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FEKETE-SZEGÖ INEQUALITIES FOR CLASSES OF BI-STARLIKE AND BI-CONVEX FUNCTIONS

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Abstract. We investigate the Fekete-Szegö inequalities for two comprehensive classes of bi-starlike and bi-convex functions. The coefficient bounds obtained in this article, in some cases, improve some of the previously published results.

1. Introduction

Let $\mathcal{A}$ denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1}$$

which are analytic in the open unit disc $\mathbb{U} = \{z : z \in \mathbb{C} \text{ and } |z| < 1\}$ and let $\mathcal{S}$ denote the class of functions in $\mathcal{A}$ that are univalent in $\mathbb{U}$.

For two functions f and g, analytic in $\mathbb{U}$, we say that the function f is subordinate to g in $\mathbb{U}$, and write $f \prec g$, if there exists a Schwarz function w, analytic in $\mathbb{U}$, with $w(0) = 0$ and $|w(z)| < 1$ such that $f(z) = g(w(z))$; $z, w \in \mathbb{U}$. In particular, if the function g is univalent in $\mathbb{U}$, the above subordination is equivalent to $f(0) = g(0)$ and $f(\mathbb{U}) \subset g(\mathbb{U})$.

It is well known (e.g. see Duren [5]) that every function $f \in \mathcal{S}$ has an inverse map f^{-1}, defined by

$$f^{-1}(f(z)) = z \qquad (z \in \mathbb{U})$$

and

$$f(f^{-1}(w)) = w \qquad \left(|w| < r_0(f);\ r_0(f) \geq \frac{1}{4}\right),$$

where

$$f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2 a_3 + a_4)w^4 + \dots. \tag{2}$$

A function $f \in \mathcal{A}$ is said to be bi-univalent in $\mathbb{U}$ if both f and f^{-1} are univalent in $\mathbb{U}$. We let Σ denote the class of bi-univalent functions in $\mathbb{U}$ given by (1). For a history and examples of functions which are (or which are not) in the class Σ,

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together with various other properties of classes of bi-univalent functions refer to [1, 2, 3, 4, 6, 7, 8, 9, 10, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22]).

Two of the most famous subclasses of univalent functions are the class $\mathcal{S}^*(\alpha)$ of starlike functions of order α and the class $\mathcal{K}(\alpha)$ of convex functions of order α. By definition, we have

$$\mathcal{S}^*(\alpha) := \left\{ f \in \mathcal{S} \, : \Re\left(\frac{zf'(z)}{f(z)}\right) > \alpha; \;\; z \in \mathbb{U}; \;\; 0 \leq \alpha < 1 \right\}$$

and

$$\mathcal{K}(\alpha) := \left\{ f \in \mathcal{S} \, : \Re\left(1 + \frac{zf''(z)}{f'(z)}\right) > \alpha; \;\; z \in \mathbb{U}; \;\; 0 \leq \alpha < 1 \right\}.$$

For $0 \leq \alpha < 1$, a function $f \in \Sigma$ is in the class $S^*_\Sigma(\alpha)$ of bi-starlike functions of order α, or $\mathcal{K}_{\Sigma,\alpha}$ of bi-convex functions of order α if both f and its inverse map f^{-1} are, respectively, starlike or convex of order α. For $0 < \beta \leq 1$, a function $f \in \Sigma$ is strongly bi-starlike function of order β, if both the functions f and its inverse map f^{-1} are strongly starlike of order β. We denote the class of all such functions is denoted by $S^*_{\Sigma,\beta}$.

Let φ be an analytic and univalent function with positive real part in $\mathbb{U}$, $\varphi(0) = 1$, $\varphi'(0) > 0$ and φ maps the unit disk $\mathbb{U}$ onto a region starlike with respect to 1 and symmetric with respect to the real axis. The Taylor's series expansion of such function is

$$\varphi(z) = 1 + B_1 z + B_2 z^2 + B_3 z^3 + \ldots, \tag{3}$$

where all coefficients are real and $B_1 > 0$. Throughout this paper we assume that the function φ satisfies the above conditions unless otherwise stated.

By $\mathcal{S}^*(\varphi)$ and $\mathcal{K}(\varphi)$ we denote the following classes of functions

$$\mathcal{S}^*(\varphi) := \left\{ f \in \mathcal{S} : \; \frac{zf'(z)}{f(z)} \prec \varphi(z); \;\; z \in \mathbb{U} \right\}$$

and

$$\mathcal{K}(\varphi) := \left\{ f \in \mathcal{S} : 1 + \frac{zf''(z)}{f'(z)} \prec \varphi(z); \;\; z \in \mathbb{U} \right\}.$$

The classes $\mathcal{S}^*(\varphi)$ and $\mathcal{K}(\varphi)$ are the extensions of a classical set of starlike and convex functions (e.g. see Ma and Minda [13]). A function f is said to be bi-starlike of Ma-Minda type or bi-convex of Ma-Minda type if both f and f^{-1} are, respectively, of Ma-Minda starlike or convex type. These classes are denoted, respectively, by $\mathcal{S}^*_\Sigma(\varphi)$ and $\mathcal{K}_\Sigma(\varphi)$ (see [1]).

In order to derive our main results, we shall need the following lemma.

Lemma 1 (see [5] or [11]) Let $p(z) = 1 + p_1 z + p_2 z^2 + \cdots \in \mathcal{P}$, where $\mathcal{P}$ is the family of all functions p, analytic in $\mathbb{U}$, for which $\Re\{p(z)\} > 0 \quad (z \in \mathbb{U})$. Then

$$|p_n| \leq 2; n = 1, 2, 3, ...,$$

and

$$\left| p_2 - \frac{1}{2}p_1^2 \right| \leq 2 - \frac{1}{2}|p_1|^2.$$

Motivated by the recent publications (especially [1, 19, 22]), we consider the following comprehensive class of functions in Σ.

A function $f \in \Sigma$ given by (1) is said to be in the class $\mathcal{M}_{\Sigma}^{\lambda}(\varphi)$ if the following conditions are satisfied:

$$\frac{zf'(z)}{(1-\lambda)z+\lambda f(z)} \prec \varphi(z) \qquad (0 \le \lambda \le 1,\ z \in \mathbb{U})$$

and for $g = f^{-1}$ given by (2)

$$\frac{wg'(w)}{(1-\lambda)w+\lambda g(w)} \prec \varphi(w) \qquad (0 \le \lambda \le 1,\ w \in \mathbb{U}).$$

A function $f \in \Sigma$ given by (1) is said to be in the class $\mathcal{K}_{\Sigma}(\varphi, \lambda)$ if the following conditions are satisfied:

$$\frac{f'(z)+zf''(z)}{f'(z)+\lambda zf''(z)} \prec \varphi(z) \qquad (0 \le \lambda < 1;\ z \in \mathbb{U})$$

and for $g = f^{-1}$ given by (2)

$$\frac{g'(w)+wg''(w)}{g'(w)+\lambda wg''(w)} \prec \varphi(w) \qquad (0 \le \lambda < 1;\ w \in \mathbb{U}).$$

Remark 1 The following 8 special cases demonstrate the significance of comprehensiveness of the defined classes $\mathcal{M}_{\Sigma}^{\lambda}(\varphi)$ and $\mathcal{K}_{\Sigma}(\varphi, \lambda)$:

(1) $\mathcal{M}_{\Sigma}^{0}(\varphi) = \mathcal{H}_{\Sigma}^{\varphi}$ [1].

(2) $\mathcal{M}_{\Sigma}^{0}\left(\left(\frac{1+z}{1-z}\right)^{\beta}\right) = \mathcal{H}_{\Sigma}^{\beta}$ $(0 < \beta \le 1)$ and $\mathcal{M}_{\Sigma}^{0}\left(\frac{1+(1-2\alpha)z}{1-z}\right) = \mathcal{H}_{\Sigma}^{\alpha}$ $(0 \le \alpha < 1)$ [19].

(3) $\mathcal{M}_{\Sigma}^{1}(\varphi) = \mathcal{S}_{\Sigma}^{*}(\varphi)$ [1].

(4) $\mathcal{M}_{\Sigma}^{1}\left(\left(\frac{1+z}{1-z}\right)^{\beta}\right) = \mathcal{S}_{\Sigma,\beta}^{*}$ $(0 < \beta \le 1)$ and $\mathcal{M}_{\Sigma}^{1}\left(\frac{1+(1-2\alpha)z}{1-z}\right) = \mathcal{S}_{\Sigma}^{*}(\alpha)$ $(0 \le \alpha < 1)$.

(5) $\mathcal{M}_{\Sigma}^{\lambda}\left(\left(\frac{1+z}{1-z}\right)^{\beta}\right) = \mathcal{M}_{\Sigma}^{\lambda}(\beta)$ $(0 \le \lambda \le 1; 0 < \beta \le 1)$
and
$\mathcal{M}_{\Sigma}^{\lambda}\left(\frac{1+(1-2\alpha)z}{1-z}\right) = \mathcal{M}_{\Sigma}^{\lambda}(\alpha)$ $(0 \le \lambda \le 1; 0 \le \alpha < 1)$ [14].

(6) $\mathcal{K}_{\Sigma}(\varphi, 0) = \mathcal{K}_{\Sigma}(\varphi)$ [1].

(7) $\mathcal{K}_{\Sigma}\left(\left(\frac{1+z}{1-z}\right)^{\beta}, \lambda\right) = \mathcal{K}_{\Sigma}(\beta, \lambda)$ $(0 \le \lambda < 1; 0 < \beta \le 1)$

(8) $\mathcal{K}_{\Sigma}\left(\frac{1+(1-2\alpha)z}{1-z}, \lambda\right) = \mathcal{K}_{\Sigma}(\alpha, \lambda)$ $(0 \le \lambda < 1; 0 \le \alpha < 1)$.

In this paper we shall obtain the Fekete-Szegö inequalities for $\mathcal{M}_{\Sigma}^{\lambda}(\varphi)$ and $\mathcal{K}_{\Sigma}(\varphi, \lambda)$ as well as its special classes. Some of the coefficient estimates obtained in this paper prove to be better than these obtained in [1, 14, 19].

2. Fekete-Szegö inequalities

Theorem 1 Let f of the form (1) be in $\mathcal{M}_{\Sigma}^{\lambda}(\varphi)$. Then

$$|a_2| \le \begin{cases} \sqrt{\frac{B_1}{\lambda^2-3\lambda+3}}, & \text{if} \quad |B_2| \le B_1; \\ \sqrt{\frac{|B_2|}{\lambda^2-3\lambda+3}}, & \text{if} \quad |B_2| \ge B_1 \end{cases} \tag{4}$$

and

$$\left|a_3 - \frac{\lambda(2-\lambda)}{3-\lambda}a_2^2\right| \le \begin{cases} \frac{B_1}{3-\lambda}, & \text{if} \quad |B_2| \le B_1; \\ \frac{|B_2|}{3-\lambda}, & \text{if} \quad |B_2| \ge B_1. \end{cases} \tag{5}$$

Proof. Since $f \in \mathcal{M}_\Sigma^\lambda(\varphi)$, there exist two analytic functions $r, s : \mathbb{U} \to \mathbb{U}$, with $r(0) = 0 = s(0)$, such that

$$\frac{zf'(z)}{(1-\lambda)z + \lambda f(z)} = \varphi(r(z)) \tag{6}$$

and

$$\frac{wg'(w)}{(1-\lambda)w + \lambda g(w)} = \varphi(s(w)). \tag{7}$$

Define the functions p and q by

$$p(z) = \frac{1+r(z)}{1-r(z)} = 1 + p_1 z + p_2 z^2 + p_3 z^3 + \dots$$

and

$$q(w) = \frac{1+s(w)}{1-s(w)} = 1 + q_1 w + q_2 w^2 + q_3 w^3 + \dots$$

or equivalently,

$$r(z) = \frac{p(z)-1}{p(z)+1} = \frac{1}{2}\left(p_1 z + \left(p_2 - \frac{p_1^2}{2}\right)z^2 + \left(p_3 + \frac{p_1}{2}\left(\frac{p_1^2}{2} - p_2\right) - \frac{p_1 p_2}{2}\right)z^3 + \dots\right) \tag{8}$$

and

$$s(w) = \frac{q(w)-1}{q(w)+1} = \frac{1}{2}\left(q_1 w + \left(q_2 - \frac{q_1^2}{2}\right)w^2 + \left(q_3 + \frac{q_1}{2}\left(\frac{q_1^2}{2} - q_2\right) - \frac{q_1 q_2}{2}\right)w^3 + \dots\right). \tag{9}$$

Using (8) and (9) in (6) and (7), we have

$$\frac{zf'(z)}{(1-\lambda)z + \lambda f(z)} = \varphi\left(\frac{p(z)-1}{p(z)+1}\right) \tag{10}$$

and

$$\frac{wg'(w)}{(1-\lambda)w + \lambda g(w)} = \varphi\left(\frac{q(w)-1}{q(w)+1}\right). \tag{11}$$

Again using (8) and (9) along with (3), it is evident that

$$\varphi\left(\frac{p(z)-1}{p(z)+1}\right) = 1 + \frac{1}{2}B_1 p_1 z + \left(\frac{1}{2}B_1\left(p_2 - \frac{1}{2}p_1^2\right) + \frac{1}{4}B_2 p_1^2\right)z^2 + \dots \tag{12}$$

and

$$\varphi\left(\frac{q(w)-1}{q(w)+1}\right) = 1 + \frac{1}{2}B_1 q_1 w + \left(\frac{1}{2}B_1\left(q_2 - \frac{1}{2}q_1^2\right) + \frac{1}{4}B_2 q_1^2\right)w^2 + \dots. \tag{13}$$

It follows from (10), (11), (12) and (13) that

$$(2-\lambda)a_2 = \frac{1}{2}B_1 p_1$$

$$(3-\lambda)a_3 + (\lambda^2 - 2\lambda)a_2^2 = \frac{1}{2}B_1\left(p_2 - \frac{1}{2}p_1^2\right) + \frac{1}{4}B_2 p_1^2 \tag{14}$$

$$-(2-\lambda)a_2 = \frac{1}{2}B_1 q_1$$

and

$$(\lambda^2-4\lambda+6)a_2^2-(3-\lambda)a_3 = \frac{1}{2}B_1\left(q_2-\frac{1}{2}q_1^2\right)+\frac{1}{4}B_2q_1^2. \tag{15}$$

Dividing (14) by $(3-\lambda)$ and taking the absolute values we obtain

$$\left|a_3-\frac{\lambda(2-\lambda)}{3-\lambda}a_2^2\right| \le \frac{B_1}{2(3-\lambda)}\left|p_2-\frac{1}{2}p_1^2\right|+\frac{|B_2|}{4(3-\lambda)}|p_1|^2.$$

Now applying Lemma 1 yields

$$\left|a_3-\frac{\lambda(2-\lambda)}{3-\lambda}a_2^2\right| \le \frac{B_1}{3-\lambda}+\frac{|B_2|-B_1}{4(3-\lambda)}|p_1|^2.$$

Therefore

$$\left|a_3-\frac{\lambda(2-\lambda)}{3-\lambda}a_2^2\right| \le \begin{cases} \frac{B_1}{3-\lambda}, & \text{if} \quad |B_2|\le B_1; \\ \\ \frac{|B_2|}{3-\lambda}, & \text{if} \quad |B_2|\ge B_1. \end{cases}$$

Adding (14) and (15), we have

$$2(\lambda^2-3\lambda+3)a_2^2 = \frac{B_1}{2}(p_2+q_2)-\frac{(B_1-B_2)}{4}(p_1^2+q_1^2). \tag{16}$$

Dividing (16) by $2(\lambda^2-3\lambda+3)$ and taking the absolute values we obtain

$$|a_2|^2 \le \frac{1}{2(\lambda^2-3\lambda+3)}\left[\frac{B_1}{2}\left|p_2-\frac{1}{2}p_1^2\right|+\frac{|B_2|}{4}|p_1|^2+\frac{B_1}{2}\left|q_2-\frac{1}{2}q_1^2\right|+\frac{|B_2|}{4}|q_1|^2\right].$$

Once again, apply Lemma 1 to obtain

$$|a_2|^2 \le \frac{1}{2(\lambda^2-3\lambda+3)}\left[\frac{B_1}{2}\left(2-\frac{1}{2}|p_1|^2\right)+\frac{|B_2|}{4}|p_1|^2+\frac{B_1}{2}\left(2-\frac{1}{2}|q_1|^2\right)+\frac{|B_2|}{4}|q_1|^2\right].$$

Upon simplification we obtain

$$|a_2|^2 \le \frac{1}{2(\lambda^2-3\lambda+3)}\left[2B_1+\frac{|B_2|-B_1}{2}\left(|p_1|^2+|q_1|^2\right)\right].$$

Therefore

$$|a_2| \le \begin{cases} \sqrt{\frac{B_1}{\lambda^2-3\lambda+3}}, & \text{if} \quad |B_2|\le B_1; \\ \\ \sqrt{\frac{|B_2|}{\lambda^2-3\lambda+3}}, & \text{if} \quad |B_2|\ge B_1 \end{cases}$$

which completes the proof.

Remark 2 Taking

$$\varphi(z) = \left(\frac{1+z}{1-z}\right)^\beta = 1+2\beta z+2\beta^2z^2+\dots \qquad (0<\beta\le 1) \tag{17}$$

the inequalities (4) and (5) become

$$|a_2| \le \sqrt{\frac{2\beta}{\lambda^2-3\lambda+3}} \qquad \text{and} \qquad \left|a_3-\frac{\lambda(2-\lambda)}{3-\lambda}a_2^2\right| \le \frac{2\beta}{3-\lambda}. \tag{18}$$

The bound on $|a_2|$ given in (18) is better than that given in [14, Corollary 2.2].

For

$$\varphi(z) = \frac{1+(1-2\alpha)z}{1-z} = 1 + 2(1-\alpha)z + 2(1-\alpha)z^2 + \dots \qquad (0 \le \alpha < 1) \quad (19)$$

the inequalities (4) and (5) become

$$|a_2| \le \sqrt{\frac{2(1-\alpha)}{\lambda^2 - 3\lambda + 3}} \qquad \text{and} \qquad \left| a_3 - \frac{\lambda(2-\lambda)}{3-\lambda} a_2^2 \right| \le \frac{2(1-\alpha)}{3-\lambda}. \quad (20)$$

The bound on $|a_2|$ given in (20) is coincides with that in [14, Corollary 2.3].

Theorem 2 Let f of the form (1) be in $\mathcal{K}_\Sigma(\varphi, \lambda)$. Then

$$|a_2| \le \begin{cases} \sqrt{\frac{B_1}{2(1-\lambda)(1-2\lambda)}}, & \text{if } |B_2| \le B_1; \\ \\ \sqrt{\frac{|B_2|}{2(1-\lambda)(1-2\lambda)}}, & \text{if } |B_2| \ge B_1 \end{cases} \quad (21)$$

and

$$\left| a_3 - \frac{2(1+\lambda)}{3} a_2^2 \right| \le \begin{cases} \frac{B_1}{6(1-\lambda)}, & \text{if } |B_2| \le B_1; \\ \\ \frac{|B_2|}{6(1-\lambda)}, & \text{if } |B_2| \ge B_1. \end{cases} \quad (22)$$

The proof is omitted as it is similar to the proof of Theorem 1.

Remark 3 For $\varphi(z)$ as given in (17) the inequalities (21) and (22) become

$$|a_2| \le \sqrt{\frac{\beta}{(1-\lambda)(1-2\lambda)}} \qquad \text{and} \qquad \left| a_3 - \frac{2(1+\lambda)}{3} a_2^2 \right| \le \frac{\beta}{3(1-\lambda)}.$$

Taking $\varphi(z)$ as given in (19) the inequalities (21) and (22) become

$$|a_2| \le \sqrt{\frac{1-\alpha}{(1-\lambda)(1-2\lambda)}} \qquad \text{and} \qquad \left| a_3 - \frac{2(1+\lambda)}{3} a_2^2 \right| \le \frac{1-\alpha}{3(1-\lambda)}.$$

Corollary 1 If $f \in \mathcal{H}_\Sigma^\varphi$ then

$$|a_2| \le \begin{cases} \sqrt{\frac{B_1}{3}}, & \text{if } |B_2| \le B_1; \\ \\ \sqrt{\frac{|B_2|}{3}}, & \text{if } |B_2| \ge B_1 \end{cases} \quad (23)$$

and

$$|a_3| \le \begin{cases} \frac{B_1}{3}, & \text{if } |B_2| \le B_1; \\ \\ \frac{|B_2|}{3}, & \text{if } |B_2| \ge B_1. \end{cases} \quad (24)$$

Remark 4 The bounds on $|a_2|$ and $|a_3|$ given by (23) and (24) are better than those given in [1, Theorem 2.1] and [17, Theorem 2.1].

Corollary 2 If $f \in \mathcal{S}_\Sigma^*(\varphi)$ then

$$|a_2| \le \begin{cases} \sqrt{B_1}, & \text{if } |B_2| \le B_1; \\ \\ \sqrt{|B_2|}, & \text{if } |B_2| \ge B_1 \end{cases} \quad (25)$$

and

$$\left|a_3 - \frac{1}{2}a_2^2\right| \leq \begin{cases} \frac{B_1}{2}, & \text{if} \quad |B_2| \leq B_1; \\ \frac{|B_2|}{2}, & \text{if} \quad |B_2| \geq B_1. \end{cases} \tag{26}$$

Remark 5 For $\varphi(z)$ as given in (17) the inequalities (25) and (26) become

$$|a_2| \leq \sqrt{2\beta} \qquad \text{and} \qquad \left|a_3 - \frac{1}{2}a_2^2\right| \leq \beta. \tag{27}$$

The bound on $|a_2|$ given in (27) is better than that given in [17, Remark 2.2].

Taking $\varphi(z)$ as given in (19), the inequalities (25) and (26) become

$$|a_2| \leq \sqrt{2(1-\alpha)} \qquad \text{and} \qquad \left|a_3 - \frac{1}{2}a_2^2\right| \leq 1-\alpha. \tag{28}$$

The bound on $|a_2|$ given in (28) is better than that given in [17, Remark 2.2].

Corollary 3 If $f \in \mathcal{K}_\Sigma(\varphi)$ then

$$|a_2| \leq \begin{cases} \sqrt{\frac{B_1}{2}}, & \text{if} \quad |B_2| \leq B_1; \\ \sqrt{\frac{|B_2|}{2}}, & \text{if} \quad |B_2| \geq B_1 \end{cases}$$

and

$$\left|a_3 - \frac{2}{3}a_2^2\right| \leq \begin{cases} \frac{B_1}{6}, & \text{if} \quad |B_2| \leq B_1; \\ \frac{|B_2|}{6}, & \text{if} \quad |B_2| \geq B_1. \end{cases}$$

Remark 6 The bound on $|a_2|$ given in Corollary 3 is better than that given in [1, Corollary 2.2].

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