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FRACTIONAL ANALOGUE OF THE LYAPUNOV INEQUALITY IN CONFORMABLE SENSE

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Abstract. In this article, we derive a few Lyapunov-type inequalities for two-point conformable fractional boundary value problems associated with mixed boundary conditions. To demonstrate the applicability of established results, we obtain sufficient conditions for disconjugacy and disfocality of conformable fractional boundary value problems and estimate lower bound for eigenvalues of the corresponding fractional eigenvalue problem.

1. Introduction

In 1907, Lyapunov [7] proved the following result, which provides a necessary condition for the existence of a nontrivial solution of Hill's equation associated with Dirichlet boundary conditions.

Theorem 1.1. [7] *If the boundary value problem*

$$\begin{cases} y''(t) + p(t)y(t) = 0, \quad a < t < b, \\ y(a) = 0, \ y(b) = 0, \end{cases} \tag{1.1}$$

has a nontrivial solution, where $p : [a, b] \to \mathbb{R}$ is a continuous function, then

$$\int_a^b |p(s)|ds > \frac{4}{(b-a)}. \tag{1.2}$$

The inequality (1.2), known as Lyapunov inequality, has several applications in various problems related to differential equations, including oscillation theory, asymptotic theory, eigenvalue problems, disconjugacy, etc. Due to its importance, the Lyapunov inequality has been generalized in many forms. For more details on Lyapunov-type inequalities and their applications, we refer [3, 8, 9, 11, 12, 13] and the references therein.

On the other hand, Abdeljawad [2] and Gholami et al. [4] independently generalized Theorem 1.1 to the case where the classical second-order derivative in (1.1) is replaced by an α^{th}-order $(1 < \alpha \le 2)$ conformable fractional derivative.

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Theorem 1.2. [2] *If the boundary value problem*

$$\begin{cases} \big(T^{\alpha}_{a+}y\big)(t) + p(t)y(t) = 0, \quad a < t < b, \\ y(a) = 0, \ y(b) = 0, \end{cases} \tag{1.3}$$

has a nontrivial solution, where $p : [a, b] \to \mathbb{R}$ *is a continuous function, then*

$$\int_a^b |p(s)|ds > \frac{\alpha^{\alpha}}{(\alpha-1)^{\alpha-1}(b-a)^{\alpha-1}}. \tag{1.4}$$

Here T^{α}_{a+} denotes the α^{th}-order conformable differential operator. Motivated by these works, in this article, we derive Lyapunov-type inequalities for the following two-point conformable fractional boundary value problems:

$$\begin{cases} \big(T^{\alpha}_{a+}y\big)(t) + p(t)y(t) = 0, \quad 1 < \alpha \leq 2, \quad a < t < b, \\ y'(a) = 0, \ y(b) = 0, \end{cases} \tag{1.5}$$

and

$$\begin{cases} \big(T^{\alpha}_{a+}y\big)(t) + p(t)y(t) = 0, \quad 1 < \alpha \leq 2, \quad a < t < b, \\ y(a) = 0, \ y'(b) = 0. \end{cases} \tag{1.6}$$

2. Preliminaries

Throughout, we shall use the following notations, definitions and known results of conformable fractional calculus [1, 6].

Definition 2.1. [1] Let $y : [a, \infty) \to \mathbb{R}$ and $0 < \alpha \leq 1$. The α^{th}-order conformable fractional derivative of y starting from a is defined by

$$\big(T^{\alpha}_{a+}y\big)(t) = \lim_{\varepsilon \to 0}\Big[\frac{y\big(t + \varepsilon(t-a)^{1-\alpha}\big) - y(t)}{\varepsilon}\Big], \quad t \in (a, \infty).$$

If $\big(T^{\alpha}_{a+}y\big)$ exists on (a, b) then,

$$\big(T^{\alpha}_{a+}y\big)(a) = \lim_{t \to a^+}\big(T^{\alpha}_{a+}y\big)(t).$$

Definition 2.2. [1] Let $y : [a, \infty) \to \mathbb{R}$, $\alpha > 0$ and choose $n \in \mathbb{N}_1$ such that $n - 1 < \alpha \leq n$. Assume that $y^{(n-1)}$ exists on (a, ∞). The α^{th}-order conformable fractional derivative of y starting from a is defined by

$$\begin{aligned} \big(T^{\alpha}_{a+}y\big)(t) &= \big(T^{\alpha-n+1}_{a+}y^{(n-1)}\big)(t) \\ &= \lim_{\varepsilon \to 0}\Big[\frac{y^{(n-1)}\big(t + \varepsilon(t-a)^{n-\alpha}\big) - y^{(n-1)}(t)}{\varepsilon}\Big], \quad t \in (a, \infty). \end{aligned}$$

If $y^{(n)}$ exists on (a, ∞), we have

$$\big(T^{\alpha}_{a+}y\big)(t) = (t-a)^{n-\alpha}y^{(n)}(t), \quad t \in (a, \infty).$$

Also, we define

$$\big(T^{0}_{a+}y\big)(t) = y(t), \quad t \in (a, \infty).$$

Definition 2.3. [1] Let $y : [a, b] \to \mathbb{R}$, $\alpha > 0$ and choose $n \in \mathbb{N}_1$ such that $n - 1 < \alpha \leq n$. The α^{th}-order conformable fractional integral of y starting from a is defined by

$$\big(I^{\alpha}_{a+}y\big)(t) = \frac{1}{(n-1)!}\int_a^t (t-s)^{n-1}(s-a)^{\alpha-n}y(s)ds, \quad t \in [a, b].$$

Theorem 2.1. [1] *Let $y : [a,b] \to \mathbb{R}$, $\alpha > 0$ and choose $n \in \mathbb{N}_1$ such that $n-1 < \alpha \le n$. If $y^{(n-1)}$ exists on (a,b) then,*

$$\big(I^{\alpha}_{a+} T^{\alpha}_{a+} y\big)(t) = y(t) - \sum_{k=0}^{n-1} \frac{y^{(k)}(a)(t-a)^k}{k!}, \quad t \in (a,b).$$

3. Boundary Value Problem 1.5

In this section, we derive a few properties of the Green's function for the boundary value problem (1.5) and obtain the corresponding Lyapunov-type inequality.

Theorem 3.1. *Let $1 < \alpha \le 2$ and $h : [a,b] \to \mathbb{R}$ is a continuous function. The conformal fractional boundary value problem*

$$\begin{cases} \big(T^{\alpha}_{a+} y\big)(t) + h(t) = 0, & a < t < b, \\ y'(a) = 0, \ y(b) = 0, \end{cases} \tag{3.1}$$

has the unique solution

$$y(t) = \int_a^b G(t,s) h(s) ds, \tag{3.2}$$

where

$$G(t,s) = \begin{cases} (b-s)(s-a)^{\alpha-2}, & a < t \le s \le b, \\ (b-t)(s-a)^{\alpha-2}, & a < s \le t \le b. \end{cases} \tag{3.3}$$

Proof. Applying I^{α}_{a+} on both sides of (3.1) and using Theorem 2.1, we have

$$y(t) = C_1 + C_2(t-a) - \int_a^t (t-s)(s-a)^{\alpha-2} h(s) ds. \tag{3.4}$$

Further, we have

$$y'(t) = C_2 - \int_a^t (s-a)^{\alpha-2} h(s) ds. \tag{3.5}$$

Using $y'(a) = 0$ in (3.5) we get $C_2 = 0$. Using $y(b) = 0$ in (3.4) we get

$$C_1 = \int_a^b (b-s)(s-a)^{\alpha-2} h(s) ds. \tag{3.6}$$

Then, from (3.4) and (3.6), we have

$$\begin{aligned} y(t) &= \int_a^b (b-s)(s-a)^{\alpha-2} h(s) ds - \int_a^t (t-s)(s-a)^{\alpha-2} h(s) ds \\ &= \int_a^t \big[(b-s) - (t-s)\big](s-a)^{\alpha-2} h(s) ds + \int_t^b (b-s)(s-a)^{\alpha-2} h(s) ds \\ &= \int_a^t (b-t)(s-a)^{\alpha-2} h(s) ds + \int_t^b (b-s)(s-a)^{\alpha-2} h(s) ds \\ &= \int_a^b G(t,s) h(s) ds. \end{aligned}$$

□

Lemma 3.2. *The Green's function $G(t,s)$ defined in (3.3) satisfies the following properties:*

(1) $G(t,s) \ge 0$, $(t,s) \in [a,b] \times (a,b]$ *and* $\big[(s-a)^{2-\alpha} G(t,s)\big]_{s=a} = 0$, $t \in [a,b]$.

(2) $G(t,s) \leq G(s,s),\ (t,s) \in [a,b] \times [a,b]$.
(3) $(s-a)^{2-\alpha} G(s,s) \leq (b-a),\ s \in [a,b]$.
(4) $\int_a^b G(t,s)ds \leq \frac{(b-a)^\alpha}{\alpha(\alpha-1)},\ t \in [a,b]$.
(5) $\int_a^b (s-a)^{2-\alpha} G(t,s)ds \leq \frac{(b-a)^2}{2},\ t \in [a,b]$.
(6) $\int_a^b \big|G'(t,s)\big|ds \leq \frac{(b-a)^{\alpha-1}}{(\alpha-1)},\ t \in [a,b]$.
(7) $\int_a^b \big|(s-a)^{2-\alpha} G'(t,s)\big|ds \leq (b-a),\ t \in [a,b]$.

Proof. The proofs of (1) and (3) are trivial. To prove (2), consider

$$\frac{G(t,s)}{G(s,s)} = \begin{cases} 1, & a \leq t \leq s \leq b, \\ \frac{(b-t)}{(b-s)}, & a \leq s \leq t \leq b. \end{cases}.$$

Clearly,

$$\frac{G(t,s)}{G(s,s)} \leq 1 \text{ for all } (t,s) \in [a,b] \times [a,b].$$

Hence the proof of (2). Now, consider

$$\begin{aligned} \int_a^b G(t,s)ds &= \int_a^t (b-t)(s-a)^{\alpha-2}ds + \int_t^b (b-s)(s-a)^{\alpha-2}ds \\ &= \frac{(b-a)^\alpha}{\alpha(\alpha-1)} - \frac{(t-a)^\alpha}{\alpha(\alpha-1)}. \end{aligned} \tag{3.7}$$

We know that

$$0 \leq \frac{(t-a)^\alpha}{\alpha(\alpha-1)} \leq \frac{(b-a)^\alpha}{\alpha(\alpha-1)},\ t \in [a,b]. \tag{3.8}$$

Using (3.8) in (3.7), we have (4). To prove (5), consider

$$\int_a^b (s-a)^{2-\alpha} G(t,s)ds = \int_a^t (b-t)ds + \int_t^b (b-s)ds = (b-t)\Big(\frac{b+t}{2} - a\Big). \tag{3.9}$$

We know that

$$0 \leq (b-t)\Big(\frac{b+t}{2} - a\Big) \leq \frac{(b-a)^2}{2},\ t \in [a,b]. \tag{3.10}$$

Using (3.10) in (3.9), we get (5). Next, we consider

$$\int_a^b \big|G'(t,s)\big|ds = \int_a^t (s-a)^{\alpha-2}ds = \frac{(t-a)^{\alpha-1}}{(\alpha-1)} \leq \frac{(b-a)^{\alpha-1}}{(\alpha-1)},$$

for all $t \in [a,b]$. The proof of (6) is complete. Finally, consider

$$\int_a^b \big|(s-a)^{2-\alpha} G'(t,s)\big|ds = \int_a^t ds = (t-a) \leq (b-a),$$

for all $t \in [a,b]$. Hence the proof of (7). □

We are now able to formulate a Lyapunov-type inequality for the left focal boundary value problem.

Theorem 3.3. *If* (1.5) *has a nontrivial solution, then*

$$\int_a^b \big|(s-a)^{\alpha-2} p(s)\big|ds > \frac{1}{(b-a)}. \tag{3.11}$$

Proof. Let $C[a,b]$ be the Banach space of continuous functions y on $[a,b]$ with the norm

$$\|y\|_C = \max_{t\in[a,b]} |y(t)|.$$

It follows from Theorem 3.1 that a solution to (1.5) satisfies the equation

$$y(t) = \int_a^b G(t,s)p(s)y(s)ds.$$

Hence,

$$\begin{aligned}
|y(t)| &= \Big|\int_a^b G(t,s)p(s)y(s)ds\Big| \\
&\le \int_a^b G(t,s)|p(s)||y(s)|ds \\
&\le \|y\| \int_a^b G(s,s)|p(s)|ds \\
&= \|y\| \int_a^b \big[(s-a)^{2-\alpha}G(s,s)\big]\big|(s-a)^{\alpha-2}p(s)\big|ds,
\end{aligned}$$

implies

$$\|y\| \le \|y\| \max_{s\in[a,b]} \Big[(s-a)^{2-\alpha}G(s,s)\Big]\Big[\int_a^b \big|(s-a)^{\alpha-2}p(s)\big|ds\Big].$$

An application of Theorem 3.2 yields the result. □

4. Boundary Value Problem 1.6

In this section, we derive a few properties of the Green's function for the boundary value problem (1.6) and obtain the corresponding Lyapunov-type inequality.

Theorem 4.1. *Let $1 < \alpha \le 2$ and $h : [a,b] \to \mathbb{R}$ is a continuous function. The conformal fractional boundary value problem*

$$\begin{cases} \big(T^{\alpha}_{a+}y\big)(t) + h(t) = 0, & a < t < b, \\ y(a) = 0, \ y'(b) = 0, \end{cases} \tag{4.1}$$

has the unique solution

$$y(t) = \int_a^b G(t,s)h(s)ds, \tag{4.2}$$

where

$$G(t,s) = \begin{cases} (t-a)(s-a)^{\alpha-2}, & a \le t \le s \le b, \\ (s-a)^{\alpha-1}, & a \le s \le t \le b. \end{cases} \tag{4.3}$$

Proof. Using $y(a) = 0$ in (3.4) we get $C_1 = 0$. Using $y'(b) = 0$ in (3.5) we get

$$C_2 = \int_a^b (s-a)^{\alpha-2}h(s)ds. \tag{4.4}$$

Then, from (3.4) and (4.4), we have

$$\begin{aligned} y(t) &= \int_a^b (t-a)(s-a)^{\alpha-2}h(s)ds - \int_a^t (t-s)(s-a)^{\alpha-2}h(s)ds \\ &= \int_a^t \big[(t-a)-(t-s)\big](s-a)^{\alpha-2}h(s)ds + \int_t^b (t-a)(s-a)^{\alpha-2}h(s)ds \\ &= \int_a^t (s-a)^{\alpha-1}h(s)ds + \int_t^b (t-a)(s-a)^{\alpha-2}h(s)ds \\ &= \int_a^b G(t,s)h(s)ds. \end{aligned}$$

□

Lemma 4.2. *The Green's function $G(t,s)$ defined in (4.3) satisfies the following properties:*

(1) $G(t,s) \geq 0,\ (t,s) \in [a,b]\times[a,b]$.
(2) $G(t,s) \leq (b-a)^{\alpha-1},\ (t,s) \in [a,b]\times[a,b]$.
(3) $\int_a^b G(t,s)ds \leq \frac{(b-a)^{\alpha}}{(\alpha-1)},\ t \in [a,b]$.
(4) $\int_a^b \big|G'(t,s)\big|ds \leq \frac{(b-a)^{\alpha-1}}{(\alpha-1)},\ t \in [a,b]$.

Proof. The proof of (1) and (2) are trivial. Consider

$$\begin{aligned} \int_a^b G(t,s)ds &= \int_a^t (s-a)^{\alpha-1}ds + \int_t^b (t-a)(s-a)^{\alpha-2}ds \\ &= \frac{(t-a)^{\alpha}}{\alpha} + (t-a)\Big[\frac{(b-a)^{\alpha-1}}{(\alpha-1)} - \frac{(t-a)^{\alpha-1}}{(\alpha-1)}\Big] \\ &= \frac{(t-a)(b-a)^{\alpha-1}}{(\alpha-1)} - \frac{(t-a)^{\alpha}}{\alpha(\alpha-1)}. \end{aligned} \tag{4.5}$$

For $t \in [a,b]$, we have

$$0 \leq \frac{(t-a)^{\alpha}}{\alpha(\alpha-1)} \leq \frac{(b-a)^{\alpha}}{\alpha(\alpha-1)}, \tag{4.6}$$

and

$$0 \leq \frac{(t-a)(b-a)^{\alpha-1}}{(\alpha-1)} \leq \frac{(b-a)^{\alpha}}{(\alpha-1)}. \tag{4.7}$$

Using (4.6) and (4.7) in (4.5), we get (4). To prove (5), consider

$$\int_a^b \big|G'(t,s)\big|ds = \int_t^b (s-a)^{\alpha-2}ds = \frac{(b-a)^{\alpha-1}}{(\alpha-1)} - \frac{(t-a)^{\alpha-1}}{(\alpha-1)} \leq \frac{(b-a)^{\alpha-1}}{(\alpha-1)},$$

for all $t \in [a,b]$. The proof of (5) is complete. □

We are now able to formulate a Lyapunov-type inequality for the right focal boundary value problem.

Theorem 4.3. *If* (1.6) *has a nontrivial solution, then*

$$\int_a^b |p(s)|ds > \frac{1}{(b-a)^{\alpha-1}}. \tag{4.8}$$

5. Applications

In this section, we discuss two applications of the established results in Section 4. We begin with disconjugacy and disfocality.

Definition 5.1. The conformable fractional boundary value problem (1.3) is disconjugate on $[a, b]$ if and only if each nontrivial solution has less than $[\alpha] + 1$ zeros on $[a, b]$.

Definition 5.2. The conformable fractional boundary value problem (1.3) is left disfocal on $[a, b]$ if and only if each nontrivial solution has less than $[\alpha]$ zeros on $[a, b]$.

Definition 5.3. The conformable fractional boundary value problem (1.3) is right disfocal on $[a, b]$ if and only if each nontrivial solution has less than $[\alpha]$ zeros on $[a, b]$.

Using these definitions, we introduce a non-existence criterion for nontrivial solutions as follows:

Theorem 5.1. *The conformable fractional boundary value problem* (1.3) *is disconjugate if*

$$\int_a^b |p(s)|ds \leq \frac{\alpha^\alpha}{(\alpha-1)^{\alpha-1}(b-a)^{\alpha-1}}. \tag{5.1}$$

Proof. If possible, suppose that the conformable fractional boundary value problem (1.3) is not disconjugate on $[a, b]$. Then, there exists at least one nontrivial solution y such that $y(t)$ has at least two zeros on $[a, b]$. According to Theorem 1.2, we have (1.4). This is a contradiction to (5.1). Hence the proof. □

Theorem 5.2. *Assume that the assumptions of Theorem 5.1 are satisfied. Then, the conformable fractional boundary value problem* (1.3) *has no nontrivial solution on* $[a, b]$.

Proof. The proof is the same as of Theorem 5.1. □

Theorem 5.3. *The conformable fractional boundary value problem* (1.5) *is left disfocal if*

$$\int_a^b \left|(s-a)^{\alpha-2}p(s)\right|ds \leq \frac{1}{(b-a)}. \tag{5.2}$$

Proof. If possible, suppose that the conformable fractional boundary value problem (1.5) is not left disfocal on $[a, b]$. Then, there exists at least one nontrivial solution y such that $y(t)$ has at least one zero on $[a, b]$. According to Theorem 3.3, we have (3.11). This is a contradiction to (5.2). Hence the proof. □

Theorem 5.4. *Assume that the assumptions of Theorem 5.3 are satisfied. Then, the conformable fractional boundary value problem* (1.5) *has no nontrivial solution on* $[a, b]$.

Proof. The proof is the same as of Theorem 5.3. □

Theorem 5.5. *The conformable fractional boundary value problem* (1.6) *is right disfocal if*

$$\int_a^b |p(s)|ds \leq \frac{1}{(b-a)^{\alpha-1}}. \tag{5.3}$$

Proof. The proof is the same as of Theorem 5.3. □

Theorem 5.6. *Assume that the assumptions of Theorem 5.5 are satisfied. Then, the conformable fractional boundary value problem* (1.6) *has no nontrivial solution on* $[a, b]$.

Proof. The proof is the same as of Theorem 5.3. □

Next, we estimate a lower bound for the eigenvalues of the conformable fractional eigenvalue problem corresponding to the conformable fractional boundary value problems (1.3), (1.5) and (1.6).

Theorem 5.7. *Assume that* $1 < \alpha < 2$ *and* y *is a nontrivial solution of the conformable fractional eigenvalue problem*

$$\begin{cases} \big(T^{\alpha}_{a+}y\big)(t) + \lambda y(t) = 0, \quad a < t < b, \\ y(a) = 0, \ y(b) = 0, \end{cases} \tag{5.4}$$

where $y(t) \neq 0$ *for each* $t \in (a, b)$. *Then,*

$$|\lambda| > \frac{\alpha^{\alpha}}{(\alpha - 1)^{\alpha-1}(b - a)^{\alpha}}. \tag{5.5}$$

Theorem 5.8. *Assume that* $1 < \alpha < 2$ *and* y *is a nontrivial solution of the conformable fractional eigenvalue problem*

$$\begin{cases} \big(T^{\alpha}_{a+}y\big)(t) + \lambda y(t) = 0, \quad a < t < b, \\ y'(a) = 0, \ y(b) = 0, \end{cases} \tag{5.6}$$

where $y(t) \neq 0$ *for each* $t \in (a, b)$. *Then,*

$$|\lambda| > \frac{(\alpha - 1)}{(b - a)^{\alpha}}. \tag{5.7}$$

Theorem 5.9. *Assume that* $1 < \alpha < 2$ *and* y *is a nontrivial solution of the conformable fractional eigenvalue problem*

$$\begin{cases} \big(T^{\alpha}_{a+}y\big)(t) + \lambda y(t) = 0, \quad a < t < b, \\ y(a) = 0, \ y'(b) = 0, \end{cases} \tag{5.8}$$

where $y(t) \neq 0$ *for each* $t \in (a, b)$. *Then,*

$$|\lambda| > \frac{1}{(b - a)^{\alpha}}. \tag{5.9}$$

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