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UNIFIED INTEGRALS ASSOCIATED WITH H-FUNCTIONS AND M-SERIES

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Abstract. The paper presents two new unified integral formulas involving the Fox H-function and M-Series. The results are expressed in terms of the H-function. Some interesting special cases of main results are also considered in form of many corollaries. Due to general nature of H-function and M-Series, a number of results involving different special functions can be obtained merely by specializing the parameters in the presented formulas.

1. Introduction and Preliminaries

The importance of the H-Function and M-Series are realized by scientists, engineers and statisticians (Caputo [7] , Glockle and Nonnenmacher [14], Mainardi et al. [17], Hilfer [15] etc.) due to vast potential of their applications in diversified fields of science and engineering such as fluid flow, rheology, diffusion in porous media, propagation of seismic waves, anomalous diffusion and turbulence etc.

In view of importance and popularity of the H-function a large number of integral formulas involving this function have been developed by many authors. For example, Garg and Mittal [13] obtained an interesting unified integral involving Fox H-function, Choi and Agarwal [8] derived unified integrals associated with Bessel functions. Further, Ali [5] gave three interesting unified integrals involving the hypergeometric function. Recently, many useful integral formulas associated with the Bessel functions of several kinds and Hypergeometric functions have been studied by Agarwal ([1]-[3]), Agarwal et al. [4], Choi and Agarwal [9] and Choi et al. [10].

Fox [12] introduced the H-function, via a Mellin-Barnes type integral for integers m, n, p, q such that $0 = m = q$, $0 = n = p$, for $a_i, b_j \in \mathbb{C}$ and for $\alpha_i, \beta_j \in \mathbb{R}_+ = (0, \infty)$ $(i = 1, 2, \ldots, p; j = 1, 2 \ldots, q)$, as

$$H_{p,q}^{m,n}(z) = H_{p,q}^{m,n}\left[z \,\middle|\, \begin{matrix} (a_i, \alpha_i)_1^p \\ (b_j, \beta_j)_1^q \end{matrix}\right] = \frac{1}{2\pi i}\int_{\mathcal{L}} \mathcal{H}_{p,q}^{m,n}(s)\, z^{-s} ds \qquad (1)$$

with

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$$\mathcal{H}_{p,q}^{m,n}(s)=\mathcal{H}_{p,q}^{m,n}\left[\begin{array}{l}(a_i,\alpha_i)_1^p\\(b_j,\beta_j)_1^q\end{array}\mid s\right]=\frac{\prod_{j=1}^{m}\Gamma(b_j+\ \beta_j\ s)\prod_{i=1}^{n}\Gamma(1-a_i-\ \alpha_i s)}{\prod_{i=n+1}^{p}\Gamma(a_i+\alpha_i s)\prod_{j=m+1}^{q}\Gamma(1-b_j-\beta_j s)} \tag{2}$$

with all convergence conditions as given by Braaksma [6], Mathai [18], Kilbas and Saigo [16]. On putting $\alpha_j=\beta_j=1$ in H-function, we obtain the Meijer's G-functions $G_{p,q}^{m,n}(z)$ (Fox [12]).

Wright [25] defined generalized hypergeometric function by means of the series representation in the form

$$ {}_p\psi_q(z)={}_p\psi_q\left[\begin{array}{l}(a_1,A_1),\ldots,(a_p,A_p)\\(b_1,B_1),\ldots,(b_q,B_q)\end{array}\mid z\right]=\sum_{n=0}^{\infty}\frac{\prod_{i=1}^{p}\Gamma(a_i+nA_i)}{\prod_{j=1}^{q}\Gamma(b_j+nB_j)}\frac{z^n}{n!} \tag{3}$$

where $z,a_i,b_j\in\mathbb{C},A_i,B_j\in\mathbb{R}_+,A_i\neq0,B_j\neq0;i=1,\ldots,p;j=1,\ldots,q,$

$$\sum_{j\ =\ 1}^{q}B_j-\sum_{i=\ 1}^{p}A_i>-1$$

Sharma and Jain [23] introduced the generalized M-series as the function defined by means of the power series:

$$ {}_p^{\alpha}M_q^{\beta}(a_1,a_2,\ldots,a_p;b_1,b_2,\ldots b_q;z)={}_p^{\alpha}M_q^{\beta}(z)={}_p^{\alpha}M_q^{\beta}\left((a_j)_1^p;(b_j)_1^q;z\right)$$

$$=\sum_{n=0}^{\infty}\frac{(a_1)_n\ldots(a_p)_n}{(b_1)_n\ldots(b_q)_n}\frac{z^n}{\Gamma(\alpha n+\beta)},z,\alpha,\beta\in\mathbb{C},R(\alpha)>0 \tag{4}$$

where, $(a_j)_n,(b_j)_n$ are the known Pochammer symbols. The series (4) is defined when none of the parameters $b_j's, j=1,2,\ldots,q$ is a negative integer or zero; if any numerator parameter a_j is a negative integer or zero, then the series terminates to a polynomial in z. The series in (4) is convergent for all z if $p\leq q$, it is convergent for $|\mathrm{z}|<\delta=\alpha^{\alpha}$ if $p=q+1$ and divergent, if $p>q+1$. When $p=q+1$ and $|z|=\delta$, the series can converge on conditions depending on the parameters. Properties of M-series are further studied by Saxena [22], Chouhan and Sarswat [11] etc.

The generalized Mittag-Leffler function, introduced by Prabhakar [21], may be obtained from (4) for $p=q=1$; $a=\gamma\in\mathbb{C}$; b = 1, as

$$E_{\alpha,\beta}^{\gamma}(z)=\sum_{m=0}^{\infty}\frac{(\gamma)_m}{\Gamma(\alpha m+\beta)}\frac{z^m}{m!}=\sum_{m=0}^{\infty}\frac{(\gamma)_m}{(1)_m}\frac{z^m}{\Gamma(\alpha m+\beta)}={}_1^{\alpha}M_1^{\beta}(;1;z) \tag{5}$$

The generalized M-series (4) can be represented as a special case of Fox H-function (2) and Wright generalized hypergeometric function (3), as

$$ {}_p^{\alpha}M_q^{\beta}\left((a_j)_1^p;(b_j)_1^q;z\right)=k\ {}_{p+1}\psi_{q+1}\left[\begin{array}{l}(a_1,1),\ldots,(a_p,1),(1,1);\\(b_1,1),\ldots,(b_q,1),(\beta,\alpha);\end{array}z\right]$$

$$=kH_{p+1,q+2}^{1,p+1}\left[-z\left|\begin{array}{l}(1-\alpha_j,1;1)_1^p,(0,1)\\(0,1),\ (1-\beta_j,\ 1)_1^q,(1-\beta,\alpha)\end{array}\right.\right] \tag{6}$$

where $k=\frac{\prod_{j=1}^{q}\Gamma(b_j)}{\prod_{j=1}^{p}\Gamma(a_j)}$.

Oberhettinger [19] established the following interesting integral formula

$$\int_0^{\infty}x^{\mu-1}\left(x+a+\sqrt{x^2+2ax}\right)^{-\lambda}dx=2\lambda a^{-\lambda}\left(\frac{a}{2}\right)^{\mu}\frac{\Gamma(2\mu)\Gamma(\lambda-\mu)}{\Gamma(1+\lambda+\mu)} \tag{7}$$

where $0 < Re\,(\mu) < Re\,(\lambda)$.

We aim at presenting two generalized integral formulas involving Fox H-function and M-Series. Results derived in this paper are in terms of H-function and due to generous nature of H-function, several particular cases are considered in the form of corollaries.

2. Unified Integral Involving H-Function

In this section we established a unified integral formula involving H-function under theorem 1. Certain particular cases are discussed in the form of corollaries.

Theorem 2.1

$$\int_0^\infty x^{\mu-1}\Big(x+a+\sqrt{x^2+2ax}\Big)^{-\lambda} H^{m,n}_{p,q}\left[\frac{y^k}{\omega^k\ (\ x+a+\sqrt{x^2+2ax})^k}\middle| \begin{matrix}(a_i,\ \alpha_i)_1^p\\ (\ b_j,\ \beta_j)_1^q\end{matrix}\right]dx$$

$$= 2^{1-\mu}a^{\mu-\lambda}\Gamma\,(2\mu)\,H^{m,n+2}_{p+2,q+2}\left[\Big(\frac{y}{\omega a}\Big)^k\middle| \begin{matrix}(-\lambda,k)\,,(1-\lambda+\mu,k)\,,(a_i,\ \alpha_i)_1^p\\ (1-\lambda,k)\,,(-\lambda-\mu,k)\,,(\ b_j,\ \beta_j)_1^q\end{matrix}\right] \quad (8)$$

The convergence conditions for the validity of (8) are as follows:

(i) $0\le m\le q, 0\le n\le p$, for $a_i, b_j\in\mathbb{C}$ and for $\alpha_i,\beta_j\in\mathbb{R}_+=(0,\infty)$ $(i=1,2,\ldots,p; j=1,2\ldots,q)$ $\lambda,\mu\in\mathbb{C}$ with $0<R\,(\mu)<R(\lambda-sk)$ and $k>0, x>0$.

(ii) $\mathcal{L}=\mathcal{L}_{i\gamma\infty}$ is a contour starting at the point $\gamma-i\infty$ and going to $\gamma+i\infty$ where $\gamma\in R(-\infty,+\infty)$ such that all the poles of $\Gamma\,(b_j+\beta_j s)\,, j=1,\ldots,m$ are separated from those of $\Gamma\,(1-a_\theta-\alpha_\theta s)\,,\theta=1,\ldots,n$. The integral converges if $\alpha>0, |argz|<\frac{1}{2}\pi\alpha, a\neq 0$.

The integral also converges if $\sigma=0, \gamma\mu+R\,(\delta)<-1, argz=0$ and $z\neq 0$ where

$$\sigma=\sum_{j=1}^{n}\alpha_j-\sum_{j=n+1}^{p}\alpha_j+\sum_{j=1}^{m}\beta_j-\sum_{j=m+1}^{q}\beta_j$$

(iii) $R\,(\mu)>0,\ R\,(\mu)+k\ min_{1\le j\le m}\,R\left[\frac{b_j}{\beta_j}\right]>max\,[0,(\lambda-sk)]$.

Proof. By applying (1) to the L.H.S of (8) and then interchanging the order of integration, we get L.H.S of (8) as

$$=\frac{\omega^{ks}}{2\pi i}\int_{\mathcal{L}}\mathcal{H}^{m,n}_{p,q}(s)\left(\int_0^\infty x^{\mu-1}\Big(x+a+\sqrt{x^2+2ax}\Big)^{-(\lambda-sk)}y^{-sk}dx\right)ds$$

by the virtue of integral formula (7) under the conditions stated in theorem 1, we obtained $=2^{1-\mu}a^{\mu-\lambda}\Gamma\,(2\mu)\,\frac{1}{2\pi i}\int_{\mathcal{L}}\mathcal{H}^{m,n}_{p,q}\,(s)\big(\frac{y}{\omega a}\big)^{-sk}\frac{\Gamma(\lambda-sk+1)\Gamma(\lambda-ks-\mu)}{\Gamma(\lambda-sk)\Gamma(1+\lambda-sk+\mu)}ds$ which, upon using definition of H-function (1), yields (8).

Corollary 2.1 Let the condition of theorem 1 be satisfied and set $\alpha_i=\beta_j=1$, the equation (8) reduces to

$$\int_0^\infty x^{\mu-1}\Big(x+a+\sqrt{x^2+2ax}\Big)^{-\lambda} G^{m,n}_{p,q}\left[\frac{y^k}{\omega^k(\ x++\sqrt{x^2+2ax})^k}\middle| \begin{matrix}(a_i)_1^p\\ (\ b_j)_1^q\end{matrix}\right]dx$$

$$=\ 2^{1-\mu}a^{\mu-\lambda}\Gamma\,(2\mu)\,G^{m,n+2}_{p+2,q+2}\left[\Big(\frac{y}{\omega a}\Big)^k\middle| \begin{matrix}(-\lambda,k)\,,(1-\lambda+\mu,k)\,,(a_i)_1^p\\ (1-\lambda,k)\,,(-\lambda-\mu,k)\,,(\ b_j)_1^q\end{matrix}\right] \quad (9)$$

where $G_{p,q}^{m,n}[.]$ is well known Meijer G-Function (Fox [12]).

Corollary 2.2 Let the condition of theorem 1 be satisfied and $R(\gamma)>0$ and on setting $m=1,n=1,p=1,q=2,k=1,\omega=1,y=-Y,a_1=1-\gamma,\alpha_1=1,b_1=0,\beta_1=1,b_2=1-\beta,\beta_2=\alpha$ then equation (8) reduces to the following integral

$$\int_0^\infty x^{\mu-1}\left(x+a+\sqrt{x^2+2ax}\right)^{-\lambda}E_{\alpha,\beta}^{\gamma}\left[\frac{Y}{\left(x+a+\sqrt{x^2+2ax}\,\right)}\right]dx$$

$$=\frac{\Gamma(2\mu)2^{1-\mu}}{\Gamma(\gamma)}a^{\mu-\ \lambda}H_{3,4}^{1,3}\left[\frac{-Y}{a}\middle|\begin{matrix}(-\lambda,1),(1-\lambda+\mu,1),(1-\gamma,1)\\(0,1),(1-\lambda,1),(-\lambda-\mu,1),(1-\beta,\alpha)\end{matrix}\right] \tag{10}$$

where, $E_{\alpha,\beta}^{\gamma}(z)$ is the generalized Mittag – Leffler function as represented by equations (5).

Corollary 2.3 Let the conditions of theorem 1 be satisfied and on setting $m=1,n=0,p=0,q=2,k=2,\omega=2,b_1=0,\beta_1=1,b_2=-\nu,\beta_2=1,\lambda=\lambda_1+\nu$ then equation (8) reduces to the known integral due to Choi and Agarwal [8]

$$\int_0^\infty x^{\mu-1}(\ x+a+\sqrt{x^2+2ax})^{-\lambda_1}J_\nu\left(\frac{y}{(\ x+a+\sqrt{x^2+2ax}}\right)dx$$

$$=2^{1-\nu-\mu}a^{\mu-\nu-\lambda_1}y^\nu\Gamma(2\mu)\,{}_{23}\left[\begin{matrix}(1+\lambda_1+\nu,2),(\lambda_1+\nu-\mu,2);\\(\lambda_1+\nu,2),(1+\lambda_1+\nu+\mu,2),(1+\nu,1);\end{matrix}\ \frac{-y^2}{4a^2}\right]. \tag{11}$$

where $J_\nu(z)$ is the ordinary Bessel function of first kind (Olver [20]).

Corollary 2.4 Let the conditions of theorem 1 be satisfied and on setting $m=1,n=1,p=1,q=2,a_1=1-a,\alpha_1=1,b_1=0,\beta_1=1,b_2=1-c,\beta_2=1,\mu=\lambda,\lambda=\nu,y=-Y,k=1,\omega=1$, then the equation (8) reduces to the known integral given by Choi and Agarwal [8]

$$\int_0^\infty x^{\lambda-1}(x+a+\sqrt{x^2+2ax})^{-\nu}{}_1F_1\left[a;c;\frac{Y}{\left(x+a+\sqrt{x^2+2ax}\,\right)}\right]dx$$

$$=2^{1-\lambda}\left[\frac{\nu a^{\lambda-\nu}\Gamma(2\lambda)\Gamma(\nu-\lambda)}{\Gamma(\nu+\lambda+1)}\right]{}_3F_3\left[\begin{matrix}a,\nu-\lambda,\nu+1;\\c,\nu+\lambda+1,\nu;\end{matrix}\ \frac{Y}{a}\right] \tag{12}$$

provided $0<R(\lambda)<R(\nu)$ and $\left|\frac{Y}{a}\right|<1$.

3. Unified Integral Involving M-Series

In this section we established a unified integral formula involving M-series under theorem 2. Certain particular cases are discussed in the form of remarks.

Theorem 3.1

$$\int_0^\infty x^{\mu-1}\left(x+a+\sqrt{x^2+2ax}\right)^{-\lambda}{}_p^{\alpha}M_q^{\beta}\left(\frac{y^k}{\omega^k\ (x+a+\sqrt{x^2+2ax})^k}\right)dx$$

$$= 2^{1-\mu} a^{\mu-\lambda} \Gamma(2\mu) H^{1,p+3}_{p+3,q+4} \left[-\left(\frac{y}{\omega a}\right)^k \middle| \begin{matrix} (-\lambda, k), (1-\lambda+\mu, k), (0,1), (1-a_i, 1)_1^p \\ (1-\lambda, k), (-\lambda-\mu, k), (0,1), (1-\beta, \alpha), (1-b_j, 1)_1^q \end{matrix} \right] \tag{13}$$

The convergence conditions for the validity of (13) are as follows

(i) $0 \le m \le q, 0 \le n \le p$ for $a_i, b_j \in \mathbb{C}$ and for $\alpha_i, \beta_j \in \mathbb{R}_+ = (0,\infty)$ $(i = 1, 2, \ldots, p; j = 1, 2 \ldots, q)$ $\lambda, \mu, \alpha \in \mathbb{C}$ with $0 < R(\mu) < R(\lambda+k)$ and $min(k, \alpha, x) > 0$.

(ii) $\mathcal{L} = \mathcal{L}_{i\gamma\infty}$ is a contour starting at the point $\gamma - i\infty$ and going to $\gamma + i\infty$ where $\gamma \in R(-\infty, +\infty)$ such that all the poles of $\Gamma(b_j + \beta_j s), j = 1, \ldots, m$ are separated from those of $\Gamma(1 - a_\theta - \alpha_\theta s), \theta = 1, \ldots, n$. The integral converges if $\alpha > 0, |argz| < \frac{1}{2}\pi\alpha, a \neq 0$. The integral also converges if $\sigma = 0, \gamma\mu + R(\delta) < -1, argz = 0$ and $z \neq 0$ where

$$\sigma = \sum_{j=1}^{n} \alpha_j - \sum_{j=n+1}^{p} \alpha_j + \sum_{j=1}^{m} \beta_j - \sum_{j=m+1}^{q} \beta_j$$

(iii) $R(\mu) > 0,\ R(\mu) + k\ min_{1\le j\le m} R\left[\frac{b_j}{\beta_j}\right] > max\,[0, (\lambda + k)]$.

Proof. By applying (4) to the L.H.S of (13) and then interchanging the order of integration and summation, we get L.H.S of (13) as

$$= \sum_{r=0}^{\infty} \frac{(a_1)_r \ldots (a_p)_r}{(b_1)_r \ldots (b_q)_r} \frac{1}{\Gamma(\alpha r + \beta)} \left(\frac{y}{\omega}\right)^{kr} \int_0^\infty x^{\mu-1} \left(x + a + \sqrt{x^2 + 2ax}\right)^{-(\lambda+kr)} dx$$

by the virtue of integral formula (7) under the conditions $0 < R(\mu) < R(\lambda+k) < R(\lambda + kr)$, we obtained

$$= 2^{1-\mu} a^{\mu-\lambda} \Gamma(2\mu) \sum_{r=0}^{\infty} \frac{(a_1)_r \ldots (a_p)_r}{(b_1)_r \ldots (b_q)_r} \frac{\Gamma(\lambda + kr + 1)\Gamma(\lambda + kr - \mu)}{\Gamma(\alpha r + \beta)\Gamma(\lambda + kr)\Gamma(1 + \lambda + kr + \mu)} \left(\frac{y}{\omega a}\right)^{kr}$$

which, upon using definition of H-function (1), yields (13).

Remark 1. Under the relation (6) theorem 2 may be considered as a particular case of theorem 1.

Remark 2. If $p = q = 0, a = \gamma \in \mathbb{C}, b = 1$ in equation (13) then we obtain corollary 2.

Several illustrative examples of theorem 2 involving appropriately chosen special values of the parameters can also be derived fairly easily.

4. Conclusion

The Fox H-function can be regarded as an extreme generalization of the generalized hypergeometric function ${}_pF_q$ beyond the Meijer G-function and M-Series. In this paper we have presented two generalized integral formulas involving Fox H-function and M-Series. Further many known and unknown results have been established in terms of special cases, which involve Meijer G-function, Mittag-Leffler function, Bessel function and hypergeometric function. The results presented in this paper are in terms of the Fox H-function which are having vast importance due to the generous nature of H-function and their applications in some other research areas. For example Srivastava and Exton [24] applied their integral involving the product of several Bessel functions to give explicit expression of a generalized random walk.

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