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THE $\bar{H}$-FUNCTION AND SRIVASTAVA'S POLYNOMIALS INVOLVING THE GENERALIZED MELLIN-BARNES CONTOUR INTEGRALS

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Abstract. The main object of the present article is to evaluate three definite integrals involving the $\bar{H}$-function together with the general class of Srivastava's polynomials. Some known results follow as special cases of our findings.

1. Introduction

The generalization of H-function is defined by Inayat-Hussain [6, 7] and represented in the following manner:

$$\bar{H}_{p,q}^{m,n}[z] = \bar{H}_{p,q}^{m,n}\left[z \left| \begin{array}{l} (a_j, \alpha_j, A_j)_{1,n}, (a_j, \alpha_j)_{n+1,p} \\ (b_j, \beta_j)_{1,m}, (b_j, \beta_j, B_j)_{m+1,q} \end{array} \right.\right] = \frac{1}{2\pi i}\int_L \bar{\phi}(\xi) z^{\xi} d\xi \tag{1}$$

where

$$\bar{\phi}(\xi) = \frac{\prod_{j=1}^{m}\Gamma(b_j - \beta_j\xi)\prod_{j=1}^{n}\{\Gamma(1 - a_j + \alpha_j\xi)\}^{A_j}}{\prod_{j=m+1}^{q}\{\Gamma(1 - b_j + \beta_j\xi)\}^{B_j}\prod_{j=n+1}^{p}\Gamma(a_j - \alpha_j\xi)} \tag{2}$$

which contains fractional powers of some of the gamma function $L = L_{i\infty}$ is a contour initially at the point $\tau - i\,\infty$, terminating at the point $\tau + i\,\infty$ with $\tau \in R = (-\infty, \infty)$. Here z may be real pr complex but is not to zero and an empty product is interpreted as unity; m, n, p, q are integers such that $1 \leq m \leq q, 1 \leq n \leq p$, are positive real numbers and $(A_j)_{1,\,n}, (B_j)_{m+1,\,q}$ may take non-integer values, which we assume to be positive for standardization purpose. $(a_j)_{1,\,p}$ and $(b_j)_{1,\,q}$ are complex numbers. The following sufficient condition for absolute convergence of the contour integral for the $\bar{H}$-function given by Bushman and Srivastava [2];

$$\mu_1 = \sum_{j=1}^{m}|\beta_j| - \sum_{j=m+1}^{q}|\beta_j B_j| + \sum_{j=1}^{n}|\alpha_j A_j| - \sum_{j=n+1}^{p}|\alpha_j| > 0, \tag{3}$$

The condition provides exponential decay if the integrand (1), and region absolute convergence of (1) is given by $|\arg z| < (\pi/2)\,\mu_1$. The behavior of the $\bar{H}$-

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function for small values of $|z|$ follows easily from a result given by Rathie [12]:

$$\bar{H}_{p,q}^{m,n}[z] = O\left(|z|^{-\alpha}\right), \tag{4}$$

where $\alpha = \min\limits_{1 \le j \le n}\left(\frac{-Re(b_j)}{B_j}\right)$.
For a detailed definition, convergence and existence conditions, and for a computable representation of the $\bar{H}$- function the reader is referred to the original paper of Buschman and Srivastava [2], Saxena [13].

Special cases of $\bar{H}$-function:

(i) On setting on exponents$A_j = B_j = 1, \forall\, i, j$, then $\bar{H}$-function reduces to the familiar Fox's H-function defined by Fox [4]; see also Mathai and Saxena [9]:

$$\bar{H}_{p,q}^{m,n}\left[z\,\middle|\,\begin{array}{l}(a_j,\alpha_j,1)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j)_{1,m},(b_j,\beta_j,1)_{m+1,q}\end{array}\right] = H_{p,q}^{m,n}\left[z\,\middle|\,\begin{array}{l}(a_1,\alpha_1),...,(a_p,\alpha_p)\\(b_1,\beta_1),...,(b_q,\beta_q)\end{array}\right] \tag{5}$$

(ii) On setting $A_j = B_j = 1, \alpha_j = \beta_j = 1, \forall\, i, j$, then $\bar{H}$-function reduces to the general type of the G-function [10];

$$\bar{H}_{p,q}^{m,n}\left[z\,\middle|\,\begin{array}{l}(a_j,1,1)_{1,n},(a_j,1)_{n+1,p}\\(b_j,1)_{1,m},(b_j,1,1)_{m+1,q}\end{array}\right] = G_{p,q}^{m,n}\left[z\,\middle|\,\begin{array}{l}(a_j,1)_{1,p}\\(b_j,1)_{1,q}\end{array}\right] \tag{6}$$

(iii) On setting $n = p, m = 1, q = q+1, b_1 = 0, \beta_1 = 1, a_j = 1 - a_j, b_j = 1 - b_j$, then reduces to generalized Wright hypergeometric function [19]

$$\bar{H}_{p,\,q+1}^{1,p}\left[-z\,\middle|\,\begin{array}{l}(1-a_j,\alpha_j,A_j)_{\;1,p}\\(0,1),(1-b_j,\beta_j,B_j)_{1,q}\end{array}\right] = {}_p\bar{\psi}_q\left[\begin{array}{r}(a_j,\alpha_j,A_j)_{1,p};\\z\\(b_j,\beta_j,B_j)_{1,q};\end{array}\right] \tag{7}$$

The general class of polynomials is defined by Srivastava ([16], p. 1, Eq. 1) in the following manner :

$$S_w^u[x] = \sum_{l=0}^{[w/u]} \frac{(-w)_{ul}}{l!} A_{w,l}\, x^l, \quad w = 0, 1, 2, ... \tag{8}$$

where u is an arbitrary positive integer and the coefficients $A_{w,l}\,(w, l \ge 0)$ are arbitrary constants, real or complex.

Results required in the sequel:

For $(a > 0,\ b \ge 0,\ c + 4ab > 0,\ (\Re(p) + 1/2) > 0$, the following formula is defined by Qureshi et al. ([11], p.77, eq. (3.1)-(3.3)).

$$\int_0^\infty \left[\left(ax + \frac{b}{x}\right)^2 + c\right]^{-p-1} dx = \frac{\sqrt{\pi}}{2a\,(4ab+c)^{p+\frac{1}{2}}} \frac{\Gamma\left(p+\frac{1}{2}\right)}{\Gamma(p+1)}, \tag{9}$$

For $(a \ge 0,\ b > 0,\ c + 4ab > 0,\ (\Re(p) + 1/2) > 0$,

$$\int_0^\infty \frac{1}{x^2}\left[\left(ax + \frac{b}{x}\right)^2 + c\right]^{-p-1} dx = \frac{\sqrt{\pi}}{2b\,(4ab+c)^{p+\frac{1}{2}}} \frac{\Gamma\left(p+\frac{1}{2}\right)}{\Gamma(p+1)}, \tag{10}$$

For $(a > 0,\ b > 0,\ c + 4ab > 0,\ (\Re(p) + 1/2) > 0$,

$$\int_0^\infty \left(a + \frac{b}{x^2}\right)\left[\left(ax + \frac{b}{x}\right)^2 + c\right]^{-p-1} dx = \frac{\sqrt{\pi}}{(4ab+c)^{p+\frac{1}{2}}} \frac{\Gamma\left(p+\frac{1}{2}\right)}{\Gamma(p+1)}, \tag{11}$$

The following formulas ([15], p.75) will be required in our investigation

$$(1-y)^{a+b-c}\,{}_2F_1(2a,2b;2c;y)=\sum_{r=0}^{\infty}a_r y^r, \tag{12}$$

and

$${}_2F_1\left(a,\,b;c+\frac{1}{2};x\right){}_2F_1\left(c-a,\,c-b;c+\frac{1}{2};x\right)=\sum_{r=0}^{\infty}\frac{(c)_r}{\left(c+\frac{1}{2}\right)_r}a_r\,x^r. \tag{13}$$

In several centuries, applications with several properties of special functions have been used in applied mathematics and computing sciences. Under different definite integral, the illustration of image formulas involving, one or more variable of special functions, are significant in the assessment of generalized integrals, applied physics and in many branches of engineering. Motivated essentially by diverse applications of these results, a number of integrals involving Srivastava's polynomials and verity of special functions have been developed by many authors. Recently, certain generalized Gradshteyn–Ryzhil type integrals have been established, see [3, 17, 18]. Thus, the main object of this investigation is to found three definite Gradshteyn–Ryzhil type integrals involving $\bar{H}$-function (1) and general class of Srivastava's polynomials defined by (8). By virtue of the unified nature of our results, a large amount of new results can be constructing as particular cases from our leading results.

2. **Main Integral formulas**

Here, we establish three theorems as:

Theorem 2.1 Let $a>0$; $b\geq 0; c+4ab>0$; k is a positive integer, such that $-\frac{1}{2}<(a-b-c)<\frac{1}{2}$; and $|\arg z|<\frac{1}{2}\mu_1\pi$, then the following formula holds:

$$\begin{aligned}&\int_0^{\infty}X^{-\lambda-1}{}_2F_1\left(a,b;c+\frac{1}{2};X\right){}_2F_1\left(c-a,c-b;c+\frac{1}{2};X\right)\\&\qquad\times S_w^u[y/X]\bar{H}_{p,q}^{m,n}\left[z/X^k\right]dx\\&=\frac{\sqrt{\pi}}{2a\,(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\frac{(c)_r}{\left(c+\frac{1}{2}\right)_r}\frac{a_r}{(4ab+c)^{-r}}\sum_{l=0}^{[w/u]}\frac{(-w)_{ul}}{l!}A_{w,l}\left(\frac{y}{4ab+c}\right)^l\\&\times\bar{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4\mathrm{ab}+\mathrm{c})^k}\;\middle|\;\begin{matrix}\left(\frac{1}{2}+r-l-\lambda,k;1\right),(a_j,\alpha_j,A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j,B_j)_{1,m},(b_j,\beta_j)_{m+1,q},(r-l-\lambda,k;1)\end{matrix}\right],\end{aligned}\tag{14}$$

where $X=\left(ax+\frac{b}{x}\right)^2+c$.

Proof. By virtue of equation (1), (8), (9) and (13), we have the following

$$\begin{aligned}&\int_0^{\infty}X^{-\lambda-1}{}_2F_1\left(a,b;c+\frac{1}{2};X\right){}_2F_1\left(c-a,c-b;c+\frac{1}{2};X\right)\\&\qquad\times S_w^u[y/X]\bar{H}_{p,q}^{m,n}\left[z/X^k\right]dx\\&=\int_0^{\infty}\left\{X^{-\lambda-1}\sum_{r=0}^{\infty}\frac{(c)_r}{\left(c+\frac{1}{2}\right)_r}a_rX^r\sum_{l=0}^{[w/u]}\frac{(-w)_{ul}}{l!}A_{w,l}y^lX^{-l}\right.\end{aligned}$$

$$\times \frac{1}{(2\pi i)} \int_L \bar{\phi}(\xi) z^{\xi} X^{-k\xi} d\xi \Big\} dx,$$

$$= \sum_{r=0}^{\infty} \frac{(c)_r}{\left(c+\frac{1}{2}\right)_r} a_r \sum_{l=0}^{[w/u]} \frac{(-w)_{ul}}{l!} A_{w,l} y^l \frac{1}{2\pi i} \int_L \bar{\phi}(\xi) \left\{ \int_0^{\infty} X^{-\lambda-1+r-l-k\xi} dx \right\} z^{\xi} d\xi \,,$$

$$= \sum_{r=0}^{\infty} \frac{(c)_r}{\left(c+\frac{1}{2}\right)_r} a_r \sum_{l=0}^{[w/u]} \frac{(-w)_{ul}}{l!} A_{w,l} y^l$$

$$\times \frac{1}{2\pi i} \int_L \bar{\phi}(\xi) \left\{ \frac{\sqrt{\pi}}{2a\,(4ab+c)^{\lambda+k\xi-r+l+\frac{1}{2}}} \frac{\Gamma\left(\lambda+k\xi-r+l+\frac{1}{2}\right)}{\Gamma\left(\lambda+k\xi-r+l+1\right)} \right\} z^{\xi} d\xi \,,$$

$$= \frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}} \sum_{r=0}^{\infty} \frac{(c)_r}{\left(c+\frac{1}{2}\right)_r} \frac{a_r}{(4ab+c)^{-r}} \sum_{l=0}^{[w/u]} \frac{(-w)_{ul}}{l!} A_{w,l} \left(\frac{y}{4ab+c}\right)^l$$

$$\times \frac{1}{2\pi i} \int_L \bar{\phi}(\xi) \left\{ \frac{\Gamma\left(\frac{1}{2}-r+l+\lambda+k\xi\right)}{\Gamma\left(\lambda-r+l+1+k\xi\right)} \right\} \left(\frac{z}{(4ab+c)^k}\right)^{\xi} d\xi$$

which in view of (1) reduces to the aimed outcome (14). This concludes the proof of result (14).

Theorem 2.2 Let $a \geq 0$; $b > 0; c + 4ab > 0$; k is a positive integer, such that $-\frac{1}{2} < (a-b-c) < \frac{1}{2}$; and $|\arg z| < \frac{1}{2}\mu_1 \pi$, then the following formula holds:

$$\int_0^{\infty} \frac{1}{x^2} X^{-\lambda-1} {}_2F_1\left(a,\, b; c+\frac{1}{2}; X\right) {}_2F_1\left(c-a, c-b; c+\frac{1}{2}; X\right)$$

$$\times S_w^u[y/X] \bar{H}_{p,q}^{m,n}\left[z/X^k\right] dx$$

$$= \frac{\sqrt{\pi}}{2b\,(4ab+c)^{\lambda+1/2}} \sum_{r=0}^{\infty} \frac{(c)_r}{\left(c+\frac{1}{2}\right)_r} \frac{a_r}{(4ab+c)^{-r}} \sum_{l=0}^{[w/u]} \frac{(-w)_{ul}}{l!} A_{w,l} \left(\frac{y}{4ab+c}\right)^l$$

$$\times \bar{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4\mathrm{ab}+\mathrm{c})^k} \,\middle|\, \begin{matrix} \left(\frac{1}{2}+r-l-\lambda, k; 1\right), (a_j, \alpha_j, A_j)_{1,n}, (a_j, \alpha_j)_{n+1,p} \\ (b_j, \beta_j, B_j)_{1,m}, (b_j, \beta_j)_{m+1,q}, (r-l-\lambda, k; 1) \end{matrix} \right], \tag{15}$$

Theorem 2.3 Let $a > 0$; $b > 0; c + 4ab > 0$; k is a positive integer, such that $-\frac{1}{2} < (a-b-c) < \frac{1}{2}$; and $|\arg z| < \frac{1}{2}\mu_1 \pi$, then the following formula holds:

$$\int_0^{\infty} \left(a+\frac{b}{x^2}\right) X^{-\lambda-1} {}_2F_1\left(a, b; c+\frac{1}{2}; X\right) {}_2F_1\left(c-a, c-b; c+\frac{1}{2}; X\right)$$

$$\times S_w^u[y/X] \bar{H}_{p,q}^{m,n}\left[z/X^k\right] dx$$

$$= \frac{\sqrt{\pi}}{(4ab+c)^{\lambda+1/2}} \sum_{r=0}^{\infty} \frac{(c)_r}{\left(c+\frac{1}{2}\right)_r} \frac{a_r}{(4ab+c)^{-r}} \sum_{l=0}^{[w/u]} \frac{(-w)_{ul}}{l!} A_{w,l} \left(\frac{y}{4ab+c}\right)^l$$

$$\times \bar{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4\mathrm{ab}+\mathrm{c})^k} \,\middle|\, \begin{matrix} \left(\frac{1}{2}+r-l-\lambda, k; 1\right), (a_j, \alpha_j, A_j)_{1,n}, (a_j, \alpha_j)_{n+1,p} \\ (b_j, \beta_j, B_j)_{1,m}, (b_j, \beta_j)_{m+1,q}, (r-l-\lambda, k; 1) \end{matrix} \right], \tag{16}$$

Following the similar procedure, as in Theorem 2.1, one can easily prove the above integral Theorem 2.2 and Theorem 2.3. Therefore, we omit the detailed proof of these results.

3. SPECIAL CASES

In this section, we represent certain cases of the $\bar{H}$- function (1).

(I) If we put $A_j = B_j = 1$, in Theorem (2.1-2.3) and making use of the relation (5), using same assumptions in the equations (14), (15) and (16), then we have the following results.

Corollary 3.1.

$$\int_0^\infty X^{-\lambda-1} {}_2F_1\left(a, b; c+\frac{1}{2}; X\right) {}_2F_1\left(c-a, c-b; c+\frac{1}{2}; X\right) \times S_w^u[y/X] H_{p,q}^{m,n}\left[z/X^k\right] dx$$

$$= \frac{\sqrt{\pi}}{2a\,(4ab+c)^{\lambda+1/2}} \sum_{r=0}^{\infty} \frac{(c)_r}{\left(c+\frac{1}{2}\right)_r} \frac{a_r}{(4ab+c)^{-r}} \sum_{l=0}^{[w/u]} \frac{(-w)_{ul}}{l!} A_{w,l} \left(\frac{y}{4ab+c}\right)^l$$

$$\times H_{p+1,q+1}^{m,\,n+1}\left[\frac{z}{(4\mathrm{ab}+\mathrm{c})^k}\ \middle|\ \begin{matrix} \left(\frac{1}{2}+r-l-\lambda, k; 1\right), (a_j, \alpha_j)_{1,p} \\ (b_j, \beta_j)_{1,q}, (r-l-\lambda, k; 1) \end{matrix}\right], \tag{17}$$

Corollary 3.2.

$$\int_0^\infty \frac{1}{x^2} X^{-\lambda-1} {}_2F_1\left(a, b; c+\frac{1}{2}; X\right) {}_2F_1\left(c-a, c-b; c+\frac{1}{2}; X\right) \times S_w^u[y/X] H_{p,q}^{m,n}\left[z/X^k\right] dx$$

$$= \frac{\sqrt{\pi}}{2b\,(4ab+c)^{\lambda+1/2}} \sum_{r=0}^{\infty} \frac{(c)_r}{\left(c+\frac{1}{2}\right)_r} \frac{a_r}{(4ab+c)^{-r}} \sum_{l=0}^{[w/u]} \frac{(-w)_{ul}}{l!} A_{w,l} \left(\frac{y}{4ab+c}\right)^l$$

$$\times H_{p+1,\,q+1}^{m,\,n+1}\left[\frac{z}{(4\mathrm{ab}+\mathrm{c})^k}\ \middle|\ \begin{matrix} \left(\frac{1}{2}+r-l-\lambda, k; 1\right), (a_j, \alpha_j)_{1,p} \\ (b_j, \beta_j)_{1,q}, (r-l-\lambda, k; 1) \end{matrix}\right], \tag{18}$$

Corollary 3.3.

$$\int_0^\infty \left(a+\frac{b}{x^2}\right) X^{-\lambda-1} {}_2F_1\left(a, b; c+\frac{1}{2}; X\right) {}_2F_1\left(c-a, c-b; c+\frac{1}{2}; X\right) \times S_w^u[y/X] H_{p,q}^{m,n}\left[z/X^k\right] dx$$

$$= \frac{\sqrt{\pi}}{(4ab+c)^{\lambda+1/2}} \sum_{r=0}^{\infty} \frac{(c)_r}{\left(c+\frac{1}{2}\right)_r} \frac{a_r}{(4ab+c)^{-r}} \sum_{l=0}^{[w/u]} \frac{(-w)_{ul}}{l!} A_{w,l} \left(\frac{y}{4ab+c}\right)^l$$

$$\times H_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4\mathrm{ab}+\mathrm{c})^k}\ \middle|\ \begin{matrix} \left(\frac{1}{2}+r-l-\lambda, k; 1\right), (a_j, \alpha_j)_{1,p} \\ (b_j, \beta_j)_{1,q}, (r-l-\lambda, k; 1) \end{matrix}\right]. \tag{19}$$

The conditions of validity of corollary (17), (18) and (19), easily follow from those given in Theorem (14), (15) and (16).

(II) If we put $A_j = B_j = 1$, $\alpha_j = \beta_j = 1$, in Theorem (2.1-2.3) and making use of the relation (6), using same assumptions in the equations (14), (15) and (16), we have the following form.

Corollary 3.4.

$$\int_0^\infty X^{-\lambda-1}{}_2F_1\left(a,b;c+\frac{1}{2};X\right){}_2F_1\left(c-a,c-b;c+\frac{1}{2};X\right)$$
$$\times S_w^u[y/X]G_{p,q}^{m,n}\left[z/X^k\right]dx$$
$$=\frac{\sqrt{\pi}}{2a\,(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\frac{(c)_r}{\left(c+\frac{1}{2}\right)_r}\frac{a_r}{(4ab+c)^{-r}}\sum_{l=0}^{[w/u]}\frac{(-w)_{ul}}{l!}A_{w,l}\left(\frac{y}{4ab+c}\right)^l$$
$$\times G_{p+1,\,q+1}^{m,\,n+1}\left[\frac{z}{(4\mathrm{ab}+\mathrm{c})^k}\middle|\begin{array}{l}\left(\frac{1}{2}+r-l-\lambda,k;1\right),(a_j,1)_{1,p}\\(b_j,1)_{1,q},(r-l-\lambda,\,k;\,1)\end{array}\right],\tag{20}$$

Corollary 3.5.

$$\int_0^\infty \frac{1}{x^2}X^{-\lambda-1}{}_2F_1\left(a,b;c+\frac{1}{2};X\right){}_2F_1\left(c-a,c-b;c+\frac{1}{2};X\right)$$
$$\times S_w^u[y/X]G_{p,q}^{m,n}\left[z/X^k\right]dx$$
$$=\frac{\sqrt{\pi}}{2b\,(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\frac{(c)_r}{\left(c+\frac{1}{2}\right)_r}\frac{a_r}{(4ab+c)^{-r}}\sum_{l=0}^{[w/u]}\frac{(-w)_{ul}}{l!}A_{w,l}\left(\frac{y}{4ab+c}\right)^l$$
$$\times G_{p+1,q+1}^{m,\,n+1}\left[\frac{z}{(4\mathrm{ab}+\mathrm{c})^k}\middle|\begin{array}{l}\left(\frac{1}{2}+r-l-\lambda,k;1\right),(a_j,1)_{1,p}\\(b_j,1)_{1,q},(r-l-\lambda,k;1)\end{array}\right],\tag{21}$$

Corollary 3.6.

$$\int_0^\infty \left(a+\frac{b}{x^2}\right)X^{-\lambda-1}{}_2F_1\left(a,b;c+\frac{1}{2};X\right){}_2F_1\left(c-a,c-b;c+\frac{1}{2};X\right)$$
$$\times S_w^u[y/X]G_{p,q}^{m,n}\left[z/X^k\right]dx$$
$$=\frac{\sqrt{\pi}}{(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\frac{(c)_r}{\left(c+\frac{1}{2}\right)_r}\frac{a_r}{(4ab+c)^{-r}}\sum_{l=0}^{[w/u]}\frac{(-w)_{ul}}{l!}A_{w,l}\left(\frac{y}{4ab+c}\right)^l$$
$$\times G_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4\mathrm{ab}+\mathrm{c})^k}\middle|\begin{array}{l}\left(\frac{1}{2}+r-l-\lambda,k;1\right),(a_j,1)_{1,p}\\(b_j,1)_{1,q},(r-l-\lambda,k;1)\end{array}\right].\tag{22}$$

The conditions of validity of corollary (20), (21) and (22), easily follow from those given in Theorem (14), (15) and (16).

(III) If we put $p=n$, $q=q+1$, $m=1$, $B_1=1$, $b_1=0$, $a_j=1-a_j$, $b_j=1-b_j$, in Theorem (2.1-2.3) and making use of the relation (7), using same assumptions in the equations (14), (15) and (16), we have the following form.

Corollary 3.7.

$$\int_0^\infty X^{-\lambda-1}{}_2F_1\left(a,b;c+\frac{1}{2};X\right){}_2F_1\left(c-a,c-b;c+\frac{1}{2};X\right)$$
$$\times S_w^u[y/X]\,{}_p\bar{\psi}_q\left[-z/X^k\right]dx$$
$$=\frac{\sqrt{\pi}}{2a\,(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\frac{(c)_r}{\left(c+\frac{1}{2}\right)_r}\frac{a_r}{(4ab+c)^{-r}}\sum_{l=0}^{[w/u]}\frac{(-w)_{ul}}{l!}A_{w,l}\left(\frac{y}{4ab+c}\right)^l$$

$$\times {}_{p+1}\bar{\psi}_{q+1}\left[\begin{array}{c}\left(\frac{1}{2}+r-l-\lambda,k;\,1\right),\left(a_j,\alpha_j,A_j\right)_{1,p}\\ \left(b_j,\beta_j,B_j\right)_{1,q},\left(r-l-\lambda,k;\,1\right)\end{array};-\frac{z}{\left(4\mathrm{ab}+\mathrm{c}\right)^k}\right], \tag{23}$$

Corollary 3.8.

$$\int_0^\infty \frac{1}{x^2}X^{-\lambda-1}{}_2F_1\left(a,b;c+\frac{1}{2};X\right){}_2F_1\left(c-a,c-b;c+\frac{1}{2};X\right)$$
$$\times S_w^u[y/X]_p\bar{\psi}_q\left[-z/X^k\right]dx$$
$$=\frac{\sqrt{\pi}}{2b\left(4ab+c\right)^{\lambda+1/2}}\sum_{r=0}^{\infty}\frac{(c)_r}{\left(c+\frac{1}{2}\right)_r}\frac{a_r}{\left(4ab+c\right)^{-r}}\sum_{l=0}^{[w/u]}\frac{(-w)_{ul}}{l!}A_{w,l}\left(\frac{y}{4ab+c}\right)^l$$
$$\times {}_{p+1}\bar{\psi}_{q+1}\left[\begin{array}{c}\left(\frac{1}{2}+r-l-\lambda,k;1\right),\left(a_j,\alpha_j,A_j\right)_{1,p}\\ \left(b_j,\beta_j,B_j\right)_{1,q},\left(r-l-\lambda,k;1\right)\end{array};-\frac{z}{\left(4\mathrm{ab}+\mathrm{c}\right)^k}\right], \tag{24}$$

Corollary 3.9.

$$\int_0^\infty \left(a+\frac{b}{x^2}\right)X^{-\lambda-1}{}_2F_1\left(a,b;c+\frac{1}{2};X\right){}_2F_1\left(c-a,c-b;c+\frac{1}{2};X\right)$$
$$\times S_w^u[y/X]_p\bar{\psi}_q\left[-z/X^k\right]dx$$
$$=\frac{\sqrt{\pi}}{\left(4ab+c\right)^{\lambda+1/2}}\sum_{r=0}^{\infty}\frac{(c)_r}{\left(c+\frac{1}{2}\right)_r}\frac{a_r}{\left(4ab+c\right)^{-r}}\sum_{l=0}^{[w/u]}\frac{(-w)_{ul}}{l!}A_{w,l}\left(\frac{y}{4ab+c}\right)^l$$
$$\times {}_{p+1}\bar{\psi}_{q+1}\left[\begin{array}{c}\left(\frac{1}{2}+r-l-\lambda,k;1\right),\left(a_j,\alpha_j,A_j\right)_{1,p}\\ \left(b_j,\beta_j,B_j\right)_{1,q},\left(r-l-\lambda,k;1\right)\end{array};-\frac{z}{\left(4\mathrm{ab}+\mathrm{c}\right)^k}\right]. \tag{25}$$

The conditions of validity of corollary (23), (24) and (25), easily follow from those given in Theorem (14), (15) and (16).

4. **CONCLUSION**

We have established three main definite integrals of Gradshteyn–Ryzhil type, involving Srivastava's polynomials and the $\bar{H}$-function. We have also derived analogous result of these theorems in H-function, G-function and generalized wright hypergeometric functions, which have been depicted in corollaries. Since Srivastava's polynomials is a generalization of different type of classical orthogonal polynomials, hence we can find a number of results by assigning the suitable values to the parameter.

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