

Blind Estimation of PN Sequence of DS-CDMA Signals over Time Varying Flat Fading Channel

By

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Abstract:

Spread spectrum signals are widely used for secure communications, as well as for multiple access interference (MAI). They have many interesting properties, including low probability of interception. Indeed, direct sequence spread spectrum systems (DSSSS) use a periodical pseudo-random sequence to modulate the baseband signal before transmission. Interception of DSSS signals is a difficult task due to the hiding of the PN sequence, especially if the accompanied noise is non-Gaussian and the channel is time varying.

In this paper, a receiver is proposed to intercept asynchronous direct sequence code division multiple access (DS-CDMA) signals received from different users and estimate the PN sequences of each one of them. The signals are assumed to be transmitted over time varying flat fading channel and contaminated with Class-A impulsive noise. The proposed interception receiver is based on Independent Component Analysis (ICA). The proposed receiver is tested for different types of PN sequences including m sequence, Gold sequence, and Walsh codes. The results show that the interception receiver is not sensitive to the time variation of the channel and to the type of PN sequence. Moreover it is found that the receiver performs well for low signal to noise ratio and for any type of PN codes.

Keywords: PN sequence; DS-CDMA; ICA, Impulsive noise; time varying flat fading channel.

I. Introduction

Direct Sequence Spread Spectrum (DSSS) communication has many characteristics, such as wide spectrum, low power spectral density, confidentiality, anti-interference and easy to use in multiple access communication. Combined with the double merits of multiple access and direct sequence spread spectrum, direct sequence code division multiple access (DS-CDMA) has been widely used in both military and civil communications. The PN sequences are very important parameters in DS-CDMA communication system, from which multi-user can share the same frequency band at the same time [1].

At present, the researchers has conducted in-depth study of blind estimation of the PN sequence of DSSS signal, and put forward a series of methods, such as in [2], this method proposed a cipher analysis method of m-sequence utilizing the logic character of linear feedback shift register (LFSR). This method made use of only 1th order statistic therefore it was sensitive to noise. In [3] the previous method has adopted with maximum vote criterion to find the primitive polynomial of m-sequence. When information modulation existed, the method was not available any more. In [4] it is proposed a method of pseudo random estimation based on principal component analysis (PCA), extended the method to the situation of two users in [5], and analysis the performance from the idea of eigen value decomposition (EVD) in [6]. But it was not available in multiuser situations. The study of blind estimation of PN sequences of DS-CDMA signal was started later, and had fewer achievements. In DS-CDMA system, information sequences of different users remained statistically independent and the pseudo random sequences of different users remained none correlation, so the DS-CDMA signals were fit for the theory model of blind source separation (BSS)[7]. ICA, the typical technology of BSS, therefore could be used to achieve the blind separation of DS-CDMA signals in non-cooperative situations, only when few parameters were estimated. With the development of DS-CDMA signal processing technology, some efficient methods of parameter estimation for the carrier frequency [8], chip interval[9], and symbol interval[10] of the DS-CDMA signals had appeared. In [11] it is used ICA method to detect the information of DS-CDMA system blindly. If the period of PN sequence was known, we could get the information sequence of each user from asynchronous DS-CDMA signal using ICA, and estimating the true PN code for each user.

From the above survey we note that researchers have not focused on blind estimation of PN sequence over time varying flat fading channel, although most of wireless communication systems work on time varying flat fading channel. Typically wireless transceiver designs are based on the assumption that noise is additive, white and Gaussian (AWG) [12]. These transceivers perform fine in normal environments (where optimum reception can be achieved with Gaussian channel assumption) but their applicability in noise intensive electricity substation environment is not risk free and needs thorough investigation [13]. Partial discharges and sferic radiation (from fault and switching transients) are major sources of impulsive noise, in electricity transmission substations and if the risks of deploying wireless communications equipment are to be properly assessed the impact of such impulsive processes requires thorough evaluation [14]. Accordingly, a method based on ICA was proposed here to achieve the estimation of PN sequence in the presence of impulsive noise and time varying flat fading channel. Simulation results proved the validity of the method proposed here and give a good performance under high effects of impulsive noise and time varying flat fading channel.

The rest of the paper is organized as follows: In Section II, mathematical model is presented. The analysis method of ICA is introduced Section III. Simulation results are presented in Section V.

II. SYSTM MODEL

A. Asynchronous DS-CDMA Model

In asynchronous DS-CDMA, each of the K users uses his own PN sequence to spread his narrowband information signal. Consider a BPSK modulation the received baseband signal can be written as:

$$s(t) = \sum_{k=1}^K s_k(t) \quad (1)$$

where $s_k(t)$ is the k^{th} user's transmitted signal, its complex baseband representation could be modeled as:

$$s_k(t) = \sqrt{\xi_k} \sum_{m=1}^N [b_k(m) c_k(t - mT_b - \tau_k)] \quad (2)$$

where ξ_k is the signal energy per bit; N is the number of transmitted data bits; b_{km} is the m th data bit of the k th user; $c_k(t)$ is the PN sequence of the k th user; T_b is the bit period; T_c is the PN sequence chip period; τ_k is the delay of the k th user and τ_k is integer times the T_c .

B. Middleton Class A Model

It refers to impulsive noise with a spectrum that is narrow compared to the receiver bandwidth and includes all pulses which do not produce transients in the receiver front end [15]. Its probability density function (pdf), given by:

$$f_x(x) = e^{-A} \sum_{m=0}^{\infty} \frac{A^m}{m! \sqrt{2\pi\sigma_m^2}} e^{-\frac{x^2}{2\sigma_m^2}} \quad (3)$$

where

$$\sigma_m^2 = \frac{m}{A} + \zeta \quad (4)$$

is noise variance, $A = \zeta t T_s$ is impulse index, ζ is mean impulse rate and T_s is mean impulse duration. Equation (3) is a weighted sum of Gaussian distributions. By increasing impulse index, A , the noise can be made arbitrarily close to Gaussian and by decreasing A it can be made arbitrarily close to a conventional Poisson process. The model assumes that the individual impulses are Poisson distributed in time. The scale factor ζ is the ratio of powers in the Gaussian and Poisson (non-Gaussian) components, i.e.:

$$= \frac{(X_G^2)}{(X_P^2)} \quad (5)$$

Fig.1 shows the pdf of Middleton class A noise with various values of A for $\zeta = 0.001$.

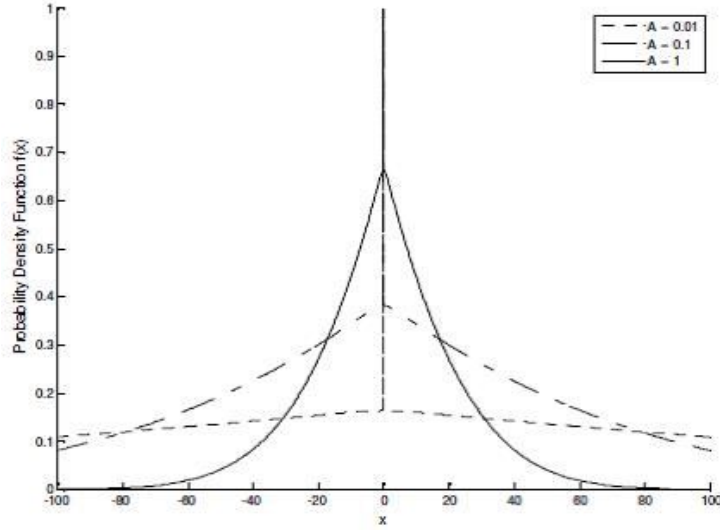


Figure (1): Probability density of the amplitude of Class-A noise with different values of A and $C = 0.001$ [16]

C. Channel Model

The dynamics of the channel state \mathbf{h}_k of the time-varying flat Rayleigh fading channel are modeled by a first-order Gauss–Markov process [17]

$$\mathbf{u}_i \stackrel{i.i.d}{\sim} N(0, (1 - a^2)\sigma_h^2), \quad \mathbf{h}_i = a\mathbf{h}_{i-1} + \mathbf{u}_i \quad (6)$$

where $\mathbf{h}_i \sim N(0, \sigma_h^2)$ the zero mean complex Gaussian channel state with variance σ_h^2 , \mathbf{u}_i is the white Gaussian driving noise at time i . Parameter $a \in [0, 1]$ is the fading correlation coefficient that characterizes the degree of time variation; small a models fast fading, and large a corresponds to slow fading. The Gauss–Markov model is widely adopted as a simple and effective model to characterize the fading process [18][19]. The first-order Gauss–Markov model is parameterized by the fading correlation coefficient. The value of a can be determined by the channel Doppler spread and the transmission bandwidth, where the relation among the three is found in [19]. It can be accurately obtained at the receiver for a variety of channels [20]. Here, we assume that is known.

III. BLIND SEPARATION OF DS-CDMA SIGNALS WITH ICA

The received CDMA signal over time varying flat fading channel and additive impulse noise can be written as:

$$r(t) = \sum_{k=1}^K \sum_{m=1}^M \sum_{p=1}^P \sqrt{\xi_k} b_{k,m} \mathbf{h}_{k,m}(p) C_k(t - mT_b - pT_c - (k)) + n(t) \quad (7)$$

where $h_{k,m}(p)$ is the attenuation factor coefficient of every chip bit of m th bit of k th user of time varying flat fading channel; P is the length of the PN code; $n(t)$ is the class A impulsive noise.

This DS-CDMA system is an asynchronous system, that is to say C_k is different for each user. But we assume that the length of PN sequence of each user is equal to P , T_c and T_b are known and have the relationship of $T_b = PT_c$. Samples are taken from the observed data in T_c cycle.

Define a sampled data vector r_m of length $2P \times 1$,

$$r_m = [r_p(t - (m-1)P + 1) \ r_p(t - (m-1)P + 2) \ \dots \ r_p(t - (m+1)P)]^T \quad (8)$$

For asynchronous DS-CDMA system r_m can be written as:

$$r_m = \sqrt{E_b} \left[\sum_{k=1}^K \left(\sqrt{b_k(k,m-1)} h_{k,m-1}(t - (\text{mod}(P - (k + p - 1, P) + 1) T_c)) g_k^L(t - pT_c) \right) \mathbf{1}_{(1)} + \sum_{k=1}^K \left(\sqrt{b_k(k,m)} h_{k,m}(t - (\text{mod}(P - (k + p - 1, P) + 1) T_c)) g_k^C(t - pT_c) \right) \mathbf{1}_{(2)} + \sum_{k=1}^K \left(\sqrt{b_k(k,m+1)} h_{k,m+1}(t - (\text{mod}(P - (k + p - 1, P) + 1) T_c)) g_k^F(t - pT_c) \right) \mathbf{1}_{(3)} \right] + n(t)$$

where g_k^L is $2P \times 1$ vector contains last C_k chips of the k th PN sequence; g_k^C is $2P \times 1$ vector contains a complete period of the k th PN sequence; g_k^F is $2P \times 1$ vector contains first $(P - C_k)$ chips of the k th PN sequence; $n(t)$ is the $2P \times 1$ vector of impulsive noise samples.

$$g_k^L = [C_k(P - (k + 1)) \ \dots \ C_k(P) \ 0 \ \dots \ 0]^T \quad (10)$$

$$g_k^C = [0 \ \dots \ 0 \ C_k(1) \ \dots \ C_k(P) \ 0 \ \dots \ 0]^T \quad (11)$$

$$g_k^F = [0 \ \dots \ 0 \ C_k(1) \ \dots \ C_k(P - (k))]^T \quad (12)$$

Fig.2 shows that the received signal r_m which must contain the complete periods of the three users codes and a beginning and ending of the three codes.

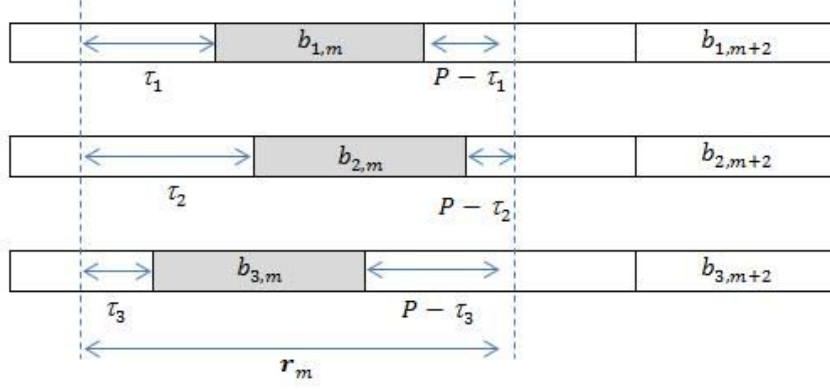


Figure (2): Sketch map of \mathbf{r}_m

If we want to express (9) in a matrix notation, we can replace \mathbf{g}_k^L , \mathbf{g}_k^C and \mathbf{g}_k^F with a matrix \mathbf{G} and can be written as:

$$\mathbf{Q} = \mathbf{GB} + \mathbf{N}_m \quad (13)$$

If the number of observed bits is N , we can define $\mathbf{Q} = [\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N]$ with dimension $2C \times N$ and $\mathbf{B} = [\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_N]$ with dimension $3K \times N$ and \mathbf{b}_m is $3K \times 1$ vector composed of the $3K$ statistically independent data bit with

$[\mathbf{b}_m = [b_{1,m-1}, b_{1,m}, b_{1,m+1}, \dots, b_{K,m-1}, b_{K,m}, b_{1,m+1}]^T]$ with dimension and

$\mathbf{G} = [\mathbf{g}_1^L, \mathbf{g}_1^C, \mathbf{g}_1^F, \dots, \mathbf{g}_k^L, \mathbf{g}_k^C, \mathbf{g}_k^F]$ with dimension $2C \times 3K$ where

$$\mathbf{g}_k^L = \sum_{p=1}^{2P} \mathbf{h}_{k,m-1}(t - MT_c) \mathbf{g}_k^L(t - pT_c) \quad (14)$$

$$\mathbf{g}_k^C = \sum_{p=1}^{2P} \mathbf{h}_{k,m}(t - MT_c) \mathbf{g}_k^C(t - pT_c) \quad (15)$$

$$\mathbf{g}_k^F = \sum_{p=1}^{2P} \mathbf{h}_{k,m+1}(t - MT_c) \mathbf{g}_k^F(t - pT_c) \quad (16)$$

In the regular ICA process, the observed data should be preprocessed, including centering and whitening.

A. Centering

The received baseband signal $\mathbf{r}(t)$ is centered by removing its mean,[21]

$$\mathbf{r}_c(t) = \mathbf{r}(t) - E[\mathbf{r}(t)] \quad (17)$$

B. Whitening

Whiteness of a Zero mean random vector means that its components are uncorrelated and their variances equal unity. Whitening using the principal component analysis (PCA) method

can reduce dimension and make ICA process more accurate and faster. The whitening method is as follows:

Eigen value decomposition (EVD) of the covariance matrix \mathbf{R} of the sampled observations is:

$$\mathbf{R} = \mathbf{U}\mathbf{D}\mathbf{U}^T = \mathbf{G}\mathbf{G}^T + \sigma_n^2\mathbf{I} = [\mathbf{U}_S \quad \mathbf{U}_N] \begin{bmatrix} \lambda_S + \sigma_n^2\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \sigma_n^2\mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{U}_S^T \\ \mathbf{U}_N^T \end{bmatrix} \quad (18)$$

where \mathbf{U}_S is the signal subspace with dimension of $2P \times 3K$; \mathbf{U}_N is the noise subspace with dimension of $2P \times (2P - 3K)$; λ_S is the diagonal matrix of eigenvalues of the signal subspace; σ_n^2 is the variance of noise.

Then whitening is applied by projecting to the signal subspace so the correlation of different user's signal could be removed. The whitening matrix is:

$$\mathbf{V} = (\lambda_S + \sigma_n^2\mathbf{I})^{-\frac{1}{2}}\mathbf{U}_S^T \quad (19)$$

The data after whitening is:

$$\mathbf{Z} = \mathbf{V}\mathbf{Q} = (\mathbf{V}\mathbf{G})\mathbf{B} + \mathbf{V}\mathbf{N} = \mathbf{A}\mathbf{B} + \mathbf{N}' \quad (20)$$

$$\mathbf{A} = \mathbf{V}\mathbf{G} = (\lambda_S + \sigma_n^2\mathbf{I})^{-\frac{1}{2}}\mathbf{U}_S^T\mathbf{G} \quad (21)$$

\mathbf{A} is the mixing matrix which is a full rank and square matrix. The dimension of the signal after whitening is reduced from $2P$ to $3K$. Using ICA method, an orthogonal matrix $\mathbf{W} = \mathbf{A}^{-1}$ can be found to estimate the data \mathbf{B} .

$$\hat{\mathbf{B}} = \mathbf{W}\mathbf{Z} \quad (22)$$

C. ICA algorithm

Through the preprocessing steps above, data \mathbf{Z} are ready to be processed by ICA. The fastICA algorithm is based on a fixed point iteration scheme for finding the maximum of the none-Gaussianity [22]. From the information theory, it is rational to use negentropy as the measurement of the none-Gaussianity of stochastic variables. In [23], two none linear functions f are recommended to estimate the negentropy.

$$f_2(u) = u \exp(-u^2/2), \quad f_1(u) = u^3 \quad (23)$$

ICA based on the negentropy criterion achieved the separation of the signal through following steps:

1. Choose m , the number of ICs to estimate. Set $l \leftarrow 1$.
2. Initialize \mathbf{w}_l (e.g. randomly)
3. Do an iteration of a one-unit algorithm on \mathbf{w}_l .

$$\mathbf{w}_l \leftarrow \frac{1}{N} \sum_{m=1}^N [\mathbf{z}_m (\mathbf{w}_l^T \mathbf{z}_m)^2] - 3\mathbf{w}_l \quad (24)$$

4. Do the following orthogonalization:

$$\mathbf{w}_l \leftarrow \mathbf{w}_l - \sum_{j=1}^{l-1} (\mathbf{w}_l^T \mathbf{w}_j) \mathbf{w}_j \quad (25)$$

5. Normalize \mathbf{w}_l by dividing it by its norm.

6. If \mathbf{w}_l not converged, i.e. $|\mathbf{w}_l^T \mathbf{w}_{l-1}|$ is not close enough to 1, go back to 3.

7. Set $l \leftarrow l + 1$. If l is not greater than the desired number of ICs, go back to step 2.

When iteration ended, the unmixing matrix $\mathbf{W} \approx \mathbf{A}^{-1}$ is calculated out. When left multiplied by \mathbf{W} and right multiplied by the matrix of pseudo random sequences, \mathbf{G}^T , expression (21) can be written as:

$$\mathbf{W} (\lambda_s + \sigma_n^2 \mathbf{I})^{-\frac{1}{2}} \mathbf{U}_s^T \mathbf{G} \mathbf{G}^T = \mathbf{G}^T \quad (26)$$

From expression (18), $\mathbf{G}^T \approx \mathbf{U}_s (\lambda_s + \sigma_n^2 \mathbf{I}) \mathbf{U}_s^T$, thus the matrix of pseudo random sequence could be estimated by :

$$\mathbf{G}^T = \mathbf{W} (\lambda_s + \sigma_n^2 \mathbf{I})^{-\frac{1}{2}} \mathbf{U}_s^T \mathbf{U}_s (\lambda_s + \sigma_n^2 \mathbf{I}) \mathbf{U}_s^T \quad (27)$$

$$\mathbf{G} = \left(\mathbf{W} (\lambda_s + \sigma_n^2 \mathbf{I})^{-\frac{1}{2}} \mathbf{U}_s^T \right)^T \quad (28)$$

From the expression above, the pseudo random sequence could be recovered directly.

V. SIMULATION

In order to test the performance of the method proposed above, several experiments are performed in this section. The simulation parameters are as follows: DS-CDMA signal, BPSK modulation, the number of bits is 10^5 bits, the PN sequence is Gold sequence with the length of 63, the SNR is -8dB, there are three users in this asynchronous DS-CDMA system with delays $d_1 = 40$, $d_2 = 30$, $d_3 = 20$ chips, the signals are assumed to transmitted over time varying flat fading channel with the first-order Gauss–Markov model with fading correlation coefficient for the three users channels is $\alpha = 0.1$ and contaminated with Class-A impulsive noise with $A = 0.1$ and $(= 0.5 \times \mathbf{[10]}^T(-3))$.

Now we estimate the PN sequence of each user using the method proposed above. Fig. 3 shows the original PN sequence waveforms of the 3 users. Fig.4 shows time realization of highly impulsive ($A = 0.1$ and $(= 0.5 \times \mathbf{[10]}^T(-3))$) Middleton class A impulsive noise. Fig.5 shows the signal of each user under time varying flat fading channels and the total received signal at the receiver under class-A impulsive noise with SNR= -8 dB. Fig.6 shows the estimated codes which appears that code of user 1 is estimated at code of user (c) with delay 40 chips, and the code of user 2 is estimated at code of user (b) with delay 30 chips, and the code of user 3 is estimated at code of user (a) with delay 20 chips. Fig.7 shows that the

comparison between the original codes and the estimated codes which shows a good performance of the algorithm at SNR= -8 db. Fig. 8 shows that the comparison between the original codes and the estimated codes which shows a good performance of the algorithm at SNR= -13 dB .

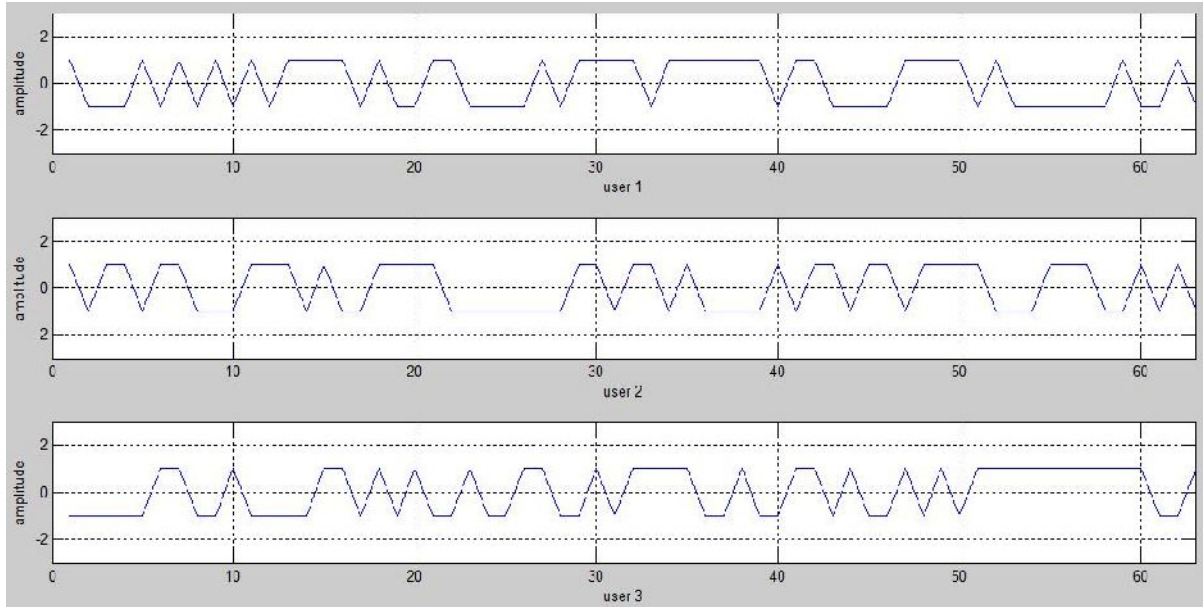


Figure (3): The original PN sequence waveforms of the 3 users

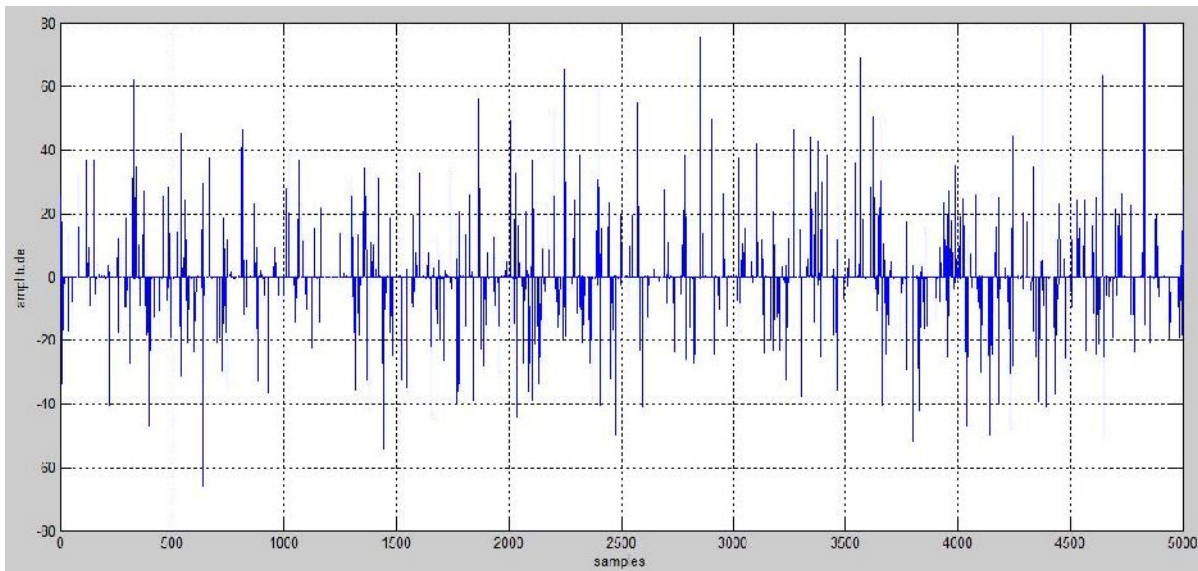


Figure (4): Time realization of highly impulsive ($\alpha = 0.1$ and $\beta = 0.5 \times 10^{-3}$) Middleton class A noise

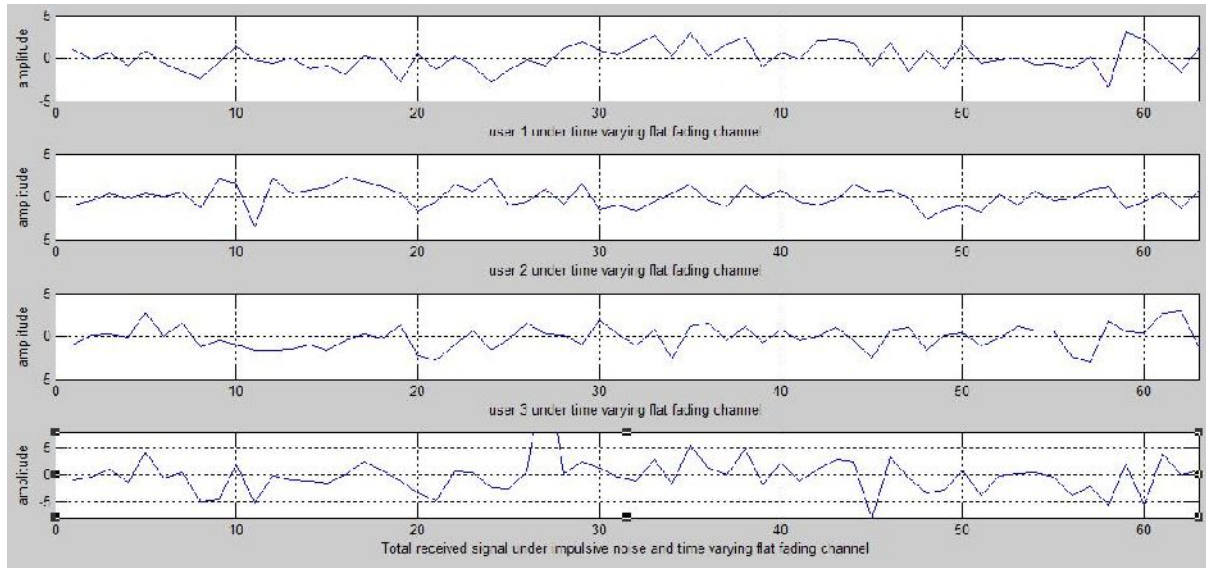


Figure (5): The total received data of the three users under impulsive noise and time varying flat fading channel

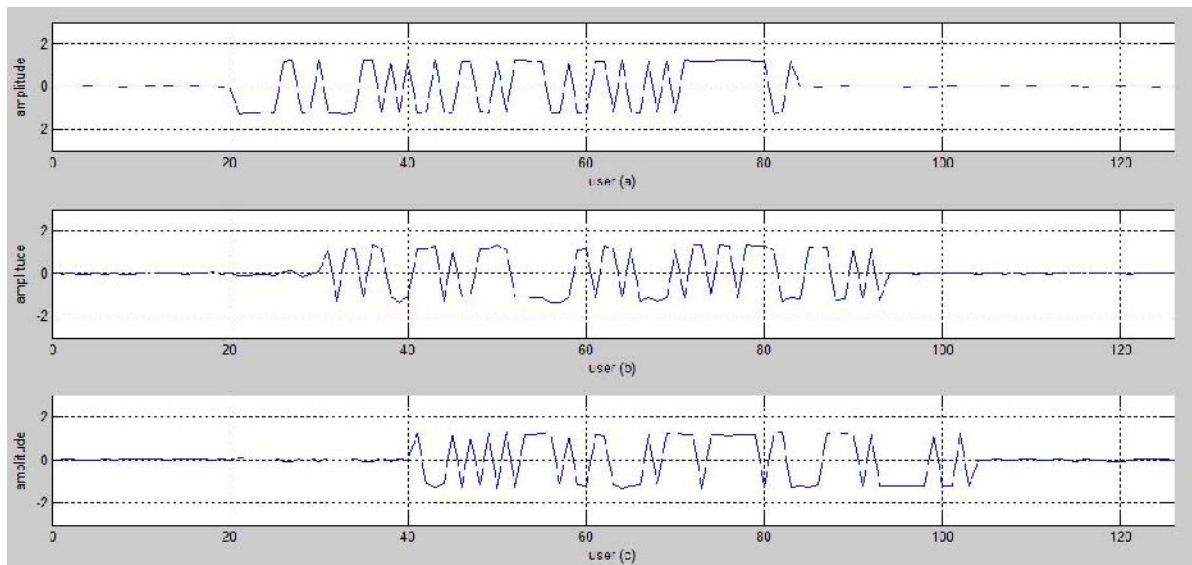


Figure (6): The estimated results of ICA at SNR= - 8 dB

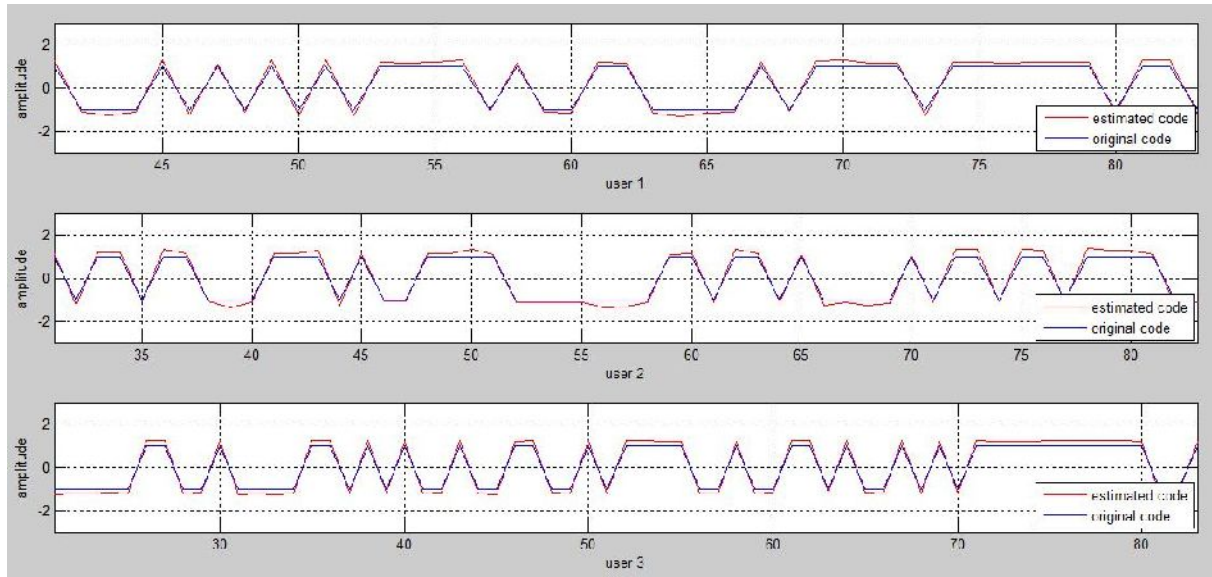


Figure (7): The estimated three codes at SNR= -8 dB

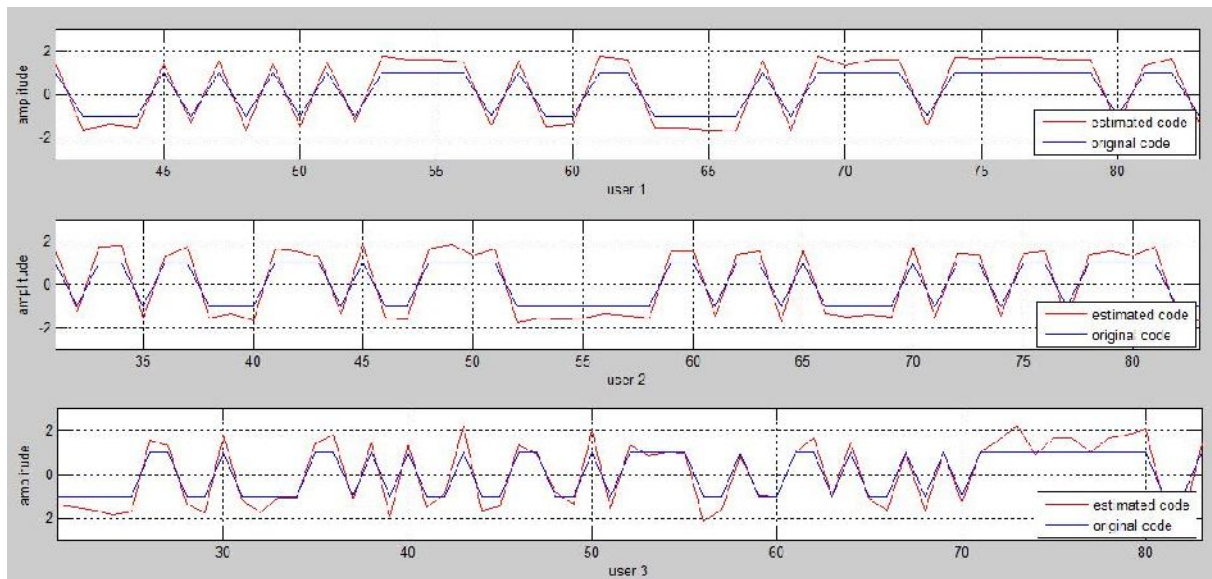


Figure 8: The estimated three codes at SNR= -13 dB

6. Conclusion

Technology ICA could be used to achieve the blind separation of DS-CDMA signals. When the subspace of pseudo random sequence is multiplied by the unmixing matrix which is calculated from ICA, the estimated waveform of pseudo random could be acquired.

In this paper, the DS-CDMA signal model under impulsive noise and time-varying flat fading is presented; we also give the theoretical derivation and computer simulation. The proposed method can correct estimate the PN sequences of each user in low SNR with no restriction for the type of PN sequence.

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