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EXTENDED BINARY SIMPLEST EQUATION METHOD TO STUDY A GENERALIZED SINH-GORDON EQUATION

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Abstract. The generalized sinh-Gordon equation for describing generic properties of string dynamics for strings and multi-strings in constant-curvature space is studied. Extended binary simplest equation method, which is based on the assumptions that the exact solutions can be expressed by a polynomial in F, G such that $F = F(\xi)$, $G = G(\eta)$ satisfy in the equations of Bernoulli and Riccati which are well known nonlinear ordinary differential equations and their solutions can be expressed by elementary functions, is used for obtaining new exact solutions of this equation. This method is a powerful tool for searching exact travelling solutions in closed form.

1. Introduction

In this paper, we consider the generalized sinh-Gordon equation [1-9],

$$u_{tt} - au_{xx} + b\sinh(nu) = 0, \tag{1}$$

where a, b are two constants and n is a positive integer.
The study of nonlinear partial differential equations (NPDEs) is extremely important in various branches of applied sciences. These NPDEs form the fabric of various physical phenomena in nonlinear optics, plasma physics, nuclear physics, mathematical biology, fluid dynamics, and many other areas in physical and biological sciences. Recently many new approaches for finding the exact solutions to NPDEs have been proposed [1-34]. One of the most effective mathematical methods for finding exact solutions of NPDEs is the simplest equation method. This method was introduced by Kudryashov [10, 11] and modified by Vitanov [12, 13]. The simplest equations we use in this paper are the Bernoulli and Riccati equations. Their solutions can be written in elementary functions. In the present work, we would like to extend the simplest equation method to solve the generalized sinh-Gordon equation. The outline of this paper is organized as follows: In Section 2, we give the description of the extended binary simplest equation method. In Section 3, we apply this method to the generalized sinh-Gordon equation. Conclusions are given in Section 4.

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2. EXTENDED BINARY SIMPLEST EQUATION METHOD

Consider a general nonlinear partial differential equation (PDE) in the form

$$H\left(g\left(u\right), u_x, u_t, u_{xx}, u_{xt}, ...\right) = 0. \tag{2}$$

where $g(u)$ is a composite function which is similar to $\sin\left(nu\right)$ or $\sinh\left(nu\right)$, $(n = 1, 2, ...)$ etc.
Using a transformation [1]

$$u = \phi\left(\frac{U(\xi)}{V(\eta)}\right) \tag{3}$$

where $u = \frac{4}{n}\tanh^{-1}\left(\frac{U(\xi)}{V(\eta)}\right)$, $u = \frac{4}{n}\tan^{-1}\left(\frac{U(\xi)}{V(\eta)}\right)$ are two special cases of Eq. (3), Eq. (2) can be converted to an ordinary differential equation (ODE) as

$$H\left(U,\ U',\ U'', ..., V,\ V',\ V'', ...\right) = 0, \tag{4}$$

where $\xi = \lambda_1\left(x + c_1 t\right)$, $\eta = \lambda_2\left(x + c_2 t\right)$ and $\lambda_1,\ \lambda_2,\ c_1,\ c_2$ are unknown parameters which are to be determined later.
Then, we look for the solution of Eq. (4) in the form

$$U = \sum_{k=0}^{M} A_k F^k\left(\xi\right), \tag{5}$$

$$V = \sum_{k=0}^{N} B_k G^k\left(\eta\right), \tag{6}$$

where $F\left(\xi\right)$, $G\left(\eta\right)$ satisfy the Bernoulli and Riccati equations, $M,\ N$ are a positive integer that can be determined by balancing procedure as in [14] and $A_k,\ B_k$ are parameters to be determined. The Bernoulli and Riccati equations are well-known the simplest equations whose solutions can be expressed in terms of elementary functions. In this paper, we use the following Riccati equations as simplest equations

$$F'(\xi) = \alpha F^2(\xi) + \beta F(\xi) + \gamma,\ G'(\eta) = \alpha G^2(\eta) + \beta G(\eta) + \gamma \tag{7}$$

where α, β and γ are independent on ξ. Eqs. (7) admit the following exact solutions

$$\begin{aligned} F(\xi) &= -\frac{\beta}{2\alpha} - \frac{\theta}{2\alpha}\tanh\left\{\frac{\theta}{2}\left(\xi + \xi_0\right)\right\} \\ G(\eta) &= -\frac{\beta}{2\alpha} - \frac{\theta}{2\alpha}\tanh\left\{\frac{\theta}{2}\left(\eta + \eta_0\right)\right\} \end{aligned} \tag{8}$$

and

$$\begin{aligned} F(\xi) &= -\frac{\beta}{2\alpha} - \frac{\theta}{2\alpha}\tanh\left(\frac{\theta}{2}\xi\right) + \frac{\operatorname{sech}\left(\frac{\theta}{2}\xi\right)}{C\cosh\left(\frac{\theta}{2}\xi\right) - \frac{2\alpha}{\theta}\sinh\left(\frac{\theta}{2}\xi\right)} \\ G(\eta) &= -\frac{\beta}{2\alpha} - \frac{\theta}{2\alpha}\tanh\left(\frac{\theta}{2}\eta\right) + \frac{\operatorname{sech}\left(\frac{\theta}{2}\eta\right)}{C\cosh\left(\frac{\theta}{2}\eta\right) - \frac{2\alpha}{\theta}\sinh\left(\frac{\theta}{2}\eta\right)} \end{aligned} \tag{9}$$

with $\theta^2 = \beta^2 - 4\alpha\gamma > 0$ and C is a constant of integration.
Substituting Eqs. (5) and (6) into Eq. (4) by using Eqs. (7) yields a set of algebraic equations for $F^k\left(\xi\right)$, $G^k\left(\eta\right)$ and all coefficients of $F^k\left(\xi\right)$, $G^k\left(\eta\right)$ have to vanish. After this separated algebraic equations, we can find coefficients

$\lambda_1,\ \lambda_2,\ c_1,\ c_2,\ A_k,\ B_k$.

Remark1. If $\gamma = 0$, then the Riccati Eqs. (7) reduce to the Bernoulli equations

$$F'(\xi) = \alpha F^2(\xi) + \beta F(\xi),\ G'(\eta) = \alpha G^2(\eta) + \beta G(\eta), \tag{10}$$

The solution of the Bernoulli Eqs. (10) can be written in the following form [15]:

$$\begin{aligned} F(\xi) &= \beta \left\{ \frac{\cosh\left(\beta(\xi+\xi_0)\right) + \sinh\left(\beta(\xi+\xi_0)\right)}{1 - \alpha\cosh\left(\beta(\xi+\xi_0)\right) - \alpha\sinh\left(\beta(\xi+\xi_0)\right)} \right\}, \\ G(\eta) &= \beta \left\{ \frac{\cosh\left(\beta(\eta+\eta_0)\right) + \sinh\left(\beta(\eta+\eta_0)\right)}{1 - \alpha\cosh\left(\beta(\eta+\eta_0)\right) - \alpha\sinh\left(\beta(\eta+\eta_0)\right)} \right\} \end{aligned} \tag{11}$$

Remark2. when $\alpha = 1,\ \beta = 0$, the Riccati Eqs. (7) reduce to the following Riccati equations

$$F'(\xi) = F^2(\xi) + \gamma,\ G'(\eta) = G^2(\eta) + \gamma. \tag{12}$$

The Riccati Eqs. (12) have the following solutions:

$$\begin{aligned} F(\xi) &= -\sqrt{-\gamma}\ \tanh\left(\sqrt{-\gamma}\xi\right),\ G(\eta) = -\sqrt{-\gamma}\ \tanh\left(\sqrt{-\gamma}\eta\right),\ \gamma < 0, \\ F(\xi) &= \sqrt{\gamma}\ \tan\left(\sqrt{\gamma}\xi\right),\ G(\eta) = \sqrt{\gamma}\ \tan\left(\sqrt{\gamma}\eta\right),\ \gamma > 0. \end{aligned} \tag{13}$$

3. APLLICATION TO THE GENERALIZED SINGH-GORDON EQUATION

First, considering the following traveling wave variable:

$$\xi = \lambda\left(x + ct\right),\ \eta = \lambda\left(x + \frac{a}{c}t\right),\ a \neq c^2, \tag{14}$$

where $\lambda,\ c$ are two parameters to be determined later, under the traveling wave variables (14), Eq. (1) can be rewritten as

$$\lambda^2 c^2 (c^2 - a) u_{\xi\xi} + \lambda^2 a (a - c^2) u_{\eta\eta} + bc^2 \sinh(nu) = 0. \tag{15}$$

By means of a similar ansatz as given in Refs. [16, 17], letting

$$u = \frac{4}{n} \tanh^{-1}\left(\frac{U(\xi)}{V(\eta)}\right), \tag{16}$$

for Eq. (15), yields

$$\begin{aligned} &\lambda^2 c^2 (c^2 - a)\left((U^2 - V^2)\frac{U_{\xi\xi}}{U} - 2U_\xi^2\right) \\ &+\lambda^2 a(a - c^2)\left((V^2 - U^2)\frac{V_{\eta\eta}}{V} - 2V_\eta^2\right) = nbc^2(U^2 + V^2). \end{aligned} \tag{17}$$

Successive differentiation of this result with respect to both ξ and η results in

$$\frac{c^2}{UU_\xi}\left(\frac{U_{\xi\xi}}{U}\right)_\xi - \frac{a}{VV_\eta}\left(\frac{V_{\eta\eta}}{V}\right)_\eta = 0, \tag{18}$$

and from (18), one has

$$\frac{c^2}{UU_\xi}\left(\frac{U_{\xi\xi}}{U}\right)_\xi = \frac{a}{VV_\eta}\left(\frac{V_{\eta\eta}}{V}\right)_\eta = \omega, \tag{19}$$

where ω is a parameter to be determined later, i.e.

$$U_{\xi}^2 = \frac{\omega}{4c^2}U^4 + \mu_1 U^2 + v_1, \tag{20}$$

$$V_{\eta}^2 = \frac{\omega}{4a}V^4 + \mu_2 V^2 + v_2, \tag{21}$$

where $v_1,\ v_2,\ \mu_1,\ \mu_2$ are integral constants. Considering (17) and (20), (21) we have the corresponding constraint conditions

$$\lambda^2 c^2 (c^2 - a)\mu_1 + \lambda^2 a(a - c^2)\mu_2 + nbc^2 = 0,\ c^2 v_1 - a v_2 = 0. \tag{22}$$

When balancing U_{ξ}^2 with U^4 and V_{ξ}^2 with V^4 then gives

$$2M + 2 = 4M \Longrightarrow M = 1,$$

$$2N + 2 = 4N \Longrightarrow N = 1.$$

Therefore, we may choose

$$U(\xi) = A_0 + A_1 F(\xi), \tag{23}$$

$$V(\eta) = B_0 + B_1 G(\eta). \tag{24}$$

Substituting Eqs. (23), (24) into Eqs. (20), (21) using (7) and setting all the coefficients of powers $F(\xi),\ G(\eta)$ to be zero, then we obtain a system of nonlinear algebraic equations and by solving it, we obtain

$$\begin{aligned}
&A_0 = \pm\frac{cB_0}{\sqrt{a}},\ A_1 = \pm\frac{2cB_0\alpha}{\sqrt{a}\beta},\ B_1 = \frac{2\alpha B_0}{\beta},\\
&\mu_1 = -\frac{1}{2}\beta^2 + 2\gamma\alpha,\ \mu_2 = -\frac{1}{2}\beta^2 + 2\gamma\alpha,\\
v_1 \quad = \quad &\frac{c^2 B_0^2 (16\alpha^2\gamma^2 + \beta^4 - 8\beta^2\gamma\alpha)}{4\beta^2 a},\\
&v_2 = \frac{B_0^2(16\alpha^2\gamma^2 + \beta^4 - 8\beta^2\gamma\alpha)}{4\beta^2},\ \omega = \frac{a\beta^2}{B_0^2},
\end{aligned} \tag{25}$$

where B_0 is an arbitrary constant.
Substituting (25) into the constraint conditions (22), the parameters $c,\ \lambda$ can be determined as

$$c^2 = -a,\ \lambda^2 = \frac{nb}{2a\left(4\gamma\alpha - \beta^2\right)}. \tag{26}$$

Therefore, using solutions (8) and (9) of Eqs. (7), ansatzs (23) and (24), we obtain the exact solutions of Eq. (15) and then the new exact traveling-wave solutions to Eq. (1) can be written as

$$\begin{aligned}
u(x,t) &= \frac{4}{n}\tanh^{-1}\left\{\pm\frac{c}{\sqrt{a}}\frac{1+\frac{2\alpha}{\beta}\left(-\frac{\beta}{2\alpha}-\frac{\theta}{2\alpha}\tanh\left\{\frac{\theta}{2}\left(\lambda\left(x+ct\right)+\xi_0\right)\right\}\right)}{1+\frac{2\alpha}{\beta}\left(-\frac{\beta}{2\alpha}-\frac{\theta}{2\alpha}\tanh\left\{\frac{\theta}{2}\left(\lambda\left(x+\frac{a}{c}t\right)+\eta_0\right)\right\}\right)}\right\},\\
u(x,t) &= \frac{4}{n}\tanh^{-1}\left\{\pm\frac{c}{\sqrt{a}}\frac{1+\frac{2\alpha}{\beta}\left(-\frac{\beta}{2\alpha}-\frac{\theta}{2\alpha}\tanh\left(\frac{\theta\lambda}{2}\left(x+ct\right)\right)+\frac{\operatorname{sech}\left(\frac{\theta\lambda}{2}(x+ct)\right)}{C\cosh\left(\frac{\theta\lambda}{2}(x+ct)\right)-\frac{2\alpha}{\theta}\sinh\left(\frac{\theta\lambda}{2}(x+ct)\right)}\right)}{1+\frac{2\alpha}{\beta}\left(-\frac{\beta}{2\alpha}-\frac{\theta}{2\alpha}\tanh\left(\frac{\theta\lambda}{2}\left(x+\frac{a}{c}t\right)\right)+\frac{\operatorname{sech}\left(\frac{\theta\lambda}{2}\left(x+\frac{a}{c}t\right)\right)}{C\cosh\left(\frac{\theta\lambda}{2}\left(x+\frac{a}{c}t\right)\right)-\frac{2\alpha}{\theta}\sinh\left(\frac{\theta\lambda}{2}\left(x+\frac{a}{c}t\right)\right)}\right)}\right\},
\end{aligned} \tag{27}$$

where

$$c^2 = -\frac{nb}{2\lambda^2\left(4\gamma\alpha - \beta^2\right)},\ \theta^2 = \beta^2 - 4\alpha\gamma > 0. \tag{28}$$

Substituting Eqs. (23), (24) into Eqs. (20), (21) using (10), then equating the coefficients of the functions $F^k(\xi),\ G^k(\eta)$ to zero, we obtain an algebraic system of equations. Solving this system with the aid of Mathematica, we obtain

$$\begin{aligned} A_0 &= \pm\frac{B_1 c\beta}{2\alpha\sqrt{a}},\ A_1 = \pm\frac{cB_1}{\sqrt{a}},\ B_0 = \frac{\beta B_1}{2\alpha},\ \mu_1 = -\frac{1}{2}\beta^2,\ \mu_2 = -\frac{1}{2}\beta^2, \\ v_1 &= \frac{c^2 B_1^2 \beta^4}{16a\alpha^2},\ v_2 = \frac{B_1^2\beta^4}{16a\alpha^2},\ \omega = \frac{4a\alpha^2}{B_1^2}, \end{aligned} \tag{29}$$

where B_1 is an arbitrary constant.
Substituting (29) into the constraint conditions (22), the parameters $c,\ \lambda$ can be determined as

$$c^2 = -a,\ \lambda^2 = -\frac{nb}{2a\beta^2}. \tag{30}$$

Therefore, using solutions (11) of Eqs. (10), ansatzs (23) and (24), we obtain the exact solutions of Eq. (15) and then the new exact solutions to Eq. (1) can be written

$$u(x,t) = \frac{4}{n}\tanh^{-1}\left\{\pm\frac{c}{\sqrt{a}}\frac{\frac{1}{2\alpha} + \frac{\cosh(\beta(\lambda(x+ct)+\xi_0))+\sinh(\beta(\lambda(x+ct)+\xi_0))}{1-\alpha\cosh(\beta(\lambda(x+ct)+\xi_0))-\alpha\sinh(\beta(\lambda(x+ct)+\xi_0))}}{\frac{1}{2\alpha} + \frac{\cosh\left(\beta(\lambda\left(x+\frac{a}{c}t\right)+\eta_0)\right)+\sinh\left(\beta(\lambda\left(x+\frac{a}{c}t\right)+\eta_0)\right)}{1-\alpha\cosh\left(\beta(\lambda\left(x+\frac{a}{c}t\right)+\eta_0)\right)-\alpha\sinh\left(\beta(\lambda\left(x+\frac{a}{c}t\right)+\eta_0)\right)}}\right\} \tag{31}$$

where

$$c^2 = \frac{nb}{2\lambda^2\beta^2}. \tag{32}$$

Substituting Eqs. (23), (24) into Eqs. (20), (21) using (12) and setting all the coefficients of powers $F(\xi),\ G(\eta)$ to be zero, then we obtain a system of nonlinear algebraic equations and by solving it, we obtain

$$\begin{aligned} A_0 &= 0,\ A_1 = \pm\frac{B_1 c}{\sqrt{a}},\ B_0 = 0,\ \mu_1 = 2\gamma,\ \mu_2 = 2\gamma, \\ v_1 &= \frac{\gamma^2 c^2 B_1^2}{a},\ v_2 = B_1^2\gamma^2,\ \omega = \frac{4a}{B_1^2}, \end{aligned} \tag{33}$$

where B_1 is an arbitrary constant.
Substituting (33) into the constraint conditions (22), the parameters $c,\ \lambda$ can be determined as

$$c^2 = -a,\ \lambda^2 = \frac{nb}{8a\gamma}. \tag{34}$$

Therefore, using solutions (13) of Eqs. (12), ansatzs (23) and (24), we obtain the exact solutions of Eq. (15) and then the new exact traveling-wave solutions to Eq.

(1) can be written as

$$\begin{aligned} u(x,t) &= \frac{4}{n}\tanh^{-1}\left(\pm\frac{c}{\sqrt{a}}\frac{\tanh\left(\sqrt{-\gamma}\lambda\,(x+ct)\right)}{\tanh\left(\sqrt{-\gamma}\lambda\left(x+\frac{a}{c}t\right)\right)}\right) \\ u(x,t) &= \frac{4}{n}\tanh^{-1}\left(\pm\frac{c}{\sqrt{a}}\frac{\tan\left(\sqrt{\gamma}\lambda\,(x+ct)\right)}{\tan\left(\sqrt{\gamma}\lambda\left(x+\frac{a}{c}t\right)\right)}\right) \end{aligned} \tag{35}$$

where

$$c^2 = -\frac{nb}{8\lambda^2\gamma}. \tag{36}$$

4. CONCLUSIONS

Many powerful methods are used in solitary waves theory to examine exact soliton solutions for nonlinear PDEs [1-34]. In this paper, we studied the new application of the simplest equation method to derive new solitary wave solutions of the generalized sinh-Gordon equation. This method is not only efficient, but also has the merit of being widely applicable. The results show that the proposed method is direct, effective and can be applied to many other nonlinear PDEs in mathematical physics.

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