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APPLICATION OF SUMUDU DECOMPOSITION METHOD FOR SOLVING NONLINEAR WAVE-LIKE EQUATIONS WITH VARIABLE COEFFICIENTS

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ABSTRACT. In this paper, we propose a new approximate method, namely Sumudu decomposition method (SDM) to find exact solutions of nonlinear Wave-like equations with variable coefficients, in some cases, if the exact solution has a closed form. The Sumudu decomposition method is a combined form of the Sumudu transform method and the Adomian decomposition method. The nonlinear term can easily be handled with the help of Adomian polynomials which is considered to be a clear advantage of this technique over the decomposition method. The results reveal that the proposed algorithm is very efficient, simple and can be applied to other nonlinear problems.

1. INTRODUCTION

Many important phenomena occurring in various fields of engineering and science are frequently modeled through linear and nonlinear differential equations. However, it is still very difficult to obtain closed-form solutions for most models of real-life problems. A broad class of analytical methods and numerical methods were used to handle such problems. In recent years, various methods have been proposed such as finite difference method [1], Adomian decomposition method [2], the variational iteration method [3], integral transform [4], weighted finite difference techniques [5], Laplace decomposition method [6] and homotopy perturbation method [7].

Inspired and motivated by the ongoing research in this area, we describe the application of Sumudu decomposition method for nonlinear wave-like equation with initial condition based on the analysis by previous researchers.

Consider the wave-like equation.

$$\begin{aligned} U_{tt} &= \textstyle\sum_{i,j=1}^{n} F_{1ij}(X,U,t)\frac{\partial^{k+m}}{\partial x_i^k \partial x_j^m} F_{2ij}(U_{x_i},U_{x_j}) \\ &\quad + \textstyle\sum_{i=1}^{n} G_{1i}(X,U,t)\frac{\partial^p}{\partial x_i^p} G_{2i}(U_{x_i}) + H(X,t,U) + S(X,t) \end{aligned} \tag{1}$$

with the initial conditions

$$U(X,0) = a_0(X), \ \ U_t(X,0) = a_1(X) \tag{2}$$

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Here $X = (x_1, x_2, x_3, ..., x_n)$ and F_{1ij},G_{1i} are nonlinear functions of X, t and U. F_{2ij} , G_{2i} are nonlinear functions of derivatives of $x_i$and $x_j$whilst $H, S$are nonlinear functions. $k, m, p$are integers.

2. Sumudu Transform

In the early 90's, Watugala in [8] introduced a new integral transform, named the Sumudu transform and applied it to the solution of ordinary differential equation in control engineering problems. The Sumudu transform, is defined over the set of functions.

$$A = \left\{ f(t) \middle| \exists M, \tau_1, \tau_2 > 0, |f(t)| < Me^{\left(\frac{|t|}{\tau_j}\right)}, \; if \;\; t \in (-1)^j \times [0, \infty) \right\} \tag{3}$$

by the following formula

$$\bar{f}(u) = S[f(t)] = \int_0^\infty f(ut)e^{-t}dt, \quad u \in (-\tau_1, \tau_2). \tag{4}$$

Some of the properties were established in [9, 10]. In [11], further fundamental properties of this transform were also established. Similarly, this transform was applied to the one-dimensional neutron transport equation in [12]. In fact it was shown that there is a strong relationship between Sumudu and other integral transforms; see [13]. In particular the relation between Sumudu transform and Laplace transforms was proved in [14].

Further, in [15], the Sumudu transform was extended to the distributions and some of their properties were also studied in [16]. Recently, this transform is applied to solve the system of differential equations; see [17].

Note that a very interesting fact about Sumudu transform is that the original function and its Sumudu transform have the same Taylor coefficients except the factor n; see [18].

Thus if $f(t) = \sum_{n=0}^\infty a_n t^n$ then $F(u) = \sum_{n=0}^\infty n! a_n u^n$, see [13].

Similarly, the Sumudu transform sends combinations, $c(m, n)$, into permutations, $p(m, n)$ and hence it will be useful in the discrete systems.

3. Basic idea of Sumudu decomposition method

To illustrate the basic idea of this method, we consider a general nonlinear non-homogeneous partial differential equation [19, 20]:

$$DU(x,t) + RU(x,t) + NU(x,t) = g(x,t) \tag{5}$$

with initial conditions

$$U(x,0) = h(x), \quad U_t(x,0) = f(x), \tag{6}$$

where D is the second order linear differential operator $D = \frac{\partial^2}{\partial t^2}$, R is the linear differential operator of less order than D, N represents the general nonlinear differential operator and $g(x,t)$ is the source term. Taking the Sumudu transform (denoted throughout this paper by S) on both sides of Eq. (5), we get:

$$\mathrm{S}[\mathrm{U(x,t)}] + S[RU(x,t)] + S[NU(x,t)] = S[g(x,t)]. \tag{7}$$

Using the differentiation property of the Sumudu transform and above initial conditions, we have

$$\mathrm{S}[\mathrm{U(x,t)}] = Su^2[g(x,t)] + h(x) + uf(x) - Su^2[RU(x,t) + NU(x,t)]. \tag{8}$$

Now, applying the inverse Sumudu transform on both sides of Eq. (8), we get

$$\mathrm{U(x,t)} = G(x,t) - S^{-1}\left[Su^2[RU(x,t) + NU(x,t)]\right] \tag{9}$$

where $G(x,t)$ represents the term arising from the source term and the prescribed initial conditions. The second step in Sumudu decomposition method is that we represent solution as an infinite series given below

$$\mathrm{U(x,t)}=\sum_{\mathrm{n=0}}^{\infty}\mathrm{U_n(x,t)}, \tag{10}$$

and the nonlinear term can be decomposed as:

$$\mathrm{NU(x,t)}=\sum_{\mathrm{n=0}}^{\infty}A_{\mathrm{n}}, \tag{11}$$

where A_n are Adomian polynomials [21] of $U_0,U_1,U_2,...,U_n$ and it can be calculated by formula

$$A_{\mathrm{n}}=\frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{0}^{\infty}\lambda^i U_i\right)\right]_{\lambda=0}, \quad n=0,1,2,.... \tag{12}$$

Using Eq. (10) and Eq. (11) in Eq. (9), we get

$$\sum_{n=0}^{\infty}U_n(x,t)=G(x,t)-S^{-1}\left[Su^2\left[R\sum_{n=0}^{\infty}U_n(x,t)+\sum_{n=0}^{\infty}A_n(U)\right]\right]. \tag{13}$$

On comparing both sides of the Eq. (13), we get

$$\begin{aligned}
U_0(x,t)&=G(x,t),\\
U_1(x,t)&=-S^{-1}\left[Su^2\left[RU_0(x,t)+A_0\right]\right],\\
U_2(x,t)&=-S^{-1}\left[Su^2\left[RU_1(x,t)+A_1\right]\right],\\
U_3(x,t)&=-S^{-1}\left[Su^2\left[RU_2(x,t)+A_2\right]\right].\\
&\vdots
\end{aligned} \tag{14}$$

In general the recursive relation is given by

$$\begin{aligned}
U_0(x,t)&=G(x,t),\\
U_{n+1}(x,t)&=-S^{-1}\left[Su^2\left[RU_n(x,t)+A_{\mathrm{n}}\right]\right], \quad n\geq 0.
\end{aligned} \tag{15}$$

Now first of all applying the Sumudu transform of the right hand side of Eq.(15) then applying the inverse Sumudu transform, we get the values of $U_0,U_1,U_2,...,U_{\mathrm{n}}$ respectively.

4. Numerical Application

In this section, to achive the validity, the accuracy and support our theoretical discussion of the proposed method, we give some examples and results.

Example 4.1. *Consider the 2-dimensional nonlinear wave-like equation with variable coefficients*

$$U_{tt}=\frac{\partial^2}{\partial x\partial y}(U_{xx}U_{yy})-\frac{\partial^2}{\partial x\partial y}(xyU_xU_y)-U \tag{16}$$

with initial conditions

$$U(x,y,0)=e^{xy}, \qquad U_t(x,y,0)=e^{xy}, \tag{17}$$

By taking Sumudu transform for (16)*, and* (17)*, we obtain*

$$S[U(x,y,t)]=e^{xy}+ue^{xy}+u^2S\left[\frac{\partial^2}{\partial x\partial y}(U_{xx}U_{yy})-\frac{\partial^2}{\partial x\partial y}(xyU_xU_y)-U\right]. \tag{18}$$

By applying the inverse Sumudu transform for (18)*, we get*

$$U(x,y,t) = e^{xy} + te^{xy} + S^{-1}\left[u^2 S\left[\frac{\partial^2}{\partial x \partial y}(U_{xx}U_{yy}) - \frac{\partial^2}{\partial x \partial y}(xyU_xU_y) - U\right]\right]. \tag{19}$$

Following the technique, if we assume an infinite series solution of the form (10) *and* (11)*, we obtain*

$$\sum_{n=0}^{\infty} U_n(x,y,t) = e^{xy} + te^{xy} + S^{-1}\left[u^2 S\left[\frac{\partial^2}{\partial x \partial y}\sum_{n=0}^{\infty} A_n(U) - \frac{\partial^2}{\partial x \partial y}\left(xy\sum_{n=0}^{\infty} B_n(U)\right) - \sum_{n=0}^{\infty} U_n(x,y,t)\right]\right]. \tag{20}$$

In (20)*, $A_n(U)$, $B_n(U)$are Adomian polynomials that represent nonlinear term. So Adomian polynomials are given as follows:*

$$A_n(U) = U_{xx}U_{yy}, B_n(U) = U_xU_y \tag{21}$$

Using (12)*. The few components of the Adomian polynomials are given as follows:*

$$\begin{aligned} A_0 &= (U_0)_{xx}(U_0)_{yy}, \\ A_1 &= (U_1)_{xx}(U_0)_{yy} + (U_0)_{xx}(U_1)_{yy}, \\ A_2 &= (U_2)_{xx}(U_0)_{yy} + (U_1)_{xx}(U_1)_{yy} + (U_0)_{xx}(U_2)_{yy}, \\ &\vdots \end{aligned} \tag{22}$$

$$\begin{aligned} B_0 &= (U_0)_x(U_0)_y, \\ B_1 &= (U_1)_x(U_0)_y + (U_0)_x(U_1)_y, \\ B_2 &= (U_2)_x(U_0)_y + (U_1)_x(U_1)_y + (U_0)_x(U_2)_y, \\ &\vdots \end{aligned} \tag{23}$$

From the relationship in (15)*, we obtain*

$$\begin{aligned} U_0(x,y,t) &= G(x,t) = e^{xy} + te^{xy}, \\ U_1(x,y,t) &= S^{-1}\left[u^2 S\left[\frac{\partial^2}{\partial x \partial y}A_0(U) - \frac{\partial^2}{\partial x \partial y}(xyB_0(U)) - U_0(x,y,t)\right]\right] = -\frac{t^2e^{xy}}{2} - \frac{t^3e^{xy}}{6}, \\ U_2(x,y,t) &= S^{-1}\left[u^2 S\left[\frac{\partial^2}{\partial x \partial y}A_1(U) - \frac{\partial^2}{\partial x \partial y}(xyB_1(U)) - U_1(x,y,t)\right]\right] = \frac{t^4e^{xy}}{24} + \frac{t^5e^{xy}}{120}, \\ &\vdots \end{aligned} \tag{24}$$

which in closed form gives exact solution

$$U(x,y,t) = \sum_{n=0}^{\infty} U_n(x,y,t) = e^{xy}\left(1 + t - \frac{t^2}{2!} - \frac{t^3}{3!} + \frac{t^4}{4!} + \frac{t^5}{5!} - ...\right) = e^{xy}(\sin t + \cos t). \tag{25}$$

which is the same result obtained by Adomain decomposition method [22] *and Homotopy perturbation transform method* [23] *for the same test problem.*

Table 4.1: Comparison of the absolute errors for the obtained results and the exact solution for Example 1, where $n = 3$

$t/x,y$	0.1	0.3	0.5	0.7
0.1	1.42264 e-9	1.54112 e-9	1.80852 e-9	2.29908 e-9
0.3	1.06481 e-6	1.15349 e-6	1.35364 e-6	1.72081 e-6
0.5	2.33822 e-5	2.53296 e-5	2.97246 e-5	3.77873 e-5
0.7	1.80000 e-4	1.94991 e-4	2.28825 e-4	2.90893 e-4
0.9	8.29627 e-4	8.98724 e-4	1.05466 e-3	1.34074 e-3

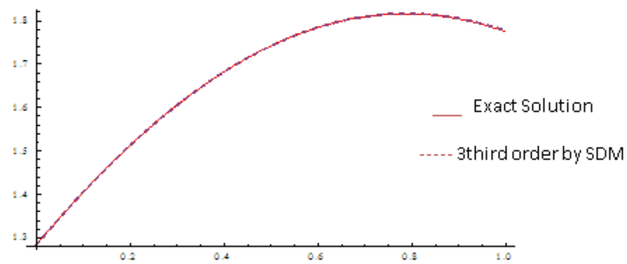

FIGURE 1. The behavior of the exact solution and the approximate solution of $U(x,y,t)$ in case $x = y = 0.5$, $t \in [0,1]$.

Example 4.2. *Let us consider the nonlinear partial differential equation*

$$U_{tt} = U^2 \frac{\partial^2}{\partial x^2}\left(U_x U_{xx} U_{xxx}\right) - U_x^2 \frac{\partial^2}{\partial x^2}\left(U_{xx}^3\right) - 18U^5 + U, \quad 0 < x < 1, \quad t > 0 \tag{26}$$

with initial conditions

$$U(x,0) = e^x, \qquad U_t(x,0) = e^x, \tag{27}$$

By taking Sumudu transform for (26), *we obtain*

$$S[U(x,t)] = e^x + ue^x + u^2 S\left[U^2 \frac{\partial^2}{\partial x^2}\left(U_x U_{xx} U_{xxx}\right) - U_x^2 \frac{\partial^2}{\partial x^2}\left(U_{xx}^3\right) - 18U^5 + U\right]. \tag{28}$$

By applying the inverse Sumudu transform for (28), *we get*

$$U(x,t) = e^x + te^x + S^{-1}\left[u^2 S\left[U^2 \frac{\partial^2}{\partial x^2}\left(U_x U_{xx} U_{xxx}\right) - U_x^2 \frac{\partial^2}{\partial x^2}\left(U_{xx}^3\right) - 18U^5 + U\right]\right]. \tag{29}$$

Following the technique, if we assume an infinite series solution of the form (10) *and* (11), *we obtain*

$$\sum_{n=0}^{\infty} U_n(x,t) = e^x + te^x + S^{-1}\left[u^2 S\left[\sum_{n=0}^{\infty} A_n(U) - \sum_{n=0}^{\infty} B_n(U) - 18\sum_{n=0}^{\infty} C_n(U) + \sum_{n=0}^{\infty} U_n(x,t)\right]\right]. \tag{30}$$

In (30), $A_n(U)$, $B_n(U)$, $C_n(U)$ *are Adomian polynomials that represent nonlinear term. So Adomian polynomials are given as follows:*

$$A_n(U) = U^2 \frac{\partial^2}{\partial x^2}\left(U_x U_{xx} U_{xxx}\right), B_n(U) = U_x^2 \frac{\partial^2}{\partial x^2}\left(U_{xx}^3\right), C_n(U) = U^5. \tag{31}$$

Using (12). *The few components of the Adomian polynomials are given as follows:*

$$A_0 = U_0^2 \frac{\partial^2}{\partial x^2}\left((U_0)_x (U_0)_{xx} (U_0)_{xxx}\right),$$

$$A_1 = U_0 U_1 \frac{\partial^2}{\partial x^2}\left((U_0)_x (U_0)_{xx} (U_0)_{xxx}\right) + U_0^2 \frac{\partial^2}{\partial x^2}\left((U_1)_x (U_0)_{xx} (U_0)_{xxx}\right) \\ + \left((U_0)_x (U_1)_{xx} (U_0)_{xxx}\right) + \left((U_0)_x (U_0)_{xx} (U_1)_{xxx}\right) , \tag{32}$$

$$\vdots$$

$$B_0 = (U_0)_x^2 \frac{\partial^2}{\partial x^2}\left((U_0)_{xx}^3\right),$$

$$B_1 = 2(U_0)_x (U_1)_x \frac{\partial^2}{\partial x^2}\left((U_0)_{xx}^3\right) + (U_0)_x^2 \frac{\partial^2}{\partial x^2}\left(3(U_0)_{xx}^2 (U_1)_{xx}\right), \tag{33}$$

$$\vdots$$

$$C_0 = U_0^5, C_1 = 5U_0^4 U_1,$$

From the relationship in (15)*, we obtain*

$$U_0(x,t) = G(x,t) = e^x + te^x,$$

$$U_1(x,t) = S^{-1}\left[u^2 S[A_0(U) - B_0(U) - 18C_0(U) + U_0(x,t)]\right] = \frac{t^2 e^x}{2} + \frac{t^3 e^x}{6},$$

$$U_2(x,t) = S^{-1}\left[u^2 S[A_1(U) - B_1(U) - 18C_1(U) + U_1(x,t)]\right] = \frac{t^4 e^x}{24} + \frac{t^5 e^x}{120}, \tag{34}$$

$$\vdots$$

which in closed form gives exact solution

$$U(x,t) = \sum_{n=0}^{\infty} U_n(x,t) = e^{xy}\left(1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \frac{t^5}{5!} + \ldots\right) = e^{x+t}, \tag{35}$$

which is the same result obtained , for the same test problem, by Adomain decomposition method [22] *and Homotopy perturbation transform method* [23].

Table 4.2: Comparison of the absolute errors for Example 4.2, where $n = 3$

t/x	0.1	0.3	0.5	0.7
0.1	1.55716 e-9	1.90193 e-9	2.32302 e-9	2.83734 e-9
0.3	1.16880 e-6	1.42758 e-6	1.74365 e-6	2.12970 e-6
0.5	2.58102 e-5	3.15246 e-5	3.85043 e-5	4.70292 e-5
0.7	2.00357 e-4	2.44717 e-4	2.98898 e-4	3.65075 e-4
0.9	9.33716 e-4	1.14044 e-3	1.39294 e-3	1.70134 e-3

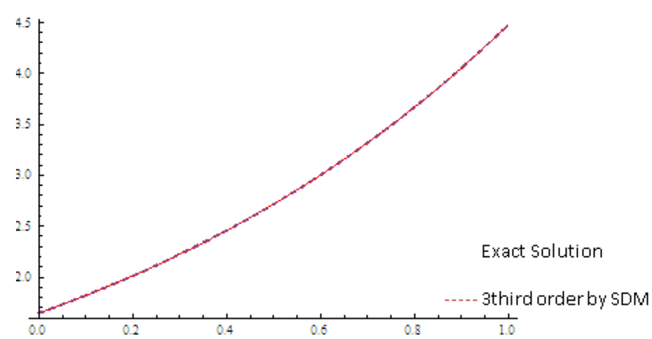

FIGURE 2. The behavior of the exact solution and the approximate solution of $U(x,y,t)$ in case $x = y = 0.5$, $t \in [0,1]$

Example 4.3. *Consider the following one dimensional nonlinear wave-like equation*

$$U_{tt}=x^2\frac{\partial}{\partial x}\left(U_xU_{xx}\right)-x^2\left(U_{xx}\right)^2-U,\quad 0\leq x\leq 1,\ 0\leq t, \tag{36}$$

with initial conditions

$$U(x,0)=0,\qquad U_t(x,0)=x^2 \tag{37}$$

By taking Sumudu transform for (36)*, we obtain*

$$S\left[U(x,t)\right]=ux^2+u^2S\left[x^2\frac{\partial}{\partial x}\left(U_xU_{xx}\right)-x^2\left(U_{xx}\right)^2-U\right]. \tag{38}$$

Now, applying the inverse Sumudu transform for (38)*, we get*

$$U(x,t)=tx^2+S^{-1}\left[u^2S\left[x^2\frac{\partial}{\partial x}\left(U_xU_{xx}\right)-x^2\left(U_{xx}\right)^2-U\right]\right]. \tag{39}$$

Following the technique, if we assume an infinite series solution of the form (10) *and* (11)*, we obtain*

$$\sum_{n=0}^{\infty}U_n(x,t)=tx^2+S^{-1}\left[u^2S\left[x^2\frac{\partial}{\partial x}\sum_{n=0}^{\infty}A_n(U)-x^2\sum_{n=0}^{\infty}B_n(U)-\sum_{n=0}^{\infty}U_n(x,t)\right]\right]. \tag{40}$$

In (40)*, $A_n(U)$, $B_n(U)$are Adomian polynomials that represent nonlinear term. So Adomian polynomials are given as follows:*

$$A_n(U)=\left(U_xU_{xx}\right),B_n(U)=\left(U_{xx}\right)^2. \tag{41}$$

Using (12)*. The few components of the Adomian polynomials are given as follows:*

$$\begin{aligned}A_0&=\left(U_0\right)_x\left(U_0\right)_{xx},\\A_1&=\left(U_0\right)_x\left(U_1\right)_{xx}+\left(U_1\right)_x\left(U_0\right)_{xx},\\A_2&=\left(U_0\right)_x\left(U_2\right)_{xx}+\left(U_1\right)_x\left(U_1\right)_{xx}+\left(U_2\right)_x\left(U_0\right)_{xx},\\&\vdots\end{aligned} \tag{42}$$

$$\begin{aligned}B_0&=\left(U_0\right)_{xx}^2,\\B_1&=2\left(U_0\right)_{xx}\left(U_1\right)_{xx},\\B_2&=\left(U_1\right)_{xx}^2+2\left(U_0\right)_{xx}\left(U_2\right)_{xx},\\&\vdots\end{aligned} \tag{43}$$

From the relationship in (15)*, we obtain*

$$\begin{aligned}U_0(x,t)&=G(x,t)=tx^2,\\U_1(x,t)&=S^{-1}\left[u^2S\left[x^2\frac{\partial}{\partial x}A_0(U)-x^2B_0(U)-U_0(x,t)\right]\right]=-\frac{t^3}{6}x^2,\\U_2(x,t)&=S^{-1}\left[u^2S\left[x^2\frac{\partial}{\partial x}A_1(U)-x^2B_1(U)-U_1(x,t)\right]\right]=\frac{t^5}{120}x^2,\\U_3(x,t)&=S^{-1}\left[u^2S\left[x^2\frac{\partial}{\partial x}A_2(U)-x^2B_2(U)-U_2(x,t)\right]\right]=-\frac{t^7}{5040}x^2,\\&\vdots\end{aligned} \tag{44}$$

which in closed form gives exact solution

$$U(x,t)=\sum_{n=0}^{\infty}U_n(x,t)=x^2\left(1-\frac{t^3}{3!}-\frac{t^5}{5!}-\frac{t^7}{7!}+...\right)=x^2\left(\sin t\right). \tag{45}$$

which is the same result obtained by Adomain decomposition method [22] *and Homotopy perturbation transform method* [23] *for the same test problem.*

Table 4.3: Comparison of the absolute errors for Example 3, where $n = 3$

t/x	0.1	0.3	0.5	0.7
0.1	0.	0.	0.	0.
0.3	0.	0.	0.	0.
0.5	1.96024 e-16	1.76248 e-15	4.89886 e-15	9.60343 e-15
0.7	1.55232 e-14	1.39708 e-13	3.88078 e-13	7.60614 e-13
0.9	4.06630 e-13	3.65968 e-12	1.01658 e-11	1.99248 e-11

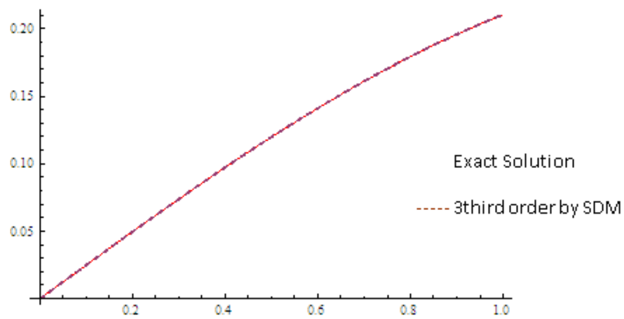

FIGURE 3. The behavior of the exact solution and the approximate solution of $U(x,y,t)$in case $x = y = 0.5$, $t \in [0,1]$.

5. CONCLUSION

In this paper, the SDM has been applied to solving nonlinear Wave-like equations with variable coefficient. Three examples have been presented. The results show that the SDM is powerful and efficient technique in finding exact and approximate solutions for nonlinear differential equations amounts to an improvement of the performance of the approach. The fact that the SDM solves nonlinear problems without using He's polynomials is a clear advantage of this technique over the decomposition method. In conclusion, the SDM may be considered as a nice refinement in existing numerical techniques and might find the wide applications.

REFERENCES

[1] L. Debnath, Nonlinear partial differential equations for scientists and engineers, Birkhauser, 1997.
[2] G. Adomian, Solving frontier problems of physics: The decomposition method, Kluwer Acad. Publ, 1994.
[3] J.H. He, Variational iteration method-a kindofnon linear analytical technique :some examples, International Journal of Non-Linear Mechanics, Vol. 34, pp. 699-708, 1999.
[4] J. Biazer and H. Ghazvini, He's homotopy perturbation method for solving systems of volterra integral equations, Chaos, Solitons and Fractals, Vol. 39, 370-377, 2009.
[5] M. Dehghan, Weighted finite difference techniques for the one-dimensional advection-diffusion equation, Applied Mathematics and Computation, Vol. 147, No. 2, 307-319, 2004.
[6] Y. Khan and F. Austin, Application of the Laplace decomposition method to nonlinear homogeneous and non-homogeneous advection equations , Zeitschrift fuer Naturforschung A, Vol. 65, 1-5, 2010.
[7] J.H. He, "Homotopy perturbation technique", Computer Methods in Applied Mechanics and Engineering, Vol. 178, 257-262, 1999.
[8] G. K.Watugala, Sumudu transform-a new integral transform to solve differential equations and control engineering problems, Mathematical Engineering in Industry,Vol. 6, No. 4, 319-329, 1998.

[9] S. Weerakoon, Application of Sumudu transform to partial differential equations, International Journal of Mathematical Education in Science and Technology, Vol. 25, No. 2, 277-283, 1994.
[10] S. Weerakoon, Complex inversion formula for Sumudu transform", International Journal of Mathematical Education in Science and Technology, Vol. 29, No. 4, 618-621, 1998.
[11] M. A. Asiru, Further properties of the Sumudu transform and its applications, International Journal of Mathematical Education in Science and Technology, Vol. 33, No. 3, 441-449, 2002.
[12] A. Kadem, Solving the one-dimensional neutron transport equation using Chebyshev polynomials and the Sumudu transform, Analele Universitatii dinOradea, Vol. 12, pp. 153-171, 2005.
[13] A. Kilicman, H. Eltayeb, and K. A. M. Atan, A note on the comparison between Laplace and Sumudu transforms, Iranian Mathematical Society, Vol. 37, No. 1, pp. 131-141, 2011.
[14] A. Kilicman and H. E. Gadain, On the applications of Laplace and Sumudu transforms, Journal of the Franklin Institute, Vol. 347, No. 5, pp. 848-862, 2010.
[15] H. Eltayeb, A. Kilicman, and B. Fisher, A new integral transform and associated distributions, Integral Transforms and Special Functions, Vol. 21, No. 5-6, pp. 367- 379, 2010.
[16] A. Kilicman and H. Eltayeb, A note on integral transforms and partial differential equations, Applied Mathematical Sciences, Vol. 4, No. 1-4, pp. 109-118, 2010.
[17] A. Kilicman, H. Eltayeb, and R. P. Agarwal, On Sumudu transform and system of differential equations, Abstract and Applied Analysis, Article ID598702, 11 pages, 2010.
[18] J. Zhang, A Sumudu based algorithm for solving differential equations, Academy of Sciences of Moldova, Vol. 15, No. 3, 303-313, 2007.
[19] H. Eltayeb and A. Kilicman, Application of Sumudu Decomposition Method to Solve Nonlinear System of Partial Differential Equations, Journal of Abstract and Applied Analysis, Vol. 2012, Article ID 412948, 13 pages, 2012.
[20] M. A. Ramadan and M. S. Al-luhaibi, Application of Sumudu Decomposition Method for Solving Linear and Nonlinear Klein-Gordon Equations, International Journal of Soft Computing and Engineering, , Vol. 3, Issue-6, 2231-2307,January 2014.
[21] A. M. Wazwaz, A new algorithm for calculating Adomian polynomials for nonlinear operators, Applied Mathematics and Computation , Vol. 111, 53-69, 2000.
[22] M. Ghoreishi, A.I.B. Ismail, N.H.M.Ali, Adomain decomposition method for nonlinear wave- like equation with variable coefficients, Applied Mathematical Sciences, Vol. 4 ,No. 49, 2431- 2444, 2010.
[23] V.G. Gupta, S. Gupta, Homotopy perturbation transform method for solving nonlinear wave- like equations of variable coefficients, Journal of Information and Computing Science, Vol. 8, No.3,163-172, 2013.

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