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GENERALIZED FRACTIONAL INTEGRAL FORMULAS ASSOCIATED WITH THE SRIVASTAVA -TOMOVSKI MITTAG-LEFFLER FUNCTION AND THE SRIVASTAVA POLYNOMIALS

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Abstract. The aim of the present paper is to study two Generalized fractional integrals due to Saigo involving Srivastava-Tomovski Mittag-Leffler function and Srivastava Polynomials in its kernel. The functions and Polynomials involved herein are general in nature so the results obtained in the present paper are capable of unifying scores of hitherto scattered results in the concerned literature.

1. Introduction

Recently Mittag-Leffler function and its further generalizations have become a topic of interest for the researchers due to its importance in the problems of physics, chemistry, engineering and applied sciences.

In 1903, G. Mittag-Leffler [1] introduced and defined the Mittag- Leffler function as

$$E_\alpha(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n + 1)}; (z \in \mathbb{C}; \Re(\alpha) > 0) \tag{1}$$

In 1905, generalization of $E_\alpha(z)$ was given by Wiman [2] in terms of the following series

$$E_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n + \beta)}; (z, \beta \in \mathbb{C}; \Re(\alpha) > 0) \tag{2}$$

In particular $E_{\alpha,1}(z) = E_\alpha(z)$

In 1971, Prabhakar [3] generalized the Mittag-Leffler function by means of the following series presentation

$$E^\gamma_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_n}{\Gamma(\alpha n + \beta)} \frac{z^n}{n!}; (z, \beta, \gamma \in \mathbb{C}; \Re(\alpha) > 0) \tag{3}$$

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where $(\gamma)_n$ denotes the Pochhammer symbol [4].

Clearly $E^1_{\alpha,\beta}(z) = E_{\alpha,\beta}(z)$ and $E^1_{\alpha,1}(z) = E_\alpha(z)$

In 2009, an important generalization of the Mittag-Leffler function $E^\gamma_{\alpha,\beta}(z)$ was introduced by Srivastava and Tomovski [5] as follows

$$E^{\gamma,k}_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_{kn}}{\Gamma(\alpha n+\beta)} \frac{z^n}{n!} \tag{4}$$

Where $z, \beta, \gamma \in C; \Re(\alpha) > \max\{0, \Re(k) - 1\}, \Re(k) > 0$

Which in particular for $k = q(q \in (0,1) \cup N); \min\{\Re(\beta), \Re(\gamma)\} > 0$, reduced to the another generalization of Mittag- Leffler function, considered earlier by Shukla and Prajapati [6].

In 1987, Hussain ([7], [8]) introduced the $\bar{H}$-function as

$$\bar{H}^{A,C}_{B,D}[z] = \bar{H}^{A,C}_{B,D}\left[z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,C}(a_j,\alpha_j)_{C+1,B}\\(b_j,\beta_j)_{1,A}(b_j,\beta_j;B_j)_{A+1,D}\end{matrix}\right.\right] = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\Phi(\xi)z^\xi d\xi \tag{5}$$

where $\Phi(\xi) = \dfrac{\prod_{j=1}^{A}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{C}\left\{\Gamma(1-a_j+\alpha_j\xi)\right\}^{A_j}}{\prod_{j=A+1}^{D}\{\Gamma(1-b_j+\beta_j\xi)\}^{B_j}\prod_{j=N+1}^{B}\Gamma(a_j-\alpha_j\xi)}$

The convergence and existence condition, basic properties of this function were given by Buschman and Srivastava [9] .

In 1972, Srivastava [10] introduced and studied Srivastava polynomials as

$$S^M_N[x] = \sum_{k=0}^{[N/M]} (-N)_{Mk}\, A_{N,k}\, \frac{x^k}{k!}, N = 0, 1, 2, . \tag{6}$$

where m is an arbitrary positive integer and the coefficients $A_{N,k}(N, k \geq 0)$ are arbitrary constants, real or complex. Here $(\lambda)_n$ denotes the Pochhammer symbol defined by

$(\lambda)_n = \frac{\Gamma(\lambda+n)}{\Gamma(\lambda)} = \begin{cases} 1, & \text{if } n = 0 \\ \lambda(\lambda+1)\dots(\lambda+n-1), & \forall n \in \{i = 1, ..., r\}. \end{cases}$

The H-function of several complex variables is introduced by Srivastava and Panda [11] in terms of Mellin-Barnes type contour integral as

$H[z_1, \dots, z_r]$

$$= H^{0,M:(u',v');\dots;(u^{(r)},v^{(r)})}_{P,Q:[B',D'];\dots;[B^{(r)},D^{(r)}]}\left[\begin{matrix}z_1\\\vdots\\z_r\end{matrix}\left|\begin{matrix}[a_j:\theta^1_j;\dots;\theta^{(r)}_j]_{1,P}:[b^1_j:\Phi^1_j];\dots;[b^{(r)}_j:\Phi^{(r)}_j]\\ [c_j:\psi^1_j;\dots;\psi^{(r)}_j]:[d^1_j:\delta^1_j];\dots;[d^{(r)}_j:\delta^{(r)}_j]\end{matrix}\right.\right] \tag{7}$$

$$= \frac{1}{(2\pi i)^r}\int_{L_1}\cdots\int_{L_r}\Psi(\xi_1,\dots,\xi_r)\varphi_1(\xi_1)\cdots\varphi_r(\xi_r)z_1^{\xi_1}\cdots z_r^{\xi_r}d\xi_1\cdots d\xi_r \tag{8}$$

The convergence conditions and other details of the multivariable H-function $H[z_1, \dots, z_r]$ are given by Srivastava, Gupta and Goyal [12]. The multivariable H-function is an extension of multivariable G-function. It includes Foxs H-function, Apell function, Whittaker function and so on.

In recent years Fractional calculus has become a rapidly growing area and has found many applications in diverse field ranging from physical sciences to biological sciences and economics [13-17]. The most important reason for growing popularity of fractional calculus is its nonlocal property or global dependency [18]. The modeling accuracy of many phenomena in natural science have been improved by using fractional calculus. Many researchers worked on fractional differential and integral operators and gave many important results.

In 1978, Saigo M. [19] introduced and study the integral operator $I_{0,z}^{\alpha,\beta,\gamma} f(z)$ for real numbers $\alpha > 0, \beta$ and γ, as

$$I_{0,z}^{\alpha,\beta,\gamma} f(z) = \frac{z^{-\alpha-\beta}}{\Gamma(\alpha)} \int_0^z (z-t)^{\alpha-1} {}_2F_1(\alpha+\beta, -\gamma; \alpha; 1-\frac{t}{z}) f(t) dt \tag{9}$$

In 1996, Saigo M.[20] introduced a generalization of this operator as

$$I_{0,z}^{\alpha,\alpha',\beta,\beta',\gamma} f(z) = \frac{z^{-\alpha}}{\Gamma(\gamma)} \int_0^z (z-\xi)^{\gamma-1} \xi^{-\alpha'} F_3(\alpha,\alpha',\beta,\beta';\gamma;1-\frac{\xi}{z},1-\frac{z}{\xi}) f(\xi) d\xi \tag{10}$$

where $\alpha, \alpha', \beta, \beta', \gamma \in C; \mathrm{Re}(\gamma) > 0, F_3$ denote the Appell third function.

In 1998, Saigo and Maeda[21] studied several relations of this operator with the Mellin Transforms, Hypergeometric operators, their decompositions and properties in the McBride spaces $F_{p,\mu}$. This operator is known to reduce to the fractional integral operator (9) as

$I_{0,z}^{\alpha,0,\beta,\beta',\gamma} f(z) = I_{0,z}^{\gamma,\alpha-\gamma,-\beta} f(z); (\gamma \in C)$

The following auxiliary lemma ([21], p.394) will be required in the sequel

Lemma: Let

$\alpha, \alpha', \beta, \beta', \gamma, k \in C; \mathrm{Re}(\gamma) > 0, \mathrm{Re}(k) > \max\{0, \mathrm{Re}(\alpha+\alpha'+\beta-\gamma), \mathrm{Re}(\alpha'-\beta')\}$

then

$$I_{0,z}^{\alpha,\alpha',\beta,\beta',\gamma} z^k = \frac{\Gamma(k+1)\Gamma(-\alpha-\alpha'-\beta+\gamma+k+1)\Gamma(-\alpha'+\beta'+k+1)}{\Gamma(-\alpha-\alpha'+\gamma+k+1)\Gamma(-\alpha'-\beta+\gamma+k+1)\Gamma(\beta'+k+1)} z^{k-\alpha-\alpha'+\gamma} \tag{11}$$

2. MAIN RESULT-I

$$\left\{ I_{0,x}^{\alpha,\alpha',\beta,\beta',\gamma} \left\{ t^{\rho-1}(b-at)^{-\mu} S_B^A \left(ct^\lambda (b-at)^{-\upsilon} \right) E_{p,q}^{\xi,k} \left(rt^n (b-at)^{-w} \right) \bar{H}_{P,Q}^{M,N} \left(zt^\sigma (b-at)^{-u} \right) \right\} \right\} (x)$$

$$= b^{-\mu} x^{\rho-\alpha-\alpha'+\gamma-1} \sum_{e=0}^{\infty} \sum_{m=0}^{\infty} \sum_{l=0}^{[B/A]} \frac{(-B)_{Al} c^l A'_{B,l} (\xi)_{mk} r^m a^e b^{-e-\upsilon l - wm}}{l! m! e! \Gamma(pm+q)} x^{e+\lambda l + mn}$$

$$\bar{H}_{P+4,Q+4}^{M,N+4} \left[\frac{zx^\sigma}{b^u} \middle| \begin{matrix} (1-e-\lambda l - mn - \rho; 1), (1-e-\lambda l - mn - \rho - \gamma + \alpha + \alpha' + \beta, \sigma; 1) \\ (f_j, F_j)_{1,M}, (1-e-\lambda l - mn - \rho - \gamma + \alpha + \alpha', \sigma; 1)(1-e-\lambda l - mn - \rho - \gamma + \alpha' + \beta, \sigma; 1) \end{matrix} \right.$$

$$\left. \begin{matrix} (1-\mu-\upsilon l - wm - e, u; 1), (1-e-\lambda l - mn - \rho - \beta' + \alpha', \sigma; 1), (e_j, E_j; a_j)_{1,N}, (e_j, E_j)_{N+1,P} \\ (1-\mu-\upsilon l - wm, u; 1)(1-e-\lambda l - mn - \rho - \beta', \sigma; 1)(f_j, F_j; b_j)_{M+1,Q} \end{matrix} \right] \tag{12}$$

Where $\alpha, \alpha', \beta, \beta', \gamma, p, q, \mu, \xi, u, a, b \in \mathrm{C}, \mathrm{Re}(\gamma) > 0, \lambda > 0, \sigma > 0$

$$|\arg(z)| < \frac{\pi}{2}\Omega; \Omega > 0$$

$$;\Omega = \sum_{j=1}^{M} F_j + \sum_{j=1}^{N} |a_j E_j| - \sum_{j=M+1}^{Q} |b_j F_j| - \sum_{j=N+1}^{P} E_j > 0$$

$$\mathrm{Re}(p) > \max\{0, \mathrm{Re}(k)-1\}; \mathrm{Re}(k) > 0$$

$$\mathrm{Re}(\rho) + \sigma \min_{1\le j\le M} \mathrm{Re}\left(\frac{f_j}{F_j}\right) > \max\{0, \mathrm{Re}(\alpha+\alpha'+\beta-\gamma), \mathrm{Re}(\alpha'-\beta')\}$$

Proof: In order to prove (12), we first express the Srivastava polynomials in series form (6) , the $\bar{H}$ function in terms of Mellin-Barnes type of contour integrals (5), Srivastava- Tomovski Mittag-Leffler function in series form (4) and then interchange the order of summations, integration and fractional integral operator, which is permissible under the stated conditions. Now using the lemma (11) we arrive at the desired result after a little simplification.

Special Cases:

(i) on taking

$$M=1, N=2, P=2=Q, e_1=0, e_2=1, E_1=E_2=1, a_1=1, a_2=\varepsilon$$

$$f_1=0, F_1=1, f_2=0, F_2=1, b_2=\varepsilon-1$$

result (12) takes the form

$$\left\{I_{0,x}^{\alpha,\alpha',\beta,\beta',\gamma}\left\{t^{\rho-1}(b-at)^{-\mu}S_B^A\left(ct^{\lambda}(b-at)^{-\upsilon}\right)E_{p,q}^{\xi,k}\left(rt^n(b-at)^{-w}\right)L^{\varepsilon}\left(-zt^{\sigma}(b-at)^{-u}\right)\right\}\right\}(x)$$

$$= b^{-\mu}x^{\rho-\alpha-\alpha'+\gamma-1}\sum_{e=0}^{\infty}\sum_{m=0}^{\infty}\sum_{l=0}^{[B/A]}\frac{(-B)_{Al}c^l A'_{B,l}(\xi)_{mk}r^m a^e b^{-e-\upsilon l-wm}}{l!m!e!\Gamma(pm+q)}x^{e+\lambda l+mn}$$

$$\bar{H}_{6,6}^{1,6}\left[\frac{zx^{\sigma}}{b^u}\middle|\begin{matrix}(1-e-\lambda l-mn-\rho;1),(1-e-\lambda l-mn-\rho-\gamma+\alpha+\alpha'+\beta,\sigma;1)\\(0,1),(1-e-\lambda l-mn-\rho-\gamma+\alpha+\alpha',\sigma;1)(1-e-\lambda l-mn-\rho-\gamma+\alpha'+\beta,\sigma;1)\end{matrix}\right.$$

$$\left.\begin{matrix}(1-\mu-\upsilon l-wm-e,u;1),(1-e-\lambda l-mn-\rho-\beta'+\alpha',\sigma;1),(0,1;1),(1,1;\varepsilon)\\(1-\mu-\upsilon l-wm,u;1)(1-e-\lambda l-mn-\rho-\beta',\sigma;1)(0,1;\varepsilon-1)\end{matrix}\right] \tag{13}$$

valid under the conditions mentioned with result (12).Here $L^{\varepsilon}(z)$ is the polylogarithm of complex order $\in$

(ii) On taking $a_j=1=b_j$ result (12) takes the form

$$\left\{I_{0,x}^{\alpha,\alpha',\beta,\beta',\gamma}\left\{t^{\rho-1}(b-at)^{-\mu}S_B^A\left(ct^{\lambda}(b-at)^{-\upsilon}\right)E_{p,q}^{\xi,k}\left(rt^n(b-at)^{-w}\right)H_{P,Q}^{M,N}\left(zt^{\sigma}(b-at)^{-u}\right)\right\}\right\}(x)$$

$$= b^{-\mu}x^{\rho-\alpha-\alpha'+\gamma-1}\sum_{e=0}^{\infty}\sum_{m=0}^{\infty}\sum_{l=0}^{[B/A]}\frac{(-B)_{Al}c^l A'_{B,l}(\xi)_{mk}r^m a^e b^{-e-\upsilon l-wm}}{l!m!e!\Gamma(pm+q)}x^{e+\lambda l+mn}$$

$$H_{P+4,Q+4}^{M,N+4}\left[\frac{zx^{\sigma}}{b^u}\middle|\begin{matrix}(1-e-\lambda l-mn-\rho;1),(1-e-\lambda l-mn-\rho-\gamma+\alpha+\alpha'+\beta,\sigma;1)\\(f_j,F_j)_{1,M},(1-e-\lambda l-mn-\rho-\gamma+\alpha+\alpha',\sigma;1)(1-e-\lambda l-mn-\rho-\gamma+\alpha'+\beta,\sigma;1)\end{matrix}\right.$$

$$\left.\begin{matrix}(1-\mu-\upsilon l-wm-e,u;1),(1-e-\lambda l-mn-\rho-\beta'+\alpha',\sigma;1),(e_j,E_j;1)_{1,P}\\(1-\mu-\upsilon l-wm,u;1)(1-e-\lambda l-mn-\rho-\beta',\sigma;1)(f_j,F_j;1)_{M+1,Q}\end{matrix}\right] \tag{14}$$

valid under the same conditions surrounding result (12).

3. MAIN RESULT-II

$$\left\{I_{0,x}^{\alpha,\alpha',\beta,\beta',\gamma}\left\{t^{\rho-1}(b-at)^{-\mu}S_B^A\left(ct^{\lambda}(b-at)^{-\upsilon}\right)E_{p,q}^{\xi,k}\left(vt^n(b-at)^{-w}\right)\right.\right.$$

$$\left.\left. H_{P,Q:[B',D'];...;[B^{(r)},D^{(r)}]}^{0,M:(u',v');...;(u^{(r)},v^{(r)})}\begin{pmatrix} z_1t^{\sigma_1}(b-at)^{-u_1}\\ \vdots \\ z_rt^{\sigma_r}(b-at)^{-u_r}\end{pmatrix}\right\}\right\}(x)$$

$$=b^{-\mu}x^{\rho-\alpha-\alpha'+\gamma-1}\sum_{e=0}^{\infty}\sum_{m=0}^{\infty}\sum_{l=0}^{[B/A]}\frac{(-B)_{Al}c^lA'_{B,l}(\xi)_{mk}v^ma^eb^{-e-\upsilon l-wm}}{l!m!e!\Gamma(pm+q)}x^{e+\lambda l+mn}$$

$$H_{P+4,Q+4:[B',D'];...;[B^{(r)},D^{(r)}]}^{0,M+4:(u',v');...;(u^{(r)},v^{(r)})}\left[\begin{matrix} z_1x^{\sigma_1}/b^{u_1}\\ \vdots \\ z_rx^{\sigma_r}/b^{u_r}\end{matrix}\left|\begin{matrix}(1-e-\lambda l-mn-\rho;\sigma_1,...,\sigma_r)\\ (c_j,\psi_j^{(1)},...,\psi_j^{(r)})_{1,Q},(1-e-\lambda l-mn-\rho-\beta';\sigma_1,...,\sigma_r)\end{matrix}\right.\right.$$

$$,(1-e-\lambda l-mn-\rho-\gamma+\alpha+\alpha'+\beta;\sigma_1,...,\sigma_r),(1-e-\lambda l-mn-\rho-\beta'+\alpha';\sigma_1,...,\sigma_r)$$
$$,(1-e-\lambda l-mn-\rho-\gamma+\alpha+\alpha';\sigma_1,...,\sigma_r),(1-e-\lambda l-mn-\rho-\gamma+\alpha'+\beta;\sigma_1,...,\sigma_r)$$

$$\left.\begin{matrix},(1-\mu-\upsilon l-wm-e;u_1,...,u_r),(a_j,\theta_j^1,...,\theta_j^{(r)})_{1,P};(b_j^1,\Phi_j^1)_{1,B'},...,(b_j^{(r)},\Phi_j^{(r)})_{1,B^{(r)}}\\ ,(1-\mu-\upsilon l-wm;u_1,...,u_r);(d_j^1,\delta_j^1)_{1,D'},...,(d_j^{(r)},\delta_j^{(r)})_{1,D^{(r)}}\end{matrix}\right] \tag{15}$$

where $\alpha,\alpha',\beta,\beta',\gamma,\rho,q,\mu,\xi,u_i,z_i,a,b\in\mathrm{C},\mathrm{Re}(\gamma)>0,\lambda>0,\sigma_i>0$

$$\mathrm{Re}(p)>\max\{0,\mathrm{Re}(k)-1\};\mathrm{Re}(k)>0$$

$$\mathrm{Re}(\rho)+\sum_{i=1}^{r}\sigma_j\min_{1\le j\le u^{(r)}}\mathrm{Re}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)>\max\{0,\mathrm{Re}(\alpha+\alpha'+\beta-\gamma),\mathrm{Re}(\alpha'-\beta')\}$$

$$\mathrm{Re}(\mu)+\sum_{i=1}^{r}u_j\min_{1\le j\le u^{(r)}}\mathrm{Re}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)>\max\{0,\mathrm{Re}(\alpha+\alpha'+\beta-\gamma),\mathrm{Re}(\alpha'-\beta')\}$$

$$\left|\frac{a}{b}x\right|<1$$

,

$$\sum_{j=1}^{P}\theta_j^{(i)}-\sum_{j=1}^{Q}\psi_j^{(i)}+\sum_{j=1}^{B^{(i)}}\Phi_j^{(i)}-\sum_{j=1}^{D^{(i)}}\delta_j^{(i)}\le 0$$

$$|\arg(z_i)|<\frac{1}{2}\pi T_i\ (i=1,2,...,r)$$

$$T_i=\sum_{j=0}^{M}\theta_j^{(i)}-\sum_{j=\lambda+1}^{P}\theta_j^{(i)}-\sum_{j=1}^{Q}\psi_j^{(i)}+\sum_{j=1}^{v^{(i)}}\Phi_j^{(i)}-\sum_{j=v^{(i)}+1}^{B^{(i)}}\Phi_j^{(i)}+\sum_{j=1}^{u^{(i)}}\delta_j^{(i)}-\sum_{j=u^{(i)}+1}^{D^{(i)}}\delta_j^{(i)}-u_{\mathrm{i}}+\sigma_i>0$$

Proof: In order to prove (15), we first express the Srivastava polynomials in series form (6) , the multivariable H-function in terms of Mellin-Barnes type of contour integrals (8), Srivastava- Tomovski Mittag-Leffler function in series form (4) and then interchange the order of summations, integration and fractional integral operator, which is permissible under the stated conditions. Now using the lemma (11) we arrive at the desired result after a little simplification.

Special Case:

On setting M = P = Q = 0 , the main result (15) takes the form

$$\left\{I_{0,x}^{\alpha,\alpha',\beta,\beta',\gamma}\left\{t^{\rho-1}(b-at)^{-\mu}S_B^A\left(ct^{\lambda}(b-at)^{-\upsilon}\right)E_{p,q}^{\xi,k}\left(vt^{n}(b-at)^{-w}\right)\right.\right.$$

$$\left.\left.\cdot\prod_{i=1}^{r}H_{B^{(i)},D^{(i)}}^{u^{(i)},v^{(i)}}\left(z_i t^{\sigma_i}(b-at)^{-u_i}\right)\right\}\right\}(x)$$

$$= b^{-\mu}x^{\rho-\alpha-\alpha'+\gamma-1}\sum_{e=0}^{\infty}\sum_{m=0}^{\infty}\sum_{l=0}^{[B/A]}\frac{(-B)_{Al}c^{l}A'_{B,l}(\xi)_{mk}v^{m}a^{e}b^{-e-\upsilon l-wm}}{l!m!e!\Gamma(pm+q)}x^{e+\lambda l+mn}$$

$$H_{4,4:[B',D'];...;[B^{(r)},D^{(r)}]}^{0,4:(u',v');...;(u^{(r)},v^{(r)})}\left[\begin{matrix}z_1x^{\sigma_1}/b^{u_1}\\ \vdots\\ z_rx^{\sigma_r}/b^{u_r}\end{matrix}\middle|\begin{matrix}(1-e-\lambda l-mn-\rho;\sigma_1,...,\sigma_r)\\(1-e-\lambda l-mn-\rho-\beta';\sigma_1,...,\sigma_r)\end{matrix}\right.$$

$$,(1-e-\lambda l-mn-\rho-\gamma+\alpha+\alpha'+\beta;\sigma_1,...,\sigma_r),(1-e-\lambda l-mn-\rho-\beta'+\alpha';\sigma_1,...,\sigma_r)$$
$$,(1-e-\lambda l-mn-\rho-\gamma+\alpha+\alpha';\sigma_1,...,\sigma_r),(1-e-\lambda l-mn-\rho-\gamma+\alpha'+\beta;\sigma_1,...,\sigma_r)$$

$$\left.\begin{matrix},(1-\mu-\upsilon l-wm-e;u_1,...,u_r),;(b_j^1,\Phi_j^1)_{1,B'},...,(b_j^{(r)},\Phi_j^{(r)})_{1,B^{(r)}}\\ ,(1-\mu-\upsilon l-wm;u_1,...,u_r);(d_j^1,\delta_j^1)_{1,D'},...,(d_j^{(r)},\delta_j^{(r)})_{1,D^{(r)}}\end{matrix}\right] \qquad (16)$$

which holds true under the conditions followed by result (15).

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