**Military Technical College
Kobry El-Kobbah,
Cairo, Egypt**

**8th International Conference
on Electrical Engineering
1. ICEENG 2012**

Control of Industrial Servo System in Presence of Motor-Load Inertia Mismatch using µ-synthesis

Farid A. Tolbah [*], Magdy M. Abdelhameed [†] and Ahmed Ramadan[‡]

Abstract: There are a number of challenging factors that influence the performance of the industrial servo systems. One of these factors is the inertia mismatch between the motor and the load. Another problem is the variable-inertia load. This paper applies a robust control tool called µ-synthesis to design a controller which deals with the uncertainty of the inertia in the load. Different practical experiments are presented to investigate the effect of some parameters in the synthesis process on the performance of the synthesized controller. Also simulations are included to introduce different values of inertia mismatch and study the effect of increasing inertia mismatch on the servo system performance. It has been found the robust controller can deal with variable parameter control problem with adequate weighting functions. Also the increased inertia mismatch has been found to slow the servo system down and demand higher control action.

Keywords: Inertia mismatch, variable inertia, servo system, robust control, µ-synthesis, PMSM

2. Introduction

The machine tool servo system has been developed over the last years so rapidly to enhance the performance of the system. Enhancement includes higher accuracy demand on the dimensions of the final product. One of the factors that influence this performance is the inertia mismatch between the motor and the load. The industrial servo motor manufacturers have a rule of thumb to keep the inertia mismatch in the range from one to ten. It's desirable to make the motor inertia low to get more torque to inertia ratio and hence faster response. On the other side, reducing the motor inertia for a given load induces higher inertia mismatch between load and motor.

One of the other problems that face the servo system industry is variable-inertia load. It's very common in servo applications like loading-unloading or connecting a variable inertia mechanism to the servo motor. To handle this case, robust control techniques are the most suitable, especially if the variation of the inertia is not a function of time or not periodic over a cycle. If the variation of the inertia is deterministic, then nonlinear control theory can be used. In this paper, focus is set on applying a robust control tool, µ-synthesis to handle the problem of mismatched, variable inertia load connected to servo motor.

[*] Professor of Automatic Control, Ain Shams University, Email: tolbah1@yahoo.com
[†].Professor of Mechatronics and Automatic Control, Ain Shams University, Email: magdyma@gmx.net
[‡] Teaching Assistant, Ain Shams University, Email: ahmed_ramadan@eng.asu.edu.eg

3. Ahmed_Ramadan@eng.asu.edu.eg**Servo System Modeling**

The servo system under study is a PMSM (Permanent Magnet Synchronous Motor) equipped with a gear box and a resolver from SEW Eurodrive. It is connected to a four-bar mechanism which is the variable inertia load. The motor is driven by an industrial inverter from SEW Eurodrive too. The model of the PMSM can be written as [1]:

$$v_d = i_d R_s - \omega_r \lambda_q + \frac{d}{dt}\lambda_d \tag{1}$$

$$v_q = i_q R_s + \omega_r \lambda_d + \frac{d}{dt}\lambda_q \tag{2}$$

Where v_d, v_q are dq voltages, i_d, i_q are the dq currents, λ_d, λ_q are the flux linkages in the dq axes, R_s is the stator resistance, and ω_r is the electrical rotating speed of the rotor. The equivalent electric circuits of the d and q axes are shown in Fig. 1.

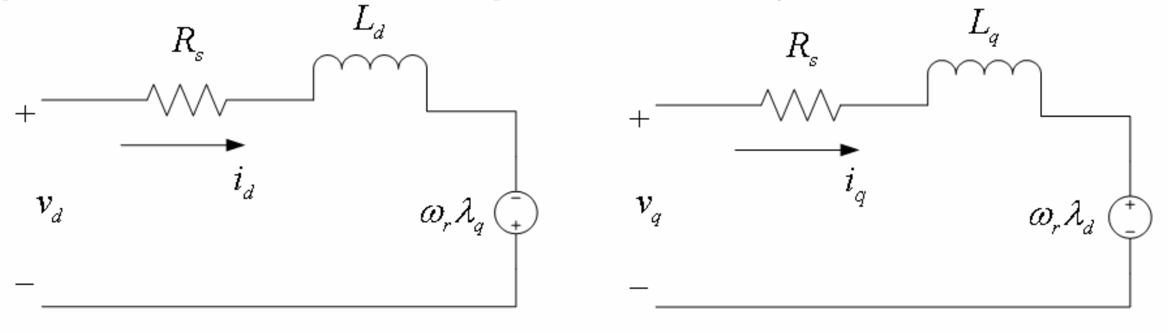

Fig. 1 Equivalent electric circuits of PMSM (redrawn from [2])

Flux Linkages are given by:

$$\lambda_d = L_d i_d + \lambda_f \tag{3}$$

$$\lambda_q = L_q i_q \tag{4}$$

Where L_d, L_q are the dq inductivity and λ_f is the permanent magnet flux linkage. Substituting the flux linkages, Equations (3) and (4) into the voltage Equations (1) and (2) yields:

$$v_d = i_d R_s - \omega_r L_q i_q + \frac{d}{dt}(L_d i_d + \lambda_f) \tag{5}$$

$$v_q = i_q R_s + \omega_r \left(L_d i_d + \lambda_f\right) + \frac{d}{dt}(L_q i_q) \tag{6}$$

The developed electric torque (T_e) is given by:

$$T_e = \frac{3}{2} p\left(\lambda_d i_q - \lambda_q i_d\right) = \frac{3}{2} p\left(\left(L_d i_d + \lambda_f\right) i_q - \left(L_q i_q\right) i_d\right) \tag{7}$$

$$T_e = \frac{3}{2}p\left(L_d i_d i_q + \lambda_f i_q - L_q i_q i_d\right) = \frac{3}{2}p\left(\left(L_d - L_q\right)i_d i_q + \lambda_f i_q\right) \quad (8)$$

p is the number of rotor pole pairs. On a smooth air gap PMSM, L_d=L_q [3]
In field Orientation Control, the set point of d-component of the current is to be zero, thus the electromagnetic torque T_e is dependent on i_q and proportional to it. The torque equation is then reduced to the form [3]:

$$T_e = k_t i_q \quad (9)$$

Where $T_e = \frac{3}{2}p\lambda_f$ and k_t is the torque constant
The mechanical system is modeled as variable inertia load (J_{total}) and viscous damping (B):

$$T_e = T_l + B\omega_m + J_{total}\frac{d}{dt}\omega_m \quad (10)$$

Where ω_m is the mechanical speed of the rotor and T_l is the load torque.
The relation between ω_r and ω_m is given by [4]:

$$\omega_r = p\omega_m \quad (11)$$

The load is a four bar mechanism modeled as an uncertain atom J, of nominal value $J_{nominal}$, and uncertainty percentage. This can be defined on MATLAB as follows [5]:

```
J=ureal('J', Jnominal,'Percentage', [-15.8  25.7]);
```

4. Design of Proposed Controller by µ-Synthesis:

The classical control scheme used in industrial servo systems is a cascaded control loop. It consists of inner current loop, speed loop and position loop as shown in Fig. 2. To introduce a robust controller, this control loop is broken to insert the desired controller in the speed loop as shown in Fig. 3. Now the proposed controller can be speed controller or both speed and position controllers cascaded.

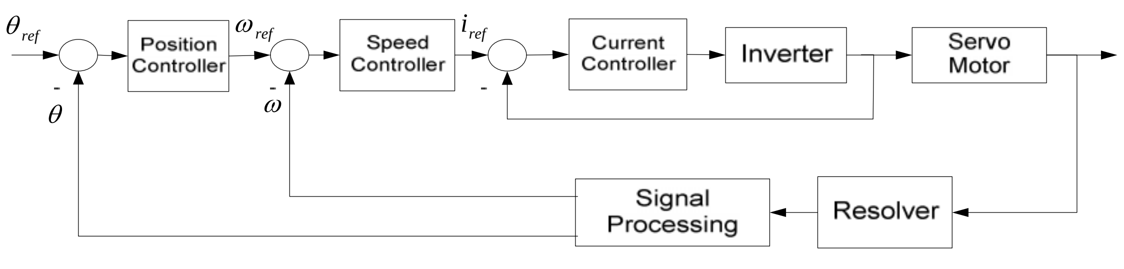

Fig. 2 Cascaded Control Scheme for servo systems

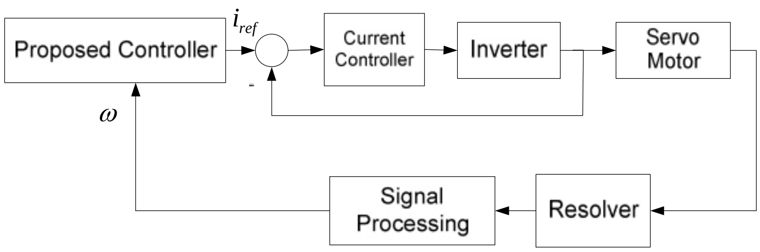

Fig. 3 Proposed Control Scheme

The µ-synthesis framework requires reformulating the plant to be MIMO (Multi-Input, Multi-Output) system as shown in Fig. 4. This configuration implies weighting the inputs and outputs to be normalized. The blocks of 6 and 60/(3000*2π) are used to weight the set point current and motor speed respectively. More weighting functions are added to define the performance indices to be minimized. W_u and W_S penalize the control action and the tracking error respectively.

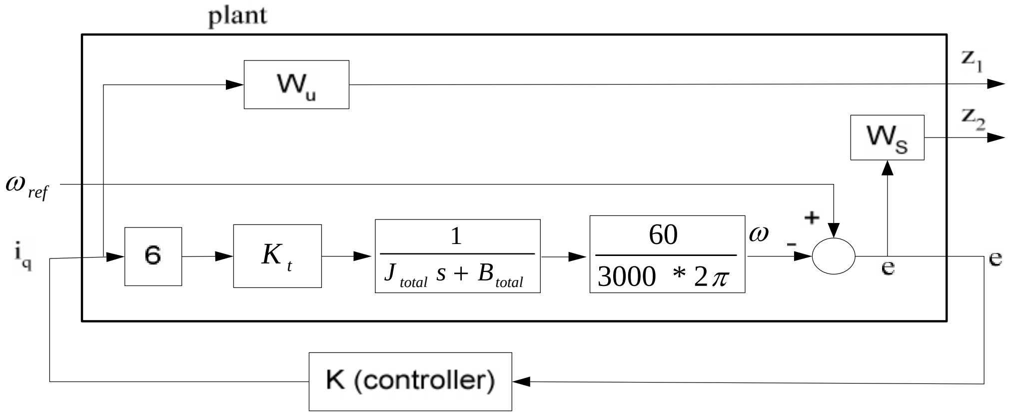

Fig. 4 µ-synthesis Framework of the servo system

To define this plant on MATLAB, it needs to be on the form of system matrix which is an augmentation of the state space matrices into one partitioned matrix [6] as shown in Fig. 5. The signal w is the reference and/or disturbance inputs, z is the penalized performance indices, u is the control action, and y is the feedback measurement. This is done on MATLAB by one of the two commands (from robust control toolbox) [5]:

```
P=pck(A, [B1 B2], [C1; C2], [D11 D12; D21 D22]);
P=sysic;
```

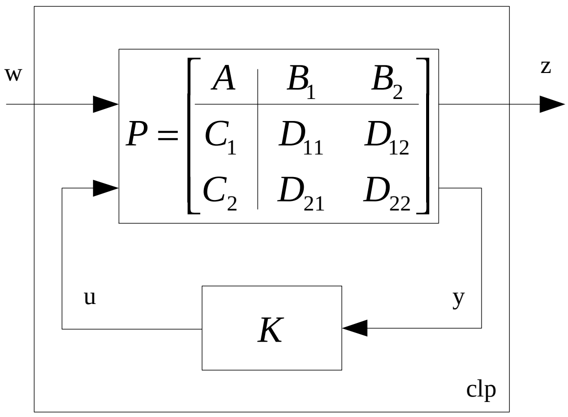

Fig. 5 General Form of Robust control Problem (redrawn from [7])

It should be noticed that this configuration is used for synthesizing the controller, K. Once the controller is obtained, it is inserted in the usual SISO (Single-Input, Single-Output) configuration which is shown in Fig. 3.
The robust control is the art of selecting the weighting functions: W_u and W_S. The widely used weighting functions are given by [8]:

$$W_S = \frac{s/M + \omega_0}{s + \omega_0 A} \tag{12}$$

$$W_u = 1 \tag{13}$$

Weighting the sensitivity function with the weight W_S, implies that the H_∞-norm should be less than the bound γ; i.e. $\left\| SW_S \right\|_\infty < \gamma$. If γ is kept below unity; $\gamma \le 1$, then the performance specification is translated to the form $S \le W_S^{-1}$. The singular values diagram of W_S^{-1} is shown in Fig. 6. It means that the tracking error magnitude should be kept under a low level at low frequencies. This bound is released at higher frequencies.

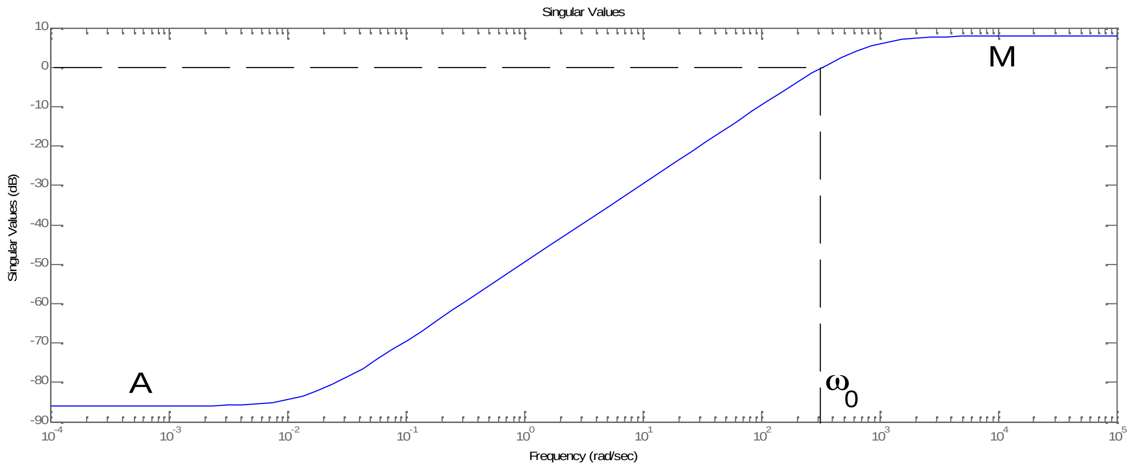

Fig. 6 Singular values diagram of W_S^{-1}

The necessary and sufficient condition for a successful controller is that the maximum singular value of the closed loop is less than one [7]:

$$\sigma_{\max}\left(clp\left(j\omega\right)\right) \le 1, \qquad \forall \omega \in \mathbf{R} \tag{14}$$

MATLAB introduces the function, `dksyn`, which computes the μ-synthesis controller based on DK-iteration.

5. Simulation of the Controllers

In the simulation process three different values of inertia mismatch have been studied. Each of them has been set as an uncertain parameter as shown in Table 1. J_{mot} is the motor inertia which is 0.85×10^{-4} kg.m^2. The controllers are built on the speed loop to control the speed of the motor.

Table 1 Proposed Inertia Values

Inertia	**Nominal value (kg.m^2)**	**Variability**
J1[1]	1.3285×10^{-4}	[-15.8% 25.7%]
J2	$J_{mot}\times5=4.25\times10^{-4}$	[-25% 25%]
J3	$J_{mot}\times10=8.5\times10^{-4}$	[-25% 25%]

For each of the three inertia values mentioned in Table 1, a different controller is synthesized with the weighting functions given by Equations (12) and (13). The W_S parameters are selected to be on the form

$$W_S = \frac{s/1.5+50}{s+50\times10^{-4}} \tag{15}$$

The controller produced is inserted into the SIMULINK model shown in Fig. 7 as a state space block. The sine wave block is used to simulate the inertias J2 and J3 while the "Inertia Synthesizer" block is to simulate the four bar mechanism inertia J1. The simulation parameters are listed in Table 2.

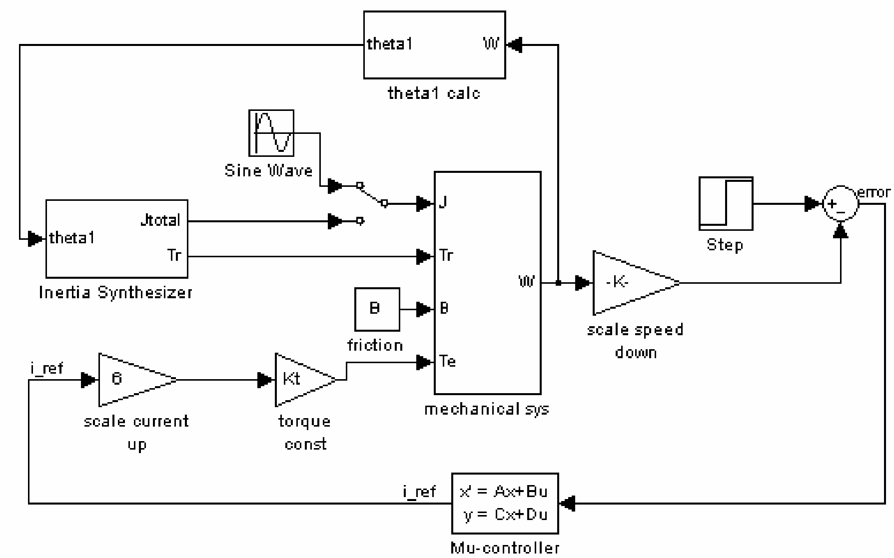

Fig. 7 SIMULINK model to simulate speed μ-synthesized controller with different inertia mismatch values

Table 2 Simulation Parameters

Start time	0
Stop time	5
Solver Type	Variable step
Solver	Ode45

The speed response due to different controllers is shown in Fig. 8 and Fig. 9. The speed set point is 0.5 which is normalized, i.e. it corresponds to half the nominal speed of the motor which is 3000 rpm. Fig. 10 shows the corresponding control effort from the controllers.

[1] J1 is the same as the implemented four bar mechanism.

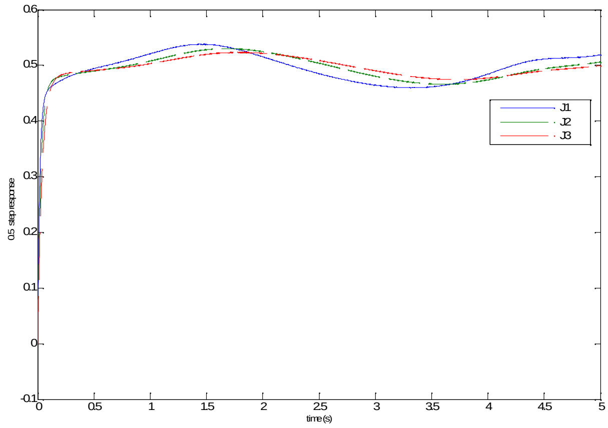

Fig. 8 Speed Response with μ-synthesized controller

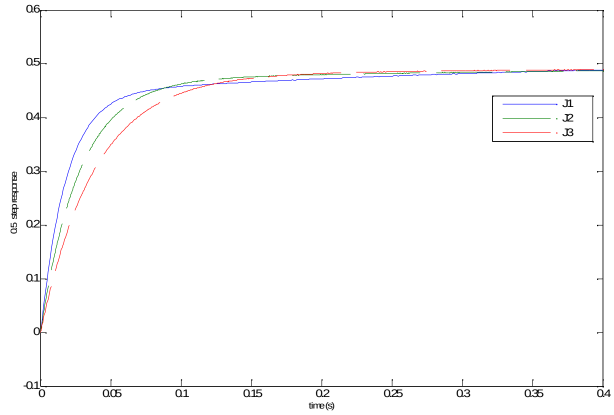

Fig. 9 Zoomed-in view of speed response with μ-synthesized controller

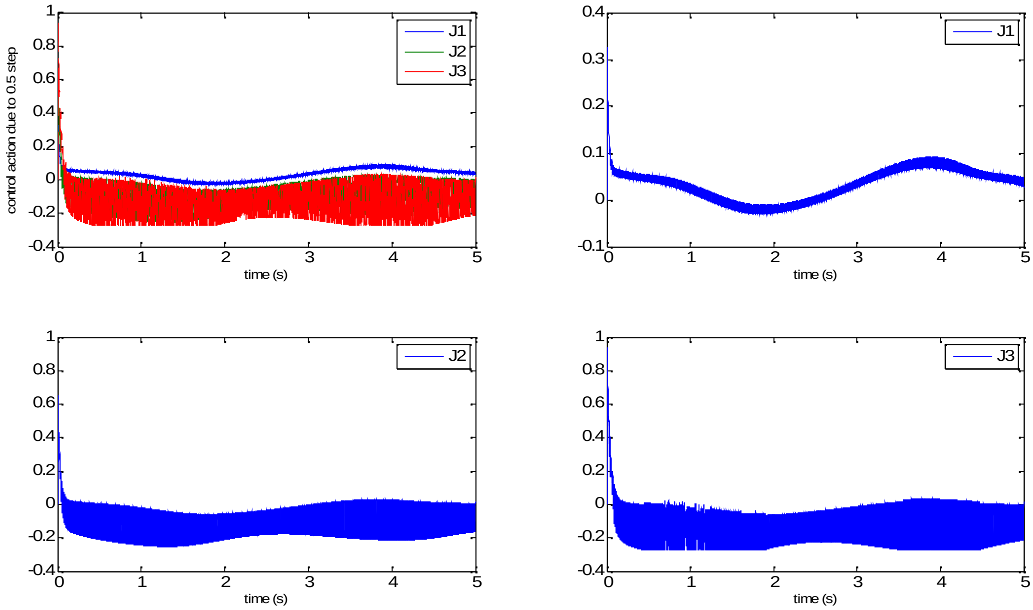

Fig. 10 Control Action with μ-synthesized controller

6. Practical Implementation of Proposed Controller

The same control laws were applied on the test rig. The controller was built on SIMULINK and then downloaded to DS1104 R&D controller board. The board is connected to the inverter via analog lines to read the speed of the motor and to write the set point current. The board executes the control law in real time with sampling time set to 100μs. The four bar mechanism has inertia given by J1 in Table 1.

The controller has been generated as illustrated before, but additional steps have been added. The controller is given in continuous-time, so it needs to be transferred to discrete-time with the sampling time of the controller board. Different values of weighting function W_S have been applied to find out the effect of changing the weighting function on the controller performance. Table 3 summarizes these experiments. The SIMULINK model used for control is shown in Fig. 11. The real time data has been captured by Control Desk software from dSPACE. The experimental system setup is shown in Fig. 12.

Table 3 μ-synthesized speed control experiments

	W_S

	M	ω_0	A
Experiment 1	1.5	10	1e-4
Experiment 2	1.5	50	1e-4
Experiment 3	1.5	100	1e-4

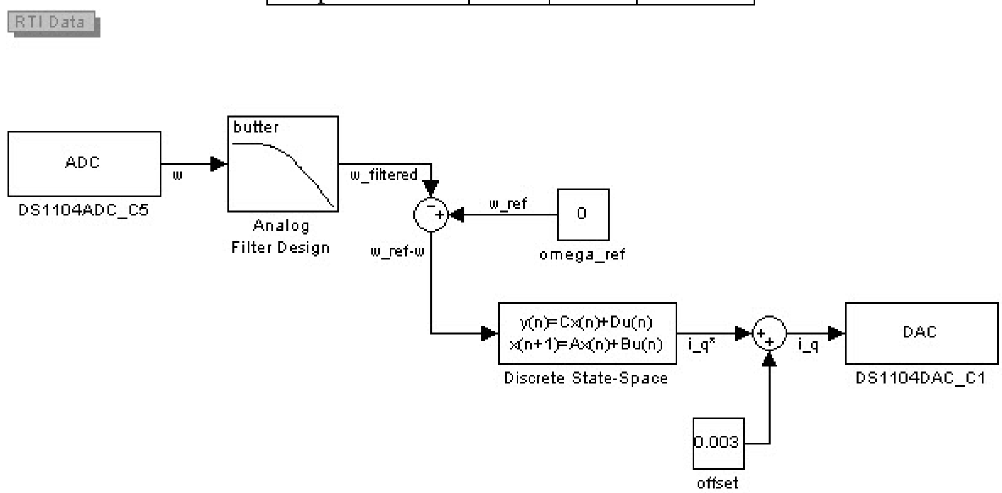

Fig. 11 Proposed SIMULINK model for µ-synthesized speed control

7. Results and Discussion

The response and control action of experiment 1 are shown in Fig. 13 and Fig. 14. The tracking of the reference set point is poor since the cross over frequency ω_0 is low. To achieve better tracking, ω_0 has been increased to 50. The response and control action are shown in Fig. 15 and Fig. 16. The weighting function in this experiment is the same as the one used in simulation.

Comparing the practical experiment 2 results with simulation of inertia J1 (Fig. 8 and Fig. 10) shows that the response and control effort of the practical experiment resemble that of the simulation. Some oscillations exist in the practical system while it doesn't appear in the simulation. One of the reasons could be the noise on the analog lines due to the high frequency switching job of the inverter. A filter has been set on the line of the actual speed read from the inverter as shown in Fig. 11. But it couldn't be done on the inverter side to filter out the analog signal of the current set point.

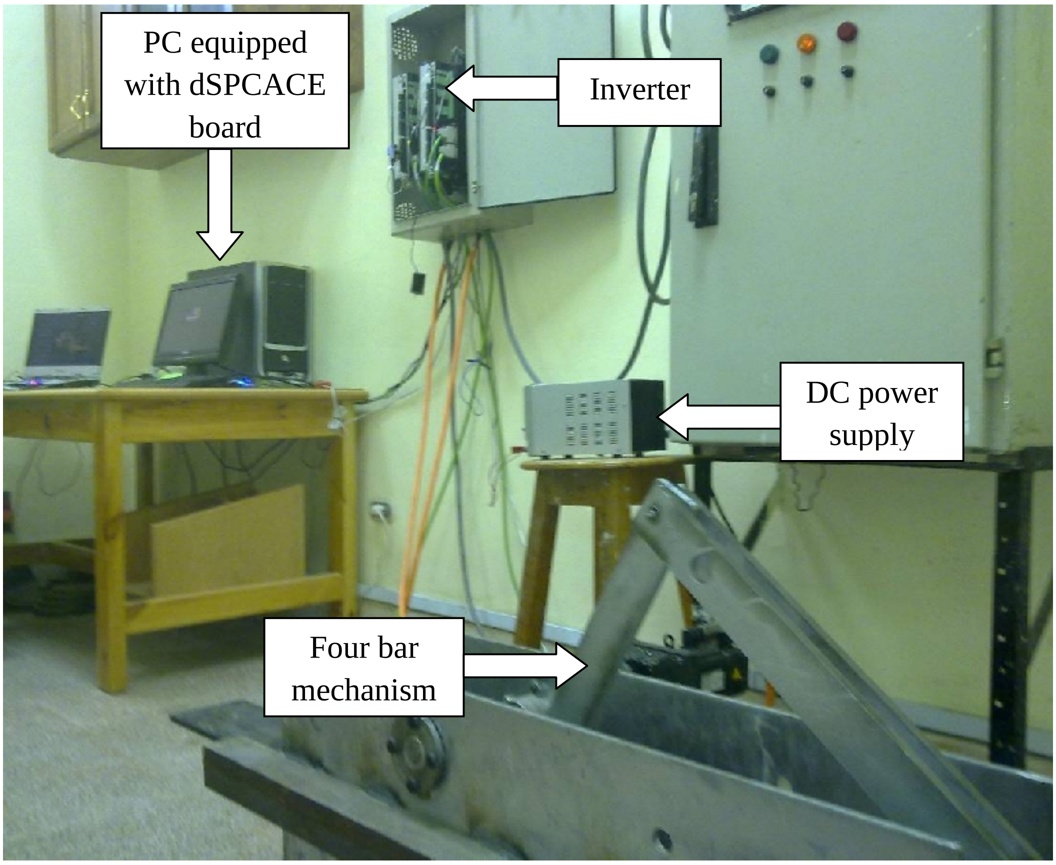

Fig. 12 System Integration

Increasing ω_0 to 100 makes the response more oscillatory as shown in Fig. 17. Fig. 18 shows that control action is oscillatory too. Thus increasing ω_0 improves the performance and pushing it more forwards makes the response worse than before.
In the simulations, it's obvious that increasing the inertia mismatch led to longer rising time which means the system response becomes slower. Also the control action increases as the inertia mismatch increases which is logic.
The response in both simulations and practical experiment seems to be periodic after the transient region in the beginning. This is due to the cyclic variation of the attached load over one revolution. Consequently, the control action is periodic.

8. Conclusions

From the previous analysis, it can e concluded that:

1. Robust control can present a good control law for variable inertia load which maintains stability and achieves good performance.
2. The tuning process of the robust control is dependent on the talent of selecting the weighting function that produces better performance. However there is no guarantee that the selected weighting function is the optimal one.
3. Increased inertia mismatch makes the system response slower and demands large control actions which may be not feasible.

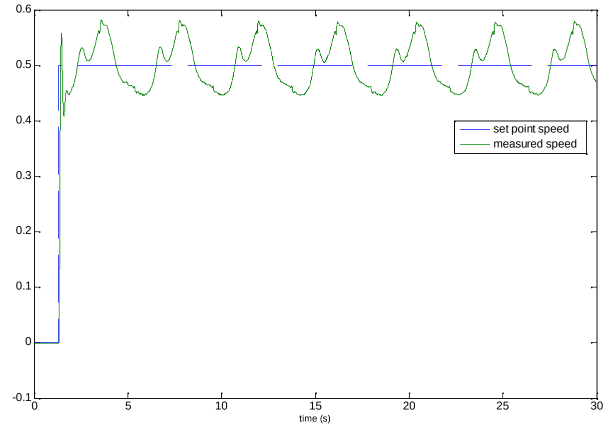

Fig. 13 Step Response of experiment 1

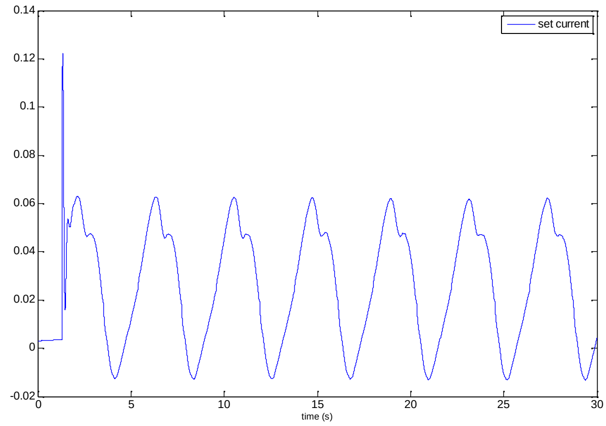

Fig. 14 Control action of experiment 1

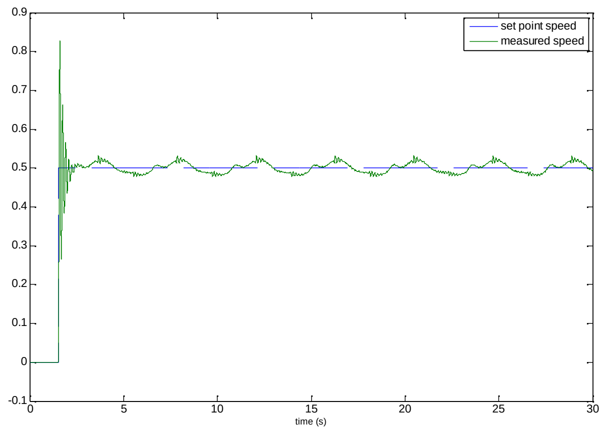

Fig. 15 Step Response of experiment 2

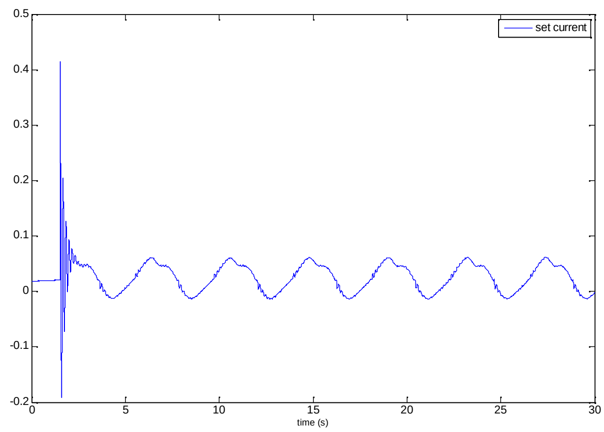

Fig. 16 Control action of experiment 2

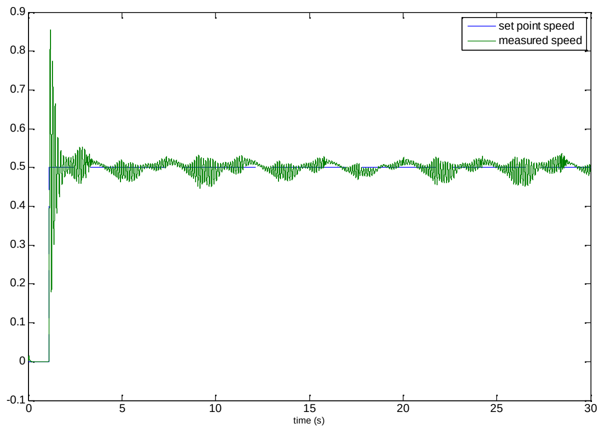

Fig. 17 Step Response of experiment 3

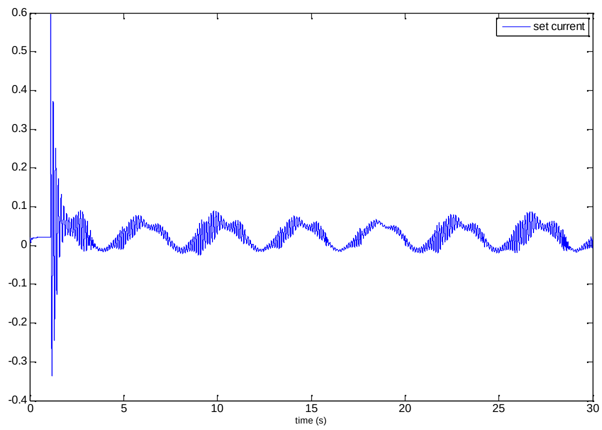

Fig. 18 Control action of experiment 3

9. References

[1] Arroyo, Enrique L. Carrillo. Modeling and Simulation of Permanent Magnet Synchronous Motor Drive System. IEEE Croatia Section. [Online] 2006. http://www.ieee.hr/_download/repository/lres_tema5_2.pdf.

[2] *Modeling of Permanent Magnet Motor Drives.* **Pillay, Pragasan and Krishnan, R.** 4, s.l. : IEEE, 1988, Vol. 35.

[3] *Design of robust controllers for PMSM drive fed with PWM inverter with inertia load variation.* **Alexandre, Marcus.** Toulouse : IEEE, 2006.

[4] *Nonlinear Torque Control for PMSM: A Lyapunov Technique Approach.* **Ouassaid, M., M. Cherkaoui, A. Nejmi and Maaroufi, M.** s.l. : World Academy of Science, Engineering and Technology, 2005.

[5] **Balas, Gary, et al.** *Robust Control Toolbox User's Guide.* s.l. : The MathWorks, 2012.

[6] **Zhou, Kemin and Doyle, John C.** *Essentials of Robust Control.* s.l. : Prentice Hall, 1999.

[7] **Madden, Ryan J.** Identification of Unmodeled Dynamics in Rotor Systems Using Mu-sythesis Approach. *OhioLINK ETD Center.* [Online] 9 2009. http://etd.ohiolink.edu/send-pdf.cgi/Madden%20Ryan%20J.pdf?csu1294695084.

[8] **Skogestad, Sigurd and Postlethwaite, Ian.** *Mltivariable Feedback Control: Analysis and Design.* s.l. : JOHN WILEY & SONS, 2001.