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EFFICIENT METHODS FOR THE ANALYTICAL SOLUTION OF THE FRACTIONAL GENERALIZED FISHER EQUATION

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Abstract. In this paper, analytical approximate solutions of the generalized fractional Fisher equation (GFFE) are given using the Laplace Adomain decomposition method (LADM) and the reduced differential transform method (RDTM). The two proposed methods are effective and easy to implement. The approximate solutions of the two proposed schemes give better results when compared with the exact solutions or the numerical solutions in the existing literature.

1. Introduction

Fractional partial differential equations (FPDEs) have an increased widespread position in many scientific applications due to their accuracy in modeling many phenomena's in different fields of chemistry, biology, applied science, engineering...etc.([1]-[5])
Fisher's equation is first introduced by Fisher as a deterministic model of the wave propagation of favored gene in population [6]. Also it arises in many physical, biological, chemical, and engineering problems that are described by the interaction of diffusion and reaction process. For example, it plays a significant role include flame propagation, neutron flux in a nuclear reactor and the dynamics of defects in nematic liquid crystal [7]. The general form of the Fisher equation which is termed as "generalized Fisher equation" (GFE) is of great interest for many researchers and scientists. Many articles ([8]-[21]) have presented various analytical and numerical methods to solve GFE; In [11] a modified scheme based on the hybridization of Exp function method with nature inspired algorithm was used to find the approximate solution of GFE. Ismail et al. [15] used Adomian decomposition method (ADM), Rashidi et al. [16] employed homotopy perturbation method (HPM), Nawaz et al. [13] applied optimal homotopy asymptotic method (OHAM) for obtaining approximate solutions of the generalized Burgers -Fisher Equation (GBFE). Mittal and Tripathi [12] applied the modified cubic B-spline functions for the numerical solution of GBFE and Burgers-Huxley equations. Khattak [17] used collocation based radial base functions method (CBRBF) for numerical solution of the GBFE. Javidi

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[18] applied a modified pseudo spectral method for GBFE.
Recently many researchers are interested to provide different numerical methods for solving GFFE ([22]-[28]).The fractional time Fisher equation was solved by using Hemotopy perturbation method in [27], while the fractional space Fisher equation and the fractional time Fisher equation were solved by the modified Adomain decomposition method in [28]. In [25] the Haar wavelet method and optimal homotopy asymptotic method were used to find the approximate solution to the fractional Fisher type equation. To the best of our knowledge the fractional time GFFE has not been treated yet by using LADM or by RDTM.
The main objective of the present paper is to offer two numerical techniques based upon LADM and RDTM to solve the fractional time GFFE:

$$\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} = u_{xx} - \mu u^\delta u_x + \gamma u(1-u^\delta),\ x \in [0,1],\ t > 0,\ 0 < \alpha \leq 1. \tag{1}$$

Which is subject to the initial condition

$$u(x,0) = f(x) = (0.5\ - 0.5\tanh[\frac{\mu\delta}{2(\delta+1)}x])^{\frac{1}{\delta}},\ x \in [0,1]. \tag{2}$$

This equation reduced to the classical generalized Fisher equation at $\alpha = 1$ and has exact solution [14]

$$u(x,t) = (0.5\ - 0.5\tanh[\frac{\mu\delta}{2(\delta+1)}(x - (\frac{\mu}{\delta+1} + \frac{\mu(\delta+1)}{\mu})t)])^{\frac{1}{\delta}} \tag{3}$$

Where $\frac{\partial^\alpha u(x,t)}{\partial t^\alpha}$ is the fractional time derivative operator in sense of Caputo.

The LADM was offered by Khuri who applied the scheme to a class of nonlinear differential equations [29]. The achieved solutions are expressed as infinite series which converge rabidly to the exact solutions. It was shown that LADM is easily to implement and accurately to approximate solutions of wide classes of linear and nonlinear ODEs and PDEs of integer order ([30]-[36]).
Recently, Keskin and Oturanc [37] developed the reduced differential transform method (RDTM) for the fractional differential equations and showed that RDTM is an easily useable semi analytical method and gives the exact solution for both the linear and nonlinear differential equations. RDTM can be well-thought-out as an iterative process for obtaining Taylor series solution of differential equations.

This paper is outlined as follows: In the next section, basic definition of the Caputo fractional derivative and its Laplace transform are informed. The proposed numerical techniques of RDTM and LADM are explained in section 3 and section 4, respectively. Numerical results which validate the applicability of the anticipated techniques are set in section 5. Finally, the main conclusion ends the paper in the last section.

2. Basic Definitions

Definition 1 A real function $f(t),\ t >\ 0$, is said to be in the space $C_\mu,\ \mu \in R$, if there exists a real number $p\ > \mu$ such that $f(t) = t^p g(t)$ where $g(t) \in C(0,\infty)$, and is said to be in space C^m_μ if and only if $f^n \in C,\ n \in N$.

Definition 2 The time fractional derivative D^{α}_{*t} of $u(x,t)$ in the Caputo sense is defined as

$$D^{\alpha}_{*t}u(x,t) = \frac{\partial^{\alpha}u(x,t)}{\partial t^{\alpha}} \;=\; \int_0^t \frac{(t-s)^{n-\alpha-1}}{\Gamma(n-\alpha)}u^n(x,s)ds. \tag{4}$$

For $n-1<\alpha\leq n,\ n\in N,\ t>0,\ u(x,t)\in C^n_{-1}$
Definition 3 The Laplace transform of the fractional Caputo derivative is

$$\mathfrak{L}[D^{\alpha}_{*t}u(x,t)] \;=\; s^{\alpha}U(x,s) - \sum_{k=1}^{n-1} s^{\alpha-k-1}u^{(k)}(x,0),\ n-1\ <\alpha\ \leq n. \tag{5}$$

We refer to ([38]-[39]) for more details about fractional operators.

3. Reduced Differential Transform Method

3.1. **Basic idea of RDTM.** The basic definition of RDTM is defined as follows.
Definition 4 If $u(x,t)$ be an analytic and continuously differentiable with respect to space variable x and time t in the domain of interest, then the t-dimensional spectrum function

$$U_k(x) = \frac{1}{\Gamma(k\alpha+1)}[\frac{\partial^{k\alpha}}{\partial t^{k\alpha}}u(x,t)]_{t=t_0}, \tag{6}$$

is the reduced transformed function of $u(x,t)$, where α is a parameter which describes the order of time-fractional derivative. Throughout this paper $u(x,t)$ represents the original function and $U_k(x)$ represents the reduced transformed function. Inverse transformation of the set values $(U_k(x))^n_{k=0}$ gives the approximation solution in the following form

$$\tilde{u_n}(x,t) = \sum_{k=0}^{n} U_k(x)(t-t_0)^{\alpha k}. \tag{7}$$

When $t_0=0$, Eq.(7) take the form

$$\tilde{u_n}(x,t) = \sum_{k=0}^{n} U_k(x)\ t^{\alpha k}. \tag{8}$$

Where n is the order of the approximation, then the exact solution is given by

$$u(x,t) = lim_{n\to\infty}\tilde{u_n}(x,t).$$

So we can deduce that the concept of RDTM is derived from the power series expansion of a function.
The sufficient conditions for the convergence of the generalized differential transform method when applied to fractional differential equations and the estimation of the maximum absolute errors are discussed and proved in [40].
The mathematical operations performed by the RDTM are listed in Table 1 ([41]-[42]).

Functional Form	Transformed Form
$u(x,t)$	$\frac{1}{\Gamma(k\alpha+1)}[\frac{\partial^{k\alpha}}{\partial t^{k\alpha}}u(x,t)]_{t=0}$
$\gamma u(x,t)\pm\beta v(x,t)$	$\gamma U_k(x)\pm\beta V_k(x),\ \gamma,\ \beta$ are constant
$u(x,t)v(x,t)$	$\sum_{i=0}^{k}U_i(x)V_{k-i}(x)$
$u(x,t)v(x,t)w(x,t)$	$\sum_{k=0}^{n}\sum_{i=0}^{k}U_i(x)V_{k-i}(x)W_{n-k}(x)$
$\frac{\partial^{n\alpha}}{\partial t^{n\alpha}}u(x,t)$	$\frac{\Gamma(k\alpha+n\alpha+)}{\Gamma(k\alpha+1)}U_{k+n}$
$\frac{\partial^{n}}{\partial x^{n}}u(x,t)$	$\frac{\partial^{n}}{\partial x^{n}}U_k(x)$
$x^m t^n u(x,t)$	$x^m U_{k-n}(x)$
$x^m t^n$	$x^m\delta(k\alpha-n),\ where\ \delta(k\alpha-n)=\{\begin{array}{l}1\ for\ k\alpha\ =\ n\\ 0\ for\ k\alpha\ \neq\ n\end{array}$

TABLE 1. Basic operations of RDTM.

3.2. **Procedure solution to GFFE by using RDTM.** By operating RDTM to Eq.(1) with the initial condition (2) and by using the related properties of the differential transform, the following recurrence relation is obtained

$$\begin{aligned}\frac{\Gamma(k\alpha+\alpha+1)}{\Gamma(k+1)}U_{k+1}(x)&=\frac{\partial^2}{\partial x^2}U_k(x)\\ &-\mu\sum_{r_1}^{k}\sum_{r_2=0}^{k-r_1}\cdots\sum_{r_\delta}^{k-\sum_{i=1}^{\delta-1}r_i}\frac{\partial}{\partial x}(U_{r_1}(x)U_{r_2}(x)\dots U_{k-\sum_{i=1}^{\delta}r_i}(x))+\gamma U_k(x)\\ &-\gamma\sum_{r_1}^{k}\sum_{r_2=0}^{k-r_1}\cdots\sum_{r}^{k-\sum_{i=1}^{\delta-1}r_i}U_{r_1}(x)U_{r_2}(x)\dots U_{k-\sum_{i=1}^{\delta}r_i}(x),\ k\ \geq\ 0.\end{aligned}\tag{9}$$

With the initial iteration

$$U_0(x)=(0.5\ -0.5\tanh[\frac{\mu\delta}{2(\delta+1)}x])^{\frac{1}{\delta}},\ x\in[0,1].\tag{10}$$

By using Eqs. (9) and (10), the nth order approximation is given by

$$u_n(x,t)=\sum_{k=0}^{n}U_k(x)t^{k\alpha}.\tag{11}$$

4. Laplace Adomain Decomposition Method

Consider the initial value GFFE (1) and (2). In order to apply LADM, at first taking the Laplace transform on both sides of Eq.(1), then by using the differentiation property of Laplace transform and initial condition (2) we get

$$\begin{aligned}\mathfrak{L}[\tfrac{\partial^\alpha u(x,t)}{\partial t^\alpha}]\ &=\tfrac{1}{s}((0.5\ -0.5\tanh[\tfrac{\mu\delta}{2(\delta+1)}x])^{\frac{1}{\delta}})+\tfrac{1}{s^\alpha}\mathfrak{L}[u_{xx}]\\ -\tfrac{1}{s^\alpha}\mathfrak{L}[\mu u^\delta u_x+\gamma u(1-u^\delta)].\end{aligned}\tag{12}$$

The LADM defines the solution as the series

$$u(x,t)=\sum_{k=0}^{\infty}u_k(x,t).\tag{13}$$

And the nonlinear function $N(u(x,t)) = \mu u^{\delta} u_x - \gamma u(1-u^{\delta})$ is decomposed as

$$N(t,u) = \sum_{k=0}^{\infty} A_k(t, u_0, u_1, , u_n). \tag{14}$$

Formally A_n is obtained by

$$A_k = \frac{1}{n!}\frac{d^k}{d\theta^k} N(t, \sum_{i=0}^{\infty}(\theta^i u_i) \mid_{\theta=0},\ k \ \geq \ 0. \tag{15}$$

Where θ is a former parameter. The first few terms of the Adomian polynomial can be derived as follows

$$A_0 = N(t,u_0),\ A_1 = u_1 N^{'}(t,u_0),\ A_2 = u_2 N^{'}(t,u_0) + \frac{1}{2}u_1^2 N^{''}(t,u_0),$$

$$A_3 = u_3 N^{'}(t,u_0) + u_1 u_2 N^{''}(t,u_0) + \frac{1}{6}u_1^3 N^{'''}(t,u_0), \cdots$$

The prime denote the partial derivatives with respect to u, more details in ([43]-[45]).
By substituting Eqs. (13) and (14) into Eq. (12) we obtain

$$\mathfrak{L}[\sum_{k=0}^{\infty} u_k(x,t)] = \frac{1}{s}f(x) + \frac{1}{s^{\alpha}}\mathfrak{L}[\sum_{k=0}^{\infty} u_k(x,t)_{xx}] + \frac{1}{s^{\alpha}}\mathfrak{L}[\sum_{k=0}^{\infty} A_k]. \tag{16}$$

Identifying the zero component; $u_0(x,t)$ by $(0.5\ -0.5\tanh[\frac{\mu\delta}{2(\delta+1)}x])^{\frac{1}{\delta}}$ and matching the two sides of Eq.(16) we have

$$\mathfrak{L}[u_0] = \frac{1}{s}(0.5\ \ - 0.5\tanh[\frac{\mu\delta}{2(\delta+1)}x])^{\frac{1}{\delta}}, \tag{17}$$

$$\mathfrak{L}[u_{k+1}(x,t)] = \frac{1}{s^{\alpha}}\mathfrak{L}[\sum_{k=0}^{\infty} u_k(x,t)_{xx}] + \frac{1}{s^{\alpha}}\mathfrak{L}[\sum_{k=0}^{\infty} A_k]. \tag{18}$$

By applying the inverse Laplace transform, we obtain

$$u_0 = \mathfrak{L}^{-1}[\frac{1}{s}(0.5\ \ - 0.5\tanh[\frac{\mu\delta}{2(\delta+1)}x])^{\frac{1}{\delta}}], \tag{19}$$

$$u_{k+1} = \mathfrak{L}^{-1}[\frac{1}{s^{\alpha}}\mathfrak{L}[\sum_{k=0}^{\infty} u_k(x,t)_{xx}] + \frac{1}{s^{\alpha}}\mathfrak{L}[\sum_{k=0}^{\infty} A_k]],\ k = 0,1,2, \tag{20}$$

The M Approximate solution is given by $\phi_M = \sum_{k=0}^{M-1} u_k$
And the exact solution is $u(x,t) = limt_{M\to\infty}\phi_M$.

In many cases the exact solution in a closed form may be found. Additionally, the decomposition series solutions generally converge very rapidly. The convergence of the decomposition series has been explored by several authors ([43]-[46]). In this respect we refer to [46] in which the authors presented a new numerical study of the Adomain method applied to linear and nonlinear diffusion models.

5. Numerical Results

To show the benefit and the precision of the proposed methods for solving the GFFE (1) and (2), we will present numerical solutions for three special cases of GFFE at different values of $\delta(\delta = 1, 2, 3)$ with different values of μ and γ. The approximate solutions are offered over the domain $x \in (0, 1)$ and $t \in (0, 2)$ for $\delta = 1, 2$ and in the domain $x \in (0, 1)$ and $t \in (0, 5)$ for $\delta = 3$. Comparisons between our approximated results by the fourth approximations of the RDTM and LADM with the exact solution at $\alpha = 1$ and the results given by OHAM [13], ADM [15], HPM [16], CBRBF [17] and [11] are held.

Special case1 ($\delta = 1$)

Solution by LADM

According to Eqs. (19) and (20), the first terms of the LADM will be

$$u_0 = \frac{1}{2} - \frac{1}{2}\ tanh[\frac{\mu x}{4}],$$

$$u_1 = \frac{t^{\alpha}(4\gamma + \mu^2) sech[\frac{x\mu}{4}]^2}{16\Gamma(1+\alpha)},$$

$$u_2 = \frac{t^{2\alpha}(4\gamma + \mu^2)^2 sech[\frac{x\mu}{4}]^2 tanh[\frac{x\mu}{4}]}{64\Gamma(1+2\alpha)}, \cdots$$

And so on, in the same way the remaining terms can be constructed. The approximate solution after five terms: $u_{LADM} = u_0 + u_1 + u_2 + u_3 + u_4$ will be used in the numerical comparisons with some existing methods.

Solution by FRDTM

According to Eqs.(9) and (10), we obtain the following terms

$$U_0 = \frac{1}{2} - \frac{1}{2} tanh[\frac{x\mu}{4}],$$

$$U_1 = \frac{(4\gamma + \mu^2) sech[\frac{x\mu}{4}]^2}{16\Gamma(1+\alpha)},$$

$$U_2 = \frac{(4\gamma + \mu^2)^2 sech[\frac{x\mu}{4}]^2 tanh[\frac{x\mu}{4}]}{64\Gamma(1+2\alpha)}, \cdots$$

And so on. The approximate solution after five terms which determined by the relation $u_{RDTM} = \sum_{k=0}^{4} U_k(x) t^{k\alpha}$ will be used in the numerical comparisons.

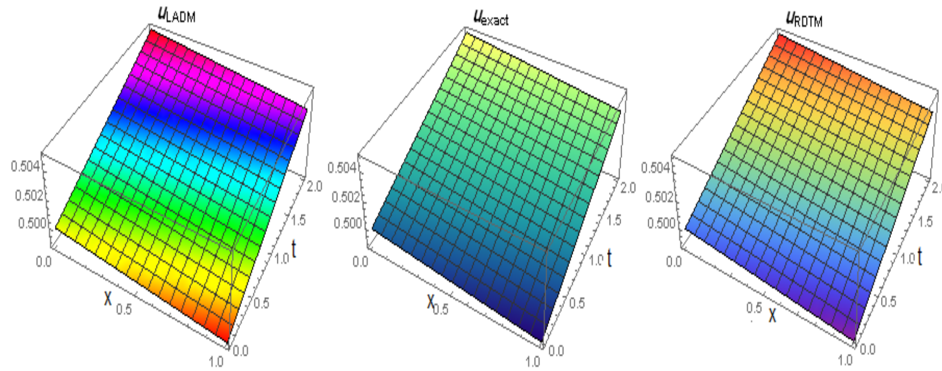

FIGURE 1. The approximate solutions, u_{LADM} and u_{RDTM}, with the exact solution, u_{exact}, at $\alpha = 1$ for special case1.

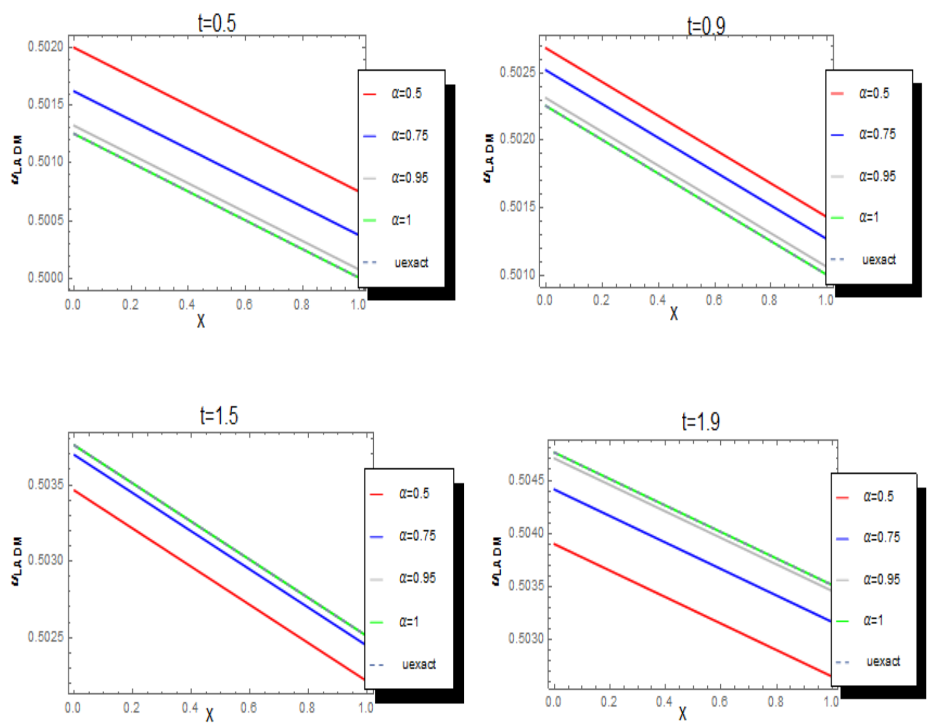

FIGURE 2. The behavior of the approximate solutions by LADM with the exact solution at $t(t = 0.5, 0.9, 1.5, 1.9)$ for different values of $\alpha(\alpha = 0.5, 0.75, 0.95, 1)$ for special case1.

The graphical representation of the evolution results of the approximate solutions of GFFE at $\delta = 1$ and $\mu = \gamma = 10^{-2}$ derived by LADM, RDTM and the exact solution at $\alpha = 1$ are displayed in Figures (1- 3). From the numerical results

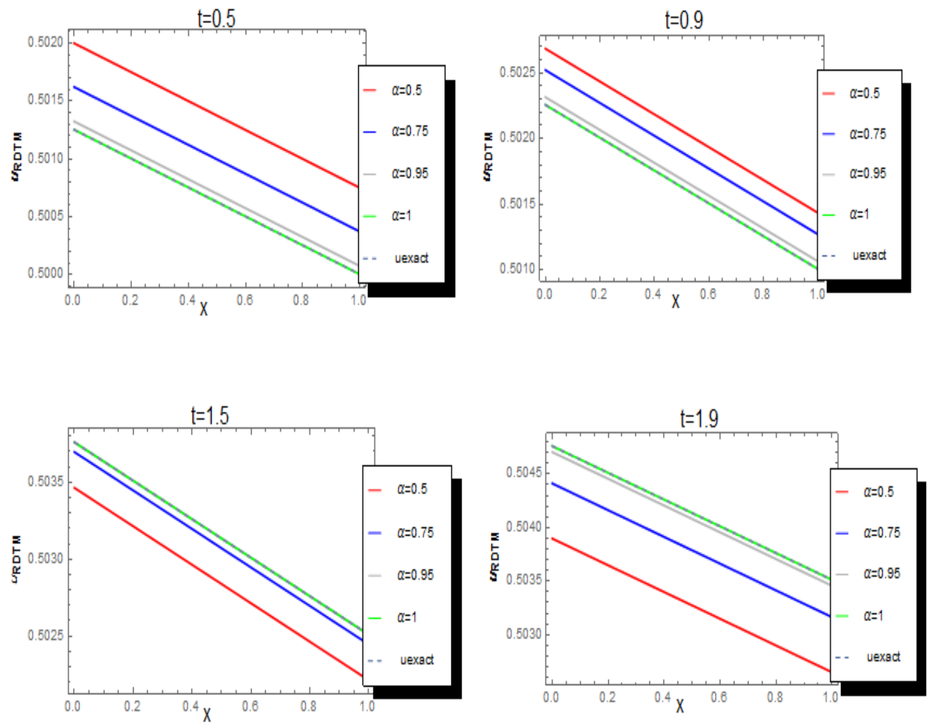

FIGURE 3. The behavior of the approximate solutions by RDTM with the exact solution at $t(t = 0.5, 0.9, 1.5, 1.9)$ for different values of $\alpha(\alpha = 0.5, 0.75, 0.95, 1)$ for special case1.

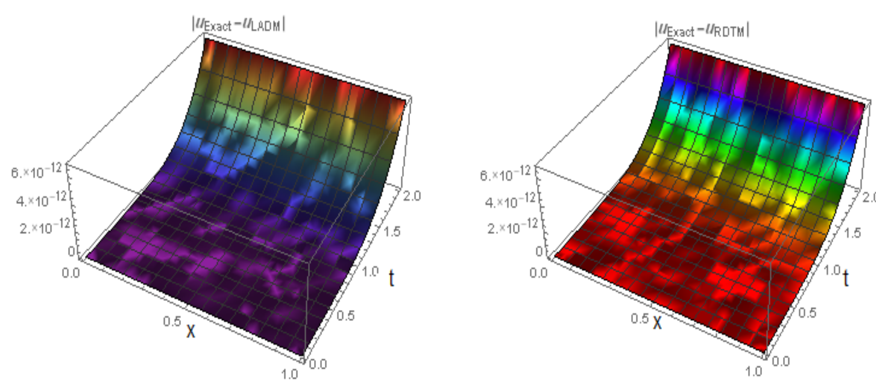

FIGURE 4. Space time surfaces of the absolute errors of GFFE (special case1) by LADM and RDTM.

in Figures 2 and 3, it is easy to conclude that the approximate solutions are continuously depended on the time-fractional derivatives and as the fractional derivative goes to unity the approximate solutions coincide with the exact solutions. Figure 4 displays the space time surfaces of the absolute errors between the two proposed

(*special case* 1) $t = 0.1,\ \gamma = \mu = 0.001$			
x	LADM	FDTM	[11]
0	0	0	2.236×10^{-8}
0.1	0	0	1.988×10^{-8}
0.2	0	0	1.706×10^{-8}
0.3	0	0	1.39×10^{-8}
0.4	0	5.55×10^{-17}	1.040×10^{-8}
0.5	5.55×10^{-17}	0	6.547×10^{-9}
0.6	5.55×10^{-17}	0	2.354×10^{-9}
0.7	0	0	2.182×10^{-9}
0.8	0	0	7.062×10^{-9}
0.9	5.55×10^{-17}	0	1.228×10^{-8}
1	0	0	1.785×10^{-8}

TABLE 2. Comparison of the absolute errors for GFFE (special case1) between LADM, RDTM and the results in [11].

(*special case* 1) $t = 0.1,\ \gamma = \mu = 0.1$			
x	LADM	FDTM	[11]
0	2.357×10^{-13}	2.357×10^{-13}	8.009×10^{-8}
0.1	2.357×10^{-13}	2.357×10^{-13}	7.001×10^{-8}
0.2	2.357×10^{-13}	2.356×10^{-13}	5.985×10^{-8}
0.3	2.356×10^{-13}	2.356×10^{-13}	4.967×10^{-8}
0.4	2.356×10^{-13}	2.355×10^{-13}	3.954×10^{-8}
0.5	2.355×10^{-13}	2.354×10^{-13}	2.954×10^{-8}
0.6	2.354×10^{-13}	2.354×10^{-13}	2.972×10^{-8}
0.7	2.352×10^{-13}	2.351×10^{-13}	1.018×10^{-8}
0.8	2.348×10^{-13}	2.349×10^{-13}	9.795×10^{-10}
0.9	2.348×10^{-13}	2.347×10^{-13}	7.780×10^{-9}
1	2.345×10^{-13}	2.345×10^{-13}	1.601×10^{-9}

TABLE 3. Comparison of the absolute errors for GFFE (special case1) between the LADM, RDTM and the results in [**?**]

methods and the exact solution in the integer order case. The comparisons between the absolute errors for the two proposed methods and the results in [11] at $\alpha = 1$ are given in Tables (2-4) for different values of γ and μ. In Table 5, the comparison between the absolute errors of the two suggested methods and the results in [11], ADM [15] and OHAM [13] at $\gamma = \mu = 0.0001$ is given. While the comparison of the absolute errors for GFFE (special case1) between the two proposed methods and the results in [11], ADM [15] and CBRBF [17] at $\mu = 1,\ \gamma = 0$ is tabulated in Table 6. It is noted that the solutions obtained by LADM are agreeable with that obtained by RDTM and more accurate than the results in CBRBF [17] and [11] in the integer order case.

special Case 2 ($\delta = 2$)

(*special case* 1) $t = 0.1,\ \gamma\ =\ \mu\ =\ 0.5$			
x	LADM	FDTM	[11]
0	$1.173\ \times 10^{-9}$	$1.173\ \times 10^{-9}$	1.669×10^{-6}
0.1	$1.172\ \times 10^{-9}$	$1.1727\ \times 10^{-9}$	1.156×10^{-6}
0.2	$1.169\ \times 10^{-9}$	$1.169\ \times 10^{-9}$	7.771×10^{-7}
0.3	$1.162\ \times 10^{-9}$	$1.1628\ \times 10^{-9}$	4.836×10^{-7}
0.4	$1.153\ \times 10^{-9}$	1.153×10^{-9}	2.670×10^{-7}
0.5	$1.139\ \times 10^{-9}$	1.139×10^{-9}	1.123×10^{-7}
0.6	$1.124\ \times 10^{-9}$	1.125×10^{-9}	6.852×10^{-9}
0.7	$1.106\ \times 10^{-9}$	1.106×10^{-9}	5.971×10^{-8}
0.8	$1.084\ \times 10^{-9}$	1.086×10^{-9}	9.571×10^{-8}
0.9	$1.060\ \times 10^{-9}$	1.064×10^{-9}	1.074×10^{-7}
1	$1.034\ \times 10^{-9}$	1.034×10^{-9}	9.900×10^{-8}

TABLE 4. Comparison of the absolute errors for GFFE (special case1) between LADM, RDTM and the results in [11].

(*special case* 1) $\gamma\ =\ \mu\ = 10^{-3}$						
x	t	LADM	RDTM	[11]	ADM [15]	OHAM [13]
0.1	0.001	5.551×10^{-17}	5.551×10^{-17}	1.97×10^{-8}	1.94×10^{-6}	4.25×10^{-8}
	0.005	0.000	5.551×10^{-17}	1.97×10^{-8}	9.69×10^{-6}	2.12×10^{-7}
	0.01	0.000	0.000	1.97×10^{-8}	1.94×10^{-6}	4.25×10^{-7}
0.5	0.001	5.551×10^{-17}	5.551×10^{-17}	3.58×10^{-9}	1.94×10^{-6}	4.58×10^{-8}
	0.005	5.551×10^{-17}	5.551×10^{-17}	3.71×10^{-9}	9.69×10^{-6}	2.29×10^{-7}
	0.01	0.000	0.000	3.88×10^{-9}	1.94×10^{-6}	4.25×10^{-7}
0.9	0.001	5.551×10^{-17}	0.000	1.80×10^{-8}	1.94×10^{-6}	4.58×10^{-8}
	0.005	0.000	0.000	1.77×10^{-8}	9.69×10^{-6}	2.29×10^{-7}
	0.01	0.000	5.551×10^{-17}	1.74×10^{-8}	1.94×10^{-6}	4.25×10^{-7}

TABLE 5. Comparison of the absolute errors for GFFE (special case1) between the LADM, RDTM, and the results in [11], ADM [15] and OHAM [13].

Solution by LADM

Permitting to Eqs. (19) and (20). The first few terms of the approximate solutions are

$$u_0 = (\frac{1}{2} - \frac{1}{2}tanh[\frac{\mu x}{3}])^{\frac{1}{2}},$$

$$u_1 = \frac{(e^{\frac{2x\mu}{3}}(1+e^{\frac{2x\mu}{3}})^{(-\frac{3}{2})}t^{\alpha}(9\gamma+\mu^2))}{9\Gamma(1+\alpha)},$$

$$u_2 = \frac{e^{\frac{2x\mu}{3}}(-2+e^{\frac{2x\mu}{3}})(1+e^{\frac{2x\mu}{3}})^{(-\frac{5}{2})}t^{2\alpha}(9\gamma+\mu^2)^2}{81\Gamma(1+2\alpha)}, \cdots$$

(*special case* 1) $\gamma\ = 0\ \mu\ = 1$						
t	x	LADM	RDTM	[11]	ADM [15]	CBRBF [17]
0.5	0.1	6.342×10^{-8}	6.342×10^{-8}	1.14×10^{-7}	6.34×10^{-8}	2.00×10^{-6}
	0.5	5.667×10^{-8}	5.667×10^{-8}	1.13×10^{-7}	5.66×10^{-8}	1.00×10^{-6}
	0.9	4.1280×10^{-8}	4.1280×10^{-8}	1.65×10^{-6}	4.12×10^{-8}	9.00×10^{-6}
1	0.1	2.029×10^{-6}	2.029×10^{-6}	1.17×10^{-6}	2.02×10^{-6}	3.00×10^{-6}
	0.5	1.847×10^{-6}	1.847×10^{-6}	3.79×10^{-8}	1.84×10^{-6}	2.00×10^{-6}
	0.9	1.380×10^{-6}	1.380×10^{-6}	1.28×10^{-6}	1.37×10^{-6}	9.00×10^{-6}
2	0.1	6.428×10^{-5}	6.428×10^{-5}	8.44×10^{-7}	6.42×10^{-5}	4.00×10^{-6}
	0.5	6.069×10^{-5}	6.069×10^{-5}	1.16×10^{-7}	6.06×10^{-5}	2.00×10^{-6}
	0.9	4.753×10^{-5}	4.753×10^{-5}	9.72×10^{-7}	4.755×10^{-5}	9.00×10^{-6}

TABLE 6. Comparison of the absolute errors for GFFE (special case1) between LADM, RDTM and the results in ADM [15], CBRBF [17] and [11].

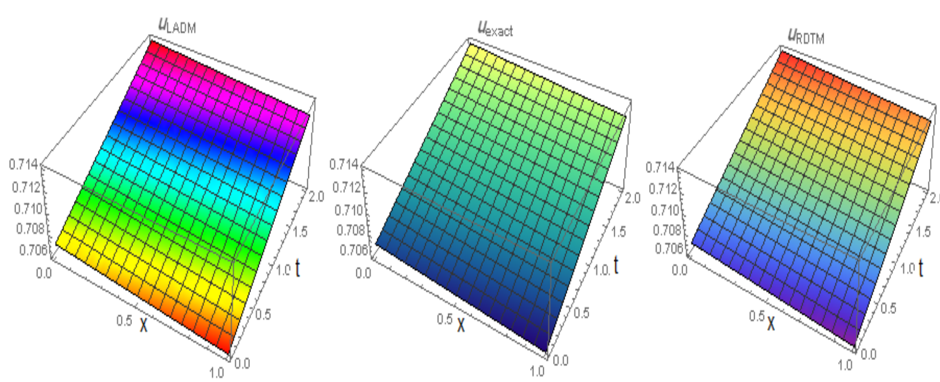

FIGURE 5. The approximate solutions, u_{LADM} *and* u_{RDTM}, with the exact solution, u_{exact}, at $\alpha = 1$ for special case2.

And so on, the approximate solution after five terms which determined by the relation $u_{LADM} = \sum_{i=0}^{4} u_i$ are used for the numerical analysis.

Solution by RDTM

By using Eqs. (9) and (10), we obtain

$$U_0 = \sqrt{(\frac{1}{2} - \frac{1}{2} tanh[\frac{\mu x}{3}])},$$

$$U_1 = \frac{(e^{\frac{2x\mu}{3}}(1 + e^{\frac{2x\mu}{3}})^{(-\frac{3}{2})} t^{\alpha}(9\gamma + \mu^2))}{9\Gamma(1 + \alpha)},$$

$$U_2 = \frac{e^{\frac{2x\mu}{3}}(-2 + e^{\frac{2x\mu}{3}})(1 + e^{\frac{2x\mu}{3}})^{(-\frac{5}{2})} t^{2\alpha}(9\gamma + \mu^2)^2}{81\Gamma(1 + 2\alpha)}, \cdots$$

and so on. The approximate solution after the fifth term will be $u_{RDTM} = \sum_{i=0}^{4} U_i t^{\alpha i}$.

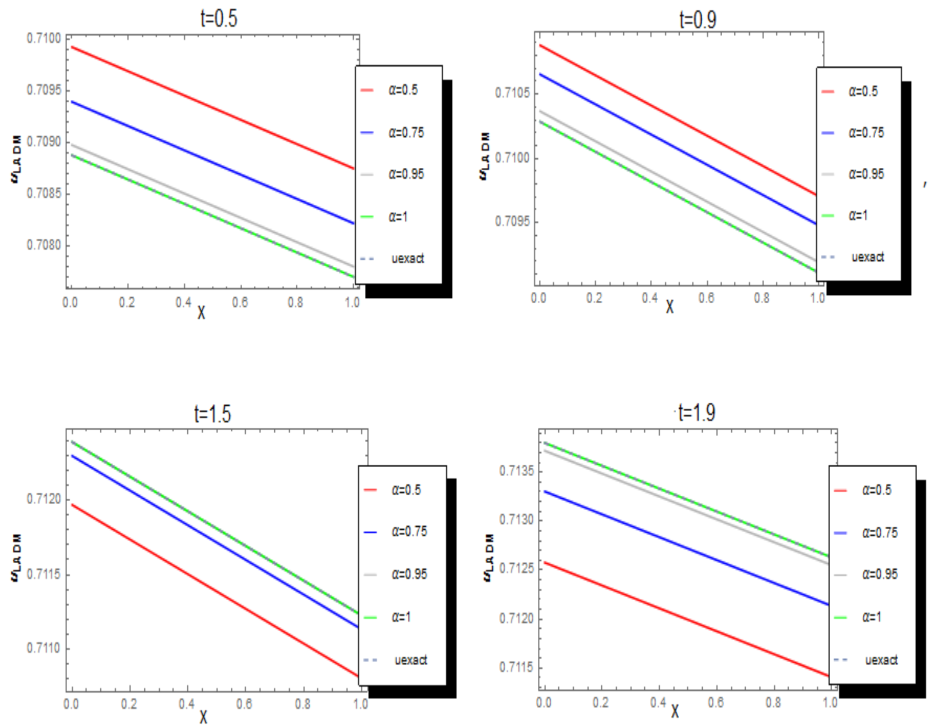

FIGURE 6. The behavior of u_{LADM} at different values of $\alpha(\alpha = 0.5, 0.75, 0.95, 1)$ with the exact solution at $t(t = 0.5, 0.9, 1.5, 1.9)$ for special case2.

(*special case* 2) $t = 0.1,\ \gamma = \mu = 1$			
x	LADM	RDTM	[11]
0	3.550×10^{-7}	3.550×10^{-7}	1.396×10^{-6}
0.1	3.927×10^{-7}	3.927×10^{-7}	8.651×10^{-7}
0.2	4.241×10^{-7}	4.241×10^{-7}	3.226×10^{-7}
0.3	4.486×10^{-7}	4.486×10^{-7}	2.146×10^{-7}
0.4	4.658×10^{-7}	4.658×10^{-7}	7.568×10^{-7}
0.5	4.752×10^{-7}	4.752×10^{-7}	1.303×10^{-6}
0.6	4.771×10^{-7}	4.771×10^{-7}	1.859×10^{-6}
0.7	4.714×10^{-7}	4.714×10^{-7}	2.436×10^{-6}
0.8	4.587×10^{-7}	4.587×10^{-7}	3.047×10^{-6}
0.9	4.394×10^{-7}	4.394×10^{-7}	3.704×10^{-6}
1	4.144×10^{-7}	4.144×10^{-7}	4.818×10^{-6}

TABLE 7. Comparison of the absolute errors for GFFE (special case2) between LADM, RDTM and the results in [11].

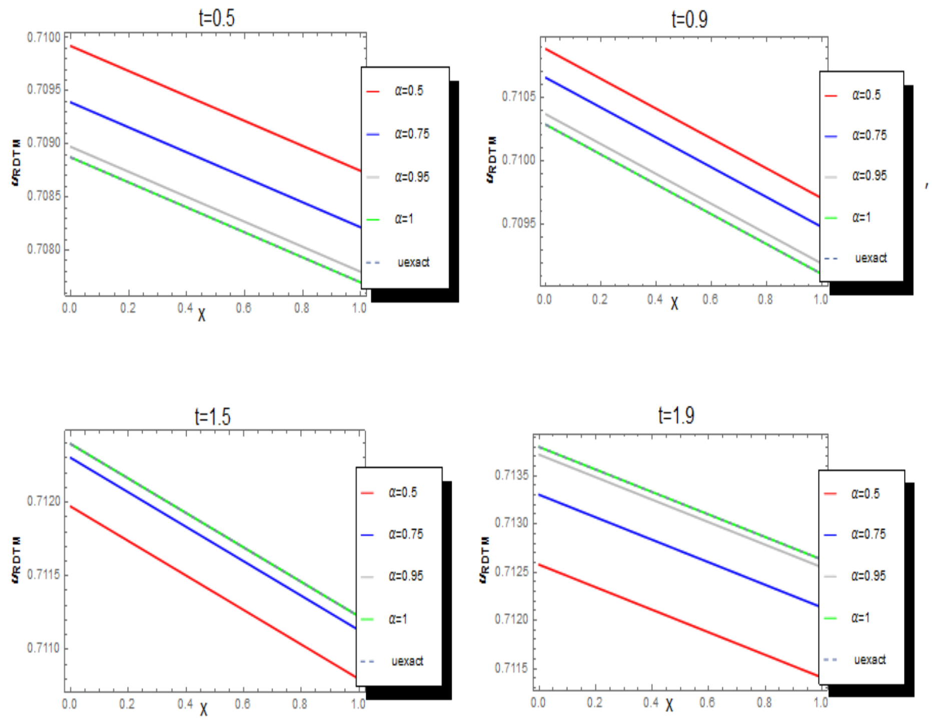

FIGURE 7. The behavior of u_{RDTM} at different values of $\alpha(\alpha = 0.5, 0.75, 0.95, 1)$ with the exact solution at $t(t = 0.5, 0.9, 1.5, 1.9)$ for especial case2.

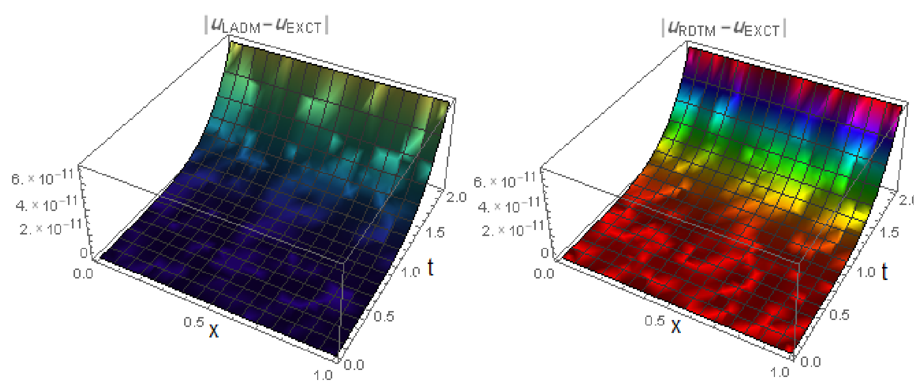

FIGURE 8. The space-time surfaces of the absolute errors of LADM and RDTM for special case2 ($\delta = 2$).

The space time surfaces of the approximate solutions of GFFE at $\delta = 2$ and $\mu = \gamma = 10^{-2}$ derived by LADM, RDTM and the exact solution at $\alpha = 1$ are displayed in Figure 5. The behaviors of the approximate solutions for different values of at α by LADM and RDTM with the exact solution at $\alpha = 1$ are shown

(*special case* 2) $\gamma = \mu = 1$						
x	t	LADM	FDTM	[11]	ADM [15]	OHAM[13]
0.1	0.0001	0.000	0.000	1.08×10^{-6}	2.80×10^{-4}	1.17×10^{-5}
	0.0005	1.110×10^{-16}	1.110×10^{-16}	1.08×10^{-6}	1.40×10^{-3}	5.87×10^{-5}
	0.01	1.110×10^{-16}	0.000	1.08×10^{-6}	2.80×10^{-3}	1.17×10^{-4}
0.5	0.0001	0.000	0.000	1.14×10^{-6}	2.69×10^{-4}	5.33×10^{-5}
	0.0005	0.000	0.000	1.14×10^{-6}	1.34×10^{-3}	1.06×10^{-5}
	0.01	2.220×10^{-16}	2.220×10^{-16}	1.14×10^{-6}	2.69×10^{-3}	1.06×10^{-5}
0.9	0.0001	0.000	0.000	4.12×10^{-6}	2.55×10^{-4}	9.29×10^{-6}
	0.0005	1.110×10^{-16}	0.000	4.12×10^{-6}	1.27×10^{-3}	4.64×10^{-5}
	0.01	1.1102×10^{-16}	1.110×10^{-16}	4.12×10^{-6}	2.55×10^{-3}	9.29×10^{-4}

TABLE 8. Comparison of the absolute errors for GFFE (special case2) between the two proposed methods and the results in [11], ADM [15] and OHAM [13].

(*special case* 2) $\gamma = 0$ $\mu = 1$						
t	x	LADM	FDTM	[11]	ADM[15]	CBRBF [17]
0.5	0.1	1.258×10^{-8}	1.258×10^{-8}	7.43×10^{-7}	1.25×10^{-8}	1.00×10^{-6}
	0.5	1.491×10^{-8}	1.491×10^{-8}	1.16×10^{-6}	1.49×10^{-8}	2.00×10^{-6}
	0.9	1.380×10^{-8}	1.355×10^{-8}	2.38×10^{-6}	1.39×10^{-8}	$----$
1	0.1	3.927×10^{-7}	3.927×10^{-7}	2.94×10^{-6}	1.25×10^{-8}	——
	0.5	4.752×10^{-7}	4.752×10^{-7}	3.22×10^{-6}	4.75×10^{-7}	1.00×10^{-6}
	0.9	4.394×10^{-7}	4.394×10^{-7}	4.20×10^{-6}	4.39×10^{-7}	2.00×10^{-6}
2	0.1	1.186×10^{-5}	1.186×10^{-5}	1.18×10^{-5}	2.00×10^{-6}	2.00×10^{-6}
	0.5	1.500×10^{-5}	1.500×10^{-5}	1.49×10^{-5}	5.00×10^{-6}	5.00×10^{-6}
	0.9	1.436×10^{-5}	1.436×10^{-5}	1.43×10^{-5}	1.000×10^{-6}	1.00×10^{-6}

TABLE 9. Comparison of the absolute errors for GFFE (special case2) between the two proposed methods and the results in [11], ADM[15] and CBRBF [17].

in Figures 6 and 7 respectively. The absolute error surfaces of LADM and RDTM are shown in Figure 8. The comparisons between the absolute errors for our anticipated methods and the results in [11], ADM [15] and OHAM [13] at $\alpha = 1$ are given in Tables 7 and 8 for $\mu = \gamma = 1$. Table 9 contains the numerical results of the comparison between the absolute errors for our anticipated methods and the results in [11], ADM [15] and CBRBF [17] at $\alpha = 1$ and $\mu = 1, \gamma = 0$. These numerical results demonstrate that the approximate results of the two suggested implementations are in a good agreement with the exact solution at $\alpha = 1$. As the order of the fractional derivatives approaches the unity the approximate results are in a good agreement with each other and with the exact solution. It is noted that the solutions obtained by the two proposed techniques are more accurate than the results in [11], ADM [15] and OHAM [13] in the integer order case.

Special Case 3 ($\delta = 3$)
Solution by LADM
By using Eqs. (19) and (20), we have

(*special case* 3) $\gamma\ = 0\ \mu\ = 1$						
t	x	LADM	FDTM	[11]	ADM [15]	CBRBF [17]
0.0001	0.1	0.000	0.000	4.55×10^{-7}	4.46×10^{-4}	——
	0.5	0.000	0.000	5.66×10^{-7}	1.86×10^{-3}	− − − − −
	0.9	0.0000	0.000	7.00×10^{-7}	9.32×10^{-4}	− − −−
0.0005	0.1	0.000	0.000	4.57×10^{-7}	4.45×10^{-4}	6.00×10^{-6}
	0.5	0.000	0.000	5.63×10^{-7}	1.85×10^{-3}	5.00×10^{-6}
	0.9	0.000	0.000	6.98×10^{-7}	9.20×10^{-4}	4.00×10^{-6}
0.001	0.1	0.000	0.000	4.60×10^{-7}	4.44×10^{-4}	1.90×10^{-5}
	0.5	0.000	0.000	5.61×10^{-7}	1.85×10^{-3}	1.60×10^{-5}
	0.9	0.000	0.000	6.95×10^{-7}	19.05×10^{-4}	1.50×10^{-5}

Table 10. Absolute errors for GFFE at $\delta = 3$ between the two proposed methods and [11], ADM[15] and CBRBF[17].

$$u_0 = (\frac{1}{2} - \frac{1}{2}tanh[\frac{3x\mu}{8}])^{\frac{1}{3}},$$

$$u_1 = \frac{(e^{\frac{3x\mu}{4}}(1+e^{\frac{3x\mu}{4}})^{(-\frac{4}{3})}t^{\alpha}(16\gamma+\mu^2))}{16\Gamma(1+\alpha)},$$

$$u_2 = \frac{e^{\frac{3x\mu}{4}}(-3+e^{\frac{3x\mu}{4}})(1+e^{\frac{3x\mu}{4}})^{(-\frac{7}{3})}t^{2\alpha}(16\gamma+\mu^2)^2}{256\Gamma(1+2\alpha)}, \cdots$$

And so on. The remaining components can be determined by the same way. In the following numerical results the approximated solution after five terms will be used.

Solution by RDTM

According to Eqs.(9) and (10), the first few transformed terms are

$$U_0 = (\frac{1}{2} - \frac{1}{2}tanh[\frac{2x\mu}{8}])^{\frac{1}{3}},$$

$$U_1 = \frac{(e^{\frac{3x\mu}{4}}(1+e^{\frac{3x\mu}{4}})^{(-\frac{4}{3})}(16\gamma+\mu^2))}{16\Gamma(1+\alpha)},$$

$$U_2 = \frac{e^{\frac{3x\mu}{4}}(-3+e^{\frac{3x\mu}{4}})(1+e^{\frac{3x\mu}{4}})^{(-\frac{7}{3})}(16\gamma+\mu^2)^2}{256\Gamma(1+2\alpha)}, \cdots$$

And So on. The approximate solution after five terms is used for the numerical comparisons and determined by $u_{RDTM} = \sum_{i=0}^{4} U_i t^{\alpha i}$.

The progress of the approximate solutions of the generalized fractional time Fisher equation at $\delta = 3$ copied by LADM, RDTM and the exact solution for $\mu = \gamma = 10^{-2}$ are presented by Figures (9-12). The comparison between the absolute errors for our two proposed methods and the results in [11] at $\alpha = 1$ is given in Table 10. The numerical comparisons of the absolute errors obtained by our suggested methods and the results obtained by [11] and ADM [15] and CBRBF [17] are tabulated in Tables 10 and 11 for $\mu = 1, \gamma = 0$.

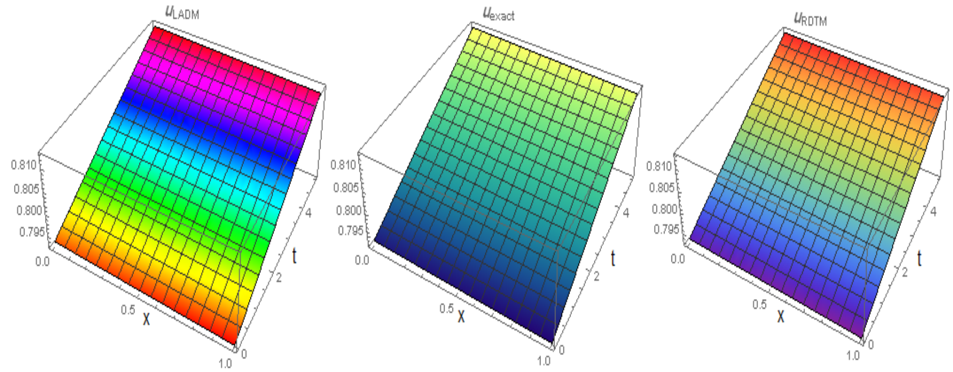

FIGURE 9. The approximate solutions of LADM and the RDTM with the exact solution at $\alpha = 1, \delta = 3$.

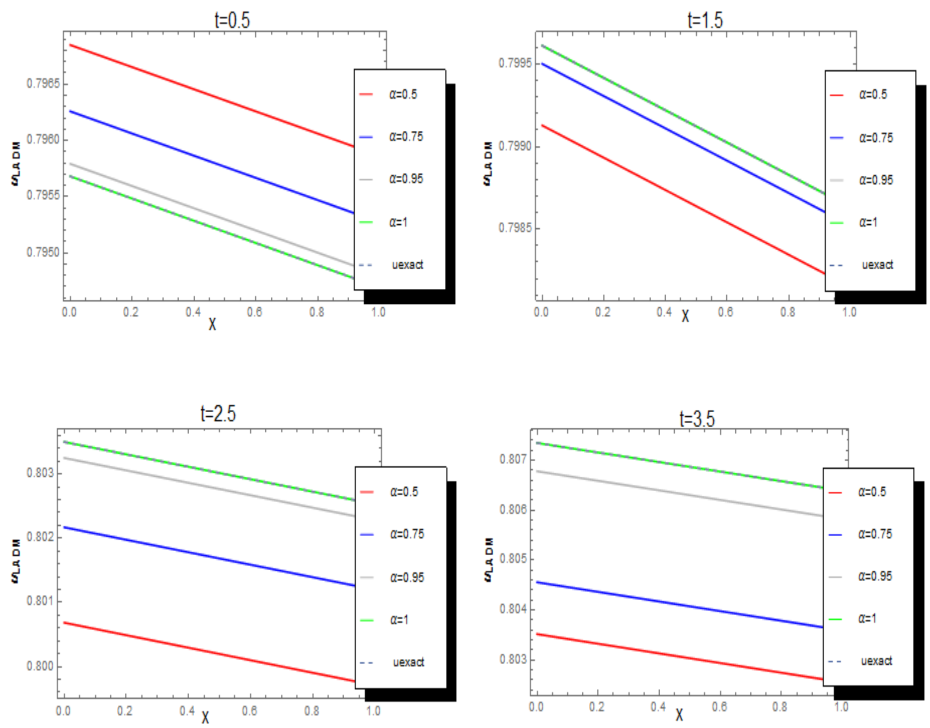

FIGURE 10. The approximate solutions of LADM for different values of α with the exact solution at $\alpha = 1$ at $t(t = 0.5, 1.5, 2.5, 3.5)$ and $(\delta = 3)$.

6. Conclusion

In this work, the LADM and the RDTM have been successfully employed to GFFE. The two suggested methods presented the solution as a convergent series

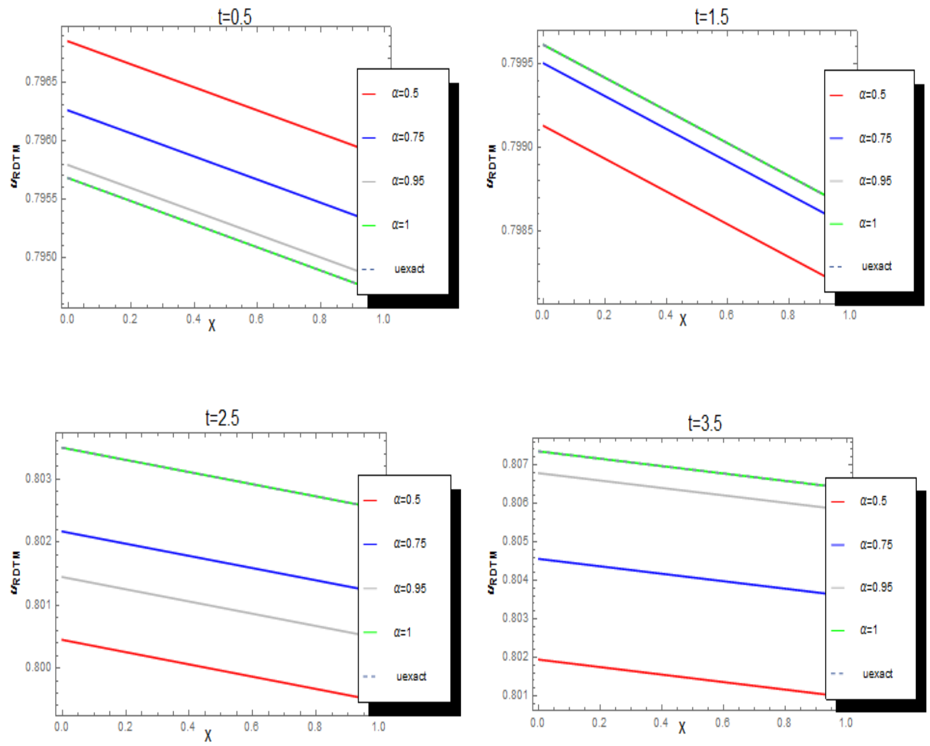

FIGURE 11. The approximate solutions of RDTM for different values of α with the exact solution at $\alpha = 1$ at $t(t = 0.5, 1.5, 2.5, 3.5)$ and $(\delta = 3)$.

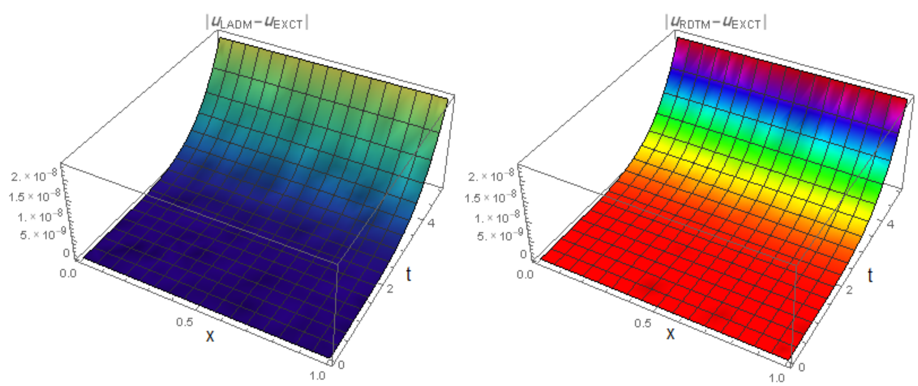

FIGURE 12. The space time surfaces of the absolute errors for the two proposed methods at $\delta = 3$.

with simply computable components. The effectiveness of the suggested techniques was confirmed by numerical comparisons with the exact solution and with the results in [11], [15], [17] and [13] in the integer order case. It is noted that only five terms of the decomposition series and only the fifth order term solutions of

(*special case* 3) $\gamma\ = 0\ \mu\ = 1$					
t	x	LADM	FDTM	[11]	CBRBF [17]
0.5	0.1	2.640×10^{-9}	2.640×10^{-9}	$1.000{\times}10^{-6}$	$3.000{\times}10^{-6}$
	0.5	3.705×10^{-9}	3.705×10^{-9}	$1.000{\times}10^{-6}$	$7.000{\times}10^{-6}$
	0.9	3.644×10^{-9}	3.644×10^{-9}	1.00×10^{-6}	1.000×10^{-6}
1	0.1	8.176×10^{-8}	8.176×10^{-8}	2.000×10^{-6}	2.000×10^{-6}
	0.5	1.176×10^{-7}	1.176×10^{-7}	2.000×10^{-6}	8.000×10^{-6}
	0.9	1.175×10^{-7}	1.175×10^{-7}	3.000×10^{-6}	1.000×10^{-6}
2	0.1	2.434×10^{-6}	2.434×10^{-6}	5.000×10^{-6}	3.000×10^{-6}
	0.5	3.681×10^{-6}	3.681×10^{-6}	5.000×10^{-6}	8.000×10^{-6}
	0.9	3.799×10^{-6}	3.799×10^{-6}	6.000×10^{-6}	1.000×10^{-6}
5	0.1	1.799×10^{-4}	1.799×10^{-4}	1.2×10^{-5}	4.000×10^{-6}
	0.5	3.244×10^{-4}	3.244×10^{-4}	1.3×10^{-5}	1.3×10^{-5}
	0.9	3.703×10^{-4}	3.703×10^{-4}	1.4×10^{-5}	3.000×10^{-6}

Table 11. Absolute errors for GFFE at ($\delta = 3$) between the two proposed methods and [11] and CBRBF[17].

the RDTM were used to evaluate the approximations for the concerned problem. It is obvious that the efficiency of the anticipated approaches can be increased by calculating more terms or further components, it is distinguished that the solutions depend on the time fractional derivatives. Moreover the solutions obtained by LADM are covenant with that obtained by RDTM and more accurate than the results in [11], [15], [17] and [13].

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