Fayoum University Faculty of Engineering, 2023, Vol: 6(2), 1-12
https://fuje.journals.ekb.eg/
ISSN Online: 2537-0634
ISSN Print: 2537-0626

Survey on Designing Fractional-Order Filters: Metaherustic Approach

Andrew Amgad[1,*], A. M. AbdelAty[1], Menna T. M. M. Elbarawy[1], Hazem A. Attia[1]

[1]Engineering Mathematics and Physics Dept., Faculty of Engineering, Fayoum University, Egypt.

*Corresponding author: Andrew Amgad (aaa55@fayoum.edu.eg).

How to cite this paper: Amgad, A. *el al.* (2023). Survey on Designing Fractional-Order Filters: Metaherustic Approach. *Fayoum University Journal of Engineering, 6,* (2) 1-12
https://dx.doi.org/10.21608/FUJE.2023.177757.1032

Abstract

International scholars have recently demonstrated significant interest in fractional order filters (FOF) due to their greater design freedom and continuously stepped stopband attenuation rate. This paper presents a literature review on designing a fractional order filter based on the metaheuristic approach, which provides an optimum design technique and less passband and stopband error. The nonlinear, non-uniform, multidimensional, and multimodal FOF design issue error landscape is heavily exploited by the metaheuristic global search method.
The algorithm's optimal coefficient values, which closely approximate the magnitude response of the ideal FOF, are reached at the completion of its repeated search routine.

Keywords

Fractional calculus; Fractional order filter; FPA.

1. Introduction

Calculus in the fractional domain [1] recently had a widespread application in several engineering areas. The developments in fractional calculus applications were slow mainly due to the absence of a universally accepted geometrical and physical interpretation; this result in the whole field is limited to just a theory [2]. However, there has been considerable interest in the topic in recent years. Researchers have found that fractional-order models can represent systems better than integer-order ones despite the need for a universally accepted interpretation. The field has seen a steep rise in publications and has grown, but it is still far from conceptual completion, particularly in the theories, applications, and implementation methods [3].

Fractional calculus is viewed as a generalization of integer order calculus. In other words, the integration and differentiation for the integer order are exceptional cases of the corresponding operations for the fractional order. Since the mid-twentieth century, this mathematical tool has been used to create better models for simple and complex physical systems. Integer order (Newtonian) differential operators are local, while most fractional calculus operators are non-local except at integer points.

This allows the fractional order operator to capture the memory of the model and even extrapolate between past and future behaviour. This leaves Newtonian calculus as a special case and a subspace of fractional calculus. Fractional differentiation is often used to model phenomena exhibiting non-standard dynamical behaviors [4].

The Riemann-Liouville (R-L) definition and the Grunwald-Letnikov (G-L) definition are the two main definitions of differentiation and integration in the fractional domain (differ-integration)[5]. The G-L formula is a numerical differentiation formula extracted from the backward finite difference formula. As per the G-L method, the fractional order differentiation of a function $f(t)$ is given by Eqn. (1) [5]

$$\boldsymbol{D_t^{\alpha} f(t)} := \lim_{h\to 0} \frac{1}{h^{\alpha}} \sum_{j=0}^{\infty} (-1)^j \binom{\alpha}{j} f(t - jh), \quad (1)$$

where, $\binom{\alpha}{j} = \frac{\Gamma(\alpha+1)}{\Gamma(j+1)\Gamma(\alpha-j+1)}$ represents the binomial coefficients, and, $\Gamma(\cdot)$ denotes the gamma function.

The formula (1) is considered a generalization case of differentiation as $\alpha \in R$. The Laplace transform of the G-L fractional differ-integration with zero initial conditions is given by Eqn. (2).

$$\int_0^{\infty} e^{-st} \, {}_0D_t^{\alpha} f(t) dt = s^{\alpha} F(s). \quad (2)$$

The R-L definition of fractional calculus is a continuation of the n-fold successive integration process. As per the R-L method, the fractional differentiation of a function $f(t)$ is given by Eqn. (3) [6]

$${}_0D_t^{\alpha} f(t) := \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_0^t \frac{f(\tau)}{(t-\tau)^{\alpha+1}} d\tau, \quad (3)$$

where $n - 1 < \alpha < n$. The Laplace transform of the R-L fractional differ-integration is given by Eqn. (4).

$$\int_0^{\infty} e^{-st} \, {}_0D_t^{\alpha} f(t) dt = s^{\alpha} F(s) - \sum_{k=0}^{n-1} s_0^k D_t^{\alpha-k-1} f(t)\Big|_{t=0}. \quad (4)$$

This paper deals with the analysis, design and applications of meta-heuristic algorithms in the design of fractional order filters and the implantation using fractance devices. A fractance device is an electrical component that has an impedance of $Z(s) = ks^{\lambda} \Rightarrow Z(j\omega) = k\omega^{\lambda}\angle(\lambda\pi/2\,)$. The fractance components response, such as the fractional order inductor (FOI) and the fractional order capacitor (FOC) can be generated by varying λ in the range of $0 < \lambda < 1$ and $-1 < \lambda < 0$, respectively. Since their phase responses are frequency independent, these elements are also known as constant phase elements (CPE). It is appropriate to mention that for the situation when λ=1 and λ=-1, respectively, the impedance of a conventional/classical inductor and the capacitor is realized.

The conventional continuous-time filter in analog signal processing is typically written as a series of rational polynomials with the Laplacian operator s^n; as a result, only concerning the power n can the stopband attenuation characteristics of these integer order filters be tuned progressively. For instance, lowpass first and second-order filters can only explicitly display a roll-off of $-20\,decibels\,(dB)/decade$ and $-40\,dB/decade$, respectively [7].

In case the rational functions are written in terms of the non-integer order Laplacian operator s^{α}, where $0 < \alpha < 1$, the limitation in the responses of the conventional filters can be removed. For instance, a (1+α) order LPF has a gradient of -20×(1+α) dB/ decade, meaning that depending on the value, the roll-off can change within slope values of (-20,-40)dB/ decade, based on the value of α. Such a generalized class of filters is called the non-integer/fractional order filter.

The integer-order Laplacian operator is merely a particular instance of the non-integer Laplacian operator when α=1, which should be emphasized [8].

2. Related Work

i- Basic concepts of FOF design:

Much research has been done using different approximations of filters into the fractional-order domain. In [9] and [10], the author suggested designing filters using fractional order capacitors; when their value increases, these systems have a better magnitude response in the passband area. Freeborn designed FLPF in [11], and a higher-order prototype equation for the filter was designed using $H_{1+\alpha}^{LP}(s)=\frac{\tau_1}{s^{\alpha+1}+\tau_2 s^{\alpha}+\tau_3}$. By modifying τ_1, τ_2 and τ3, the passband of the magnitude response can be shaped without altering the stopband region. By applying the CFE method there are equations relate These coefficients and the fractional order α. Therefore, through careful selection of τ_2 *and* τ_3, the passband region can be shaped to closely resemble the passband of a Butterworth response while maintaining the desired fractional step through the stopband.

The higher order transfer function equations, based on using the standard Butterworth polynomials, were given by $H_{n+\alpha}^{LP}(s)=\frac{H_{1+\alpha}^{LP}(s)}{B_{n-1}(s)}$, where $B_n(s)$ is a standard Butterworth polynomial of order $n \in$ N.

ii- Previous attempts on FOF design:

Many other authors suggested approximating fractional filter transfer functions using rational integer transfer functions to create FOLPFs [12, 13, 14]. Ali et al. reported a generalized method to meet FOLPF criteria in [15].

Acharya et al. proposed a design strategy and stability study using the complex w-plane in [16] to construct FOLPFs.

In [17], Freeborn used a least square error (LSE)-based optimization approach to identify the ideal coefficient values for three different transfer functions that approximate the ideal (1+α) order Butterworth filter. The prototype equation that is used is the stepped transfer function. An interpolation function between the order of the fractional filter and the coefficients of the fractional transfer function was introduced. Then the performance was measured through the least square error equation that is given by: $LSE=\sum_{I=1}^{N}[|H_{1+\alpha}(\omega_i)|-|H_{int}(\omega_i)|]^2$, where $|H_{1+\alpha}(\omega_i)|$ *is* the FO magnitude at frequency ω_i , $|H_{int}(\omega_i)|$ is the first-order HP Butterworth magnitude at frequency ω_i, and n is the total number of compared frequency points.

In [18], the author discussed a quick and easy method for obtaining a fractional order Chebyshev-like response based on the conventional integer order poles.

The core of the proposed methodology is the generation of fractional order transfer functions utilizing the poles of the well-known Chebyshev filter of integer order.

The main idea is to construct the transfer function's denominator as Eqn. (5a):

$$D(s)=\prod_{k=1}^{l}(s^k-p^k) \quad (5a)$$

$$H(s)=\frac{1}{D(s)} \quad (5b)$$

Where $l=2[\frac{n}{2}]$ and n is the integer order, p is the integer order poles, and γ is the fractional pole powers, Eqn (5b) gives the final form of the transfer function.

In Addition, the author in [19] introduces inverse Chebyshev filters in the fractional domain. Inverse

Chebyshev lowpass filters are generally realized using lowpass notch circuits of second order. Then this transfer function was extended to the fractional order domain.

The new transfer function had an order $(1+\alpha)$, and its unknown coefficients were given using a curve fitting technique to optimize the response error between the integer transfer function and the fractional one.
This result in small-size circuit implantation due to increasing the degree of freedom in the design by introducing the parameter α.

A straightforward approach to extend the Chebyshev filter into the fractional domain is done by converting the Chebyshev polynomial to the fractional domain Eqn. (6), this is achieved by converting the integer order N into fractional order σ, which results in $T_\sigma(x)$ polynomial [8].

$$T_N(x) = \begin{cases} \cos\left(N\cos^{-1}(x)\right) & |x| \leq 1 \\ \cosh\left(N\cosh^{-1}(x)\right) & |x| > 1 \end{cases} , \quad (6)$$

Inserting the generated polynomials in the magnitude response equation Eqn. (7), result in Chebyshev LPF response, where ε is the peak ripple in the pass band area.

$$H_{LP}(\omega) = \frac{1}{\sqrt{1+\epsilon^2 T_\sigma^2(\omega)}}, \quad (7)$$

Using the same technique to generate a high pass filter is done by replacing $\omega \to \frac{1}{\omega}$ in the low pass filter magnitude response equation $H_{HP}(\omega)$.

Figure 1 shows the Chebyshev LPF generated using the magnitude response equation based on polynomials Eqn. (6) in a different order. Whereas Fig. 2 shows the counterpart high pass filter. It is clear from Fig.1 that the step toward the fractional domain in filter design introduces more degree of freedom in the design parameters such that a more accurate gradient slope is realizable to be more precise with the applications.

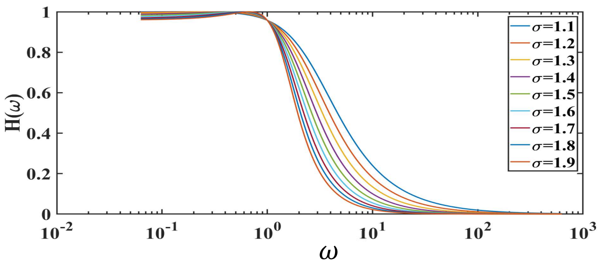

Figure 1: Fractional Chebyshev low pass filter

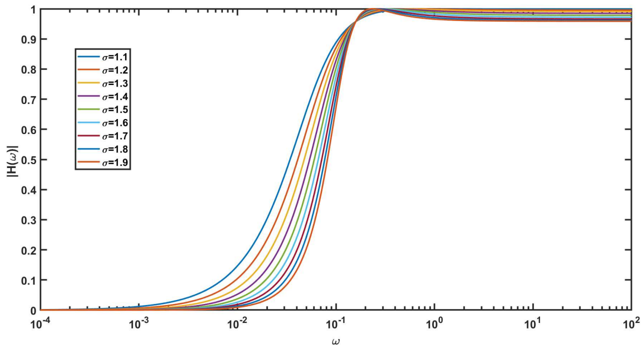

Figure 2: Fractional Chebyshev high pass filter

The fractional order bandpass filter is achieved by cascading two sections, one for fractional low pass filter and the other for fractional high pass filter; Eqn describes this. (8).

$$H_{BP}(\omega) = H_{\sigma_1}^{LP}(\omega) \times H_{\sigma_2}^{HP}(\omega) \ . \quad (8)$$

Figure 3 shows symmetric when $\sigma_1 = \sigma_2$. When $\sigma_1 \neq \sigma_2$ an asymmetric slope at band pass filter response is resulted at different orders for FLPF and FHPF.

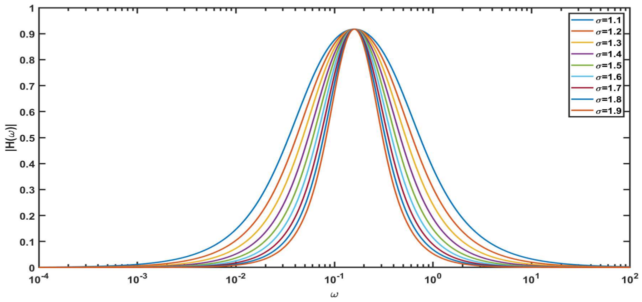

Figure3: Symmetric Fractional Chebyshev bandpass filter

It is worth noting that the techniques reported in [11],[15] and [18] employ a fractional order prototype equation to approximate the ideal FOLPF. Implementing such transfer functions can be done using fractance devices.

The fractional parameter can be viewed as a tuning key that adds flexibility to the system dynamics. Even though research is being done to physically implement the FOC [18] and [19], these fractance devices are not yet accessible on the market. As a result, using the traditional/classical (integer-order) circuit parts to create the circuit is a more straightforward design technique when using the integer order approximations to realize the FOLPFs.

The techniques described in [11],[12] approximate the FOLPF with an integer order approximation; however, the design approach uses a suboptimal method like Oustaloup and continuing fraction expansion (CFE) [20]. This should be acknowledged as well.

In [20], the fractional-step filters are designed using a synthesis approximation technique (fraction expansion (CFE)). He showed that using a maximum absolute magnitude error (MAME) analysis Eqn. (9), s.t. N represents the number of frequency point distributions, the shortcoming of CFE approximation in comparison with optimization techniques as approximation error is dependent on the fractional order, so it becomes large for higher order; this result in the design cannot be achieved at the whole design required band of frequency.

$$MAME = max_{i \in N}\{|H_{CFE}(\omega_i) - H_{exact}(\omega_i)|)\} \quad . (9)$$

The authors in [21] used curve fitting techniques to approximate FOLPF, such as the Sanathanan–Koerner (SK) least square iterative method, which approximates the integer order transfer function. However, they result in higher-order integer transfer function approximations and do not have optimal approximations.

3. Metaheuristic Algorithms:

Complex optimization problems, or those that can't be solved optimally using a deterministic approach in a reasonable amount of time, are well known for responding well to metaheuristic methods. The three main uses of metaheuristic approaches are to solve problems quickly, handle enormous issues, and create more reliable algorithms. These methods are flexible and simple to apply and design. In general, metaheuristic algorithms mimic natural events by combining rules and randomization [22].

The evolution strategy, genetic algorithms (GA), and artificial immune (AI) are a few biological systems that modify metaheuristic algorithms. Particle swarm optimization (PSO), bee colony optimization (BCO), bacterial foraging optimization algorithms (BFOA), and ant colony optimization are examples of ethnological phenomena (ACO). Swarm algorithms, micro-canonical annealing, and threshold-accepting techniques are physics phenomena [23].

Figure 4 summarizes the categorization of the meta-heuristic algorithms; it shows that swarm intelligence and evolutionary computation are two categories for metaheuristics based on population. The behaviour of social insect colonies or animal societies serves as the model for the broader phrase "swarm intelligence" another example of a metaheuristic algorithm inspired by musical phenomena is the harmony search (HS) algorithm. Single-solution-based and population-based metaheuristic algorithms can also be used to classify metaheuristic algorithms. The noising technique, TABU search, SA, and TA are a few examples of single-solution-based metaheuristics [24].

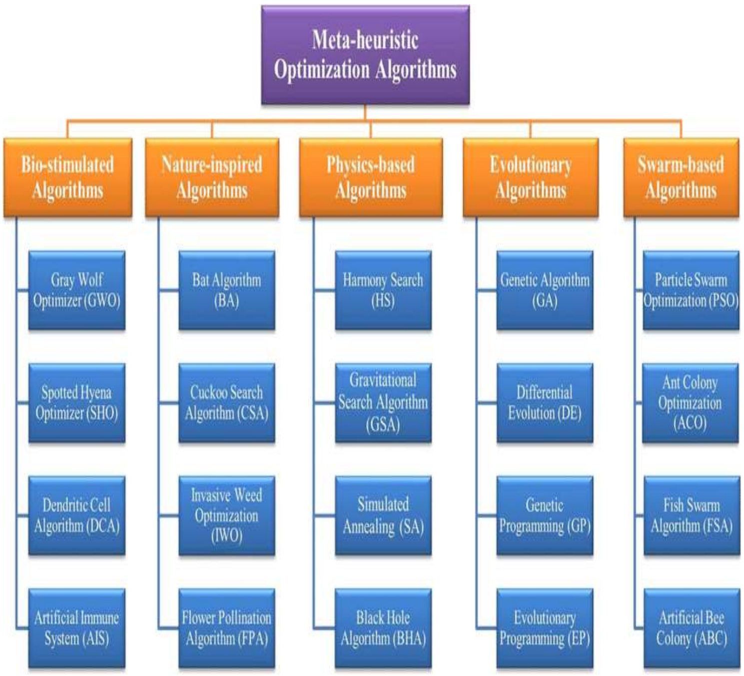

Figure 4: Metaheuristic algorithm mind map [22]

Most optimization algorithms have parameters involved that guide their direction towards the global best solution. These parameters have utmost importance since a deviation from a "reasonable" value can bring about divergence, so the convergence curve metric measures the value of the objective function versus the computation time during the minimization (model calibration) [25].

4. Filter approximation using metaheuristic algorithm:

The application of the metaheuristic algorithm on filter design is based on finding the optimal coefficient of the approximated transfer function.

The issue of approximating filter equations with integer order prototype equations is unquestionably an optimization problem, even though synthesis approximation approaches are non-optimal. It is possible to define the objective function for such an optimization issue to minimize the difference between the ideal response and the response obtained from the proposed integer order prototype equation approximation.

The suggested model's numerator and denominator polynomial coefficients serve as the optimization process's design variables. Be considerate that the denominator polynomial's existence causes the transfer function of the suggested filters to yield a very nonlinear cost function.

This optimization problem has multiple dimensions since all of the coefficients of the integer order filter are used as the decision variables. Additionally, the nature of this specific optimization problem is multimodal; therefore, a global optimization search technique can guarantee a more accurate approximation compared to the CFE-based approach presented in [20] for constructing FOLPFs.
The prototype function describes the model in the general form given by Eqn. (10) and is called a fully fractional transfer function. When $\beta = 1$, the function, in this case, is called the fractional stepped transfer function is given by Eqn. (11). It is worth mentioning that the general method to construct the denominator of Eqn. (11) for any higher order step fractional prototype equation is $D(s) = s^{\alpha} \sum_{k=0}^{n} a_k s^k + \sum_{k=1}^{n-1} a_k s^k + 1$.
In the case of equally order exponents, the prototype function is given by equation Eqn. (12). Figure 5 shows the convergence curves using different prototype equations. The algorithm reaches constant value during the optimization process for all orders at each filter type. That means the number of iterations is enough to find optimal solutions [26].

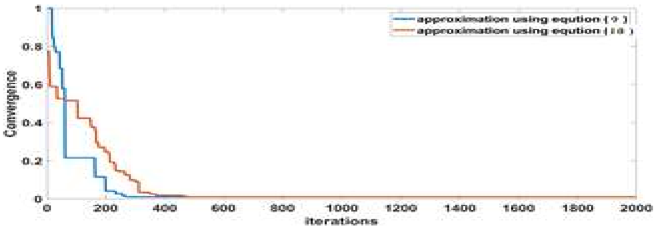

Figure 5: Convergence Curves Using a Different Approximation

Table (1) (at the end of the paper) summarizes the research works that focus on this area; the basic idea of all these works is to find the coefficients of the prototype equation that approximated the specific filter magnitude response.

$$H^{\alpha+\beta}(s)=\frac{F\times k_0}{s^{\alpha+\beta}+k_1s^{\alpha}+k_2s^{\beta}+k_0}, \quad (10)$$

$$H^{\alpha+1}(s)=\frac{F\times k_0}{s^{\alpha+1}+k_1s^{\alpha}+k_2s+k_0}, \quad (11)$$

$$H^{2\alpha}(s)=\frac{F\times k_0}{s^{2\alpha}+k_1s^{\alpha}+k_0}. \quad (12)$$

The method used in [27] approximates coefficients for various normalized FLPF transfer function situations. However, this approach concentrated on several parameters, such as transition bandwidth and maximum permitted peak, and was based on a limited search for objective functions.

In [28], the author shows that FOF can provide precise attenuation control, i.e. -3 dB frequency and stopband attenuation. Integer-order filters yield $-20n$ dB/decade stopband gradients, where n is the integer order. However, fractional order provides greater control with $-20(n+\alpha)$ dB/decade stopband attenuation, where α is any real positive value *less than 1*. The $(n+\alpha)$ FOF can provide a further degree of freedom that can provide more precise control over the attenuation slope as compared to integer-order filters of order n. In another case in [30], a fractional transfer function is approximated using an integer order transfer function; this technique is useful to reduce the circuit components used in filter implementation. For example, a 1.5th-order Butterworth filter can be realized using a single operational amplifier (op-amp), one FO capacitor, one conventional capacitor, and three resistors.

In contrast, four current feedback op-amps (CFOAs), three conventional capacitors, and eight resistors are needed to create a third-order approximation of the same filter with six decades of design bandwidth. A study case using a metaheuristic algorithm to approximate the magnitude response of the fractional Chebyshev filter was introduced in [29]. The implementation was achieved using voltage mode Sallen-Key topology. The author used two fractional Chebyshev polynomials to verify the reliability of the methods. The FOE was approximated using Valsa's approximation and verified using LT-spice circuit simulation. Eqn (10) gives the approximated transfer function used in [29]. Figures 6a and 6b compare the original magnitude response based on second and third-order fractional Chebyshev polynomial to the optimal prototype function equation Eqn. (10). The error curve is bounded, and the maximum error is *less than* 0.5% at the stopband area and *less than* 0.2% at the transition area.

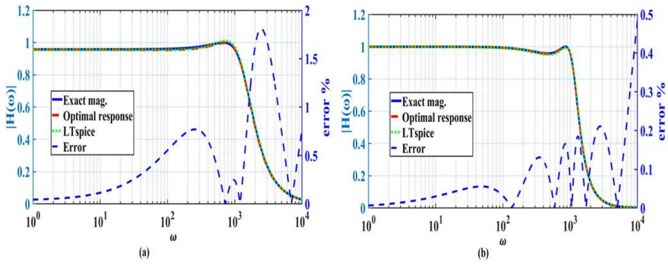

Figure 6: A comparison between MATLAB simulation and LTspice circuit simulation of the example filter implementation (a) Case second-order Chebyshev polynomial, (b) Case third-order Chebyshev polynomial

The approximation is achieved using the magnitude response equation Eqn. (6) fractional Chebyshev polynomial results from solving the Chebyshev differential equation in the fractional domain [17].

In [30], the authors target controlling some filter specifications, which are the transition bandwidth, the stop band frequency gain and the maximum allowable peak in the filter pass band. This was achieved by using a multi-objective optimization technique to design a FOLPF. The circuit realization was done through second-generation current conveyor (CCII) based fractional low-Pass filter. The design procedure proposed in this work showed the extra degree of freedom the FO introduced by fractional calculus to filter design.

It is worth mentioning that in [33], the author applied a new idea called series optimization, symbolized by: First meta-heuristic approach *to* Second meta-heuristic approach. The first population is generated from the first meta-heuristic approach and becomes an input for the second meta-heuristic approach to generate the optimal solution. This technique enabled him to benefit from the merits of two different meta-heuristic algorithms named cuckoo search algorithm (CSA) and interior search algorithm (ISA) to approximate integer order Butterworth low pass filter. Based on optimal coefficients of the prototype function $H_{LP}^{1+\alpha}(s) = \frac{a_0}{s^{1+\alpha}+a_1 s+a_2}$. The mechanism of series optimization $CSA\ to\ ISA$ is: turning the best solution from CSA to be the initial solution for ISA, this lead to more optimal and fast solutions.

In [34], the author used the genetic algorithm to perform a stochastic optimization search of the target space by artificially simulating the biological evolution process in nature [35] based on the prototype function that Eqn represents. (11). This transfer functs used for different values of n to approximate the magnitude response generated from Eqn. (7) based on polynomial represented by Eqn. (6).
The generated prototype function coefficients that approximate filter response is compared with other prototype functions generated from synthesis approximation methods or classical optimization methods based on the metrics described below:
Passband error (PE): The error observed in the passband (till 1 rad/s) when compared to the ideal magnitude response.

$$\text{PE} = 20 \times \log_{10}\left\{\sqrt{\frac{\sum_{i=1}^{K} ||H^{app.}(\omega_i)|-|\text{H}^{ideal}(\omega_i)||^2}{K}}\right\} \text{dB}\ , (13)$$

where $K = 5000$ and $0.01 \leq \omega_i \leq 1$.

Stopband error (SE): The error observed in the stopband (from 1 to 10 rad/s) when compared to the ideal magnitude response.

$$\text{SE} = 20 \times \log_{10}\left\{\sqrt{\frac{\sum_{i=1}^{K} ||H^{app.}(\omega_i)|-|\text{H}^{ideal}(\omega_i)||^2}{K}}\right\} \text{dB}\ , (14)$$

where $K = 5000$ and $1 \leq \omega_i \leq 10$.

Table2: Comparison metrics at order =1.7 using different approximations technique [29], [36].

Algorithm	SE	PE
Meta-heuristic	-95 dB	-80 dB
classical optimization	-36 dB	-26 dB

The parameter K was chosen based on the generated line space in the MATLAB simulation, as K increase the precision increase, In short, if we increase the value of K more than this limit, there will be no noticeable change [36].
Table 2 shows the difference between approximation approaches; meta-heuristic is achieved using flower pollination algorithm (FPA) algorithm, classical optimization is achieved using a curve fitting technique performs transfer function estimation using the Sanathanan–Koerner (SK) least square iterative method. The study used SE Eqn.(13) and PE Eqn.(14) error as performance metrics in the case of Chebyshev FLPF at order $\alpha = 1.7$, K= 5000 is the number of frequency points and they are logarithmically spaced in the region $\omega \in [1, 10^4] \times 2\pi$ rad/sec. The upper and lower bounds for the components of the coefficient's vector of the

transfer function Eq.(10) are 10^7 and 1, respectively, while the upper and lower bounds for the exponents are 1 and 0.7, respectively. This indicating how useful the use of metaheuristic approaches gives the best performance. The metaheuristic approach approximates the ideal magnitude response to a realizable fractional prototype function, which enables approximation all over the frequency band and decreases circuit complexity.

5. Conclusions and future work

Nature-inspired metaheuristic search techniques are useful for finding the optimal search variables (multidimensional optimization problems). The convergence speed of the meta-heuristic algorithms toward the global (or nearly global) optimal results is better than traditional techniques.

The robustness and the capability of this tool to solve multimodal systems have been verified throughout a study example that approximates the magnitude response to fully fractional prototype functions. The error at the stable output is bounded to be *less than* 2% within a few iterations, and the approximation is achieved over the whole frequency band.

The literature indicates that the filter magnitude response approximation-based meta-heuristic algorithm needs to be better covered. Future work could establish by using a similar order transfer function. This approach is useful in interpolating fractional transfer function coefficients as a function of fractional order α.

Additionally, using a metaheuristic approach, Legendre and elliptic filter types are not approximated from their magnitude response equation. Further studies should investigate the methods to generate fractional polynomial modelling for this filter type. These filter types are widely used due to their phase characteristic, which is nearly linear in the pass region. This gives a maximally-flat group delay, which becomes a good choice for pulse circuits because ringing and overshoot are minimized and have poor attenuation slopes.

Finally, the series optimization technique is useful and should be investigated in all metaheuristic algorithm types.

References:

[1] Keith Oldham, *The Fractional Calculus Theory and Applications of Differentiation and Integration to Arbitrary Order*. 1974.

[2] A. G. Radwan, A. S. Elwakil, and A. M. Soliman, “Fractional-order sinusoidal oscillators: Design procedure and practical examples,” *IEEE Trans. Circuits Syst. I Regul. Pap.*, vol. 55, no. 7, pp. 2051–2063, Aug. 2008, doi:10.1109/TCSI.2008.918196.

[3] H. Li, Y. Luo, and Y. Chen, “A Fractional Order Proportional and Derivative (FOPD) Motion Controller: Tuning Rule and Experiments,” *IEEE Trans. Control Syst. Technol.*, vol. 18, no. 2, pp. 516–520, Mar. 2010, doi:10.1109/TCST.2009.2019120.

[4] A. Elwakil, “Fractional-Order Circuits and Systems: An Emerging Interdisciplinary Research Area,” *IEEE Circuits Syst. Mag.*, vol. 10, no. 4, pp. 40– 50, 2010, doi: 10.1109/MCAS.2010.938637.

[5] S. Das and I. Pan, *Fractional Order Signal Processing*. Berlin, Heidelberg: Springer Berlin Heidelberg, 2012

[6] K. Diethelm and N. J. Ford, “Analysis of Fractional Differential Equations,” *J. Math. Anal. Appl.*, vol. 265, no. 2, pp. 229–248, Jan. 2002, doi:10.1006/jmaa.2000.7194.

[7] Y. Gao, X. Shang, L. Li, C. Guo, and Y. Wang, “Integrated Filter–Amplifiers: A Comprehensive Review,”

IEEE Microw. Mag., vol. 23, no. 6, pp. 57–75, Jun. 2022, doi: 10.1109/MMM.2022.3155034.

[8] Diniz, P. S., Simpson, M. D., Stefano, A., & Gismondi, R. C. (2003). Digital Signal Processing-Applications in Medicine. *ELECTRICAL ENGINEERING*, *1*.

[9] Radwan, A. G., Soliman, A. M., & Elwakil, A. S. (2008). First-order filters generalized to the fractional domain. *Journal of Circuits, Systems, and Computers*, *17*(01).

[10] Elwakil, A. S. (2010). Fractional-order circuits and systems: An emerging interdisciplinary research area. *IEEE Circuits and Systems Magazine*, *10*(4), 40-50.

[11] T. J. Freeborn, B. Maundy, and A. Elwakil, "Towards the realization of fractional step filters," in *Proceedings of 2010 IEEE International Symposium on Circuits and Systems*, May 2010, pp. 1037–1040, doi: 10.1109/ISCAS.2010.5537360.

[12] T. J. Freeborn, B. Maundy, and A. S. Elwakil, "Field programmable analogue array implementation of fractional step filters," *IET Circuits, Devices Syst.*, vol. 4, no. 6, p. 514, 2010, doi: 10.1049/iet-cds.2010.0141.

[13] B. Maundy, A. S. Elwakil, and T. J. Freeborn, "On the practical realization of higher-order filters with fractional stepping," *Signal Processing*, vol. 91, no. 3, pp. 484–491, Mar. 2011, doi: 10.1016/j.sigpro.2010.06.018.

[14] C. Psychalinos, G. Tsirimokou, and A. S. Elwakil, "Switched-Capacitor FractionalStep Butterworth Filter Design," *Circuits, Syst. Signal Process.*, vol. 35, no. 4, pp. 1377–1393, Apr. 2016, doi: 10.1007/s00034-015-0110-9.

[15] A. S. Ali, A. G. Radwan, and A. M. Soliman, "Fractional Order Butterworth Filter: Active and Passive Realizations," *IEEE J. Emerg. Sel. Top. Circuits Syst.*, vol. 3, no. 3, pp. 346–354, Sep. 2013, doi: 10.1109/JETCAS.2013.2266753.

[16] A. Acharya, S. Das, I. Pan, and S. Das, "Extending the concept of analog Butterworth filter for fractional order systems," *Signal Processing*, vol. 94, pp. 409–420, Jan. 2014, doi: 10.1016/j.sigpro.2013.07.012.

[17] D. Kubanek, T. Freeborn, J. Koton, and N. Herencsar, "Evaluation of (1 + α)Fractional-Order Approximated Butterworth High-Pass and Band-Pass Filter Transfer Functions," *Elektron. ir Elektrotechnika*, vol. 24, no. 2, Apr. 2018, doi:10.5755/j01.eie.24.2.20634.

[18] A. M. AbdelAty, A. Soltan, W. A. Ahmed, and A. G. Radwan, "Fractional order Chebyshev-like lowpass filters based on integer order poles," *Microelectronics J.*, vol. 90, pp. 72–81, Aug. 2019, doi: 10.1016/j.mejo.2019.05.016.

[19] T. J. Freeborn, A. S. Elwakil, and B. Maundy, "Approximated Fractional-Order Inverse Chebyshev Lowpass Filters," *Circuits, Syst. Signal Process.*, vol. 35, no. 6, pp. 1973–1982, Jun. 2016, doi: 10.1007/s00034-015-0222-2.

[20] S. Mahata, R. Kar, and D. Mandal, "Optimal Modelling of (1 + α) Order Butterworth Filter under the CFE Framework," *Fractal Fract.*, vol. 4, no. 4, p. 55, Dec. 2020, doi: 10.3390/fractalfract4040055.

[21] S. Kapoulea, C. Psychalinos, and A. S. Elwakil, "Power law filters: A new class of fractional-order filters without a fractional-order Laplacian operator," *AEU - Int. J. Electron. Commun.*, vol. 129, p. 153537, Feb. 2021, doi:10.1016/j.aeue.2020.153537.

[22] K. S. Lee and Z. W. Geem, "A new meta-heuristic algorithm for continuous engineering optimization: harmony

search theory and practice,"*Comput. Methods Appl. Mech. Eng.*, vol. 194, no. 36–38, pp. 3902–3933, Sep. 2005, doi:10.1016/j.cma.2004.09.007.

[23] I. Boussaïd, J. Lepagnot, and P. Siarry, "A survey on optimization metaheuristics," *Inf. Sci. (Ny).*, vol. 237, pp. 82–117, Jul. 2013, doi: 10.1016/j.ins.2013.02.041.

[24] D. Yousri, A. M. AbdelAty, A. G. Radwan, A. S. Elwakil, and C. Psychalinos, "Comprehensive comparison based on meta-heuristic algorithms for approximation of the fractional-order Laplacian s as a weighted sum of first-order high-pass filters," *Microelectronics J.*, vol. 87, pp. 110–120, May 2019, doi: 10.1016/j.mejo.2019.03.012.

[25] M. E. Fouda, A. M. AbdelAty, A. S. Elwakil, A. G. Radwan, and A. M. Eltawil, "Programmable constant phase element realization with crossbar arrays," *J. Adv. Res.*, vol. 29, pp. 137–145, Mar. 2021, doi: 10.1016/j.jare.2020.08.007.

[26] A. Kartci *et al.*, "Synthesis and Optimization of Fractional-Order Elements Using a Genetic Algorithm," *IEEE Access*, vol. 7, pp. 80233 80246,2019,doi:10.1109/ACCESS.2019.2923166.

[27] A. Soni and M. Gupta, "Designing of Fractional Order Bessel Filter using Optimization Techniques," *Int. J. Electron. Lett.*, vol. 10, no. 1, pp. 71–86, Jan. 2022, doi: 10.1080/21681724.2020.1870715.

[28] A. Soni and M. Gupta, "Analysis and Design of Optimized Fractional Order LowPass Bessel Filter," *J. Circuits, Syst. Comput.*, vol. 30, no. 02, p. 2150035, Feb. 2021, doi: 10.1142/S0218126621500353.

[29] A. Amgad, A. M. AbdelAty, M. T. M. M. Elbarawy, H. A. Attia, and A. G. Radwan, "Design of a Lowpass Filter from Fractional Chebyshev Polynomials," in *2021 International Conference on Microelectronics (ICM)*, Dec. 2021, pp. 144–147, doi: 10.1109/ICM52667.2021.9664937.

[30] S. Mahata, S. K. Saha, R. Kar, and D. Mandal, "Optimal design of fractional order low pass Butterworth filter with accurate magnitude response," *Digit. Signal Process.*, vol. 72, pp. 96–114, Jan. 2018, doi: 10.1016/j.dsp.2017.10.001.

[31] S. Mahata, R. Kar, and D. Mandal, "Optimal approximation of fractional-order systems with model validation using CFOA," *IET Signal Process.*, vol. 13, no. 9, pp. 787–797, Dec. 2019, doi: 10.1049/iet-spr.2019.0158.

[32] S. Shrivastava, D. K. Upadhyay, and O. P. Goswami, "Optimal Design of Fractional-order Low-pass Filter Using L 2 -method," in *2021 International Conference on Communication, Control and Information Sciences (ICCISc)*, Jun. 2021, pp. 1–5, doi: 10.1109/ICCISc52257.2021.9484936.

[33] A. Soni, N. Sreejeth, V. Saxena, and M. Gupta, "Series Optimized Fractional Order Low Pass Butterworth Filter," *Arab. J. Sci. Eng.*, vol. 45, no. 3, pp. 1733–1747, Mar. 2020, doi: 10.1007/s13369-019-04225-7.

[34] He, Xue, and Zhizhong Hu. "Optimization design of fractional-order Chebyshev lowpass filters based on genetic algorithm." *International Journal of Circuit Theory and Applications* (2022).

[35] Jha, M., Long, C., Mastorakis, N., & Bulucea, C. A. (Eds.). (2009). Proceedings of the 11th WSEAS international conference on Mathematical and computational methods in science and engineering.

[36] Freeborn, T.J.: Comparison of (1 + α) fractional-order transfer functions to approximate low pass Butterworth magnituderesponses. Circuits Syst. Signal Process. *35*(6), 1983–2002 (2016). https://doi.org/10.1007/s00034-015-0226-y

Table1 : Summary of filter design-based optimization techniques

Ref.	Prototype function	Frequency response data	Optimizer used	Absolute error	Order of filter
[27]	$H^{\alpha+\beta}(s) = \frac{k_0}{s^{\alpha+\beta} + k_1 s^{\alpha} + k_2 s^{\beta} + k_0}$	Magnitude response equation of FLPF based on fractional Chebyshev polynomial.	FPA	<2%	This work targets the orders between (1,3)
[28]	$H^{\alpha+1}(s) = \frac{k_0}{s^{\alpha+1} + k_1 s^{\alpha} + k_2 s + k_0}$	Magnitude response of first-order Butterworth filter.	modified particle swarm optimization (mPSO)	<3%	This work targets orders between (1,2)
[12][29]	$H^{\alpha+1}(s) = \frac{k_0}{s^{\alpha+1} + k_1 s^{\alpha} + k_2 s + k_0}$	Magnitude response of first-order Butterworth filter. Implement FLPF.	least squares error (LSE)	<2%	This work targets orders between (1,2)
[30]	$H^{\alpha+1}(s) = \frac{k_0}{s^{2\alpha} + k_1 s^{\alpha} + k_0}$	Flat response of LP Butterworth filter.	A multi-objective optimization technique	This work targets specific orders and the error is specified by *the* ϵ value.	
[31]	Integer order (ITF)	Magnitude response of fractional order Butterworth filter.	Gravitational Search Algorithm (GSA)	<3%	This work targets orders between (1,2)
[32]	Integer order (ITF)	Magnitude response of a fractional order system.	colliding bodies optimization (CBO)	<2%	This work targets FOS of orders α, such that $0 < \alpha < 1$
[33]	$H^{\alpha+1}(s) = \frac{k_0}{s^{\alpha+1} + k_1 s^{\alpha} + k_2 s + k_0}$	Magnitude response of fractional Order Low Pass Butterworth Filter.	Series optimization using $CSA \rightarrow ISA$.	Small error	This work targets orders between (1,2)
[34]	$H^{\alpha+1}(s) = \frac{k_0}{s^{\alpha+1} + k_1 s^{\alpha} + k_2 s + k_0}$	Magnitude response of fractional Order Chebyshev lowpass filters	Genetic algorithm	< 2%	This work targets orders between (1,4)