
		β	2.7501	1.5818	1.2881	0.2362
	S_2	α	1.1500	0.0812	1.1858	0.1169
		β	2.5467	1.2949	1.2043	0.1871
	S_3	α	1.1948	0.0966	1.2324	0.1378
		β	2.8430	2.0558	1.3223	0.2971
	S_4	α	1.1477	0.0830	1.1801	0.1157
		β	2.5802	1.5758	1.2111	0.2156
(80, 40)	S_1	α	1.1382	0.0528	1.1638	0.0737
		β	2.5070	0.7435	1.1992	0.1207
	S_2	α	1.0980	0.0438	1.1163	0.0573
		β	2.3929	0.8114	1.1451	0.1210
	S_3	α	1.1258	0.0445	1.1453	0.0593
		β	2.6213	1.1478	1.2460	0.1819
	S_4	α	1.0995	0.0466	1.1148	0.0583
		β	2.4799	1.2175	1.1733	0.1702
(80, 60)	S_1	α	1.1053	0.0363	1.1269	0.0521
		β	2.3560	0.4091	1.1412	0.0687
	S_2	α	1.0768	0.0307	1.0956	0.0430
		β	2.2506	0.3588	1.0969	0.0590

	S_3	α	1.1012	0.0332	1.1200	0.0465
		β	2.3888	0.4833	1.1545	0.0807
	S_4	α	1.0751	0.0311	1.0918	0.0422
		β	2.2625	0.4168	1.0992	0.0662
(100, 60)	S_1	α	1.1032	0.0346	1.1237	0.0492
		β	2.3530	0.4012	1.1402	0.0676
	S_2	α	1.0735	0.0294	1.0893	0.0396
		β	2.2593	0.4049	1.0982	0.0647
	S_3	α	1.0949	0.0291	1.1109	0.0397
		β	2.4049	0.5212	1.1621	0.0879
	S_4	α	1.0731	0.0303	1.0866	0.0392
		β	2.2955	0.5299	1.1097	0.0812
(100, 80)	S_1	α	1.0811	0.0257	1.0977	0.0368
		β	2.2697	0.2654	1.1075	0.0446
	S_2	α	1.0588	0.0222	1.0737	0.0312
		β	2.1867	0.2324	1.0730	0.0386
	S_3	α	1.0782	0.0239	1.0930	0.0336
		β	2.2883	0.3026	1.1150	0.0508
	S_4	α	1.0569	0.0223	1.0704	0.0306
		β	2.1896	0.2599	1.0728	0.0421

Notes: Sch. – scheme, Parm. – Parameter.

4.Real Data

In this section, we analyze a real data set which describes the monthly water capacity from the Shasta reservoir in California, USA. The data are recorded for the month of February from 1991 to 2010 see Sultana *et al.* (2018) and Sultana *et al.* (2020). The data points are listed below as follows.

0.338936, 0.431915, 0.759932, 0.724626, 0.757583, 0.811556, 0.785339, 0.783660, 0.815627, 0.847413, 0.768007, 0.843485, 0.787408, 0.849868, 0.695970, 0.842316, 0.828689, 0.580194, 0.430681, 0.742563.

To determine if the considered dataset can be appropriately analyzed using a Kum. distribution, a goodness of fit test is conducted. In addition to Kum. distribution, we also fit generalized exponential [Gen.Exp], Burr XII [Burr], and beta distributions to the data set. We judge the goodness of fit using various criteria, for example, negative log-likelihood criterion (NLC), Akaike information criterion (AIC) introduced by Akaike (1969), corrected AIC (AICc) introduced by Hurvich and Tsai (1989), and Bayesian information criterion (BIC) introduced by Schwarz (1978). The smaller the value of these criteria, the better the model fits the data. The results are shown in Table (4). Besides, the histogram and empirical cumulative distribution functions are given respectively in Figure 3.

Table (4). Goodness of fit tests for different distributions for real data

Distribution	NLS	AIC	AICc	BIC	K-S	p-Value
Kum.	-13.4747	-22.9494	-22.2435	-20.9579	0.2208	0.2446
Gen.Exp	-4.7925	-5.5851	-4.8791	-3.5936	0.2948	0.0490
Burr	-11.5059	-19.0118	-18.3059	-17.0204	0.2247	0.2276
beta	-12.5619	-21.1238	-20.4179	-19.1323	0.2359	0.1833

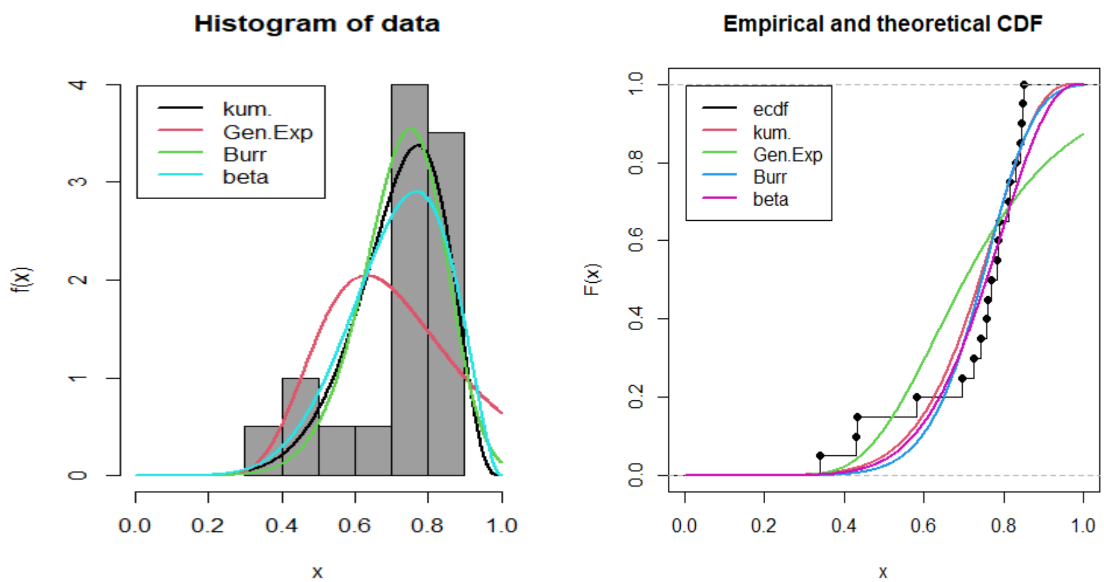


Figure 2: Goodness of fit tests for real data

Referring to thevalues reported in table (4), we conclude that the Kumaraswamy distribution fits the data set good compared to the other models. Thus, the various point and interval estimates of α and $\beta$for real data under progressive censoring schemes as following as in table(5).

In table (5), we display the estimates obtained by using MPS at $m = 10$. we computed the average estimates (Avg.) and standard deviation (SD). From Tables (5), we observed that the Avg. and MSE of the estimates are close together.

Table (5).Point and interval estimates of α and $\beta$for MPS of the Kumaraswamy distribution under progressive censoring schemes.

n	m	Sch.	Parm.	MPS	
				Avg.	SD
20	10	S_1	α	12.9105	4.3516
			β	23.4821	24.7160
		S_2	α	8.6988	3.2672
			β	3.7516	2.9224
		S_3	α	17.2251	5.2833
			β	36.6941	47.8674
		S_4	α	7.8385	3.0958
			β	2.0032	1.4750

5. Conclusion

In this paper, we have considered the problem of estimation of the unknown parameters for Kumaraswamy distribution under progressive Type-II censoring. We derived maximum product spacings estimates for the unknown parameters of a Kumaraswamy distribution. In this paper, although we have mainly considered Type-II progressive censoring cases, the same method can be extended to other censoring schemes also.

References

[1] Akaike, H. (1969). Fitting autoregressive models for regression. Annals of Institute of Statistical Mathematics. 21, 243–247.

[2] Balakrishnan, N. and Aggarwala, R. (2000). *Progressive censoring: theory, methods, and applications*.Springer Science and Business Media.

[3] Balakrishnan, N., and Sandhu, R. A. (1995). A simple simulational algorithm for generating progressive Type-II censored samples. The American Statistician, 49(2), 229-230.

[4] El-Deen, M.S., Al-Dayian, G.R. and El-Helbawy, A.A. (2014). Statistical inference for Kumaraswamy distribution based on generalized order statistics with applications. Journal of Advances in Mathematics and Computer Science, 1710-1743.

[5] Eldin, M.M., Khalil, N. and Amein, M. (2014). Estimation of Parameters of the Kumaraswamy Distribution Based on General Progressive Type-II Censoring. American Journal of Theoretical and Applied Statistics, 3(6), 217-222.

[6] El-Sagheer, R. M. (2019). Estimating the parameters of Kumaraswamy distribution using progressively censored data. Journal of Testing and Evaluation, 47(2), 905-926.

[7] El-Sherpieny, E. S. A., Almetwally, E. M., and Muhammed, H. Z. (2020). Progressive Type-II hybrid censored schemes based on maximum product spacing with application to Power Lomax distribution. Physica A: Statistical Mechanics and its Applications, 553, 124251.

[8] Erick, W.M., Kimutai, K.A. and Njenga, E.G. (2016). Parameter Estimation of Kumaraswamy Distribution Based on Progressive Type-II Censoring Scheme Using Expectation-Maximization Algorithm, American Journal of Theoretical and Applied Statistics, 5(3), 154-161.

[9] Feroze, N. and Elbatal, I. (2013). Parameter Estimations Based On Kumaraswamy Progressive Type-II Censored Data with Random Removals. Journal of Modern Applied Statistical Methods,12(2), 19-35.

[10] Ghafouri, S. and Rastogi, M.K. (2021). Reliability analysis of Kumaraswamy distribution under progressive first-failure censoring. Journal of Statistical Modelling: Theory and Applications, 2(1), 67-99.

[11] Gholizadeh, R. Khalilpor, M. and Hadian, M. (2011). Bayesian estimations in the Kumaraswamy distribution under progressively Type-II censoring data. International Journal of Engineering, Science and Technology, 3(9), 47-65.

[12] Ghosh, I. and Nadarajah, S. (2017). On the Bayesian inference of Kumaraswamy distributions based on censored samples. Communications in Statistics-Theory and Methods, 46(17), 8760-8777.

[13]Hurvich, C. M., and Tsai, C. L. (1989). Regression and time series model selection in small samples. Biometrika, 76(2), 297-307.

[14] Kohansal, A. and Bakouch, H. S. (2019). Estimation procedures for Kumaraswamy distribution parameters under adaptive Type-II hybrid progressive censoring. Communications in Statistics-Simulation and Computation, 50(12), 4059-4078.

[15] Kumaraswamy, P. (1976). Sinepower probability density function. Journal of Hydrology, 31(1-2), 181–184.

[16] Kumaraswamy, P. (1978). Extended sinepower probability density function. Journal of Hydrology, 37(1-2), 81–89.

[17] Kumaraswamy, P. (1980). A generalized probability density function for double-bounded random processes. Journal of Hydrology, 46(1-2), 79–88.

[18] Ng, H. K. T., Luo, L., Hu, Y., and Duan, F. (2012). Parameter estimation of three-parameter Weibull distribution based on progressively Type-II censored samples. Journal of Statistical Computation and Simulation, 82(11), 1661-1678.

[19] Pak, A. and Rastogi, M.K. (2018). Classical and Bayesian estimation of Kumaraswamy distribution based on Type-II hybrid censored data. Electronic Journal of Applied Statistical Analysis, 11(1), 235-252.

[20] Reyad, H. M. and Ahmed, S.O. (2016). Bayesian and E-Bayesian estimation for the Kumaraswamy distribution based on Type-II censoring. International Journal of Advanced Mathematical Sciences, 4(1), 10-17.

[21] Schwarz, G. (1978). Estimating the dimension of a model. The Annals of Statistics. 6(2), 461-464.

[22] Sindhu, T.N., Feroze, N. and Aslam, M. (2013). Bayesian analysis of the Kumaraswamy distribution under failure censoring sampling scheme. International Journal of Advanced Science and Technology, 51, 39-58.

[23] Sultana, F., Tripathi, Y. M., Wu, S. J., and Sen, T. (2020). Inference for kumaraswamy distribution based on Type-I progressive hybrid censoring. Annals of Data Science, 1-25.

[24] Sultana, F., Tripathi, Y.M., Rastogi, M.K. and Wu, S.J. (2018). Parameter estimation for kumaraswamy distribution based on hybrid censoring. American Journal of Mathematical and Management Sciences,37(3) ,1-25.

[25] Tu, J. and Gui, W. (2020). Bayesian Inference for the Kumaraswamy Distribution under GeneralizedProgressive Hybrid Censoring. Entropy, 22(9), 1032.

[26] Wu, S. J. (2002). Estimation of parameters of Weibull distribution with progressively censored data. Journal of Japan Statistics and Society, 32(2), 155-163.

الملخص:

في هذا البحث درسنا مشكلة تقدير النقاط لمعلمات توزيع كوماراسوامي على أساس الرقابة التدريجية من النوع الثاني. يتم استخدام الحد الأقصى لتباعد المنتج لتقدير معلمات النموذج. لتقييم أداء مقدر النقاط ، يتم إجراء دراسة المحاكاة. لتوضيح فائدة الدراسة في الممارسة العملية ، تم أيضًا أخذ مجموعة بيانات حقيقية في الاعتبار.

الكلمات المفتاحية:

توزيع كوماراسوامي ، الرقابة التدريجية من النوع الثاني ، تقدير المسافات القصوى للمنتج.