

ADJUSTABLE FRACTIONAL ORDER ADAPTIVE CONTROL ON SINGLE-DELAY REGENERATIVE MACHINING CHATTER

YUEQUAN WAN, HAIYAN ZHANG*, MARK FRENCH

ABSTRACT. Chatter is a harmful self-excited vibration in machining processes. When the machining condition goes beyond the stability boundary, any small disturbance could cause chatter. Rapid suppression of the chatter is always essential to the manufacturing industry. In this paper, the adjustable fractional order adaptive control (AFrAC) for machining chatter suppression is presented. This method can effectively suppress the chatter promptly without dramatically changing the cutting conditions. In comparison with the ordinary order adaptive control and fixed order fractional adaptive control, the AFrAC is proved to be superior from many perspectives based on the results of numerical simulation. The control law of the original adaptive control is derived from Lyapunov function, this ordinary control law is combined with fractional error space to yield to the new control law of AFrAC. The bias characteristics are studied and the design condition of the adjustable fractional order integrator is included in this paper. Besides the derivation of the control law, the internal energy analysis, the characteristics of fractional integration and the approximation of the fractional order integrators are also provided. The successful application of fractional order control on the ordinary time-delay system in this paper could be generalized into other related systems

Nomenclature

β , Order of fractional calculus $D^\beta y(t)$, $\beta > 0$ is for fractional derivative; \int_0^t is for fractional integral.
 $x(t)$, dynamic displacement of machining chatter (mm)
 ζ , damping ratio
 ω_n , natural frequency of elastic structure (rad/sec)
 $\Delta F(t)$, dynamic cutting force
 α , angle between cutting force and displacement
 K , cutting edge angle P, force coefficient
 w , depth of cut (mm)
 Y , force exponential
 s_0 , nominal chip thickness (mm)
 s_t instantaneous chip thickness (mm)
 T period of one work piece revolution (sec)

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1. INTRODUCTION

The regenerative chatter in machining processes results in coarse surface finishes and geometric dimensions and tolerances on the workpiece, and shortens the cutting tool life, even causes tool breakages. In the flexible manufacturing system, machining chatter occurs even more frequently due to its varying working conditions, which could bring the machining processes into unstable regions ([1]). Very often, the working conditions people selected to avoid the machining chatter make the metal-removal rate conservatively low.

Various methods have been developed to control the machining chatter without reducing the metal-removal rate. Some of them involve in modification of the cutting tool, for instance using variable-pitch cutters or bi-helical cutters, and others attach supplementary equipment to the machine tool structure in the vicinity of chip producing area, such as vibration absorbers or actuators and sensors with expensive control processors ([2][3]). In the former case, the design of cutter insert spacing is dependent upon the cutting conditions and workpiece geometry, which could be changing during the machining process. However, the insert spacing cannot be in-process adjusted to maintain the desired effect. In addition, insert spacing technique is applicable only to multi-tooth cutters, not to single-point cutters in turning or boring. In the latter case, the control laws are usually designed with the state variables, such as position variables and forces, by feedback of the error signals to control the relative vibration between the cutting tool and the workpiece. Although the research works are successful, these approaches still fall short of practical industrial applications.

Actually, there is the third approach: the control laws are designed with the cutting parameter variables, such as the spindle speed and the feed rate, because the stability of the machining process is directly affected by the cutting parameters. Adjusting the cutting parameters is a natural way to control machining chatter, and it can be easily realized without necessity of supplementary equipment in CNC environment ([4][5]).

Furthermore, a small disturbance in real production is nearly unavoidable since there are many factors in the cutting material, elastic system, and machining processes could cause small disturbances of the instantaneous cutting force. The study of chatter has been focusing on the suppression of chatter for a long time, and many approaches have showed optimistic results, such as modifying or enhancing the cutter or equipment ([6][7]) and the spindle speed variation method ([8]). However, due to the nonlinearity and time delay effect of machining chatter, finding a control approach which can suppress the chatter onset and without dramatically change on the machining process is still an on-going concern in the manufacture industry. In order to deal with the nonlinearity, intricacy, and uncertainty in the machining processes and the progressive changes of tool wear, soft computing techniques, such as artificial neural network (ANN) and fuzzy logic, were also applied to improve the machining performance. According to Chryssolouris and Guillot([9]), the machining process and its state variable estimation could be better modeled using an appropriate ANN model in comparison with other modeling techniques. This is reasonable because the ANNs exponential function itself is highly nonlinear and infinitely differentiable. The adaptive fuzzy logic controller (AFLC) was reported for precision contour machining in a 3-axis milling machine. The features of AFLC

include real-time estimation of self-tuned parameters and disturbance values, continuous monitoring of the controller performance, especially the stability assured fuzzy logic membership function adjustment. Its simulation and experimental tests demonstrated superiority over. Jee and Koren([10]), Ibrahim ([11]) generalized the differential polynomial neural network utilizing fractional calculus in the sense of the Caputo differential operator. The initial experimental results verified the effectiveness of this approach.

Stavropoulos et al. ([12]) surveyed the online monitoring and control of optimizing various manufacturing processes in terms of process efficiency, tooling life and product quality. It compared the conventional and enhanced methods of controlling the manufacturing processes, and explored the adaptive control systems implementation in a systematical manner. It will be innovative that the fractional calculus can be applied to adaptive control, because the fractional integral and derivatives are able to provide properties that cannot be properly presented by the integer-order ones. This combination is expected to produce some innovative approach.

In this paper, the authors represent a novel method which combines the adaptive control method and the adjustable fractional order integrator and this method is named as adjustable fractional order adaptive control (AFrAC). Numerical simulation shows that this method can achieve positive results from many perspectives when solving the chatter problem, also the application of fractional calculus and fractional integrators could be generalized to other relevant situation.

The history of fractional calculus could be traced back to the end of 17th century, although in those days the application of this math technology is not very clear ([13]). During the development of fractional calculus, scholars made different definitions, and among those of them three definitions have been adopted most widely. They are Riemann-Liouville definition ([14]), Grunwald-Letnikov definition ([15]) and Caputo definition ([16]). From engineering perspective, the definition by Caputo is readily applicable, since the initial condition under this definition is always obtainable. Recently, the applications of fractional calculus have been successfully found in many fields, such as in visco-elasticity ([17][18]), electro-analytical chemistry ([19][20]) and control theory ([21][22]). In the field of control theory, fractional system is just a system which could be described by the time domain equation in (1) and by the correspondingly transfer function in (2) ([23]).

$$D^{\beta_n} y(t) + a_{n-1} D^{\beta_{n-1}} y(t) + \dots + a_0 D^{\beta_0} y(t) = b_n D^{\alpha_n} y(t) + b_{n-1} D^{\alpha_{n-1}} y(t) + \dots + b_0 D^{\alpha_0} y(t) \quad (1)$$

$$G(s) = \frac{b_n s^{\alpha_n} + b_{n-1} s^{\alpha_{n-1}} + \dots + b_0 s^{\alpha_0}}{s^{\beta_n} + a_{n-1} s^{\beta_{n-1}} + \dots + a_0 s^{\beta_0}} \quad (2)$$

In the study field of fractional controllers, there have been many successful applications of fractional integrators and fractional differentiator, especially in the study of fractional $PI^\lambda D^\gamma$ controllers([23][24]). The study in this paper presents an approach integrating the traditional adaptive controller and fractional order integrators to achieve the suppression of the chatter. First of all, in this paper a regular order adaptive control method is derived from Lyapunov's second method, and this original method builds the state space and the error space based on the ordinary calculus. The updating mechanism in this adaptive control is to adjust

both the spindle speed and the feed rate. After the derivation of the ordinary control law, the fractional state space and error space has been introduced. Normally, to utilize the result involving fractional state space and fractional error space, one needs a fractional system model. However this paper presents an alternative way. The alternative way takes the ordinary state space and yield a biased control. To proceed, the characterizes of the bias are studied and a design condition to final vanish the bias is provided. Based on the design condition, the paper presents the AFrAC approach which can take advantage of the fractional control and overcome the bias issue. The section 2 of this paper includes the normal order NLDDE model, and section 3 presents the derivation of the AFrAC control law. Section 4 associates with approximation, design realization of AFrAC, optimization and simulation setup. Following the section 4, the numerical results and analysis is provided and finally in section 5 the conclusion is included.

2. THE NLDDE CHATTER MODEL AND LINEARIZATION

In this paper, only single delay regenerative effect is considered as the main source of chatter. Using the nonlinear differential-difference equation (NLDDE) with single delay, the chatter model could be described as following in Equation (3):

$$\begin{cases} \frac{d^2x}{dt^2} + 2\zeta\omega_n \frac{dx(t)}{dt} + w_n^2 x(t) = -\frac{w_n^2}{k} \Delta F(t) \cos\alpha \\ \Delta F(t) = Pw \left\{ s^Y(t) - s_0^Y + \frac{Y s_0^{Y-1}}{N} \frac{dx}{dt} \right\} \\ s(t) = \begin{cases} x(t) - x(t-T) + s_0, & \text{if } s(t) > 0; \\ 0, & \text{elsewise} \end{cases} \end{cases} \quad (3)$$

Since the direct analysis of the NLDDE model in Equation (3) is not easy, people could perform a linear approximation, and among the linearization methods, the polynomial method ([25]) is chosen in this study. Therefore the nonlinear part $s^Y(t) - s_0^Y$ could be approximated by:

$$\begin{aligned} s^Y(t) - s_0^Y &= [x(t) - x(t-T) + s_0] - s_0^Y \\ &\approx Y s_0^{Y-1} [x(t) - x(t-T)] + O([x(t) - x(t-T)]^2) \end{aligned} \quad (4)$$

When the chatter amplitude is small enough the second term of equation (4) could be neglected without losing too much precision, and this is always the case for a chatter initialized by small disturbance. Now, with the linear approximation the chatter model can be rewritten as in (5):

$$\begin{cases} \frac{d^2x}{dt^2} + a_1 \frac{dx(t)}{dt} + (\omega_n^2 + a_2)x(t) = a_2 x(t-T) \\ a_1 = 2\zeta\omega_n + \frac{\omega_n^2}{k} PwY s_0^{Y-1} C \frac{1}{N} \cos\alpha \approx 2\zeta\omega_n \\ a_2 = \frac{\omega_n^2}{k} PwY s_0^{Y-1} \cos\alpha \end{cases} \quad (5)$$

In machining process, the cutting condition are located either in stable or unstable zone while the solution of NLDDE could be categorized as stable trivial, unstable trivial and steady state non-trivial periodic. This is a duality between the stability of machining system and the solution of the NLDDE in Equation (5), and this duality is summarized in the Table 1 and showed in Figure 2.

When cutting condition locates in the unstable region demonstrated in Figure 1, which is to the right of line B-D, any small disturbance would cause chatter.

TABLE 1. Duality between machining system and the NLDDE

Machining System	NLDDE
1. No chatter	1. Stable trivial solution
2. Onset of chatter	2. Unstable trivial solution
3. Fully developed chatter, finite amplitude	3. Steady state non trivial periodic solution

Numerical simulation shows that without chatter controller a one-time small bias in the dynamic cutting force would initial the undesired vibration when cutting depth is in the unstable region. To contrast with, another simulation with the cutting condition in the stable area is also conducted, and this time the chatter is automatically absorbed even without any controller. The parameters of numerical simulation in showed in the Table 2, and these parameters as well as the unstable cutting condition are kept the same in all of the numerical simulations in this paper, for comparison purpose. Figure 2 presents the simulation results mentioned above in this section. In Figure 2, the frame A and frame B are based on the unstable chatter caused by small disturbance, and these two frames show the plot of chatter displacement and the plot of network done by dynamic force respectively. Frame C and frame D show the same content but under the situation of a stable cutting condition.

Figure 2 shows it clearly that the chatter could be self-absorbed when cutting condition locates in the stable zone. Therefore the focus of this paper is to suppress the chatter occurred in the unstable zone, although the control algorithm included in this paper will stand under all kinds of cutting condition. When the unstable

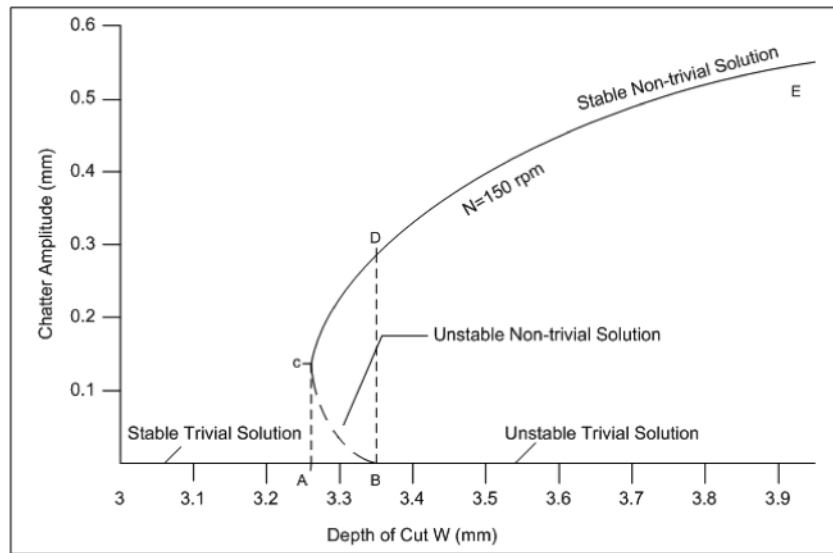


FIGURE 1. Demonstration of Duality between NLDDE and Machining System

TABLE 2. Cutting parameters for numerical simulation

Initial spindle speed:	450 rpm	Disturbance time:	0.02 sec
Initial feed rate:	0.1 mm/sec	Natural frequency:	85 Hz
Damping ratio:	0.0384	Depth of cut, unstable:	5 mm
Disturbance size	1%	Depth of cut, stable:	2 mm

chatter occurs, the internal energy will diverge, the Lissajous plots in Figure 3 demonstrates this situation. In Figure 3, the time window for frame A is $0 \sim 0.5$ sec., for B is $0.5 \sim 1.0$ sec., for C is $1.0 \sim 1.5$ sec., and for D is $1.5 \sim 2.0$ sec. The time windows for all the Lissajous plots included in this paper will keep this setup. In next section, the control law of adaptive control and the fractional integrator will be introduced, and series numerical simulation will be included latter to show the results of the controllers.

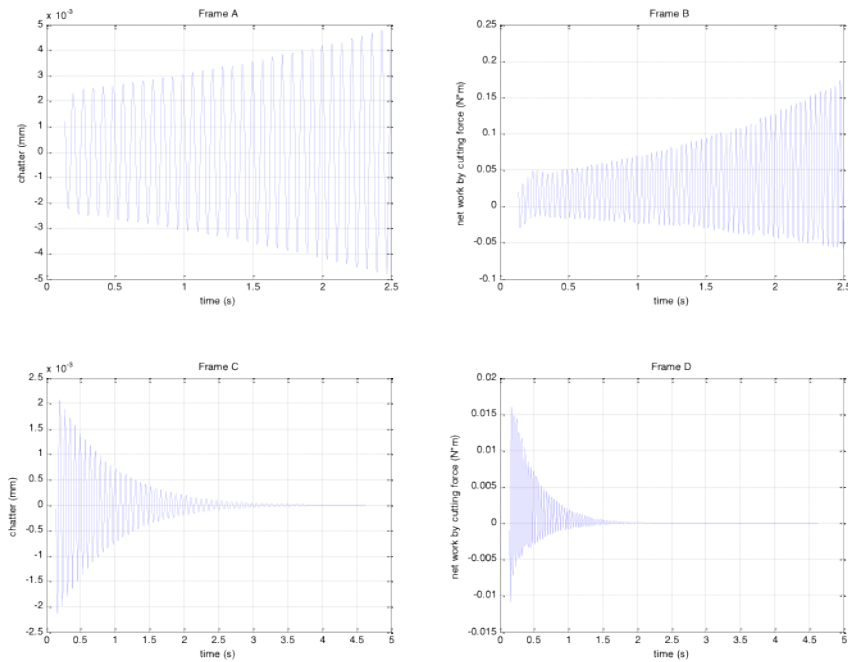


FIGURE 2. Plot of chatter on small turbulence, without control

3. METHODOLOGY

This paper is to present an approach integrated the technology of fractional integration and the adaptive control theory. We would start from the ordinary

order system, and derive a regular order model reference adaptive control law from Lyapunov function. The reference model of this controller is a stable n-dimensional time delay system which could be described by a regular order differential difference equation as in (6).

$$\dot{y}(t) = Ay(t) + By(t - T) \tag{6}$$

And the corresponding unstable n-dimensional time delay system is represented by Equation (7)

$$\dot{x}(t) = (A + \delta A)x(t) + (B + \delta B)x(t - T - h) \tag{7}$$

In (6) and (7), T is the time delay makes the reference model stable while h in (7) represents the variable time delay makes the real spindle speed equals to $\frac{60}{T+h}$. Besides, δA and δB are the parameter deviations from A and B that satisfies the stability conditions presented by Equation (8) and Equation (9), ([26]). The parameters h, δA and δB are adjustable variables.

$$P[A + Q(0)] + [A + Q(0)]'P + R = 0 \tag{8}$$

$$\dot{Q}(t) = [A + Q(0)]Q(t), t \in [0, T] \tag{9}$$

Based on Equation (6) and Equation (7), one can conduct the error state equation in (10):

$$\begin{aligned} \dot{\epsilon}(t) &= \dot{x}(t) - \dot{y}(t) \\ &= A\epsilon(t) + B\epsilon(t - T) + \delta Ax(t) + B[x(t - T + h) - x(t - T)] + \delta Bx(t - T + h) \end{aligned} \tag{10}$$

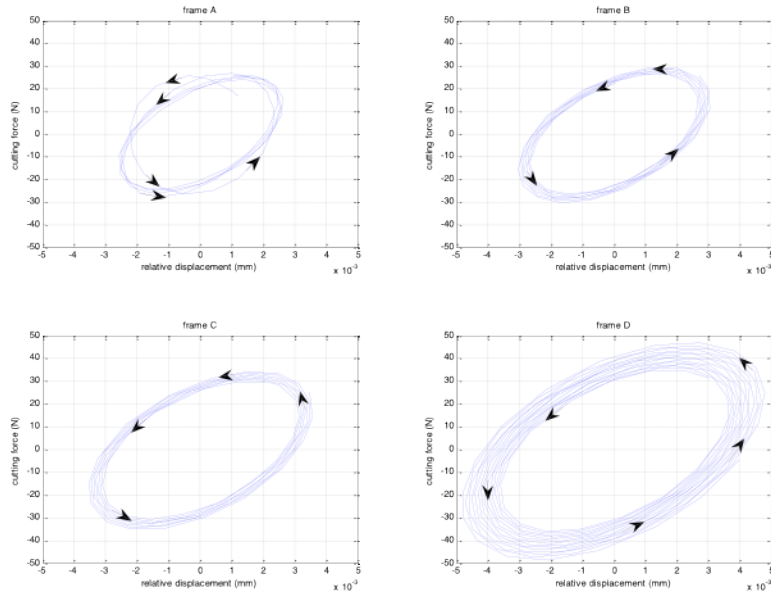


FIGURE 3. Lissajous plots on unstable chatter, without control

When the variable time delay h is sufficient small, the difference of $x(t - T + h)$ and $x(t - T)$ could be approximate by $h \frac{dx(t-T)}{dt}$, therefore the error state equation could be approximated by:

$$\dot{\epsilon}(t) = A\epsilon(t) + B\epsilon(t - T) + \phi Z \quad (11)$$

In (11), $\phi \in R^{(n \times 3n)}$ is the adjustable parameter matrix, and $Z \in R^{(3n)}$, the augmented state vector, is as the following:

$$\begin{bmatrix} Z = x(t) \\ x(t - T + h) \\ \dot{x}(t - h) \end{bmatrix} \quad (12)$$

The error space contains the four elements, which are $\epsilon(t)$, $\delta A(t)$, $\delta B(t)$, and $h(t)$. The asymptotic stability should be considered in this entire error space. The control law of the ordinary order MRAC method is to adjust the parameter matrices $\delta A(t)$, $\delta B(t)$, and the variable time delay $h(t)$ and under this control law the error space (11) would be forced to be asymptotically stable, which means all of the elements in the error space goes to zero as time goes to infinity. It can be proved for the systems described by (6) and (7), the adaptive control process presented in (13) is asymptotically stable with $\lim_{t \rightarrow \infty} \epsilon(t) = 0$ ([4]).

$$\begin{cases} \dot{\epsilon}(t) = A\epsilon(t) + B\epsilon(t - T) + \delta A x(t) + Bh\dot{x}(t - T) + \delta B x(t - T + h) \\ \text{tr}(\delta A \Lambda_1^{-1} \delta \dot{A}') = -x'(t) \delta A' P (\epsilon + Q * \epsilon) \\ \text{tr}(\delta B \Lambda_2^{-1} \delta \dot{B}') = -x'(t - T + h) \delta B' P (\epsilon + Q * \epsilon) \\ \rho \dot{\eta}(t) = -\eta(t) + \dot{x}(t - T), \rho > 0 \\ \text{tr}(Bh\Lambda_3^{-1} \dot{h} B') = -\dot{\eta}(t) B' h P (\epsilon + Q * \epsilon) \end{cases} \quad (13)$$

In (13), $\text{tr}(\cdot)$ is the trace operator, $P = [P_{ii}]$ is a positive definite symmetric matrix, ϕ is the adjustable parameter matrix, and given a positive definite matrix R , parameters A , B , P and $Q = [Q_{ii}]$ satisfy the condition in (14).

$$\begin{cases} P[A + Q(0)] + [A + Q(0)]' P + R = 0 \\ \dot{Q}(\sigma) = [A + Q(0)] Q(\sigma), \sigma \in [0, T] \\ Q(T) = B \end{cases} \quad (14)$$

Also in (13), for a positive definite symmetric matrix $\Gamma \in R^{3n \times 3n}$,

$$\Gamma = \begin{bmatrix} \Lambda_1 & 0 & 0 \\ 0 & \Lambda_2 & 0 \\ 0 & 0 & \Lambda_3 \end{bmatrix} \quad (15)$$

Next is to apply the n -dimensional ordinary order control law in equation (13) to the NLDDE model showed in (5) and then realizing the update mechanism through the adjustment of the feed rate and the spindle speed. Recall the linearized chatter model in (5), the state vector is:

$$x(t) = \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}, x(t - T) = \begin{bmatrix} x(t - T) \\ \dot{x}(t - T) \end{bmatrix}, \quad (16)$$

The parameter matrices are:

$$A = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 - a_2 & -a_1 \end{bmatrix}, A = \begin{bmatrix} 0 & 0 \\ a_2 & 0 \end{bmatrix} \quad (17)$$

And the differentials of (17) are:

$$\delta A = -\delta B = \begin{bmatrix} 0 & 0 \\ -\delta a_2 & 0 \end{bmatrix} \tag{18}$$

And for Λ_1, Λ_2 and Λ_3 in the adaptive control process described in (9), we note:

$$\Lambda_i^{-1} = \begin{bmatrix} \frac{1}{\lambda_i} & 0 \\ 0 & \frac{1}{\lambda_i} \end{bmatrix} \tag{19}$$

Then the control law of the feed rate and the spindle speed is as in (20) and (21) correspondingly.

$$\delta \dot{f}(t) = -\frac{\lambda_2 s_0 x(t-T+h)}{a_2(Y-1)\sin K} * [P_{12}(\epsilon_1 + Q_{11} * \epsilon_1 + Q_{12} * \epsilon_2) + P_{22}(\epsilon_2 + Q_{21} * \epsilon_1 + Q_{22} * \epsilon_2)] \tag{20}$$

$$\dot{h} = -\frac{\lambda_3 \dot{x}(t-T)}{a_2} * [P_{12}(\epsilon_1 + Q_{11} * \epsilon_1 + Q_{12} * \epsilon_2) + P_{22}(\epsilon_2 + Q_{21} * \epsilon_1 + Q_{22} * \epsilon_2)] \tag{21}$$

Equations (20) and (21) are the control law of the original model reference adaptive control. For the purpose of more effective suppression on the chatter, following we present a more advanced control method involving the fractional calculus To start with, recall the fundamental model structure of the ordinary order reference model in Equations (6) and (7), and if we take the advantage of fractional calculus and consider a stable model would maintain stable at any real order β rather than integer order of derivation, the reference model and the unstable model for the time delay system could be expressed as in Equation (22)

$$\begin{cases} D_t^\beta y(t) = \tilde{A}y(t) + \tilde{B}y(t-T) & (\text{stable model, fractional}) \\ D_t^\beta y(t) = (\tilde{A} + \delta\tilde{A})x(t) + (\tilde{B} + \delta\tilde{B})x(t-T-h) & (\text{unstable model, fractional}) \end{cases} \tag{22}$$

In Equation (22), the operator D_t^β is defined as the Caputos fractional derivative of order β with respect to t and with starting point t=0, one can also note it as ${}_0^C D_t^\beta$, this definition is as following:

$${}_0^C D_t^\beta = \frac{1}{\Gamma(\beta-n)} \int_0^t \frac{f^n(\tau d\tau)}{(t-\tau)^{\beta+1-n}}, \quad (n-1 < \beta < n) \tag{23}$$

As one can notice, fractional derivative as in Equation (23) includes information of $f(t)$ s integer order differentials up to the order of n. This extra information can allow more precise control. In this paper, the ordinary control law is built on the state space involving chatter displacement, $x(t)$, and the velocity of chatter, $\dot{x}(t)$. However, in reality the acceleration of chatter, $\ddot{x}(t)$, contains the forward information regarding the development trend of this chatter, thus it also needs to be included in the design of control law. Instead of expend the state space into higher dimension, which involves too complex control law, the fractional order state space is chosen to develop the new update mechanism. In this paper, the chatters displacement, velocity and the acceleration are considered to be essential to the development of the chatter, therefore in Equation (23) we take n=2 in this paper, that implies our fractional integrator has the order less than 2.

Since, the operator of fractional derivative is linear ([27]) which means:

$${}_0^C D_t^\beta [af(t) + bg(t)] = a {}_0^C D_t^\beta f(t) + b {}_0^C D_t^\beta g(t) \tag{24}$$

The error space of this fractional order system is:

$$\begin{aligned}
{}_0^C D_t^\beta \epsilon(t) &= {}_0^C D_t^\beta [x(t) - y(t)] \\
&= {}_0^C D_t^\beta x(t) - {}_0^C D_t^\beta y(t) \\
&= \tilde{A}\epsilon(t) + \tilde{B}\epsilon(t - T) + \delta\tilde{A}x(t) + \tilde{B}[x(t - T + h) - x(t - T)] + \delta\tilde{B}x(t - T + h) \\
&\approx \tilde{A}\epsilon(t) + \tilde{B}\epsilon(t - T) + \tilde{\phi}\tilde{Z}
\end{aligned} \tag{25}$$

In Equation (25), \tilde{A} , \tilde{B} , and $\tilde{\phi}$ contain scalars, and \tilde{Z} contains ${}_0^C D_t^\beta x(t - h)$ instead of $\dot{x}(t - h)$. This error space contains the information of $x(t)$ up to its order of n , which is as in Equation (23). Given the similarity between the error space (11) and (25), the whole derivation of the fractional control law could yield a fractional format of Equation (13) with the only changes are the scalar parameters and the fractional derivative operator instead of ordinary order ones, and it also means the ordinary order control law is just a special case of the fractional order control law when $\beta = 1$. The direct application of the fractional control law needs fractional NLDDE model which provides the β -order state space, However, instead of re-write the chatter model in fractional way and re-measure the constant coefficients under that situation, the following part of this paper is presenting an alternative way to take the useful characteristics of fractional order error space based on the regular chatter model.

When the system is unstable, note the bias between the fractional state and the integer state as:

$$\begin{aligned}
Bias &= {}_0^C D_t^\beta x(t) - \dot{x}(t) \\
&= (\tilde{A} + \delta\tilde{A} - A - \delta A) * x(t) + (\tilde{B} - \delta\tilde{B} - B - \delta B)x(t - T)
\end{aligned} \tag{26}$$

This bias represents difference between the actual fractional state and the ordinary order state when chatter occurs. Therefore if one applies the fractional control law to the ordinary state space as in (16) and fixed the order of through the entire working time, the results would be a biased control and lead to over adjustment when chatter presents. However, the bias will automatically vanish when the chatter suppressed, and this fact could be conducted from the definition of the Caputos fractional derivative. As defined in Equation (23), ${}_0^C D_t^\beta f(t)$ could be viewed as a linear combination of $F(f^{(n)}(\tau))$, and here $F(0)=0$, thus this bias will converge to zero when $\beta=1$ or when chatter is fully suppressed, which means $x(t) = \dot{x}(t) = \ddot{x}(t) = 0$ in this study.

Refer to Equations (20) and (21), both of the spindle speed and feed rate controllers are in the format of ordinary differentials, which also means in reality these two controllers conduct the integration control. Thus if the fractional integrator were used, the innate character of this control will be maintained, only with an enhanced control power. The tradeoff is the bias between actual fractional state and the nominal ordinary order state would bring errors which may counteract the gain from the enhanced controlling power. To get a comprehensive better control than the original adaptive control, the error caused by the bias should always be constrict to smaller the extra controlling power, as a result an adjustable order β is desired. Since the vanishing bias and the asymptotically stable characteristics, if one adjusts the β from 2 down to 1 during a small time window, the bias will absent and leaves a precise control without this errors at the final stable stage. The control law

of β is critical to the quality of this fractional adaptive control. Theoretically, β should depend on the full status of chatter, but this yields the requirement of knowing the β -order state space, which we are avoiding in this paper from reality application perspective. Approximately, if the original control method could compress the chatter monotonically in dominant time span, β could depend on the chatters absolute displacement or time. Given the error space $\lim_{t \rightarrow \infty} \epsilon(t) = 0$, one could select the control law satisfies the following condition:

$$\begin{cases} \beta(t_0) \leq 2 \\ \beta(t_{stb}) = 1 \\ \beta'(t) < 0 \\ t \in [t_0, t_{stb}] \end{cases} \quad (27)$$

In Equation (27), t_0 is the time chatter occurs, and t_{stb} is the time chatter is stabilized. Condition (27) retains the asymptotically stable characteristic of the adaptive control since it converges to the ordinary order within a finite time window. Hereby the new control law of the spindle speed and the feed rate is as in Equations (28) and (29).

$${}_0^C D_t^\beta \delta f(t) = -\frac{\lambda_2 s_0 x(t - T + h)}{a_2(Y - 1) \sin K} * G(\tilde{P}, \tilde{Q}, \tilde{\epsilon}, Bias, \beta) \quad (28)$$

$${}_0^C D_t^\beta h(t) = -\dot{x}(t - T) \frac{\lambda_3}{a_2} * H(\tilde{P}, \tilde{Q}, \tilde{\epsilon}, Bias, \beta) \quad (29)$$

Although in Equations (28) and (29) control is a biased at early stage, the tradeoff is restricted to allow a faster control and the final stable with precision no worse than the ordinary one. The left issue is to select a realization of the control law of β under condition (27). The selection is actually redundant, such as linear tuning or exponentially tuning, this paper presents a stepwise downward tuning which could be readily adopted in industry, and this control law is as:

$$\begin{cases} \text{for :} & t_0 < t_1 < \dots < t_n < t_{stb} \\ \text{set :} & 2 \geq \beta(t_0) > \beta(t_1) > \dots > \beta(t_n) > \beta(t_{stb}) = 1 \end{cases} \quad (30)$$

A demonstration chart of adjusting law (30) is provided in Figure 4. In section 5 of this paper, the numerical simulation shows the adjustable fractional order adaptive control is superior among the biased fixed order adaptive control and original order control, it achieves a quick suppression of the chatter and avoids over adjustment on the spindle speed.

4. APPROXIMATION, OPTIMIZATION AND SIMULATION

Due to the fact that the physics meaning of fractional calculus is not as straightforward as of ordinary calculus, the direct realization of fractional order controller is not easily achievable in reality. Therefore people need approximation technology to obtain a rational model approximating the original transfer function of the fractional system. In the control theory field, most of the existing approximation methodologies could be categorized into two classes, the continued fraction expansions method and the curve fitting or identification techniques. And the approach chosen in this paper is the Oustaloups method ([28]) which is one of the curve fitting techniques. The basic idea of this kind of methods is to find a rational function with the similar frequency response to the original irrational transfer function and

for the approach chosen here, it approximates the fractional integrator in Equation (31) by the function (32)

$$G(s) = \frac{1}{s^\beta} \quad (31)$$

$$\tilde{G}(s) = C \prod_{K=-N}^{K=N} \left(1 + \frac{s}{\omega_k}\right) / \left(1 + \frac{s}{\omega_{k'}}\right) \quad (32)$$

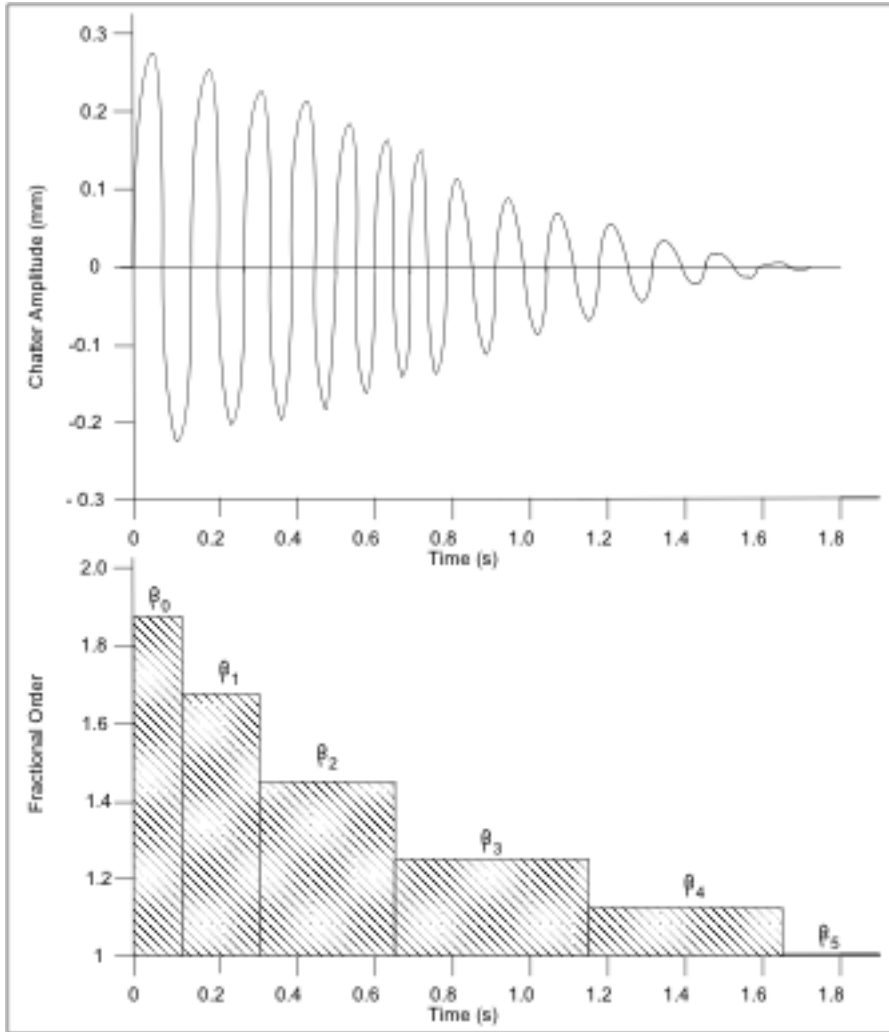


FIGURE 4. demonstration chart of the stepwise adjustment for fractional order β

TABLE 3. Approximation of Fractional Order Integrators

Fractional Order Integrators	Integer Order Approximation
$\frac{1}{s^{1.15}}$	$\frac{(944100s^3+14.19s^2+9856s+31620)}{(s^3+311.7s^2+448.8s+2.985)}$
$\frac{1}{s^{1.25}}$	$\frac{(42170s^3+8.293s^2+7534s+31620)}{(s^3+238.2s^2+262.2s+1.334)}$
$\frac{1}{s^{1.5}}$	$\frac{(5.623e6+2.164s^2+3849s+31620)}{(s^3+121.7s^2+68.45s+0.1778)}$
$\frac{1}{s^{1.75}}$	$\frac{(7.499e7+0.565s^2+1966s+31620)}{(s^3+912.8s^2+3849s+74.99)}$
$\frac{1}{s^{1.90}}$	$\frac{(2.239e7+0.2524s^2+1314s+31620)}{(s^3+41.56s^2+7.98s+0.007079)}$

Equation (32) needs the synthesis formulas as following:

$$\left\{ \begin{array}{l} \omega'_0 = \omega^{-1} = \alpha^{-0.5} \omega_u \\ (\frac{\omega_{k+1}}{\omega_k})' = \frac{\omega_{k+1}}{\omega_k} = \alpha \eta > 1 \\ \frac{\omega'_{k+1}}{\omega_k} = \eta > 0, \frac{\omega_k}{\omega'_k} = \alpha > 0 \\ N = \frac{\log(\frac{\omega_N}{\omega_0})}{\log(\alpha \eta)} \\ \mu = \frac{\log \alpha}{\log(\alpha \eta)} \end{array} \right. \quad (33)$$

The ω_u in Equation (33) is the unit gain frequency. Based on the above fundamental mechanism and the Oustaloup Recursive Approximation algorithm ([29]), one could get the integer order approximation of the fractional order integrators associated in this study. The integrators and the corresponding approximations are showed in Table 3. The fractional order integrators showed in Table 3 could construct a stepwise adjustable fractional integrator. However the step time t_i in Equation (30) is undetermined yet, in this study the optimization is used to determine these step times. The optimization uses a regular genetic algorithm([30]), and sets the optimization objective as the mean absolute chatter displacement. Other parameters setup are as in Table 4.

Note that the range of every design variable is the same and there is no constraint on their sequence. Before the first adjustment step, set $\beta=1.9$ and after time 2.4 sec, force $\beta=1$. Although no sequence constraint on the four design variable, the condition (30) requires monotonically reduction on the β , which means the expected optimization result should yield a sequence: $t_1 < t_2 < t_3 < t_4$. In every evaluation, the optimization process takes the responds from the simulation results. Figure 5 shows the system setup and Figure 6 shows the controller setup.

In the model of numerical simulation, the NLDDE model includes three modules, which are the cutting force module, the machine tool elastic structure module, and the regenerative effects module. The stable reference model is simulated by the model reference module, and the biased error state space is conducted by the reference error module. The bias will be reduced during the suppression process of chatter. The two controllers, spindle speed controller and feed rate controller, performs Equations (28) and (29) based on the biased error state space and the cutting parameters generated by the parameters module as in figure 5. The adjusted fractional integrator in Figure 6 outputs the current fractional order integrator approximated by the results in Table 3. The next section presents the simulation

TABLE 4. Optimization Parameter Setup

Design Variables: (t_1, t_2, t_3, t_4)	t_1	t_2	t_3	t_4
Order of Corresponding Integrator (β):	1.75	1.5	1.25	1.15
Bits:	6	6	6	6
Range (sec):	0.02-2.4	0.02-2.4	0.02-2.4	0.02-2.4
Termination Criterion:	Bits converge rate 99%			
Crossover Probability:	Uniform with P=0.5			
Allele Mutates:	Independently with P=0.01			

and optimization results.

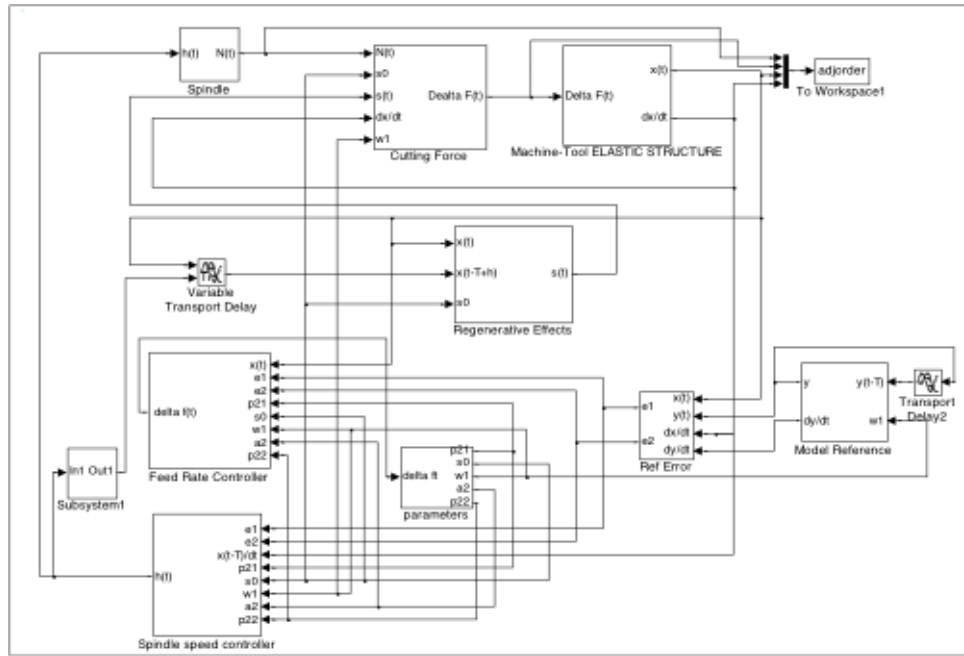


FIGURE 5. System setup of simulation

5. RESULTS AND ANALYSIS

This section shows the result from the numerical simulation, the structure of the simulation model is as in Figure 5, and the parameters are kept the same as in Table 2. For the purpose of controlling the unstable chatter, the unstable cutting condition is chosen. The numerical simulation first presents the results of the fixed order controllers and the ordinary adaptive control is considered a special case of the fractional order adaptive control with $\beta=1.0$, Figure 7 includes the chatter displacement and the spindle speed under the controller with $\beta=1.0, 1.75, 1.5,$ and 1.25

respectively.

As showed in Figure 7, the controller with $\beta=1.75$ suppresses the chatter most

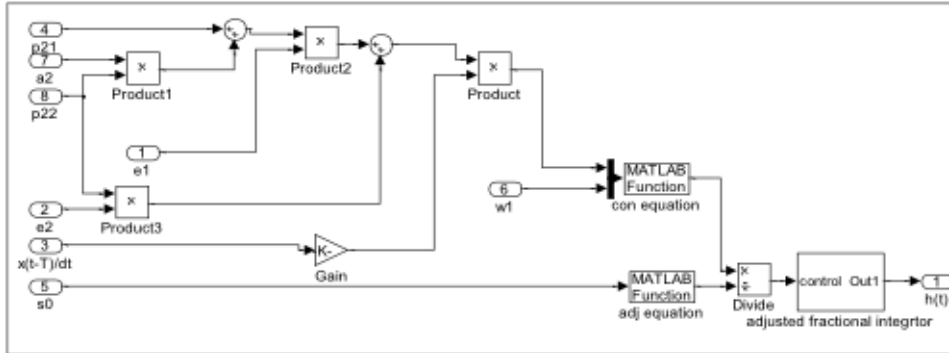


FIGURE 6. Controller setup, spindle speed

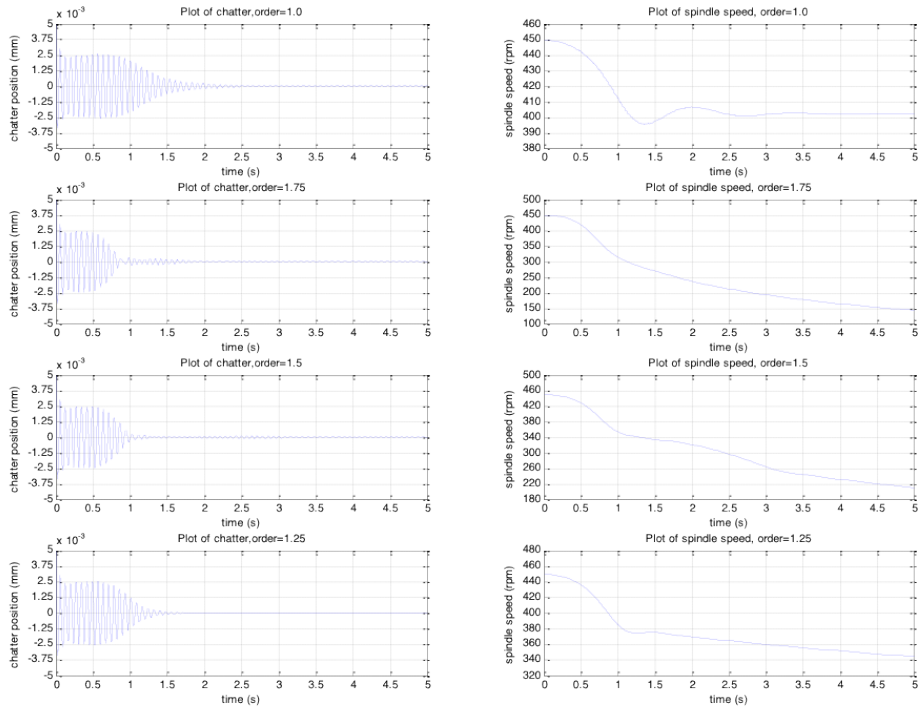


FIGURE 7. Fractional order adaptive control, fixed order

quickly, and the ordinary adaptive model takes the longest time to stable the vibration. However, the 1.75 fractional order controller adjusts the spindle speed dramatically, and this is contrast to the slight adjustment in spindle speed by the ordinary order model. This is largely due to the over adjustment characteristic of the fixed order fractional controller and this result is not desired in real industry. Although the quick suppression of chatter is the most important criteria when choosing the controller, people also hope the adjustment of spindle speed is not dramatically, in other words, it is desirable that the final spindle speed after adjustment could remain in a reasonable range of the initial spindle speed and this concern reflects the needs from real manufacture industry. Figure 8 ~ 11 contain the Lissajous plots for adaptive controller with fractional order 1.00, 1.25, 1.50 and 1.75 respectively. These Lissajous plots provide the energy absorbing process and the time windows associating to these plots are 0 ~ 0.5 sec for frame A, 0.5 ~ 1 sec for frame B, 1 ~ 1.5 sec for frame C and 1.5 ~ 2.5 sec for frame D.

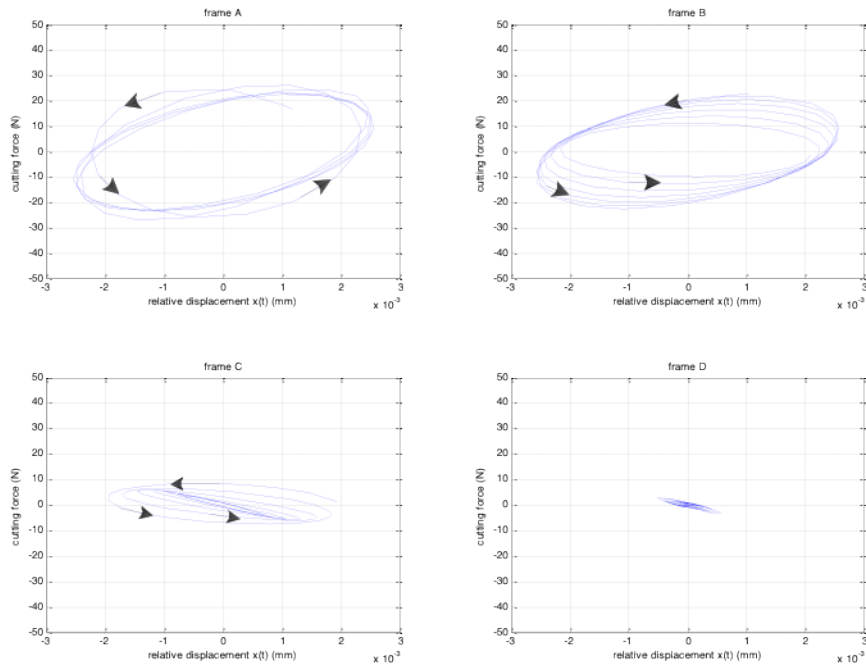


FIGURE 8. Lissajous plot for controller with $\beta=1.00$

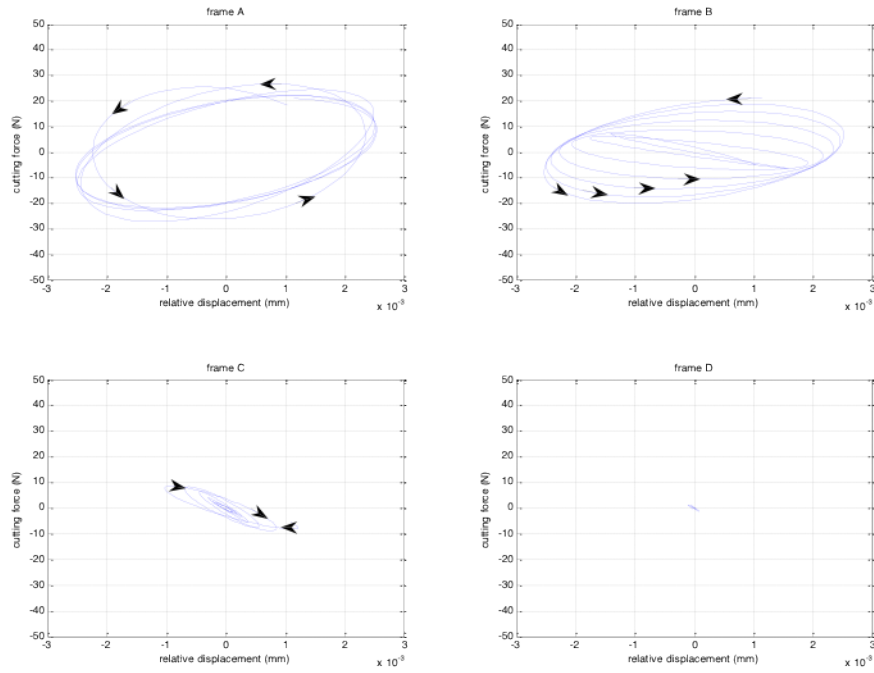
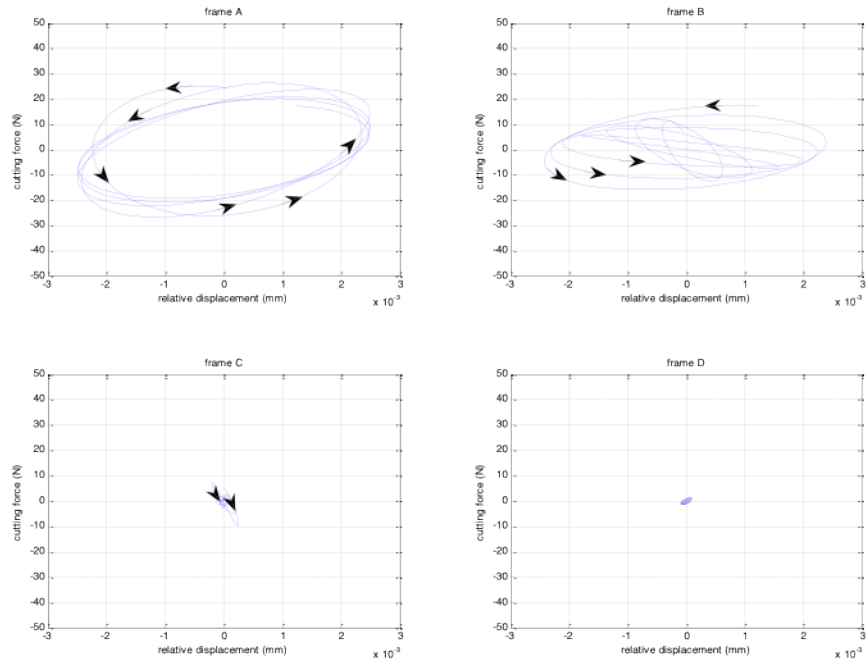


FIGURE 9. Lissajous plot for controller with $\beta=1.25$

FIGURE 10. Lissajous plot for controller with $\beta=1.5$

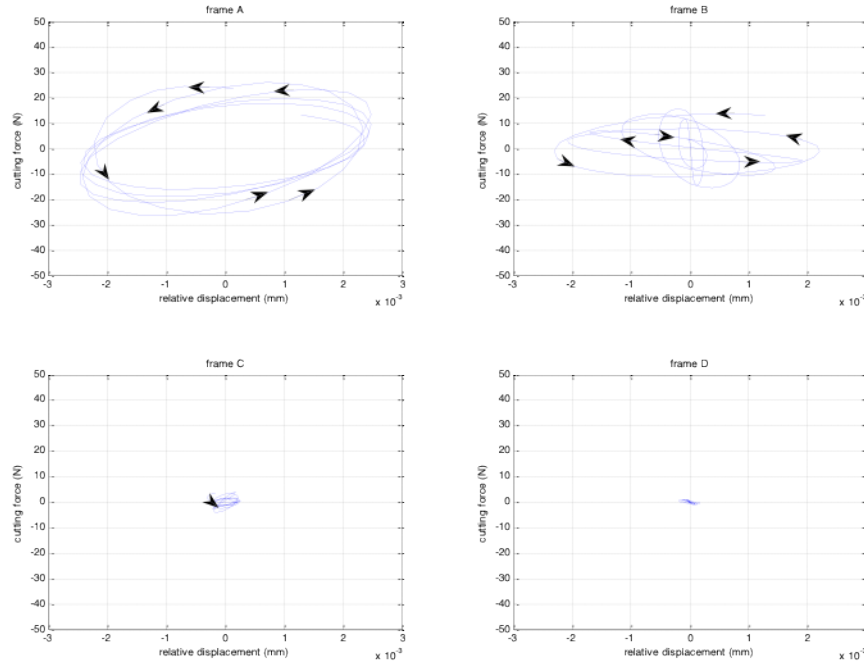


FIGURE 11. Lissajous plot for controller with $\beta=1.75$

The Lissajous plot for order 1.00 regards the ordinary adaptive controller, and it shows during each time window, except very short period at beginning, the energy is absorbed monotonically and smoothly. As showed in Figure 8, the outermost trajectory in frame A is the initial one and it shows energy turbulence since this trajectory crosses several inner ones. The initial energy turbulence is hard to avoid, because the sudden occurrence of chatter by unavoidable small disturbance. Therefore we focus more on the energy absorbing process after initial stage. One can observe that, in Figure 1 frame B, frame C and frame D do not contain the any crossing trajectory, which means the energy is suppressed monotonically. Lissajous plots for fractional order controller with $\beta > 1$ shows energy turbulence problems after initial stage. Especially in Figures 10 and 11, which associate with $\beta=1.50$ and $\beta=1.75$. Observed from frame B and frame C in those two figures, there are many crossing trajectories. These crossing trajectories is caused by the over adjustment issue of the fixed order fractional controller, and the issue could be traced back to the bias problem. In frame A the enhanced controlling power still overcome the bias, so there is no energy turbulence, but later when the chatter is suppressed partially, the bias dominate the gains from controlling power, and caused energy turbulence. For frame D, since now the chatter is almost suppressed, the bias as in Equation (26) is nearly vanished, and the fractional order controller can properly suppress the chatter now. Figure 12 shows the net working done by

TABLE 5. Optimization Results

Design Variables: (t_1, t_2, t_3, t_4)	t_1	t_2	t_3	t_4
Order of Corresponding Integrator (β):	1.75	1.5	1.25	1.15
Optimization result:	0.6935	0.7677	1.26871	1.6874

the dynamic cutting force and the chatter speed during this process.

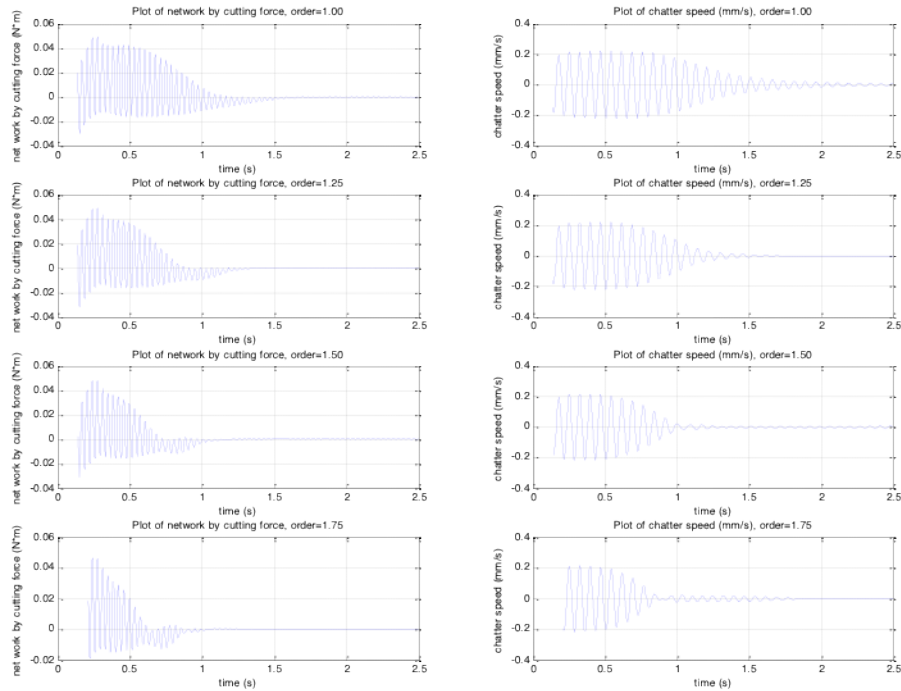


FIGURE 12. Network plots

To obtain superior chatter suppression effect, one needs to overcome the bias issue during the entire suppression process. Therefore next this study is going to show the simulation results based on adjustable fractional order controller. As mentioned in section 4, the design of the adjustable fractional controller involves the optimization. And the result of optimization is as in Table 5.

As showed in table 5, the optimization result confirms the desired sequence: $t_1 < t_2 < t_3 < t_4$, and multiple times of optimization yield the similar result. Since the optimization does not constraint the sequence of t_i in the algorithm, this global optimum result could be considered as a confirmation to the condition (27). The design of the adjustable fractional adaptive controller is based on the final optimization result included in Table 5, and the numerical simulation result is showed

in Figure 12. For comparison purpose, the controllers with other fixed order have also been included.

As showed in Figure 12, the adjustable fractional order adaptive controller has a overwhelming superior result on chatter suppression. The chatter has been fully suppressed within 1.5 sec, and the major parts of vibration have been reduced within 1 sec, all of this result is best among the controllers included in this simulation. Besides, the adjustment of spindle speed is acceptable given its quick suppression of chatter. Under this controller the spindle speed reaches the balance status more quickly as showed by the spindle speed chart for the adjustable order controller. This effect means the machine tool could gain the stable status within a short time window.

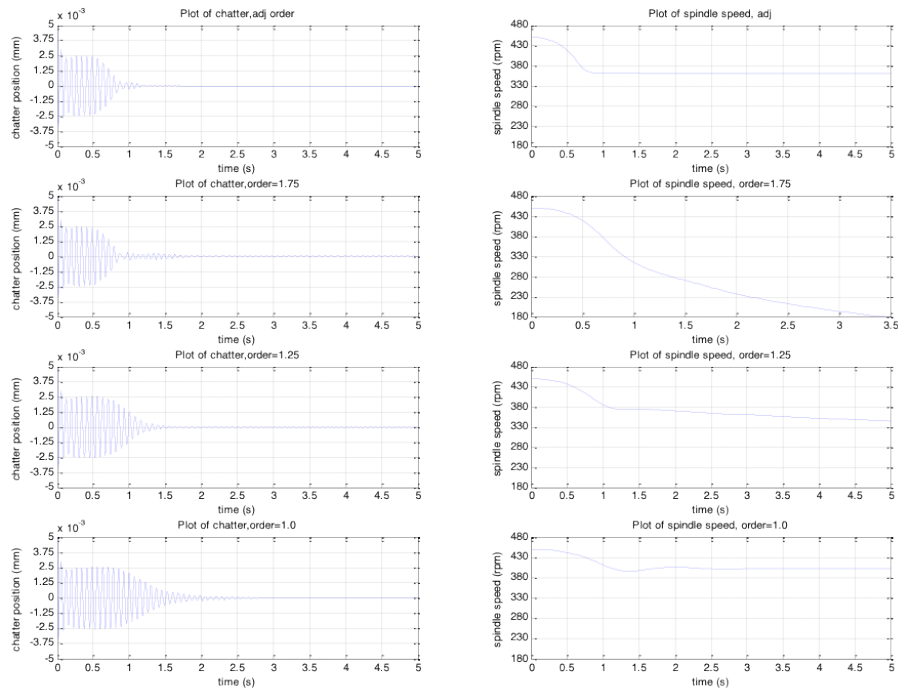


FIGURE 13. Chatter suppression and spindle speed with AFrAC

6. FUTURE WORK/RESEARCH

The novel AFrAC method is an integration of the adaptive control method based on Lyapunov's second theorem and the adjustable fractional order integrator for the benefit of stability and quick response. The numerical simulation demonstrates substantial advantages in comparison with the existing methods for regenerative machining chatter control. Further theoretical analysis will be performed as our future work, including the numerical simulations in extremely high speed cutting

conditions, in which some machine tools parameters may need reestimation.

It can be extended to broader applications model reference AFrAC control for a class of n -dimensional differential-difference systems with time delays. The AFrAC method is designed for more general systems, whose mathematical model is linear/nonlinear differential-difference equation with time-delays. Most of manufacturing processes can be described by this type of mathematical model, such as the time-delay in milling process, rolling mill, infeed grinding system, hobbing process, etc.

Another goal of this research is to apply the AFrAC in industrial production. Its significant technological impact will be at numerous levels. There are many technical details to be explored for realizing the AFrAC control on CNC machining systems. Once the realization techniques of the optimal AFrAC are available, the AFrAC method will also be transferred to the high production rate local industry for real world applications. Since this technique only requires changing the machining process parameters, no extra actuators/devices are needed for its realization, it will be much easier to upgrade the existing CNC machines in production lines.

7. CONCLUSIONS

This research is one of the earliest efforts to control regenerative machining chatter using the fractional control law, which has more potential than the integer order controller for controlling the systems with time-delays described by the nonlinear differential difference model. The fractional order treatment will reveal new perspectives and profound understanding of the manufacturing process modeling, analysis and control law design. This paper presents the adjustable fractional order adaptive control (AFrAC) to suppress the chatter happens in unstable working condition. The control law of the AFrAC has been derived based on the ordinary adaptive control and fractional error space. This paper also includes a stepwise design to the adjustable fractional order integrator, and the parameters are obtained from optimization result. The global optimum of the parameter design confirmed the condition on the AFrAC, and numerical simulation shows this controller could suppress the chatter in a more efficient way without dramatic adjustment on the cutting condition. The method itself is a successful application of fractional order integrator on ordinary order system, and this method could be generalized to other similar systems. This method can be utilized in the cases of linear/linearized nonlinear dynamic systems with time-delay. It is a new challenge to physically realize this derived control law for machining chatter control, especially for prediction of machining chatter onset.

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HAIYAN ZHANG
PURDUE UNIVERSITY, 401 N. GRANT STREET, WEST LAFAYETTE, IN47907, USA
E-mail address: hhzhang@purdue.edu