

**A COUPLE OF SOLUTIONS TO A (3+1)-DIMENSIONAL  
GENERALIZED KP EQUATION WITH VARIABLE  
COEFFICIENTS BY EXTENDED TRANSFORMED RATIONAL  
FUNCTION METHOD**

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**ABSTRACT.** In this paper, we study a (3+1)-dimensional generalized KP equation with variable coefficients. The extended transformed rational function method is applied to obtain a couple of solutions to this equation. A couple of solutions to this equation are found. This method is a powerful tool for obtaining exact solutions of nonlinear evolution equations.

1. INTRODUCTION

In this paper we consider a (3+1)-dimensional generalized Kadomtsev-Petviashvili (KP) equation in the form ([1])

$$(u_t + \alpha_1(t)u_{xxy} + 3\alpha_2(t)u_xu_y)_x + \alpha_3(t)u_{ty} - \alpha_4(t)u_{zz} + \alpha_5(t)(u_x + \alpha_3(t)u_y) = 0, \quad (1)$$

where  $\alpha_i (i = 1, 2, 3, 4, 5)$  are nonzero arbitrary analytic functions with respect to  $t$ . The Kadomtsev-Petviashvili equation is integrable and describes the evolution of quasi-one-dimensional shallow-water waves when effects of the surface tension and the viscosity are negligible ([2], [3], [4], [5], [6], [7]). The aim of this paper is to look for a couple of solutions to the Kadomtsev-Petviashvili equation. With the development of nonlinear science, nonlinear evolution equations (NLEEs) have been used as the models to describe some physical phenomena in fluid mechanics, plasma physics, optical fibers, biology, solid state physics, chemical kinematics, chemical physics, geochemistry, etc ([1]-[30]).

In order to understand the mechanisms of those physical phenomena, it is necessary to explore their solutions and properties. Solutions for the NLEEs can not only describe the designated problems, but also give more insights on the physical aspects of the problems in the related fields. In recent years, various powerful methods have been presented for finding exact solutions of the NLEEs in mathematical physics, such as multiple exp-function method ([18]), tanh-sech method ([19]-[21]), extended tanh method ([22]), ansatz method ([23]) transformed rational function

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method ([24]) and so on. The transformed rational function method, which is a direct and effective algebraic method for computing exact solutions, was first proposed by Ma and Lee ([24]). The transformed rational function method provides a systematical and convenient handling of the solution process of nonlinear equations, unifying the tanh function type methods, the homogeneous balance method, the exp-function method, the mapping method, and the F-expansion type methods. The most complete description of this method was given in ([25]) for obtaining complexiton solutions of nonlinear differential equations. In ([26]-[27]), a novel class of explicit exact solutions to the Korteweg-de Vries equation was presented through its bilinear form. Such solutions possess singularities of combinations of trigonometric function waves and exponential function waves which have different traveling speeds of new type. This type of solutions would help us in recognizing a great diversity of motions of nonlinear waves described by soliton equations. Using extended transformed rational function method (see Ref. [25]) a couple of solutions to some nonlinear differential equations were obtained. The aim of this paper is to find a couple of solutions to the Kadomtsev-Petviashvili equation by using the extended transformed rational function method ([25]).

## 2. EXTENDED TRANSFORMED RATIONAL FUNCTION METHOD

We first consider a general form of nonlinear differential equation

$$P(u, u_x, u_y, u_z, u_t, u_{xx}, \dots) = 0. \quad (2)$$

The main steps of the extended transformed rational function method ([25]) are summarized as follows.

**Step I.** Suppose Eq. (2) has a Hirota bilinear form:

$$H(D_x, D_y, D_z, D_t, \dots) f \cdot f = 0, \quad (3)$$

where  $D_x, D_y, D_z, D_t, \dots$  are Hirota's differential operators defined by

$$D_z^p f(z) \cdot g(z) = (\partial_z - \partial_{z'})^p f(z)g(z')|_{z'=z} = \partial_{z'}^p f(z+z')g(z-z')|_{z'=0}, \quad p \geq 1. \quad (4)$$

**Step II.** Suppose

$$f = \frac{p(\eta_1, \eta_2)}{q(\eta_1, \eta_2)}, \quad (5)$$

where  $p(\eta_1, \eta_2)$  and  $q(\eta_1, \eta_2)$  are polynomials and  $\eta_1$  and  $\eta_2$  admit, for example,

$$\eta_1'' = \frac{d^2 \eta_1}{d\xi_1^2} = -\eta_1, \quad (6)$$

$$\eta_2'' = \frac{d^2 \eta_2}{d\xi_2^2} = \eta_2, \quad (7)$$

where  $\xi_1 = \kappa_1 x + \kappa_1 b y + \kappa_1 c z + \omega_1 t + c_1$  and  $\xi_2 = \kappa_2 x + \kappa_2 b y + \kappa_2 c z + \omega_2 t + c_2$ ,  $\kappa_1, \kappa_2, \omega_1, b, c$  and  $\omega_2$  can be determined later and  $c_1$  and  $c_2$  are arbitrary constants.

**Step III.** Choose appropriate  $p(\eta_1, \eta_2)$  and  $q(\eta_1, \eta_2)$ , we can convert (3) into an algebra equation involving  $\kappa_i$  and  $\omega_i$ . Solving this algebra equation, we will obtain exact solutions to Eq. (2).

3. A COUPLE OF SOLUTIONS TO A (3+1)-DIMENSIONAL GENERALIZED KP EQUATION WITH VARIABLE COEFFICIENTS

Under the transformation of

$$u(x, y, z, t) = 2 \frac{\alpha_1(t)}{\alpha_2(t)} (\ln f(x, y, z, t))_x, \quad (8)$$

Eq. (1) can be rewritten in its Hirota bilinear form ([1]):

$$(\alpha_1(t)D_x^3D_y + D_tD_x + \alpha_3(t)D_tD_y - \alpha_4(t)D_z^2) f \cdot f = 0, \quad (9)$$

under the constraint

$$\alpha_1(t) = C_0\alpha_2(t)e^{-\int \alpha_5(t)dt}. \quad (10)$$

The above equation gives

$$\begin{aligned} & \alpha_1(t)f_{xxxy}f + f_{tx}f + \alpha_3(t)f_{ytf} - \alpha_4(t)f_{zz}f \\ & - 3\alpha_1(t)f_{xxy}f_x + 3\alpha_1(t)f_{xy}f_{xx} - \alpha_1(t)f_yf_{xxx} \\ & - f_t f_x - \alpha_3(t)f_t f_y + \alpha_4(t)f_z^2 = 0, \end{aligned} \quad (11)$$

where  $f = f(x, y, z, t)$ .

Using the extended transformed rational function method mentioned, we suppose that

$$f = A\eta_1 + B\eta_2, \quad (12)$$

where  $\eta_1$  and  $\eta_2$  are defined by (6) and (7) respectively, and

$$\xi_1 = \kappa_1x + \kappa_1by + \kappa_1cz + \omega_1(t)t + c_1 \quad (13)$$

and

$$\xi_2 = \kappa_2x + \kappa_2by + \kappa_2cz + \omega_2(t)t + c_2 \quad (14)$$

and  $A$ ,  $B$ ,  $\kappa_i$  and  $\omega_i$  are determined later. Based on the above assumption, we can compute that

$$f_{xxxy}f = A^2b\kappa_1^4\eta_1^2 + B^2b\kappa_2^4\eta_2^2 + ABb(\kappa_1^4 + \kappa_2^4)\eta_1\eta_2, \quad (15)$$

$$\begin{aligned} f_{tx}f &= -A^2\kappa_1 \left( t \frac{d\omega_1(t)}{dt} + \omega_1(t) \right) \eta_1^2 + B^2\kappa_2 \left( t \frac{d\omega_2(t)}{dt} + \omega_2(t) \right) \eta_2^2 \\ &+ AB \left[ \kappa_2 \left( t \frac{d\omega_1(t)}{dt} + \omega_1(t) \right) - \kappa_1 \left( t \frac{d\omega_2(t)}{dt} + \omega_2(t) \right) \right] \eta_1\eta_2, \end{aligned} \quad (16)$$

$$\begin{aligned} f_{ty}f &= -A^2b\kappa_1 \left( t \frac{d\omega_1(t)}{dt} + \omega_1(t) \right) \eta_1^2 + B^2b\kappa_2 \left( t \frac{d\omega_2(t)}{dt} + \omega_2(t) \right) \eta_2^2 \\ &+ ABb \left[ \kappa_2 \left( t \frac{d\omega_1(t)}{dt} + \omega_1(t) \right) - \kappa_1 \left( t \frac{d\omega_2(t)}{dt} + \omega_2(t) \right) \right] \eta_1\eta_2, \end{aligned} \quad (17)$$

$$f_{zz}f = -A^2c^2\kappa_1^2\eta_1^2 + B^2c^2\kappa_2^2\eta_2^2 + ABC^2(\kappa_2^2 - \kappa_1^2)\eta_1\eta_2, \quad (18)$$

$$f_{xxy}f_x = -A^2b\kappa_1^4\eta_1'^2 + B^2b\kappa_2^4\eta_2'^2 + ABb(\kappa_1\kappa_2^3 - \kappa_1^3\kappa_2)\eta_1'\eta_2', \quad (19)$$

$$f_{xy}f_{xx} = A^2b\kappa_1^4\eta_1'^2 + B^2b\kappa_2^4\eta_2'^2 - 2AB\kappa_1^2\kappa_2^2\eta_1\eta_2, \quad (20)$$

$$f_yf_{xxx} = -A^2b\kappa_1^4\eta_1'^2 + B^2b\kappa_2^4\eta_2'^2 + ABb(\kappa_1\kappa_2^3 - \kappa_1^3\kappa_2)\eta_1'\eta_2', \quad (21)$$

$$\begin{aligned} f_t f_x &= A^2\kappa_1 \left( t \frac{d\omega_1(t)}{dt} + \omega_1(t) \right) \eta_1'^2 + B^2\kappa_2 \left( t \frac{d\omega_2(t)}{dt} + \omega_2(t) \right) \eta_2'^2 \\ &+ AB \left[ \kappa_2 \left( t \frac{d\omega_1(t)}{dt} + \omega_1(t) \right) + \kappa_1 \left( t \frac{d\omega_2(t)}{dt} + \omega_2(t) \right) \right] \eta_1'\eta_2', \end{aligned} \quad (22)$$

$$\begin{aligned} f_t f_y &= A^2b\kappa_1 \left( t \frac{d\omega_1(t)}{dt} + \omega_1(t) \right) \eta_1'^2 + B^2b\kappa_2 \left( t \frac{d\omega_2(t)}{dt} + \omega_2(t) \right) \eta_2'^2 \\ &+ ABb \left[ \kappa_2 \left( t \frac{d\omega_1(t)}{dt} + \omega_1(t) \right) + \kappa_1 \left( t \frac{d\omega_2(t)}{dt} + \omega_2(t) \right) \right] \eta_1'\eta_2', \end{aligned} \quad (23)$$

$$f_z^2 = A^2c^2\kappa_1^2\eta_1'^2 + B^2c^2\kappa_2^2\eta_2'^2 + 2ABc^2\kappa_1\kappa_2\eta_1'\eta_2'. \quad (24)$$

Substituting the above into (11) with the relation between  $\eta_1^2$ ,  $\eta_2^2$  and  $\eta_1'$ ,  $\eta_2'$  we can simplify (11) into a polynomial with respect to  $\eta_1^2$ ,  $\eta_2^2$ ,  $\eta_1\eta_2$ ,  $\eta_1'\eta_2'$ . Collecting coefficients of  $\eta_1^2$ ,  $\eta_2^2$ ,  $\eta_1\eta_2$ ,  $\eta_1'\eta_2'$  and the constant term, and letting those coefficients be zero, we obtain an algebra equation for  $\kappa_i$  and  $\omega_i$ . For example, under the conditions of  $\eta_1'^2 = 1 - \eta_1^2$  and  $\eta_2'^2 = 1 + \eta_2^2$ , the following algebraic equations are obtained:

$$\begin{aligned} &\alpha_1(t)b\kappa_1^4 + \alpha_1(t)b\kappa_2^4 + \kappa_2 \left( t \frac{d\omega_2(t)}{dt} + \omega_2(t) \right) - \kappa_1 \left( t \frac{d\omega_1(t)}{dt} + \omega_1(t) \right) \\ &+ \alpha_3(t)b\kappa_2 \left( t \frac{d\omega_2(t)}{dt} + \omega_2(t) \right) - \alpha_3(t)b\kappa_1 \left( t \frac{d\omega_1(t)}{dt} + \omega_1(t) \right) \\ &- \alpha_4(t)c^2\kappa_2^2 + \alpha_4(t)c^2\kappa_1^2 - 6b\alpha_1(t)\kappa_1^2\kappa_2^2 = 0, \end{aligned} \quad (25)$$

$$\begin{aligned} &-4\alpha_1(t)(b\kappa_1\kappa_2^3 - b\kappa_2\kappa_1^3) - \left( \kappa_2 \left( t \frac{d\omega_1(t)}{dt} + \omega_1(t) \right) + \kappa_1 \left( t \frac{d\omega_2(t)}{dt} + \omega_2(t) \right) \right) \\ &2\alpha_4(t)c^2\kappa_1\kappa_2 - \alpha_3(t)(b\kappa_2 \left( t \frac{d\omega_1(t)}{dt} + \omega_1(t) \right) \\ &+ b\kappa_1 \left( t \frac{d\omega_2(t)}{dt} + \omega_2(t) \right)) = 0, \end{aligned} \quad (26)$$

$$\begin{aligned} &4\alpha_1(t)A^2b\kappa_1^4 - 4\alpha_1(t)B^2b\kappa_2^4 - A^2\kappa_1 \left( t \frac{d\omega_1(t)}{dt} + \omega_1(t) \right) - B^2\kappa_2 \left( t \frac{d\omega_2(t)}{dt} + \omega_2(t) \right) \\ &+ \alpha_4(t)A^2c^2\kappa_1^2 + \alpha_4(t)B^2c^2\kappa_2^2 - \alpha_3(t)A^2b\kappa_1 \left( t \frac{d\omega_1(t)}{dt} + \omega_1(t) \right) \\ &- \alpha_3(t)B^2b\kappa_2 \left( t \frac{d\omega_2(t)}{dt} + \omega_2(t) \right) = 0. \end{aligned} \quad (27)$$

Solving (25)-(27), we obtain

$$\begin{aligned}\omega_1(t) &= \frac{1}{t} \int_0^t \left\{ \frac{\alpha_4(t')c^2\kappa_1 - 2\alpha_1(t')b\kappa_1^3}{1 + \alpha_3(t')b} \right\} dt', \\ \omega_2 &= \frac{1}{t} \int_0^t \left\{ \frac{\alpha_4(t')c^2\kappa_1 + 2\alpha_1(t')b\kappa_1^3}{1 + \alpha_3(t')b} \right\} dt', \\ A = B, \quad \kappa_2 &= \kappa_1\end{aligned}\tag{28}$$

or

$$\begin{aligned}\omega_1(t) &= \frac{1}{t} \int_0^t \left\{ \frac{\alpha_4(t')c^2\kappa_1 - 2\alpha_1(t')b\kappa_1^3}{1 + \alpha_3(t')b} \right\} dt', \\ \omega_2 &= -\frac{1}{t} \int_0^t \left\{ \frac{\alpha_4(t')c^2\kappa_1 + 2\alpha_1(t')b\kappa_1^3}{1 + \alpha_3(t')b} \right\} dt', \\ A = B, \quad \kappa_2 &= -\kappa_1.\end{aligned}\tag{29}$$

Thus, we obtain a couple of solutions to Eq. (1) as follows:

$$u(x, y, z, t) = 2 \frac{\alpha_1(t)}{\alpha_2(t)} (\ln f(x, y, z, t))_x,\tag{30}$$

$$\begin{aligned}f(x, y, z, t) &= A \left[ \sin \left( \kappa_1 x + \kappa_1 by + \kappa_1 cz + \int_0^t \left\{ \frac{\alpha_4(t')c^2\kappa_1 - 2\alpha_1(t')b\kappa_1^3}{1 + \alpha_3(t')b} \right\} dt' + c_1 \right) \right. \\ &\quad \left. \pm \sinh \left( \kappa_1 x + \kappa_1 by + \kappa_1 cz + \int_0^t \left\{ \frac{\alpha_4(t')c^2\kappa_1 + 2\alpha_1(t')b\kappa_1^3}{1 + \alpha_3(t')b} \right\} dt' + c_2 \right) \right]\end{aligned}$$

or

$$\begin{aligned}f(x, y, z, t) &= A \left[ \cos \left( \kappa_1 x + \kappa_1 by + \kappa_1 cz + \int_0^t \left\{ \frac{\alpha_4(t')c^2\kappa_1 - 2\alpha_1(t')b\kappa_1^3}{1 + \alpha_3(t')b} \right\} dt' + c_1 \right) \right. \\ &\quad \left. \pm \sinh \left( \kappa_1 x + \kappa_1 by + \kappa_1 cz + \int_0^t \left\{ \frac{\alpha_4(t')c^2\kappa_1 + 2\alpha_1(t')b\kappa_1^3}{1 + \alpha_3(t')b} \right\} dt' + c_2 \right) \right]\end{aligned}$$

where  $A \neq 0$ ,  $\kappa_1$ ,  $c_1$ ,  $c_2$  and  $b \neq -\frac{1}{\alpha_3(t)}$  are arbitrary constants.

#### 4. CONCLUSION

In this work, we obtained a couple of solutions to a (3+1)-dimensional generalized KP equation with variable coefficients using the extended transformed rational function method. The obtained solution may be significant and important for analyzing the nonlinear phenomena arising in applied physical sciences. The results show that the proposed method is direct, effective and can be applied to many other nonlinear evolution equations in mathematical physics.

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