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CONTROLLABILITY OF SOBOLEV TYPE NONLOCAL IMPULSIVE MIXED FUNCTIONAL INTEGRODIFFERENTIAL EVOLUTION SYSTEMS

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Abstract. In the present paper, we have established a set of sufficient conditions for the controllability of Sobolev type nonlocal impulsive mixed functional integrodifferential evolution systems with finite delay. We have obtained the controllability results without assuming the compactness condition on the evolution operator and by using the semigroup theory and applying the fixed point approach.

1. Introduction

Many evolution processes are characterized by the fact that at certain moments of time they experience a change of state abruptly. These processes are subject to short-term perturbations whose duration is negligible in comparison with the duration of process. Consequently, it is natural to assume that these perturbations act instantaneously, that is, in the form of impulse. It is known, for example, that many biological phenomena involving thresholds, burning rhythm models in medicine and biological, optimal control models in economics, pharmacokinetics and frequency modulated systems, do exhibit impulsive effects. Thus, impulsive differential equations, that is, differential equations involving impulse effect, appear as a natural description of observed evolution phenomena of several real world problems. For more details on this theory and applications, see the monograph of Lakshmikantham et al. [19], Perestyuk et al. [26], Bainov and Simeonov [4] and the papers [2],[13], and [32].

The notion of controllability is of great importance in mathematical control theory. Many fundamental problems of control theory such as pole-assignment, stabilizability and optimal control may be solved under the assumption that the system is controllable. The problem of controllability is to show the existence of control function, which steers the solution of the system from its initial state to

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the final state, where the initial and final states may vary over the entire space. Controllability of nonlinear systems with and without impulse have studied by several authors, see, for instance, [3], [10], [20], and [31]. In recent years, significant progress has been made in the controllability of linear and nonlinear deterministic system [5], [7], [10], [14], [15], [16], [17], and [25]. The work on nonlocal initial value problem was first studied by Byszewski. In [9] Byszewski establish the theorems about the existence and uniqueness of solutions of a semilinear evolution nonlocal Cauchy problem. The nonlocal condition, in many cases, has a better effect than the classical condition.

On the other hand, most of the practical systems are integrodifferential equations in nature and hence the study of integrodifferential equations is very important. Many authors studied mixed type integrodifferential systems with (or without) delay conditions [11], [23], [24], [28], [29], and [30]. Recently, Machado et al. [22] establish the controllability for a class of abstract impulsive mixed type functional integrodifferential equations with finite delay in a Banach space by using the M$\ddot{o}$ nch fixed point theorem via measure of noncompactness and semigroup theory. A Sobolev-type equation constitute an important field of research due to their numerous applications such as flow of fluids through fissured rocks, thermodynamics, shear in second order fluids and propagation of long waves of small amplitude. Many researchers [6], [8], and [30] investigated the problem of controllability of Sobolev-type integrodifferential systems in Banach space. Recently, Ahmed [1] have studied the sufficient conditions for controllability of Sobolev-type fractional integrodifferential systems in a Banach space by using the compact semigroup and Schauder fixed point theorem.

Motivated by the above mentioned works and the work of Kumar et al. [18], Liu et al. [21], Balachandran et al. [6] and Radhakrishanan et al. [27], we study the controllability of nonlocal impulsive mixed Volterra-Fredholm functional integrodifferential with evolution system by using the semigroup theory and fixed point theorem. The rest of the paper is organized as follows. In section 2, we present the preliminaries and hypotheses. In sections 3, we give our main result.

2. Preliminaries

Consider the following Sobolev type nonlocal impulsive mixed functional integrodifferential evolution system

$$(Ex(t))^{'} = A(t)x(t) + Bu(t) + f\left(t, x_t, \int_0^t k(t,s,x_s)ds, \int_0^b h(t,s,x_s)ds\right),$$

$$t \neq t_i, t \in J = [0,b], \tag{1}$$

$$x(s) + [g(x_{t_1}, \cdots, x_{t_p})](s) = \phi(s), s \in [-r, 0], \tag{2}$$

$$\Delta x|_{t=t_i} = I_i\left(x(t_i^-)\right), i = 1,2,3,\cdots,m, \tag{3}$$

where $A(t), E$ are two closed operator such that $A(t)E^{-1}$ generates the strongly continuous semigroup of bounded linear operators $\{U(t,s) : 0 \leq s \leq t \leq b\}$. The state variable $x(.)$ takes values in the Banach space X with the norm $\|.\|$. The control function $u(.)$ is given in $L^2(J, V)$, a Banach space of admissible control function with V as a Banach space, and thereby $\Lambda = \{(t,s) : 0 \leq s \leq t \leq b\}$. B is a bounded linear operator from V into X. Further $f : J \times X \times X \times X \longrightarrow X, k, h : \Lambda \times X \longrightarrow X, I_i : X \longrightarrow X, \triangle x|_{t=t_i} = x(t_i^+) - x(t_i^-)$, for all $i = 1, 2, \cdots, m; 0 \leq t_0 < t_1 < \cdots, t_m < t_{m+1} \leq b$; and the nonlocal function $g : [PC([-r,0],X)]^p \longrightarrow X$ are given functions. The history x_t represents the function $x_t : (-r, 0] \longrightarrow X$ defined by $x_t(\theta) = x(t+\theta)$ for $t \in [0,b]$ and $\theta \in [-r,0]$. For the sake of simplicity, we put $J_0 = [0, t_1]$ and $J_i = (t_i, t_{i+1}], i = 1, 2, \cdots, m$. Let $PC([-r,b],X) = \{x : x \; is \; a \; function \; from \; [-r,b] \; into \; X \; such \; that \; x(t) \; is \; continuous \; at \; t \neq t_i \; and \; left \; continuous \; at \; t = t_i, \; and \; the \; right \; limit \; x(t_i^+) \; exists \; for \; i = 1, 2, \cdots, m\}$. From Machedo [22], $PC([-r,b],X)$ is a Banach space with norm

$$\|x\|_{PC} = \sup\left\{ \|x(t)\| : t \in [-r,b] \right\}.$$

For the family $\{A(t) : 0 \leq t \leq b\}$ of linear operators, we assume the following hypothesis:
(A1) $A(t)$ is closed linear operator and the domain $D(A)$ of $\{A(t) : 0 \leq t \leq b\}$ is dense in the Banach space X and independent of t.
(A2) For each $t \in [0,b]$, the resolvent $R(\lambda, A(t)) = (\lambda I - A(t))^{-1}$of $A(t)$ exists for all λ with

$$Re\lambda \leq 0 \; and \; \|R(\lambda, A(t))\| \leq C(|\lambda| + 1)^{-1}.$$

(A3) For any $t, s, \tau \in [0,b]$, there exists a $0 < \delta < 1$ and $L > 0$ such that

$$\|(A(t) - A(\tau))A^{-1}(s)\| \leq L|t-\tau|^{\delta}.$$

Statements (A1)-(A3) implies that there exists a family of evolution operators $U(t,s)$ (see [12]). The family $\{A(t) : 0 \leq t \leq b\}$ generates a unique linear evolution system $\{U(t,s) : 0 \leq s \leq t \leq b\}$ satisfying the following properties:
(a) $U(t,s) \in L(X)$, where $L(X)$ is the space of bounded linear transformation on X, whenever $0 \leq s \leq t \leq b$ and for each $x \in X$, the mapping $(t,s) \longrightarrow U(t,s)x$ is continuous.
(b) $U(t,s)U(s,\tau) = U(t,\tau)$ for $0 \leq \tau \leq s \leq t \leq b$.
(c) $U(t,t) = I$.
Let us recall the following definition.
Definition 1 A solution $x(.) \in PC([-r,b],X)$ is said to be a mild solution of (1-3) if $x(s) + \left[g(x_{t_1}, \cdots, x_{t_p}) \right](s) = \phi(s), s \in [-r,0]; \triangle x|_{t=t_i} = I_i\left(x(t_i^-) \right), i = 1, 2, 3, \cdots, m$; the restriction of $x(.)$ to the interval $J_i (i = 1, 2, \cdots, m)$ is continuous and the following conditions are satisfied:
(i)

$$x(t) = E^{-1}U(t,0)E\phi(0) - E^{-1}U(t,0)E\left[g(x_{t_1}, \cdots, x_{t_p}) \right](0)$$

$$+ \int_0^t E^{-1}U(t,s)\left[Bu(s) + f\left(s, x_s, \int_0^s k(s,\xi,x_\xi)d\xi, \int_0^b h(s,\xi,x_\xi)d\xi \right) \right] ds$$

$$+ \sum_{0<\tau_k<t} E^{-1}U(t,t_i)I_i\left(x(t_i^-) \right), t \in [0,b]$$

(ii)

$$x(s)+\left[g(x_{t_1},\cdots,x_{t_p})\right](s)=\phi(s), s\in[-r,0]$$

In order to prove our main theorem we assume the useful conditions on the operators $A(t)$ and E. Let X and Y be a Banach space with norm $|.|$ and $\|.\|$ respectively. The operators $A(t):D(A(t))\subset X\longrightarrow Y$ and $D(E)\subset X\longrightarrow Y$ satisfying the following hypotheses:
(H1) $A(t)$ and E are closed linear operators.
(H2) $D(E)\subset D(A(t))$ and E is bijective.
(H3) $E^{-1}:Y\longrightarrow D(E)$ is continuous.
The above fact and closed graph theorem imply the boundedness of the linear operator $A(t)E^{-1}:Y\longrightarrow Y$ and $A(t)E^{-1}$ generates a uniformly continuous evolution operator $U(t,s), t\geq 0$, of bounded linear operators on a Banach space Y.
To study the controllability problem we assume the following hypotheses:
(H4) $A(t)$ generates a family of evolution operator $U(t,s)$, when $t>s>0$, in X and there exists a constant $M_1>0$ such that

$$\|U(t,s)\|\leq M_1 \ for\ 0\leq s\leq t\leq b.$$

(H5) The linear operator $W:L^2(J,V)\longrightarrow X$ defined by

$$Wu=\int_0^b E^{-1}U(t,s)Bu(s)ds,$$

has an invertible operator W^{-1} which takes values in $L^2(J,V)\setminus\ker W$ and there exists positive constants M_2 such that $\|BW^{-1}\|\leq M_2$.
(H6) The nonlinear function $f:J\times X\times X\times X\longrightarrow X$ is continuous and there exist two constants $L_1,L_2>0$ such that

$$\left\|f\left(t,x_t,y_t,z_t\right)-f\left(t,u_t,v_t,w_t\right)\right\|\leq L_1\left(\|x-u\|+\|y-v\|+\|z-w\|\right),$$

for $x,y,z,u,v,w\in X, t\in J$,

$$L_2=\max_{t\in J}\left\|f(t,0,0,0)\right\|.$$

(H7) For each $(t,s)\in\Lambda$, the function $k:\Lambda\times X\longrightarrow X$ is continuous and there exist constants $K_1,K_2>0$ such that

$$\int_0^t\|k(t,s,x_s)-k(t,s,u_s)\|ds\leq K_1\|x-u\|,\ for\ x,u\in X, t,s\in J,$$

$$K_2=\max\left\{\int_0^t\|k(t,s,0)\|ds:t,s\in\Lambda\right\}.$$

(H8) For each $(t,s)\in\Lambda$, the function $h:\Lambda\times X\longrightarrow X$ is continuous and there exist constants $H_1,H_2>0$ such that

$$\int_0^b\|h(t,s,x_s)-h(t,s,u_s)\|ds\leq H_1\|x-u\|,\ for\ x,u\in X, t,s\in J,$$

$$H_2=\max\left\{\int_0^b\|h(t,s,0)\|ds:t,s\in\Lambda\right\}.$$

(H9) $I_i : X \longrightarrow X$ is continuous and there exists constant L_i such that

$$\|I_i(x) - I_i(u)\| \le L_i\|x-u\|, i = 1, 2, \cdots, m,$$

for each $x, u \in X$.
(H10) $g : [PC([-r, 0], X)]^p \longrightarrow X$ is continuous and there exists a constant $G_1 > 0$ such that

$$\left\|\left[g(x_{t_1}, \cdots, x_{t_p})\right](s) - \left[g(u_{t_1}, \cdots, u_{t_p})\right](s)\right\| \le G_1\|x-u\|_{PC},$$

for each $x, u \in PC([-r, b], X), s \in [-r, 0]$, and

$$G_2 = \max\left\{\left\|\left[g(x_{t_1}, \cdots, x_{t_p})\right](s)\right\| : x, u \in PC\left([-r, b], X\right), s \in [-r, 0]\right\}.$$

(H11) There exists a positive constant $\zeta > 0$ such that

$$\|E^{-1}\|M_1\left(1 + bM_1M_2\|E^{-1}\|\right)\left[\|E\phi(0)\| + \|E\|G_2 + b\left\{\left(L_1(1 + K_1 + H_1)r + K_2 + H_2\right) + L_2\right\} + \sum_{0<t_i<t} L_i\right] + b\|E^{-1}\|M_1M_2\|x_1\| \le \zeta.$$

Moreover, let us put $\rho = \sum_{i=1}^{m} L_i$ and

$$\mu = \|E^{-1}\|M_1\left(1 + bM_1M_2\|E^{-1}\|\right)\left[\|E\|G_1 + bL_1(1 + K_1 + H_1) + \rho\right].$$

Definition 2 The system (1-3) is said to be controllable on the interval J if for every $x_1 \in X$ and $[g(x_{t_1}, \cdots, x_{t_p})](s) \in PC([-r, b], X)$, there exists a control $u \in L^2(J, V)$ such that the mild solution $x(t)$ of (1-3) satisfies $x(0) = x_0$ and $x(b) = x_1$.

3. Controllability Result

Theorem 1: If the hypotheses (H1)-(H11) are satisfied, and if $0 \le \mu < 1$, then the system (1-3) is controllable on J.
Proof: Define an operator F on the Banach space $PC([-r, b], X)$ by the formula:

$$(Fx)(t) = \begin{cases} \phi(t) - \left[g(x_{t_1}, ., x_{t_p})\right](t), t \in [-r, 0], \\ E^{-1}U(t, 0)E\phi(0) - E^{-1}U(t, 0)E\left[g(x_{t_1}, \cdots, x_{t_p})\right](0) + \int_0^t E^{-1}U(t, \\ s)Bu(s)ds + \int_0^t E^{-1}U(t, s)f\left(s, x_s, \int_0^s k(s, \xi, x_\xi)d\xi, \int_0^b h(s, \xi, x_\xi)d\xi\right) \\ ds + \sum_{0<t_i<t} E^{-1}U(t, t_i)I_i\left(x(t_i^-)\right), t \in J \end{cases} \tag{4}$$

Using hypotheses (H5) for an arbitrary function $x(.)$ define the control

$$u(t) = W^{-1}\Bigg[x_1 - E^{-1}U(b,0)E\Bigg(\phi(0) - \Bigg(g(x_{t_1},\cdots,x_{t_p})\Bigg)(0)\Bigg)$$
$$-\int_0^b E^{-1}U(b,s)f\Bigg(s, x_s, \int_0^s k(s,\xi,x_\xi)d\xi, \int_0^b h(s,\xi,x_\xi)d\xi\Bigg)ds$$
$$-\sum_{0<t_i<b} E^{-1}U(b,t_i)I_i(x(t_i^-))\Bigg](t).$$

We shall now show that when using this control the operator

$$F : PC([-r,b],X) \longrightarrow PC([-r,b],X)$$

defined by

$$(Fx)(t) = E^{-1}U(t,0)E\Bigg[\phi(0) - \Bigg(g(x_{t_1},\cdots,x_{t_p})\Bigg)(0)\Bigg] + \int_0^t E^{-1}U(t,s)BW^{-1}$$
$$\Bigg[x_1 - E^{-1}U(b,0)E\phi(0) - E^{-1}U(b,0)E\Bigg(g(x_{t_1},\cdots,x_{t_p})\Bigg)(0) - \int_0^b E^{-1}U(b,s)$$
$$f\Bigg(s, x_s, \int_0^s k(s,\xi,x_\xi)d\xi, \int_0^b h(s,\xi,x_\xi)d\xi\Bigg)ds - \sum_{0<t_i<b} E^{-1}U(b,t_i)I_i\Bigg(x(t_i^-)\Bigg)\Bigg](s)ds$$
$$+\int_0^t E^{-1}U(t,s)f\Bigg(s, x_s, \int_0^s k\Bigg(s,\xi,x_\xi\Bigg)d\xi, \int_0^b h(s,\xi,x_\xi)d\xi\Bigg)ds$$
$$+\sum_{0<t_i<t} E^{-1}U(t,t_i)I_i\Bigg(x(t_i^-)\Bigg)$$

has a fixed point $x(.)$. To prove the controllability, it is enough to show that the operator F has a fixed point in $PC([-r,b],X)$ and since all the functions involved in the operator are continuous and therefore F is continuous.
Let S be a nonempty closed subset of $PC([-r,b],X)$ defined by

$$S = \Bigg\{x : x \in PC([-r,b],X), \|x(t)\|_{PC} \le r, 0 \le t \le b\Bigg\}.$$

First we show that F maps S into S. For $x \in S$, we have

$$\|(Fx)(t)\| \le \Bigg\|E^{-1}U(t,0)E\Bigg[\phi(0) - \Bigg(g(x_{t_1},\cdots,x_{t_p})\Bigg)(0)\Bigg]\Bigg\| + \Bigg\|\int_0^t E^{-1}U(t,s)BW^{-1}$$
$$\Bigg[x_1 - E^{-1}U(b,0)E\phi(0) - E^{-1}U(b,0)E\Bigg(g(x_{t_1},\cdots,x_{t_p})\Bigg)(0) - \int_0^b E^{-1}U(b,s)f(s,x_s,$$
$$\int_0^s k(s,\xi,x_\xi)d\xi, \int_0^b h(s,\xi,x_\xi)d\xi)ds - \sum_{0<t_i<b} E^{-1}U(b,t_i)I_i(x(t_i^-))\Bigg](s)ds\Bigg\|$$

$$
\begin{aligned}
&+\left\| \int_0^t E^{-1}U(t,s)f\left(s, x_s, \int_0^s k(s,\xi,x_\xi)d\xi, \int_0^b h(s,\xi,x_\xi)d\xi\right)ds\right\| \\
&+\left\| \sum_{0<t_i<t} E^{-1}U(t,t_i)I_i(x(t_i^-))\right\| \\
&\leq \|E^{-1}\|M_1\|E\phi(0)\| + \|E^{-1}\|\|E\|M_1G_2 + M_1M_2\int_0^t \|E^{-1}\| \\
&\Bigg[\|x_1\| + \|E^{-1}\|\|E\phi(0)\|M_1 + \|E^{-1}\|\|E\|M_1G_2 + M_1\int_0^b \|E^{-1}\| \\
&\left\|f\left(s, x_s, \int_0^s k(s,\xi,x_\xi)d\xi, \int_0^b h(s,\xi,x_\xi)d\xi\right)\right\|ds + \|E^{-1}\|M_1\sum_{0<t_i<b} L_i\Bigg]ds \\
&+M_1\int_0^t \|E^{-1}\|\left\|f\left(s, x_s, \int_0^s k(s,\xi,x_\xi)d\xi, \int_0^b h(s,\xi,x_\xi)d\xi\right)\right\|ds \\
&+\|E^{-1}\|M_1\sum_{0<t_i<t} L_i
\end{aligned}
$$

Since from assumptions (H6)-(H8), we have

$$
\begin{aligned}
&\left\|f\left(s, x_s, \int_0^s k(s,\xi,x_\xi)d\xi, \int_0^b h(s,\xi,x_\xi)d\xi\right)\right\| \\
&\leq \left\|f\left(s, x_s, \int_0^s k(s,\xi,x_\xi)d\xi, \int_0^b h(s,\xi,x_\xi)d\xi\right) - f(s,0,0,0) + f(s,0,0,0)\right\| \\
&\leq L_1\left[\|x_s\| + \left\|\int_0^s k(s,\xi,x_\xi)d\xi\right\| + \left\|\int_0^b h(s,\xi,x_\xi)d\xi\right\|\right] + L_2 \\
&\leq L_1\Bigg[\|x_s\| + \int_0^s \left\|k(s,\xi,x_\xi) - k(s,\xi,0)\right\|d\xi + \int_0^s \|k(s,\xi,0)\|d\xi \\
&+ \int_0^b \|h(s,\xi,x_\xi) - h(s,\xi,0)\|d\xi + \int_0^b \|h(s,\xi,0)\|d\xi\Bigg] + L_2 \\
&\leq L_1\Bigg[\|x_s\| + K_1\|x_s\| + K_2 + H_1\|x_s\| + H_2\Bigg] + L_2
\end{aligned}
$$

there holds

$$\|(Fx)(t)\| \leq \|E^{-1}\|M_1\|E\phi(0)\| + \|E^{-1}\|\|E\|M_1G_2 + bM_1M_2\|E^{-1}\|\Bigg[\|x_1\|$$

$$+\|E^{-1}\|\|E\phi(0)\|M_1 + \|E^{-1}\|\|E\|M_1G_2 + bM_1\|E^{-1}\|\Bigg\{L_1\Bigg[\|x_s\| + K_1\|x_s$$

$$\| + K_2 + H_1\|x_s\| + H_2\Bigg] + L_2\Bigg\} + \|E^{-1}\|M_1\sum_{i=1}^{m} L_i\Bigg] + bM_1\|E^{-1}\|$$

$$\Bigg\{L_1\Bigg[\|x_s\| + K_1\|x_s\| + K_2 + H_1\|x_s\| + H_2\Bigg] + L_2\Bigg\} + \|E^{-1}\|M_1\sum_{0<t_i<t} L_i$$

$$\leq M_1\|E^{-1}\|\Bigg(1 + bM_1M_2\|E^{-1}\|\Bigg)\Bigg[\|E\phi(0)\| + \|E\|G_2 + b\Bigg\{L_1\Bigg[(1 + K_1 + H_1)r$$

$$+K_2 + H_2\Bigg] + L_2\Bigg\} + \sum_{0<t_i<t} L_i\Bigg] + bM_1M_2\|E^{-1}\|\|x_1\|.$$

From (H11), one gets
$\|(Fx)(t)\| \leq \zeta$. Therefore F maps S into itself.
Now we shall show that F is a contraction on S. For this purpose consider two differences as follows

$$(Fx)(t) - (Fy)(t) = \Bigg[g(x_{t_1}, \cdots, x_{t_p})\Bigg](t) - \Bigg[g(y_{t_1}, \cdots, y_{t_p})\Bigg](t),$$

$$for\ x, y \in PC([-r, b], X), t \in [-r, 0), \tag{5}$$

$$(Fx)(t) - (Fy)(t) = E^{-1}U(t,0)E\Bigg[\Bigg(g(x_{t_1}, \cdots, x_{t_p})\Bigg)(0) - \Bigg(g(y_{t_1}, \cdots, y_{t_p})\Bigg)$$

$$(0)\Bigg] + \int_0^t E^{-1}U(t,\sigma)BW^{-1}\Bigg\{E^{-1}U(b,0)E\Bigg[\Bigg(g(x_{t_1}, \cdots, x_{t_p})\Bigg)(0)$$

$$-\Bigg(g(y_{t_1}, \cdots, y_{t_p})\Bigg)(0)\Bigg] + \int_0^b E^{-1}U(b,s)\Bigg[f(s, x_s, \int_0^s k(s,\xi,x_\xi)d\xi,$$

$$\int_0^b h(s,\xi,x_\xi)d\xi) - f\Bigg(s, y_s, \int_0^s k(s,\xi,y_\xi)d\xi, \int_0^b h(s,\xi,y_\xi)d\xi\Bigg)\Bigg]ds$$

$$+\sum_{0<t_1<b} E^{-1}U(b,t_i)\Bigg[I_i(x(t_i^-)) - I_i(y(t_i^-))\Bigg]\Bigg\}(\sigma)d\sigma + \int_0^t E^{-1}U(t,s)$$

$$\Bigg[f\Bigg(s, x_s, \int_0^s k(s,\xi,x_\xi)d\xi, \int_0^b h(s,\xi,x_\xi)d\xi\Bigg) - f\Bigg(s, y_s, \int_0^s k(s,\xi,y_\xi)d\xi,$$

$$\int_0^b h(s,\xi,y_\xi)d\xi\Bigg)\Bigg]ds + \sum_{0<t_1<t} E^{-1}U(t,t_i)\Bigg[I_i\Bigg(x(t_i^-)\Bigg) - I_i\Bigg(y(t_i^-)\Bigg)\Bigg],$$

$$for\ x, y \in PC([-r, b], X), t \in J. \tag{6}$$

From (5) and (H10), we have

$$\Bigg\|(Fx)(t)-(Fy)(t)\Bigg\| \leq G_1\|x-y\|_{PC}, for\ x,y\in PC([-r,b],X), t\in[-r,0). \quad (7)$$

Moreover, by (6), (H6)-(H10), we obtain

$$\Bigg\|(Fx)(t)-(Fy)(t)\Bigg\| \leq \Bigg\|E^{-1}U(t,0)E\Bigg[\Bigg(g(x_{t_1},\cdots,x_{t_p})\Bigg)(0)-\Bigg(g(y_{t_1},\cdots,y_{t_p})\Bigg)$$
$$(0)\Bigg]\Bigg\| + \Bigg\|\int_0^t E^{-1}U(t,\sigma)BW^{-1}\Bigg\{E^{-1}U(b,0)E\Bigg[\Bigg(g(x_{t_1},\cdots,x_{t_p})\Bigg)(0)-\Bigg(g(y_{t_1},$$
$$\cdots,y_{t_p})\Bigg)(0)\Bigg]+\int_0^b E^{-1}U(b,s)\Bigg[f\Bigg(s,x_s,\int_0^s k(s,\xi,x_\xi)d\xi,\int_0^b h(s,\xi,x_\xi)d\xi\Bigg)$$
$$-f\Bigg(s,y_s,\int_0^s k(s,\xi,y_\xi)d\xi,\int_0^b h(s,\xi,y_\xi)d\xi\Bigg)\Bigg]ds+\sum_{0<t_1<b}E^{-1}U(b,t_i)$$
$$\Bigg[I_i\Bigg(x(t_i^-)\Bigg)-I_i\Bigg(y(t_i^-)\Bigg)\Bigg]\Bigg\}(\sigma)d\sigma\Bigg\|+\Bigg\|\int_0^t E^{-1}U(t,s)$$
$$\Bigg[f\Bigg(s,x_s,\int_0^s k(s,\xi,x_\xi)d\xi,\int_0^b h(s,\xi,x_\xi)d\xi\Bigg)-f\Bigg(s,y_s,\int_0^s k(s,\xi,y_\xi)d\xi,$$
$$\int_0^b h(s,\xi,y_\xi)d\xi\Bigg)\Bigg]ds\Bigg\|+\Bigg\|\sum_{0<t_1<t}E^{-1}U(t,t_i)\Bigg[I_i\Bigg(x(t_i^-)\Bigg)-I_i\Bigg(y(t_i^-)\Bigg)\Bigg]\Bigg\|$$

$$\leq M_1G_1\|E^{-1}\|\|E\|\|x-y\|_{PC}+M_1M_2\int_0^t\|E^{-1}\|\Bigg[M_1G_1\|E^{-1}\|\|E\|\|x-y\|_{PC}$$
$$+M_1\int_0^b\|E^{-1}\|\Bigg\{L_1\Bigg(\|x-y\|+K_1\|x-y\|+H_1\|x-y\|\Bigg)\Bigg\}ds+M_1\sum_{i=1}^m L_i\|E^{-1}\|$$
$$\|x-y\|_{PC}\Bigg]d\sigma+M_1\int_0^t\|E^{-1}\|\Bigg\{L_1\Bigg(\|x-y\|+K_1\|x-y\|+H_1\|x-y\|\Bigg)\Bigg\}ds$$
$$+M_1\sum_{0<t_i<t}L_i\|E^{-1}\|\|x-y\|_{PC}$$

$$\leq M_1G_1\|E^{-1}\|\|E\|\|x-y\|_{PC}+bM_1M_2\|E^{-1}\|\Bigg[M_1G_1\|E^{-1}\|\|E\|\|x-y\|_{PC}$$
$$+bM_1\|E^{-1}\|\Bigg\{L_1\Bigg(\|x-y\|+K_1\|x-y\|+H_1\|x-y\|\Bigg)\Bigg\}+M_1\|E^{-1}\|\rho\|x-y\|_{PC}\Bigg]$$
$$+bM_1\|E^{-1}\|L_1\Bigg(\|x-y\|+K_1\|x-y\|+H_1\|x-y\|\Bigg)+M_1\|E^{-1}\|\rho\|x-y\|_{PC}$$

$$\leq M_1\|E^{-1}\|\left(1+bM_1M_2\|E^{-1}\|\right)\left[G_1\|E\|+bL_1\left(1+K_1+H_1\right)+\rho\right]\|x-y\|_{PC} \quad (8)$$

From (7) and (8), we get

$$\|(Fx)(t)-(Fy)(t)\| \leq \mu\|x-y\|_{PC},\ for\ x,y\in PC([-r,b],X), \quad (9)$$

where $\mu = M_1\|E^{-1}\|\left(1+bM_1M_2\|E^{-1}\|\right)\left[G_1\|E\|+bL_1\left(1+K_1+H_1\right)+\rho\right]$.
Since $\mu < 1$, then (9) shows that the operator F is a contraction on $PC([-r,b],X)$. Also, F satisfies the Banach contraction theorem. Hence there exists a unique fixed point $x \in PC([-r,b],X)$ such that $(Fx)(t) = x(t)$ and this point is the mild solution of the system (1)-(3) and $(Fx)(b) = x(b) = x_1$, which implies that the given system is controllable.

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References

[1] H. M. Ahmed, Controllability for Sobolev type fractional integrodifferential systems in a Banach space, Advance in Difference Equation , 2012:167 doi:10.1186/1687-1847-2012-167, 2012.

[2] H. Akca, A. Boucherif and V. Covachev, Impulsive functional-differential equations with nonlocal conditions, IJMMS, 29:5, 251-256, 2002.

[3] S. Baghli, M. Benchohra and K. Ezzinbi, Controllability results for semilinear functional and neutral functional evolution equations with infinite delay, Survey in Mathematics and its Applications, Vol.4, 15-39, 2009.

[4] D.D. Bainov and P.S. Simeonov, Systems with Impulse Effect, Ellis Horwood, Chichester, UK, 1989.

[5] K. Balachandran and J.P. Dauer, Controllability of nonlinear system via fixed point theorems, J. Optim. Theory and Appl., 53, 345-352, 1987.

[6] K. Balachandran and J.P. Dauer, Controllability of Sobolev-type integro differential systems in Banach spaces, Journal of Mathematical Analysis and Applications, 217, 335-348, 1998.

[7] K. Balachandran and J.P. Dauer, Controllability of nonlinear system in Banach spaces, A Survey, J. Optim. Theory and Appl., 115, 7-28, 2002.

[8] K. Balachandran and R. Sakthivel, Controllability of Sobolev type semilinear integrodifferential system in Banach spaces, Applied Mathematics Letters, 12, 63-71, 1999.

[9] L. Byszewski, Theorems about existence and uniqueness of solutions of a semi-linear evolution nonlocal Cauchy problem. J. Math. Anal. Appl. 162, 494-505, 1991.

[10] L. Chen and G. Li, Approximate controllability of impulsive differential equations with nonlocal conditions. Int. J. Nonlinear Sci. 10, 438-446, 2010.

[11] M.B. Dhakne, and K.D. Kucche, Existence of a mild solution of mixed Volterra-Fredholm functional integro-differential equation with nonlocal condition. Appl. Math. Sci. 5(8), 359-366, 2011.

[12] A. Friedman, Partial Differential Equations, Holt, Rinehart and Winston, New York, 1969.

[13] E. Hernandez, M. Rabello and H. Henriquez, Existence of solutions for impulsive partial neutral functional differential equations. J. Math. Anal. Appl. 331, 1135-1158, 2007.

[14] S. Ji, G. Li and M. Wang, Controllability of impulsive differential systems with nonlocal conditions. Appl. Math. Comput. 217, 6981-6989, 2011.

[15] J. Klamka, Schauders fixed-point theorem in nonlinear controllability problems. Control Cybern. 29(1), 153-165, 2000.

[16] J. Klamka, Constrained controllability of semilinear delayed systems. Bull. Pol. Acad. Sci., Tech. Sci. 49(3), 505-515, 2001.

[17] J. Klamka, Constrained controllability of semilinear systems. Nonlinear Anal. 47, 2939-2949, 2001.

[18] K. Kumar and R. Kumar, Nonlocal Cauchy Problem for Sobolev Type Mixed Volterra-Fredholm Functional Integrodifferential Equation, Journal of Physical Sciences, Vol. 17, 69-76, 2013.

[19] V. Lakshmikantham, D.D. Bainov and P.S. Simeonov, Theory of Impulsive Differential Equations, Series in Modern Applied Mathematics, Vol.6, World Scientific Publishing, New Jersey, 1989.

[20] M. Li, M. Wang and F. Zhang, Controllability of impulsive functional differential systems in Banach spaces. Chaos Solitons Fractals 29, 175-181, 2006.

[21] Y. Liu and D. Óregan, Controllability of impulsive functional differential systems with nonlocal conditions, EJDE, Vol. 2013, No. 194, 1-10, 2013.

[22] J.A. Machado, C. Ravichandran, M. Rivero and J.J. Trujillo, Controllability results for impulsive mixed-type functional integro-differential evolution equations with nonlocal conditions, Fixed Point Theory and Applications, 2013, 2013:66.

[23] M. Matar, Controllability of Fractional Semilinear Mixed Volterra-Fredholm Integro-differential Equations with Nonlocal Conditions, Int. Journal of Math. Analysis, Vol. 4, No. 23, 1105-1116, 2010.

[24] S.K. Ntouyas and I.K. Purnaras, Existence results for mixed Volterra-Fredholm type neutral functional integro-differential equations in Banach spaces. Nonlinear Stud. 16(2), 135-148, 2009.

[25] V. Obukhovski and P. Zecca, Controllability for systems governed by semilinear differential inclusions in a Banach space with a noncompact semigroup. Nonlinear Anal. 70, 3424-3436, 2009.

[26] N.A. Perestyuk, V.A. Plotnikov, A.M. Samoilenko and N.V. Skripnik, Differential Equation with Impulse Effects: Multivalued Right-hand Sides with discontinuities, De Gruyter Studies in Mathematics 40, Germany, (2011).

[27] B. Radhakrishnan and K. Balachandran, Controllability of nonlocal impulsive functional integrodifferential evolution systems, J. Nonlinear Sci. Appl. 4, No. 4, 281-291, 2011.

[28] C. Ravichandran and M.M. Arjunan, Existence and uniqueness results for impulsive fractional integrodifferential equations in Banach spaces, International Journal of Nonlinear Science, Vol.11, No. 4, 427-439, 2011.

[29] C. Ravichandran and J.J. Trujillo, Controllability of impulsive fractional functional integrodifferential equations in Banach spaces, Journal of Function Spaces and Applications, Vol. 2013, Article ID 812501, 8 pages.

[30] R. Sathya and K. Balachandran, Controllability of Sobolev-Type Neutral Stochastic Mixed Integro-differential Systems, European Journal of Mathematical Sciences, Vol. 1, No. 1, 68-87, 2012.

[31] S. Selvi and M.M. Arjunan, Controllability results for impulsive differential system with finite delay, Journal of Nonlinear Science and Applications, 5, 206-219, 2012.

[32] Z. Yan, Existence for a nonlinear impulsive functional integrodifferential equation with nonlocal conditions in Banach spaces, J. Appl. Math. and Informatics, Vol. 29, No. 3-4, 681-696, 2011.

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