*Electronic Journal of Mathematical Analysis and Applications*
Vol. 3(2) July 2015, pp. 249-256.
ISSN: 2090-729(online)
http://fcag-egypt.com/Journals/EJMAA/

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# FEKETE-SZEGO INEQUALITIES AND $(j, k)$-SYMMETRIC FUNCTIONS

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Abstract. In this paper sharp upper bounds of $|a_3 - \mu a_2^2|$ for functions belonging to new subclasses defined using the concept of $(j, k)$-symmetric functions are derived. Certain applications for functions defined through fractional derivatives in the sense of Riemann Liouville are discussed.

## 1. Introduction

Let $\mathcal{A}$ denote the class of functions of form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \tag{1}$$

which are analytic in the open unit disk $\mathcal{U} = \{z : z \in \mathbb{C} \ and \ |z| < 1\}$, and $\mathcal{S}$ denote the subclass of $\mathcal{A}$ consisting of all function which are univalent in $\mathcal{U}$. For $f$ and $g$ analytic in $\mathcal{U}$, we say that the function $f$ is subordinate to $g$ in $\mathcal{U}$, if there exists an analytic function $\omega$ in $\mathcal{U}$ such that $|\omega(z)| < 1$ and $f(z) = g(\omega(z))$, and we denote this by $f \prec g$. If $g$ is univalent in $\mathcal{U}$, then the subordination is equivalent to $f(0) = g(0)$ and $f(\mathcal{U}) \subset g(\mathcal{U})$ [see 8,9]. The convolution or Hadamard product of two analytic functions $f, g \in \mathcal{A}$ where $f$ is defined by (1) and $g(z) = z + \sum_{n=2}^{\infty} g_n z^n$, then

$$(f * g)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n.$$

**Definition 1** Let $k$ be a positive integer. A domain $\mathcal{D}$ is said to be $k$-fold symmetric if a rotation of $\mathcal{D}$ about the origin through an angle $\frac{2\pi}{k}$ carries $\mathcal{D}$ onto itself. A function $f$ is said to be $k$-fold symmetric in $\mathcal{U}$ if for every $z$ in $\mathcal{U}$

$$f(e^{\frac{2\pi i}{k}} z) = e^{\frac{2\pi i}{k}} f(z).$$

The family of all $k$-fold symmetric functions is denoted by $\mathcal{S}^k$ and for $k = 2$ we get class of the odd univalent functions. The notion of $(j, k)$-symmetrical functions $(k = 2, 3, ... ; j = 0, 1, 2, ..., k - 1)$ is a generalization of the notion of even, odd, $k$-symmetrical functions and also generalize the well-known result that each function

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2010 *Mathematics Subject Classification.* 30C45, 30C50.
*Key words and phrases.* Starlike Functions, Coefficient problem, Subordination, $(j, k)$-Symmetric points, Fekete-Szego Inequality.
Submitted July. 8, 2014.

defined on a symmetrical subset can be uniquely expressed as the sum of an even function and an odd function.

The theory of $(j,k)$ symmetrical functions has many interesting applications, for instance in the investigation of the set of fixed points of mappings, for the estimation of the absolute value of some integrals, and for obtaining some results of the type of Cartan,s uniqueness theorem for holomorphic mappings [6].

**Definition 2** Let $\varepsilon = (e^{\frac{2\pi i}{k}})$ and $j = 0,1,2,..,k-1$ where $k \geq 2$ is a natural number. A function $f : \mathcal{U} \mapsto \mathbb{C}$ is called $(j,k)$-symmetrical if

$$f(\varepsilon z) = \varepsilon^j f(z),\ z \in \mathcal{U}.$$

We note that the family of all $(j,k)$-symmetric functions is denoted be $\mathcal{S}^{(j,k)}$. Also, $\mathcal{S}^{(0,2)}$, $\mathcal{S}^{(1,2)}$ and $\mathcal{S}^{(1,k)}$ are called even, odd and $k$-symmetric functions respectively. We have the following decomposition theorem.

**Theorem 1** [6] For every mapping $f : \mathcal{D} \mapsto \mathbb{C}$, and $\mathcal{D}$ is a $k$-fold symmetric set, there exists exactly the sequence of $(j,k)$- symmetrical functions $f_{j,k}$,

$$f(z) = \sum_{j=0}^{k-1} f_{j,k}(z)$$

where

$$f_{j,k}(z) = \frac{1}{k}\sum_{v=0}^{k-1} \varepsilon^{-vj} f(\varepsilon^v z), \tag{2}$$

$$(f \in \mathcal{A};\ k = 1,2,...; j = 0,1,2,...,k-1).$$

From (2) we can get

$$f_{j,k}(z) = \frac{1}{k}\sum_{v=0}^{k-1} \varepsilon^{-vj} f(\varepsilon^v z) = \frac{1}{k}\sum_{v=0}^{k-1} \varepsilon^{-vj}\left(\sum_{n=1}^{\infty} a_n(\varepsilon^v z)^n\right),$$

then

$$f_{j,k}(z) = \sum_{n=1}^{\infty} \psi_n a_n z^n,\ a_1 = 1 \qquad ,\psi_n = \frac{1}{k}\sum_{v=0}^{k-1} \varepsilon^{(n-j)v} = \begin{cases} 1, & n = lk+j; \\ 0, & n \neq lk+j; \end{cases}, \tag{3}$$

**Definition 3** Let $\phi(z) = 1 + B_1 z + B_2 z^2 + ....$ be univalent starlike with respect to 1 which maps the unit disk $\mathcal{U}$ onto a region in the right half plane which is symmetric with respect to the real axis. Let $0 \leq \beta \leq \alpha \leq 1$ and $B_1 > 0$. Then the function $f(z) \in \mathcal{A}$ is in the class $\mathcal{S}^{j,k}(\phi)$ if

$$\frac{zf'(z)}{f_{j,k}(z)} \prec \phi(z).$$

We note that for suitable choices $j,k,\alpha,\beta,\phi$ we obtain the following subclasses ;

(i)$\mathcal{S}^{1,k}(\phi) = S_s^k(\phi)$ the class introduced by Al-Shaqsi and Darus in [2].

(ii)$\mathcal{S}^{1,k}(\frac{1+Az}{1+Bz}) = S_s^k[A,B]$ we get the class introduced by Al-Shaqsi and Darus in [2].

(iii)$\mathcal{S}^{1,2}(\phi) = S_s^*(\phi)$ the class introduced by Shanmugam et al. in [7].

(iv)$\mathcal{S}^{1,2}(\frac{1+z}{1-z}) = \mathcal{S}_s^*$ the famous Sakaguchi class [1].

(v)$\mathcal{S}^{1,1}(\phi) = \mathcal{S}^*(\phi)$ the class introduced by Ma and Minda [3].

## 2. Fekete-Szegö Inequality

To prove our results, we need the following lemmas.

**Lemma 1** [3] If $p(z) = 1 + c_1 z + c_2 z^2 + ...$ is analytic function with positive real part in $\mathcal{U}$ and $v$ is complex number, then

$$|c_2 - vc_1^2| \leq 2 \max\{1, |2v - 1|\},$$

the result is sharp for functions given by

$$p(z) = \frac{1 + z^2}{1 - z^2}, \qquad p(z) = \frac{1 + z}{1 - z}.$$

**Lemma 2** [3] If $p(z) = 1 + c_1 z + c_2 z^2 + ...$ is analytic function with positive real part in $\mathcal{U}$, then

$$|c_2 - vc_1^2| \leq \begin{cases} -4v + 2, & if \quad v \leq 0; \\ 2, & if \quad 0 \leq v \leq 1; . \\ 4v - 2, & if \quad v \geq 1. \end{cases} \tag{4}$$

When $v < 0$ or $v > 1$ the equality holds if and only if $p(z) = (1 + z)/(1 - z)$ or one of its rotations. If $0 < v < 1$, then the equality if and only if $p(z) = (1+z^2)/(1-z^2)$ or one of its rotations. If $v = 0$ the equality holds if and only if

$$p(z) = \left(\frac{1}{2} + \frac{1}{2}\vartheta\right) \frac{1 + z}{1 - z} + \left(\frac{1}{2} - \frac{1}{2}\vartheta\right) \frac{1 - z}{1 + z}, \ (0 \leq \vartheta \leq 1),$$

or one of its rotations. If $v = 1$, the equality holds if and only if

$$\frac{1}{p(z)} = \left(\frac{1}{2} + \frac{1}{2}\vartheta\right) \frac{1 + z}{1 - z} + \left(\frac{1}{2} - \frac{1}{2}\vartheta\right) \frac{1 - z}{1 + z}, \ (0 \leq \vartheta \leq 1).$$

Also the above upper bound is sharp and it can be improved as follows when $0 < v < 1$:

$$|c_2 - vc_1^2| + v|c_1|^2 \leq 2, \qquad (0 < v < 1/2),$$

$$|c_2 - vc_1^2| + (1 - v)|c_1|^2 \leq 2, \qquad (1/2 < v < 1),$$

**Theorem 2** Let $\phi(z) = 1 + B_1 z + B_2 z^2 + ...(B_1 > 0)$. If $f(z)$ given by (1) belongs to $\mathcal{S}^{j,k}(\phi)$, then

$$|a_3 - \mu a_2^2| \leq \begin{cases} \frac{1}{(3-\psi_3)} \left[B_2 + \frac{B_1^2 \psi_2}{(2-\psi_2)} - \mu \frac{B_1^2 (3-\psi_3)}{(2-\psi_2)^2}\right] & if \quad \mu \leq \sigma_1; \\ \frac{B_1}{(3-\psi_3)}, & if \quad \sigma_1 \leq \mu \leq \sigma_2; \\ -\frac{1}{(3-\psi_3)} \left[B_2 + \frac{B_1^2 \psi_2}{(2-\psi_2)} - \mu \frac{B_1^2 (3-\psi_3}{(2-\psi_2)^2}\right], & if \quad \mu \geq \sigma_2. \end{cases}$$

where

$$\sigma_1 = \frac{(2 - \psi_2)^2}{B_1 (3 - \psi_3)} \left[-1 + \frac{B_2}{B_1} + \frac{B_1 \psi_2}{(2 - \psi_2)}\right],$$

and

$$\sigma_2 = \frac{(2 - \psi_2)^2}{B_1 (3 - \psi_3)} \left[1 + \frac{B_2}{B_1} + \frac{B_1 \psi_2}{(2 - \psi_2)}\right],$$

where $\psi_n$ is defined by (3).
The result is sharp.

*Proof.* Let $f(z) \in \mathcal{S}^{j,k}(\phi)$, then there exists a Schwarz function $w(z)$ in $\mathcal{U}$ with $w(0) = 0$ and $|w(z)| < 1$ in $\mathcal{U}$ such that

$$\frac{zf'(z)}{f_{j,k}(z)} = \phi(w(z)).$$

If $p_1(z)$ is analytic and has positive real part in $\mathcal{U}$ and $p_1(0) = 1$ then

$$p_1(z) = \frac{1 + w(z)}{1 - w(z)} = 1 + c_1 z + c_2 z^2 + ...,\ z \in \mathcal{U}, \tag{5}$$

from (5), we have

$$w(z) = \frac{c_1}{2} z + \frac{1}{2}\left(c_2 - \frac{c_1^2}{2}\right) z^2 + ...,$$

therefore, we have

$$p(z) = \phi(w(z)) = 1 + \frac{1}{2} B_1 c_1 z + \left[\frac{1}{2} B_1 \left(c_2 - \frac{c_1^2}{2}\right) + \frac{1}{4} B_2 c_1^2\right] z^2 + .... \tag{6}$$

Now let

$$p(z) = \frac{zf'(z)}{f_{j,k}(z)} = 1 + d_1 z + d_2 z^2 + ...,$$

or

$$\frac{1 + \sum_{n=2}^{\infty} n a_n z^{n-1}}{\sum_{n=1}^{\infty} \psi_n a_n z^{n-1}} = 1 + d_1 z + d_2 z^2 + ..., \tag{7}$$

which gives

$$d_1 = \frac{(2 - \psi_2) a_2}{\psi_1}, \tag{8}$$

and

$$d_2 = \frac{(3 - \psi_3) a_3}{\psi_1} - \frac{(2 - \psi_2)\psi_2 a_2^2}{\psi_1^2}. \tag{9}$$

Now from (6),(7),(8) and (9) we get

$$a_2 = \frac{B_1 c_1 \psi_1}{2(2 - \psi_2)},$$

and

$$a_3 = \frac{B_1 \psi_1}{2(3 - \psi_3)} \left[c_2 - \frac{c_1^2}{2}\left[1 - \frac{B_2}{B_1} - \frac{B_1 \psi_2}{(2 - \psi_2)}\right]\right].$$

Therefore, we have

$$a_3 - \mu a_2^2 = \frac{B_1 \psi_1}{2(3 - \psi_3)} [c_2 - v c_1^2],$$

where

$$v = \frac{1}{2}\left[1 - \frac{B_2}{B_1} - \frac{B_1 \psi_2}{(2 - \psi_2)} + \mu \frac{B_1 \psi_1 (3 - \psi_3)}{(2 - \psi_2)^2}\right].$$

Our result now follows by an application of Lemma 2.

To show that these bounds are sharp, we define the functions $K_{\phi_m},\ m = 2, 3, ...$ by

$$\frac{zK'_{\phi_m}(z)}{K_{\phi_m(j,k)}(z)} = \phi(z^{n-1}).$$

$$K_{\phi_m}(0) = 0 = [K_{\phi_m}]'(0) - 1$$

and the function $F_\lambda$ and $G_\lambda$ $(0 \leq \lambda \leq 1)$

$$\frac{zF'_\lambda(z)}{F_{\lambda(j,k)}(z)} = \phi\left(\frac{z(z+\lambda)}{1+\lambda z}\right).$$

$$F_\lambda(0) = 0 = F'_\lambda(0) - 1,$$

and

$$\frac{zG'_\lambda(z)}{G_{\lambda(j,k)}(z)} = \phi\left(\frac{-z(z+\lambda)}{1+\lambda z}\right).$$

$$G_\lambda(0) = 0 = G'_\lambda(0) - 1.$$

Obviously the functions $K_{\phi_m}, F_\lambda, G_\lambda \in \mathcal{K}^{j,k}(\alpha, \beta, \phi)$. Also we write $K_\phi := K_{\phi_2}$ if $\mu < \sigma_1$ or $\mu > \sigma_2$, then equality holds if and only if $f$ is $K_\phi$ or one of its rotations. When $\sigma_1 < \mu < \sigma_2$, the equality holds if and only if $f$ is $K_{\phi_3}$ or one of its rotations. $\mu = \sigma_1$ then equality holds if and only if $f$ is $F_\lambda$ or one of its rotations. $\mu = \sigma_2$ then the equality holds if and only if $f$ is $G_\lambda$ or one of its rotations. If $\sigma_1 \leq \mu \leq \sigma_2$, in view of Lemma 2. this completes the proof of Theorem 2. □

**Remark 1**

- For $j = 1$ in Theorem 2 we obtain the result obtained by by Al-Shaqsi and Darus in [2].
- For $j = 1, k = 2$ in Theorem 2 we obtain the result obtained by Shanmugam et al. in [7].
- For $j = 1, k = 1$ in Theorem 2 we obtain the result obtained by see Ma and Minda [3].

If $\sigma_1 \leq \mu \leq \sigma_2$ in view Lemma 2 and Theorem 2 can be improved.

**Theorem 3** $\phi(z) = 1 + B_1 z + B_2 z^2 + ...(B_1 > 0)$. Let $f(z)$ given by (1) belongs to $\mathcal{S}^{j,k}(\phi)$ and $\sigma_3$ given by

$$\sigma_3 = \frac{(2-\psi_2)^2}{B_1\psi_1(3-\psi_3)}\left[\frac{B_2}{B_1} + B_1\frac{\psi_2}{(2-\psi_2)}\right].$$

If $\sigma_1 < \mu \leq \sigma_3$, then

$$|a_3 - \mu a_2^2| + \frac{1}{B_1^2}\left[(B_1 - B_2)\frac{(2-\psi_2)^2}{\psi_1(3-\psi_3)} - B_1^2\frac{\psi_2(2-\psi_2)}{\psi_1(3-\psi_3)} + \mu B_1^2\right]|a_2|^2 \leq \frac{B_1\psi_1}{(3-\psi_3)}.$$

If $\sigma_3 < \mu \leq \sigma_2$, then

$$|a_3 - \mu a_2^2| + \frac{1}{B_1^2}\left[(B_1 + B_2)\frac{(2-\psi_2)^2}{\psi_1(3-\psi_3)} + B_1^2\frac{\psi_2(2-\psi_2)}{\psi_1(3-\psi_3)} - \mu B_1^2\right]|a_2|^2 \leq \frac{B_1\psi_1}{(3-\psi_3)}.$$

where $\psi_n$ is defined by (3).

**Remark 2**

- For $j = 1$ we obtain the result obtained by by Al-Shaqsi and Darus [2].
- For $j = 1, k = 2$ we arrive to the result obtained by Shanmugam et al.[7].
- For $j = 1, k = 1$ we get the result obtained by Ma and Minda [3].

By Lemma 2, we can obtain the following theorem.

**Theorem 4** Let $\phi(z) = 1+B_1z+B_2z^2+...(B_1 > 0)$. If $f(z)$ given by (1) belongs to $\mathcal{S}^{j,k}(\phi)$, then

$$|a_3 - \mu a_2^2| \leq \frac{B_1\psi_1}{2(3-\psi_3)} \times \max\left\{1, \left|\frac{B_2}{B_1} + \frac{B_1}{(2-\psi_2)} - \mu\frac{B_1\psi_1(3-\psi_3)}{(2-\psi_2)^2}\right|\right\},$$

where $\psi_n$ is defined by (3).
The result is sharp.

## 3. Application to Functions Defined by Fractional Derivatives

The fractional derivatives of order $\gamma$ in the sense of Riemann Liouville [5] is defined as

$$D_z^\gamma f(z) = \frac{1}{\Gamma(1-\gamma)}\frac{d}{dz}\int_0^z \frac{f(\zeta)}{(z-\zeta)^\gamma}d\zeta.\ 0 \leq \gamma < 1,$$

where $f$ is an analytic function in a simply connected domain of the $z$-plane containing the origin and the multiplicity of $(z-\zeta)^{-\gamma}$ is removed by requiring $\log(z-\zeta)$ to be real when $z-\zeta > 0$.
Fractional derivative of higher order are defined by

$$D_z^{\gamma+\varsigma} f(z) = \frac{d^\varsigma}{dz^\varsigma}D_z^\gamma f(z),\ \varsigma \in \mathbb{N}_0.$$

Using the fractional derivatives $D_z^\gamma f$ Owa and Srivastava in [4] introduced the operator $\Omega^\gamma : \mathcal{A} \to \mathcal{A}$, which is known as an extension of fractional derivative and fractional integral as follows

$$\Omega^\gamma f(z) = \Gamma(2-\gamma)z^\gamma D_z^\gamma f(z),\ \gamma \neq 2, 3, 4, .... \tag{10}$$

The class $\mathcal{K}_\gamma^{j,k}(\phi)$ consists of functions $f \in \mathcal{A}$ for which $\Omega^\gamma f \in \mathcal{K}^{j,k}(\phi)$. The class $\mathcal{K}_\gamma^{j,k}(\phi)$ is a special case of the class $\mathcal{K}_g^{j,k}(\phi)$ when

$$g(z) = z + \sum_{n=2}^{\infty}\frac{\Gamma(n+1)\Gamma(2-\gamma)}{\Gamma(n+1-\gamma)}a_n z^k,\ z \in \mathcal{U}. \tag{11}$$

Now applying Theorem 2 for the function $(f*g)(z) = z + g_2a_2z^2 + g_3a_3z^3 + ...$ we get the following theorem.
**Theorem 5** Let $g(z) = 1 + g_1z + g_2z^2 + ...(g_n > 0)$. If $f(z)$ given by (1) belongs to $\mathcal{S}_g^{j,k}(\phi)$, then

$$|a_3 - \mu a_2^2| \leq \begin{cases} \frac{\psi_1}{g_3(3-\psi_3)}\left[B_2 + \frac{B_1^2\psi_2}{(2-\psi_2)} - \mu\frac{g_3}{g_2^2}\frac{B_1^2\psi_1(3-\psi_3)}{(2-\psi_2)^2}\right] & if \quad \mu \leq \tau_1, \\ \frac{\psi_1 B_1}{g_3(3-\psi_3)}, & if \quad \tau_1 \leq \mu \leq \tau_2\ , \\ -\frac{\psi_1}{g_3(3-\psi_3)}\left[B_2 + \frac{B_1^2\psi_2}{(2-\psi_2)} - \mu\frac{g_3}{g_2^2}\frac{B_1^2\psi_1(3)-\psi_3)}{(2-\psi_2)^2}\right], & if \quad \mu \geq \tau_2. \end{cases}$$

where

$$\tau_1 = \frac{g_2^2(2-\psi_2)^2}{g_3B_1\psi_1(3-\psi_3)}\left[-1 + \frac{B_2}{B_1} + \frac{B_1\psi_2}{(2-\psi_2)}\right],$$

and

$$\tau_2 = \frac{g_2^2(2-\psi_2)^2}{g_3B_1\psi_1(3-\psi_3)}\left[1 + \frac{B_2}{B_1} + \frac{B_1\psi_2}{(2-\psi_2)}\right],$$

where $\psi_n$ is defined by (3). The result is sharp.

**Remark 3**

- Putting $j = 1$ we obtain the result by Al-Shaqsi and Darus [2].
- Putting $j = 1, k = 2$ we get the result by Shanmugam et al [7].

Since

$$\Omega^\gamma f(z) = z + \sum_{n=2}^{\infty} \frac{\Gamma(n+1)\Gamma(2-\gamma)}{\Gamma(n+1-\gamma)} a_n z^k, \ z \in \mathcal{U},$$

we have

$$g_2 = \frac{\Gamma(3)\Gamma(2-\gamma)}{\Gamma(3-\gamma)} = \frac{2}{2-\gamma}, \tag{12}$$

and

$$g_3 = \frac{\Gamma(4)\Gamma(2-\gamma)}{\Gamma(4-\gamma)} = \frac{6}{(2-\gamma)(3-\gamma)}. \tag{13}$$

For $g_2, g_3$ given by (12) and (13), respectively, Theorem 3 reduce the following theorem.

**Theorem 6** Let $\gamma < 2$. If $f(z)$ given by (1) belongs to $\mathcal{S}_\gamma^{j,k}(\phi)$, then

$$|a_3 - \mu a_2^2| \leq \begin{cases} \frac{(2-\gamma)(3-\gamma)\psi_1}{6(3-\psi_3)} \left[B_2 + \frac{B_1^2\psi_2}{(2-\psi_2)} - \mu \frac{3(2-\gamma)}{2(3-\gamma)} \frac{B_1^2\psi_1(3-\psi_3)}{(2-\psi_2)^2}\right] & if \quad \mu \leq \tau_1^*, \\ \frac{(2-\gamma)(3-\gamma)\psi_1 B_1}{6(3-\psi_3)}, & if \quad \tau_1^* \leq \mu \leq \tau_2^*, \\ -\frac{(2-\gamma)(3-\gamma)\psi_1}{6(3-\psi_3)} \left[B_2 + \frac{B_1^2\psi_2}{(2-\psi_2)} - \mu \frac{3(2-\gamma)}{2(3-\gamma)} \frac{B_1^2\psi_1(3-\psi_3)}{(2-\psi_2)^2}\right], & if \quad \mu \geq \tau_2^*. \end{cases}$$

where

$$\tau_1 * = \left(\frac{2(3-\gamma)}{3(2-\gamma)}\right) \frac{g_2^2(2-\psi_2)^2}{g_3 B_1 \psi_1 (3-\psi_3)} \left[-1 + \frac{B_2}{B_1} + \frac{B_1\psi_2}{(2-\psi_2)}\right],$$

and

$$\tau_2^* = \left(\frac{2(3-\gamma)}{3(2-\gamma)}\right) \frac{g_2^2(2-\psi_2)^2}{g_3 B_1 \psi_1 (3-\psi_3)} \left[1 + \frac{B_2}{B_1} + \frac{B_1\psi_2}{(2-\psi_2)}\right],$$

where $\psi_n$ is defined by (3). The result is sharp.

**Remark 3**

- Putting $j = 1$ we obtain the result by Al-Shaqsi and Darus [2].
- Putting $j = 1, k = 2$ we arrive to the result by Shanmugam et al [7].

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