



Cost-Benefit Analysis of a Two-Unit Cold Standby System with Imperfect Repair Man and Abnormal Weather Conditions

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ABSTRACT

This study examines the dependability of a cold standby system made up of two similar units. One of the two units is always in use, and the other unit is on cold standby. The repairman could be present or not at the job site. The system operates in both regular and abnormal weather situations. The current systems that are in place have been impacted by recent global climatic shifts. In this study, we examine how these climate fluctuations around the world affect the two-unit standby system. Furthermore, the impact of the repairman's absence is also investigated due to the lack of a skilled crew. When the weather is normal, the device works; when the weather is abnormal, the system shuts down and the device stops working. The mean time of system failure, steady-state availability, busy periods with maintenance, and cost-benefit analysis were evaluated, among other significant dependability metrics. All of the previously mentioned analyses were done by using regenerative point technique.

Key Words:

Cold Standby; Busy period; sMTSF; Cost benefit estimated.

1. INTRODUCTION

Redundancy is known to be used in enhancing the performance and reliability of repairable systems. Therefore, stochastic models of cold standby repairable systems with identical units have repeatedly been studied by researchers. [1, 2, 3, 4] used the idea of two weather conditions (normal and abnormal) in a single-unit system. [5] discussed reliability and economic analysis of a system operating under different weather conditions. [6] discussed the reliability analysis of two-dissimilar-unit warm standby system under different weather conditions. [7] studied the reliability analysis of a cold standby device with failure of repair equipment and repairman's appearance and disappearance with correlated life time. [8] discussed the repair two phases of two-unit cold standby system. [9] dealt with a two-unit cold standby system considering hardware and human error failure with preventive maintenance (PM) and arbitrary distribution. In [10] They assumed that shocks can attack the operating unit to a cold standby system consisting of two units, the arrival times of the shocks follow a homogeneous Poisson process,

and the repair time follows a general distribution.[11]discussed a reliability analysis model for a PMS with a cold-standby system based on the GO-FLOW methodology and UGF. [12]discussed the impact of abnormal weather conditions on various reliability measures of a repairable system with inspection. [13]analyzed stochastically a cold standby system with conditional failure of server.

[14]studied the cost-benefit efficient in a two-dissimilar unit with warm units standby case subject to arbitrary repair and replacement.[15]discussed the standby redundancy system with Priority under Limited Information.

This paper aims to study the effect of the presence or absence of the repair man on cold standby system affected by the weather conditions.

2. Assumptions

- The system consists of two similar units, one unit is initially operating and the other unit is in standby state (cold standby).
- If the weather is in normal case, the unit operates, and if the weather is in abnormal case, the system stops, and the operate unit fails.
- The repair man may be present or absent.
- The connected switch is perfect.
- All times are independent and exponentially distributed.

3. Notation

E :	Set of regenerative states.
$q_{ij}(t), Q_{ij}(t)$	PDFs and CDF of time for the system transits from regenerative state V_i to V_j .
P_{ij}	Transition probability from V_i to V_j .
λ	The parameter of failure rate.
μ	The parameter of repair rate.
β, α	The parameter of abnormal weather rate/The parameter of normal weather rate.
U	The parameter of waiting repair rate.
MTSF	Mean time to system failure.
$\theta, (1 - \theta)$	Probability that the repairman is present/ probability that the repairman is absent.
η_{ij}	The mean sojourn times in state V_i , when system transits direct to V_j .
$M_i(t)$	Probability that the system stay in V_i .
M_i	Laplace transform of $M_i(t)$.
$\Omega_i(t)$	Cdf of time to system failure starting from state V_i .
$R(t)$	Cdf of repair time.
$AV_i(t)$	$p \{ \text{The system is up at time } t \text{ starting at state } V_i \}$.
$C(t)$	The net revenue of the system in $(0, t]$
$G(t)$	Probability that the unit is in repair.

⊗	Convolution.
*	Laplace transforms.

3.1 Symbols for the states of the system

- s : Unit in standby mode.
- O : Unit is operate insnormal mode.
- O_d : Unit is operate in abnormal mode.
- w_g : Normal weather.
- w_d : Abnormal weather.
- r : Unit in repair.
- wr : Waiting repair.

The system can be in anysone of the following states.

$$\begin{aligned}
 V_0 &= (O, s, w_g) & V_1 &= (O, r, w_g) & V_2 &= (O, wr, w_g), \\
 V_3 &= (O_d, r, w_d) & V_4 &= (O_d, wr, w_d) & V_5 &= (wr, r, w_g), \\
 V_6 &= (wr, r, w_d) & V_7 &= (wr, wr, w_g) & V_8 &= (wr, wr, w_d), \\
 V_9 &= (O_d, s, w_d).
 \end{aligned}$$

Up states: V_0, V_1 and V_2 , **Down states:** $V_3, V_4, V_5, V_6, V_7, V_8$ and V_9 .

All states are regenerative state.

3.2 Transition probabilities and mean sojourn time

It can be observed that the epoch of entry into any of the states $T_i \in E$ are regenerative point. Let $T_0(\equiv 0), T_1, T_2, \dots$ denote the epochs at which the system enters any state $T_i \in E$ let X_n denote the state visited at epoch T_n+ , *i.e.* just after transition at T_n . $\{X_n, T_n\}$ is a Markov renewel process with state space E and $Q_{ij}(t) = P[X_{n+1} = j, T_{n+1} - T_n \leq t | X_n = i]$, is the semi Markov kernel over E.

The transition probability matrix of embedded Markov-chain is $P = P_{ij} = Q_{ij}(\infty) = Q(\infty)$, with non-zero elements.

By probabilistic arguments, the non-zero elements P_{ij} are, $P_{ij} = Q_{ij}(\infty) = \int q_{ij}(t)dt$ as

$$P_{01} = \frac{\theta\lambda}{\beta + \lambda}, \quad P_{02} = \frac{(1 - \theta)\lambda}{\beta + \lambda}, \quad P_{03} = \frac{\theta\beta}{\beta + \lambda}, \quad P_{04} = \frac{(1 - \theta)\beta}{\beta + \lambda},$$

$$P_{01} + P_{02} + P_{03} + P_{04} = 1,$$

$$P_{10} = \frac{\mu}{\beta + \lambda + \mu}, \quad P_{15} = \frac{\lambda}{\beta + \lambda + \mu}, \quad P_{16} = \frac{\beta}{\beta + \lambda + \mu},$$

$$P_{10} + P_{15} + P_{16} = 1,$$

$$P_{21} = \frac{u}{\beta + \lambda + u}, \quad P_{27} = \frac{\lambda}{\beta + \lambda + u}, \quad P_{28} = \frac{\beta}{\beta + \lambda + u},$$

$$\begin{aligned}
 P_{21} + P_{27} + P_{28} &= 1, \\
 P_{31} &= \frac{\alpha}{\alpha + \mu}, \quad P_{39} = \frac{\mu}{\alpha + \mu}, \\
 P_{31} + P_{39} &= 1, \\
 P_{42} &= \frac{\alpha}{\alpha + u}, \quad P_{43} = \frac{u}{\alpha + u}, \\
 P_{42} + P_{43} &= 1, \\
 P_{51} &= \frac{\mu}{\beta + \mu}, \quad P_{56} = \frac{\beta}{\beta + \mu}, \\
 P_{51} + P_{56} &= 1, \\
 P_{65} &= \frac{\alpha}{\alpha + \mu}, \quad P_{63} = \frac{\mu}{\alpha + \mu}, \\
 P_{65} + P_{63} &= 1, \\
 P_{78} &= \frac{\beta}{\beta + u}, \quad P_{75} = \frac{u}{\beta + u}, \\
 P_{78} + P_{75} &= 1, \\
 P_{87} &= \frac{\alpha}{\alpha + u}, \quad P_{86} = \frac{u}{\alpha + u}, \\
 P_{87} + P_{86} &= 1, \\
 P_{90} &= 1.
 \end{aligned}$$

3.3 Mean sojourn times

The unconditional mean time taken by the system to transit for any regenerative state j , when it is counted from the epoch of entrance in to the state i , is mathematically stated as:

$$\eta_{ij} = \int_0^{\infty} t dQ_{ij}(t).$$

$$\begin{aligned}
 \eta_{01} &= \frac{\theta\lambda}{(\beta + \lambda)^2}, & \eta_{02} &= \frac{(1 - \theta)\lambda}{(\beta + \lambda)^2}, \\
 \eta_{03} &= \frac{\theta\beta}{(\beta + \lambda)^2}, & \eta_{04} &= \frac{(1 - \theta)\beta}{(\beta + \lambda)^2}, \\
 \eta_{10} &= \frac{\mu}{(\beta + \lambda + \mu)^2}, & \eta_{15} &= \frac{\lambda}{(\beta + \lambda + \mu)^2}, \\
 \eta_{16} &= \frac{\beta}{(\beta + \lambda + \mu)^2}, & \eta_{21} &= \frac{u}{(\beta + \lambda + u)^2}, \\
 \eta_{27} &= \frac{\lambda}{(\beta + \lambda + u)^2}, & \eta_{28} &= \frac{\beta}{(\beta + \lambda + u)^2}, \\
 \eta_{31} &= \frac{\alpha}{(\alpha + \mu)^2}, & \eta_{39} &= \frac{\mu}{(\alpha + \mu)^2},
 \end{aligned}$$

$$\begin{aligned} \eta_{42} &= \frac{\alpha}{(\alpha + u)^2}, & \eta_{43} &= \frac{u}{(\alpha + u)^2} \\ \eta_{51} &= \frac{\mu}{(\beta + \mu)^2}, & \eta_{56} &= \frac{\beta}{(\beta + \mu)^2}, \\ \eta_{65} &= \frac{\alpha}{(\alpha + \mu)^2}, & \eta_{63} &= \frac{\mu}{(\alpha + \mu)^2}, \\ \eta_{78} &= \frac{\beta}{(\beta + u)^2}, & \eta_{75} &= \frac{u}{(\beta + u)^2}, \\ \eta_{87} &= \frac{\alpha}{(\alpha + u)^2}, & \eta_{86} &= \frac{u}{(\alpha + u)^2}. \end{aligned}$$

Meanssojourn times in state V_i which is given by $M_i = \sum_j \eta_{ij}$.

$$\begin{aligned} M_0 &= \frac{1}{\beta + \lambda}, & M_1 &= \frac{1}{\beta + \lambda + \mu}, \\ M_2 &= \frac{1}{\beta + \lambda + u}, & M_3 &= \frac{1}{\alpha + \mu}, \\ M_4 &= \frac{1}{\alpha + u}, & M_5 &= \frac{1}{\beta + \mu}, \\ M_6 &= \frac{1}{\alpha + \mu}, & M_7 &= \frac{1}{\beta + u}, \\ M_8 &= \frac{1}{\alpha + u}, & M_9 &= \frac{1}{\alpha}. \end{aligned}$$

3.4 Mean time to system failure MTSF

Making use of arguments of the theory of regenerative processes, we obtain the following relation for $\bar{\Omega}_0(t)$

$$\bar{\Omega}_0(t) = e^{-(\beta+\lambda)} + q_{01}(t)\bar{\Omega}_1(t) + q_{02}(t)\bar{\Omega}_2(t), \tag{1}$$

$$\bar{\Omega}_1(t) = e^{-(\beta+\lambda+\mu)} + q_{10}(t)\bar{\Omega}_0(t), \tag{2}$$

$$\bar{\Omega}_2(t) = e^{-(\beta+\lambda+u)} + q_{21}(t)\bar{\Omega}_1(t). \tag{3}$$

Taking of Laplace transform (LT) for equations 1, 2 and 3 and solving for $\bar{\Omega}_0^*(s)$ considering $S = 0$, We have the mean time to systems failure MTSF as follows

$$MTSF = \frac{N_0}{D_0}, \tag{4}$$

where

$$D_0 = 1 - P_{10}(P_{01} + P_{02}P_{21}),$$

and

$$N_0 = M_0 + M_1(P_{01} + P_{02}P_{21}) + M_2(P_{02}).$$

4. Availability analysis

From the arguments used in the theory of regenerative processes, the pointwise availabilities $AV_i(t)$ where $i = 0,1,2,6,7$. we obtain the following recursive relations.

$$\begin{aligned}
 AV_0(t) = & M_0(t) + (q_{03}(t) \otimes q_{39}(t) \otimes q_{90}(t) + q_{04}(t) \otimes q_{43}(t) \otimes q_{39}(t) \otimes q_{90}(t)) \otimes AV_0(t) \\
 & + (q_{01}(t) + q_{03}(t) \otimes q_{31}(t) + q_{04}(t) \otimes q_{43}(t) \otimes q_{31}(t)) \otimes AV_1(t) \\
 & + (q_{02}(t) + q_{04}(t) \otimes q_{42}(t)) \otimes AV_2(t),
 \end{aligned} \tag{5}$$

$$\begin{aligned}
 AV_1 = & M_1(t) + q_{10}(t) \otimes AV_0(t) + q_{15}(t) \otimes q_{51}(t) \otimes AV_1(t) \\
 & + (q_{15}(t) \otimes q_{56}(t) + q_{16}(t)) \otimes AV_6(t),
 \end{aligned} \tag{6}$$

$$\begin{aligned}
 AV_2 = & M_2(t) + q_{21}(t) \otimes AV_1(t) + q_{28}(t) \otimes q_{86}(t) \otimes AV_6(t) \\
 & + (q_{27}(t) + q_{28}(t) \otimes q_{87}(t)) \otimes AV_7(t),
 \end{aligned} \tag{7}$$

$$\begin{aligned}
 AV_6(t) = & q_{63}(t) \otimes q_{39}(t) \otimes q_{90}(t) \otimes AV_0(t) + (q_{65}(t) \otimes q_{51}(t) \\
 & + q_{63}(t) \otimes q_{31}(t)) \otimes AV_1(t) + (q_{65}(t) \otimes q_{56}(t)) \otimes AV_6(t),
 \end{aligned} \tag{8}$$

$$\begin{aligned}
 AV_7(t) = & q_{75}(t) \otimes q_{51}(t) \otimes AV_1(t) + (q_{78}(t) \otimes q_{86}(t) \\
 & + q_{75}(t) \otimes q_{56}(t)) \otimes AV_6(t) + (q_{78}(t) \otimes q_{87}(t)) \otimes AV_7(t).
 \end{aligned} \tag{9}$$

where

$$M_0(t) = e^{-(\beta+\lambda)t}, M_1(t) = e^{-(\beta+\lambda+\mu)t}, M_2(t) = e^{-(\beta+\lambda+u)t}$$

Taking LT for equation 5, 6, 7, 8 and 9, and solve for $AV_0^*(s)$, then we get the steady state availability of the system AV_0 in the form,

$$AV_0 = \lim_{t \rightarrow \infty} AV_0(t) = \lim_{s \rightarrow 0} s AV_0^*(s) = \frac{N_1}{D_1} \tag{10}$$

$$L_1 = P_{03}P_{39}P_{90} + P_{04}P_{43}P_{39}P_{90},$$

$$L_2 = P_{01} + P_{03}P_{31} + P_{04}P_{43}P_{31},$$

$$L_3 = P_{02} + P_{04}P_{42},$$

$$L_4 = P_{15}P_{56} + P_{16},$$

$$L_5 = P_{27} + P_{28}P_{87},$$

$$L_6 = P_{63}P_{39}P_{90},$$

$$L_7 = P_{65}P_{51} + P_{63}P_{31},$$

$$L_8 = P_{78}P_{86} + P_{75}P_{56},$$

$$L'_1 = \eta_{03}P_{39}P_{90} + \eta_{39}P_{03}P_{90} + \eta_{90}P_{03}P_{39} + \eta_{04}P_{43}P_{39}P_{90},$$

$$+ \eta_{43}P_{04}P_{39}P_{90} + \eta_{39}P_{04}P_{43}P_{90} + \eta_{90}P_{04}P_{43}P_{39},$$

$$L'_2 = \eta_{01} + \eta_{03}P_{31} + \eta_{31}P_{03} + \eta_{04}P_{43}P_{31} + \eta_{43}P_{04}P_{31} + \eta_{31}P_{04}P_{43},$$

$$L'_3 = \eta_{02} + \eta_{04}P_{42} + \eta_{42}P_{04},$$

$$L'_4 = \eta_{15}P_{56} + \eta_{56}P_{15} + \eta_{16},$$

$$L'_5 = \eta_{27} + \eta_{28}P_{87} + \eta_{87}P_{28},$$

$$L'_6 = \eta_{63}P_{39}P_{90} + \eta_{39}P_{63}P_{90} + \eta_{90}P_{63}P_{39},$$

$$L'_7 = \eta_{65}P_{51} + \eta_{51}P_{65} + \eta_{63}P_{31} + \eta_{31}P_{63},$$

$$L'_8 = \eta_{78}P_{86} + \eta_{86}P_{78} + \eta_{75}P_{56} + \eta_{56}P_{75}.$$

$$D_1 = a_1b_1 - a_2b_2 + a_3b_3 - a_4b_4 + a_5b_5,$$

$$N_1 = M_0b_1 - M_1b_2 + M_2b_3.$$

$$a_1 = (L'_1 + L'_2 + L'_3),$$

$$a_2 = (\eta_{10} + \eta_{15}P_{51} + \eta_{51}P_{15} + L'_4),$$

$$a_3 = (\eta_{21} + \eta_{28}P_{86} + \eta_{86}P_{28} + L'_5),$$

$$a_4 = (L'_6 + L'_7 + \eta_{65}P_{56} + \eta_{56}P_{65}),$$

$$a_5 = (\eta_{75}P_{51} + \eta_{51}P_{75} - L'_8 + \eta_{78}P_{87} + \eta_{87}P_{78}).$$

$$b_1 = \begin{vmatrix} 1 - P_{15}P_{51} & 0 & -L_4 & 0 \\ -P_{21} & 1 & -(P_{28}P_{86}) & -L_5 \\ -L_7 & 0 & 1 - (P_{65}P_{56}) & 0 \\ -(P_{75}P_{51}) & 0 & -L_8 & 1 - (P_{78}P_{87}) \end{vmatrix},$$

$$b_2 = \begin{vmatrix} -L_2 & -L_3 & 0 & 0 \\ -P_{21} & 1 & -(P_{28}P_{86}) & -L_5 \\ -L_7 & 0 & 1 - (P_{65}P_{56}) & 0 \\ -(P_{75}P_{51}) & 0 & -L_8 & 1 - (P_{78}P_{87}) \end{vmatrix},$$

$$b_3 = \begin{vmatrix} -L_2 & -L_3 & 0 & 0 \\ 1 - P_{15}P_{51} & 0 & -L_4 & 0 \\ -L_7 & 0 & 1 - (P_{65}P_{56}) & 0 \\ -(P_{75}P_{51}) & 0 & -L_8 & 1 - (P_{78}P_{87}) \end{vmatrix},$$

$$b_4 = \begin{vmatrix} -L_2 & -L_3 & 0 & 0 \\ 1 - P_{15}P_{51} & 0 & -L_4 & 0 \\ -P_{21} & 1 & -(P_{28}P_{86}) & -L_5 \\ -(P_{75}P_{51}) & 0 & -L_8 & 1 - (P_{78}P_{87}) \end{vmatrix},$$

$$b_5 = \begin{vmatrix} -L_2 & -L_3 & 0 & 0 \\ 1 - P_{15}P_{51} & 0 & -L_4 & 0 \\ -P_{21} & 1 & -(P_{28}P_{86}) & -L_5 \\ -L_7 & 0 & 1 - (P_{65}P_{56}) & 0 \end{vmatrix}.$$

5. Busy period analysis

The expected busy periods of server man for repair during $(0, t]$ by probabilistic arguments, we obtain

$$\begin{aligned} G_0(t) &= (q_{03}(t) + q_{04}(t) \otimes q_{43}(t)) \otimes \bar{R}(t) + (q_{03}(t) \otimes q_{39}(t) \otimes q_{90}(t) \\ &+ q_{04}(t) \otimes q_{43}(t) \otimes q_{39}(t) \otimes q_{90}(t)) \otimes G_0(t) + (q_{01}(t) + q_{03}(t) \otimes q_{31}(t) \\ &+ q_{04}(t) \otimes q_{43}(t) \otimes q_{31}(t))G_1(t) + (q_{02}(t) + q_{04}(t) \otimes q_{42}(t)) \otimes G_2(t), \end{aligned} \tag{11}$$

$$\begin{aligned} G_1(t) &= (1 + q_{15}(t)) \otimes \bar{R}(t) + q_{10}(t) \otimes G_0(t) \\ &+ (q_{15}(t) \otimes q_{51}(t)) \otimes G_1(t) + (q_{15}(t) \otimes q_{56}(t) + q_{16}(t)) \otimes G_6(t), \end{aligned} \tag{12}$$

$$G_2(t) = q_{21}(t) \otimes G_1(t) + (q_{28}(t) \otimes q_{86}(t)) \otimes G_6(t) + (q_{27}(t) + q_{28}(t) \otimes q_{87}(t)) \otimes G_7(t), \tag{13}$$

$$\begin{aligned} G_6(t) &= (1 + q_{65}(t) + q_{63}(t)) \otimes \bar{R}(t) + (q_{63}(t) \otimes q_{39}(t) \otimes q_{90}(t)) \otimes G_0(t) \\ &+ (q_{65} \otimes q_{51}(t) + q_{63}(t) \otimes q_{31}(t)) \otimes G_1(t) + (q_{65}(t) \otimes q_{56}(t)) \otimes G_6(t), \end{aligned} \tag{14}$$

$$\begin{aligned} G_7(t) &= q_{75}(t) \otimes \bar{R}(t) + q_{75}(t) \otimes q_{51}(t) \otimes G_1(t) + (q_{78}(t) \otimes q_{86}(t) \\ &+ q_{75}(t) \otimes q_{56}(t)) \otimes G_6(t) + (q_{78}(t) \otimes q_{87}(t))G_7(t). \end{aligned} \tag{15}$$

Using LT to solve equations 11, 12, 13, 14 and 15 for $G_0^*(s)$, We have the expected busy periods with repair in steady state as follows

$$G_0 = \lim_{t \rightarrow \infty} G_0(t) = \frac{N_2}{D_1}, \tag{16}$$

where

$$N_2 = \bar{R}^*(0) \{ (P_{03} + P_{04}P_{43})b_1 - (1 + P_{15})b_2 - (1 + P_{65} + P_{63})b_4 + P_{75}b_5, \tag{17}$$

and

$$\bar{R}^*(0) = \frac{1}{\mu}.$$

6. Cost benefit analysis

This section, we calculate the expected profit to the system in the period $(0, t]$ by calculate the difference between total revenue and total cost of repair

$$C(t) = K_1 \omega_{up}(t) - K_2 \omega_r(t), \tag{18}$$

Where, K_1 is the revenue at the time the system works and K_2 is cost persunit of repair time.

$$\omega_{up}(t) = \int_0^t AV_0(t)dt, \tag{19}$$

$$\omega_r(t) = \int_0^t G_0(t)dt, \tag{20}$$

using 18, 19 and 20 we obtain

$$C^*(s) = K_1 \omega_{up}^*(s) - K_2 \omega_r^*(s).$$

Therefore the expected revenue persunit time in steadysstate is given by

$$C = \lim_{t \rightarrow \infty} \frac{C(t)}{t} = \lim_{s \rightarrow 0} s^2 C^*(s) = \frac{K_1 N_1 - K_2 N_2}{D_1}. \tag{21}$$

7. Numerical Example

By setting $K_1 = 100, K_2 = 2$, figures display the variation of MTSF, Availability, Busy period and Cost benefit, for different values of $\theta, \alpha, \beta, \mu, u$ and λ .

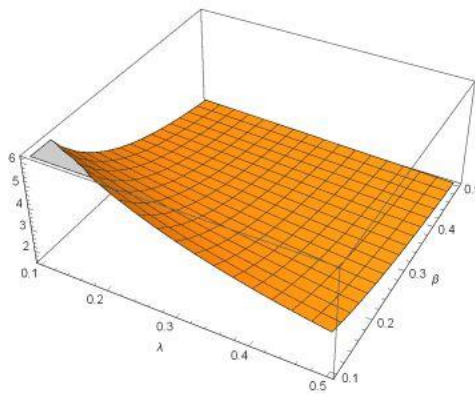


Figure 1: MTSF with $\theta = 0.99, \alpha = 0.2, \mu = 0.1, u = 0.01, \lambda = 0.1$ to 0.5 and $\beta = 0.1$ to 0.5

Table 1: MTSF with $\theta = 0.99, \mu = 0.1, u = 0.01$

λ	β				
	α	0.01	0.05	0.1	0.15
0.001	0.01	99.9066	19.9973	9.9995	6.66649
	0.10	99.9066	19.9973	9.9995	6.66649
	0.15	99.9066	19.9973	9.9995	6.66649
	0.20	99.9066	19.9973	9.9995	6.66649
0.01	0.01	92.654	19.7644	9.95439	6.65015
	0.10	92.654	19.7644	9.95439	6.65015
	0.15	92.654	19.7644	9.95439	6.65015
	0.20	92.654	19.7644	9.95439	6.65015
0.05	0.01	45.3846	16.6465	9.22708	6.36246
	0.10	45.3846	16.6465	9.22708	6.36246
	0.15	45.3846	16.6465	9.22708	6.36246
	0.20	45.3846	16.6465	9.22708	6.36246

Table 2: Availability with $\theta = 0.99, \mu = 0.1, u = 0.01$

λ	β				
	α	0.01	0.05	0.1	0.15
0.001	0.01	0.49893	0.165782	0.0902649	0.0619857
	0.10	0.900503	0.626964	0.443267	0.338515
	0.15	0.927172	0.691396	0.506976	0.393566
	0.20	0.940857	0.726623	0.542885	0.424975
0.01	0.01	0.493083	0.164291	0.0896243	0.0616213
	0.10	0.883668	0.610262	0.431577	0.330442
	0.15	0.908467	0.670211	0.491444	0.382681
	0.20	0.920974	0.702501	0.524817	0.412243
0.05	0.01	0.429263	0.150459	0.0844282	0.058925
	0.10	0.75112	0.525738	0.379603	0.295909
	0.15	0.768492	0.569752	0.425948	0.338061
	0.20	0.776704	0.592236	0.450789	0.361223

Table 3: Busy period with $\theta = 0.99, \mu = 0.1, u = 0.01$

λ	β				
	α	0.01	0.05	0.1	0.15
0.001	0.01	0.0604579	0.0956753	0.10475	0.108509
	0.10	0.150841	0.57032	0.896621	1.10405
	0.15	0.165742	0.690741	1.16063	1.48727
	0.20	0.175178	0.769698	1.34569	1.76974
0.01	0.01	0.112941	0.116491	0.117323	0.117638
	0.10	0.248593	0.655426	0.962705	1.156
	0.15	0.266711	0.784854	1.23573	1.54668
	0.20	0.277746	0.86839	1.42534	1.83284
0.05	0.01	0.349859	0.205979	0.170409	0.156051
	0.10	0.673826	0.990172	1.217	1.357
	0.15	0.702612	1.14836	1.51954	1.77301
	0.20	0.718428	1.24538	1.72319	2.07118

Table 4: Cost benefit with $\theta = 0.99, \mu = 0.1, u = 0.01$

λ	β				
	α	0.01	0.05	0.1	0.15
0.001	0.01	49.7721	16.3869	8.81699	5.98156
	0.10	89.7486	61.5558	42.5334	31.6434
	0.15	92.3858	67.7581	48.3763	36.3821
	0.20	93.7354	71.1229	51.5971	38.958
0.01	0.01	49.0825	16.1962	8.72778	5.92685
	0.10	87.8696	59.7153	41.2323	30.7322
	0.15	90.3133	65.4514	46.6729	35.1747
	0.20	91.5419	68.5134	49.631	37.5587
0.05	0.01	42.2266	14.634	8.102	5.58039
	0.10	73.7643	50.5935	35.5263	26.8769
	0.15	75.4439	54.6784	39.5557	30.26
	0.20	76.2336	56.7329	41.6325	31.9799

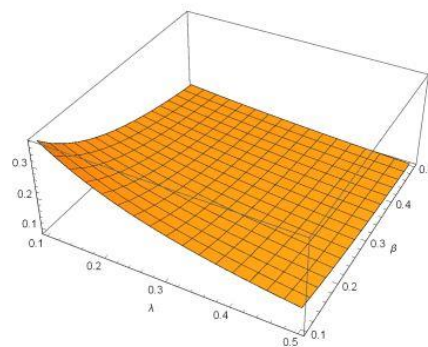


Figure 2: Availability with $\theta = 0.99, \alpha = 0.2, \mu = 0.1, u = 0.01, \lambda = 0.1$ to 0.5 and $\beta = 0.1$ to 0.5

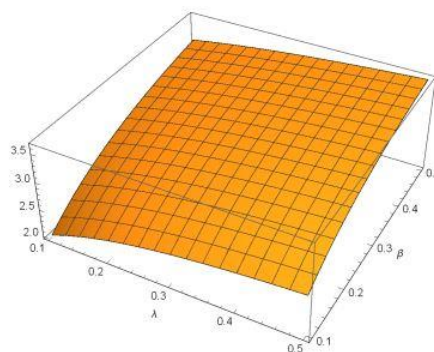


Figure 3: Busy period with $\theta = 0.99, \alpha = 0.2, \mu = 0.1, u = 0.01, \lambda = 0.1$ to 0.5 and $\beta = 0.1$ to 0.5

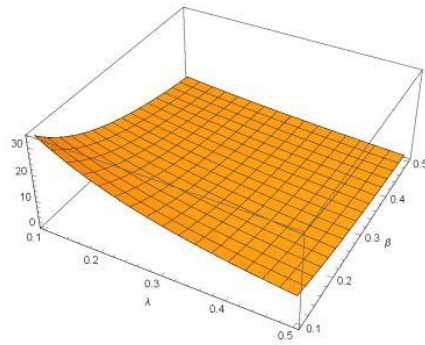


Figure 4: Cost benefit with $\theta = 0.99, \alpha = 0.2, \mu = 0.1, u=0.01, K_1 = 100, K_2 = 2, \lambda = 0.1$ to 0.5 and $\beta = 0.1$ to 0.5

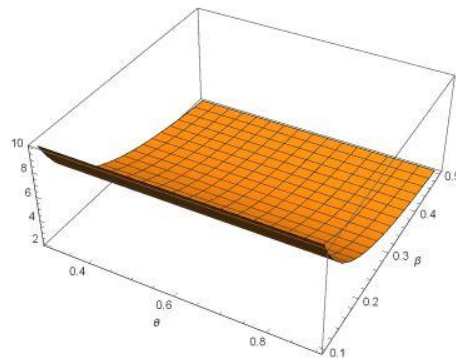


Figure 5: MTSF with $\lambda = 0.008, \alpha = 0.2, \mu = 0.1, u=0.01, \theta = 0.3$ to 0.9 and $\beta = 0.1$ to 0.5

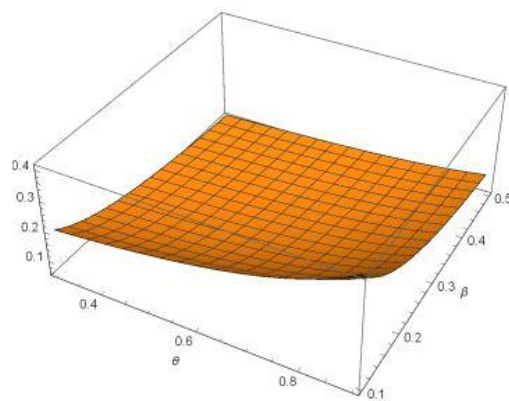


Figure 6: Availability with $\lambda = 0.008, \alpha = 0.2, \mu = 0.1, u=0.01, \theta = 0.3$ to 0.9 and $\beta = 0.1$ to 0.5

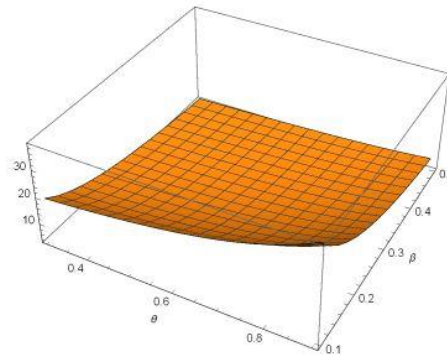


Figure 7: Cost benefit with $\lambda = 0.008$, $\alpha = 0.2$, $\mu = 0.1$, $u=0.01$, $K_1 = 100$, $K_2 = 2$, $\theta = 0.3$ to 0.9 and $\beta = 0.1$ to 0.5

8. Discussion

Figures 1, 2 and tables 1, 2 show that the MTSF decreases with the increase in the failure rate; also, availability decreases with increasing failure rate. Also, the weather conditions affect both the MTSF and the availability causing their decrease due to the increase in the abnormal weather rate, we found that the MTSF was not affected by the change in the normal weather rate and the availability increases with increasing normal weather rate. We found that MTSF = 99.9066 when $\lambda = 0.001$ and $\beta = 0.01$, MTSF = 6.36246 when $\lambda = 0.05$ and $\beta = 0.15$ and availability = 0.940857 when $\lambda = 0.001$, $\beta = 0.01$ and $\alpha = 0.20$ and availability = 0.058925 when $\lambda = 0.05$, $\beta = 0.15$ and $\alpha = 0.01$. From figure 3 and table 3, we also noticed the increase in the busy period with the increase in the failure rate and the abnormal weather rate, busy period = 0.0604579 when $\lambda = 0.001$, $\beta = 0.01$ and $\alpha = 0.01$, busy period = 2.07118 when $\lambda = 0.05$, $\beta = 0.15$ and $\alpha = 0.20$. Then by studying the effect of the failure rate and the weather rate (normal and abnormal) on the cost-benefit analysis, (figure 4 - table 4), we found that the cost-benefit decreased with the increase in the failure rate and abnormal weather rate but the cost-benefit decreased with the decrease the normal weather rate cost benefit = 93.7354 when $\lambda = 0.001$, $\beta = 0.01$ and $\alpha = 0.20$, cost benefit = 5.58039 when $\lambda = 0.05$, $\beta = 0.15$ and $\alpha = 0.01$. In figures 5 - 6, and 7 when we studied the effect of the presence of the repairman on the reliability measures, we found that the MTSF was not affected by the presence or absence of the repairman, but availability and cost-benefit analysis increased with the presence of the repairman. The MTSF, availability, and cost-benefit decreases with the increase in the repair rate but the busy period increase with the increase in the repair rate.

9. Conclusion

This paper provides the reliability analysis for a two-unit cold standby system with the presence or absence of a repairman. We assumed that the unit operates in normal weather conditions, and the unit fails in abnormal weather conditions, causing system stop. Finally, a numerical example is provided to illustrate the influence of parameters on the MTSF, availability, busy period and, cost-benefit analysis of the system.

- The abnormal weather rate together with the failure rate, both increased with the decrease of the MTSF, availability, and cost-benefit analysis.
- The abnormal weather rate together with the failure rate increased with the increase of the busy period.
- The MTSF was not affected by the normal weather rate and the presence or absence of the repairman.
- The availability and cost-benefit analysis increased with the increase in normal weather rate.
- The availability and cost-benefit analysis increased with the presence of the repairman.

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