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COMPOSITE FINITE DIFFERENCE SCHEME APPLIED TO SOME NONLINEAR EVOLUTION EQUATIONS

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Abstract. This paper is concerned with the application of the composite finite difference scheme (CFDS) to some classes of evolution equations. Three models of evolution equations are studied. The first model is the nonlinear reaction-diffusions equation (NRD) with a reaction term, while the second is modified Korteweg de Vries equation (mKdV) and the third is the Fitzhugh–Nagumo equation (FN). Numerical examples showed that the CFDS give high accuracy.

1. Introduction

Nonlinear evolution equations are widely used to describe many important phenomena and dynamic processes in physics, mechanics, chemistry, biology, etc. The study of nonlinear partial differential equations is very important. Many methods, exact, approximate and purely numerical are available for solution of nonlinear partial differential equations [1]-[30]. Reaction-diffusion equations (RD) are mathematical models which explain how the concentration of one or more substances distributed in space changes under the influence of two processes: local chemical reactions in which the substances are transformed into each other, and diffusion which causes the substances to spread out over a surface in space. A great deal of research work has been published on the development of numerical and analytical solutions of NRD equations [1]-[10]. In recent years, many physicists and mathematicians have paid much attention to the Fitzhugh– Nagumo (FN) equation due to its importance in mathematical physics. The Fitzhugh–Nagumo equation has various applications in the fields of flame propagation, logistic population growth, neurophysiology, branching Brownian motion process, autocatalytic chemical reaction and nuclear reactor theory; see, e.g. [11]-[16]. This equation is an important nonlinear reaction diffusion equation and usually used to model the transmission of nerve impulses [16]. Numerical schemes for FN equations [17]-[19] by collocation method and the "Hopscotch' finite difference scheme first proposed by Gordon [19], and further developed by Gourlay [20]-[21]. A great deal of research work has been invested during the past decades in the study of the mKdV equation [22]-[30].

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The main goal of these studies was its analyticalzz and numerical solutions. Several different approaches, such as Backland transformation, a bilinear form, and a Lax pair, have been used independently, by which Anjan et al. [22]-[25] obtain soliton and multi-soliton solutions for this equation. The aim of this paper is to apply the CFDS to obtain the solutions for the three different types of nonlinear partial differential equations such as, nonlinear Reaction–diffusion equation (NRD), Fitzhugh– Nagumo (FN) and modified Kortewege de Vries equation (mKdV) which are important equations.

2. The nonlinear reaction-diffusion equation with reaction term

An example of practical interest is known as the nonlinear reaction-diffusions equation (NRD) with a reaction term [7, 8].this equation takes the form [8].

$$u_t - u_{xx}^2 = pu - qu^2, (x,t) \ni Q_T \tag{1}$$

Here $Q_T = \Omega \times I, \Omega \equiv (a,b), I = (0,T)$, a and b are real positive constants. We consider equation (1) associated with initial condition $u(x,0) = u_0(x)$. In Finite difference method (FDM) the domain is discretized to a finite number of points forming a mesh with horizontal step size $h = \frac{b-a}{N}$, N is the number of intervals, $0 \prec i \preceq N$and k is the time step such that $T = k*j, 0 \preceq j \preceq M$. The derivatives are replaced by difference formulas [31]-[32] as follows, for $i = 1,2$ we use the forward formula

$$(u_x)_i^j = \frac{-3u_i^j + 4u_{i+1}^j - u_{i+2}^j}{2h}$$

$$(u_{xx})_i^j = \frac{2u_i^j - 5u_{i+1}^j + 4u_{i+2}^j - u_{i+3}^j}{h^2} \tag{2}$$

$$(u_{xxx})_i^j = \frac{-5u_i^j + 18u_{i+1}^j - 24u_{i+2}^j + 14u_{i+3}^j - 3u_{i+4}^j}{2h^3}$$

while for $i = 3, N-2$ we use the central formulas

$$(u_x)_i^j = \frac{u_{i+1}^j - u_{i-1}^j}{2h},\ (u_{xx})_i^j = \frac{u_{i+1}^j - 2u_i^j - u_{i-1}^j}{2h^2}, \tag{3}$$

$$(u_{xxx})_i^j = \frac{u_{i+2}^j - 2u_{i+1}^j + 2u_{i-1}^j - u_{i-2}^j}{2h^3}$$

and for $i = N-1, N$ we use the backward formulas

$$(u_x)_i^j = \frac{3u_i^j - 4u_{i-1}^j + u_{i-2}^j}{2h}$$

$$(u_{xx})_i^j = \frac{2u_i^j - 5u_{i-1}^j + 4u_{i-2}^j - u_{i-3}^j}{h^2} \tag{4}$$

$$(u_{xxx})_i^j = \frac{5u_i^j - 18u_{i-1}^j + 24u_{i-2}^j - 14u_{i-3}^j + 3u_{i-4}^j}{2h^3}$$

3. Application of CFDS to the nonlinear reaction-diffusion equation

Consider the nonlinear PDE (1), we can rewrite

$$u_t = u_{xx}^2 + pu - qu^2. \tag{5}$$

Multiply both sides of (5) by $\frac{\partial F}{\partial u}$, we have

$$\frac{\partial F}{\partial u}\frac{\partial u}{\partial t} = \frac{\partial F}{\partial u}(u_{xx}^2 + pu - qu^2) \tag{6}$$

or

$$\frac{\partial F}{\partial t} = \frac{\partial F}{\partial u}(u_{xx}^2 + pu - qu^2) \tag{7}$$

where F, is any continuous and differentiable function. If we choose $F(u) = \ln u$, we obtain the Exponential finite difference method (Exp. FDM) [33]-[36]. The Logarithmic finite difference method (Log. FDM) [32] is obtained when we set $F(u) = \exp u$.

3.1. Exponential finite difference method applied to the nonlinear reaction-diffusion equation.

In this sub-section we will apply the Exponential finite difference method to (7). The usual forward difference formula leads to, $\frac{\partial F}{\partial t} = \frac{F(u_i^{j+1})-F(u_i^j)}{k}$ is the time step. Substitute in (7)), we have

$$\frac{F(u_i^{j+1}) - F(u_i^j)}{k} = \frac{\partial F}{\partial u}(u_{xx}^2 + pu - q * u^2) \tag{8}$$

or

$$F(u_i^{j+1}) = F(u_i^j) + k(\frac{\partial F}{\partial u}(u_{xx}^2 + pu - q * u^2)) \tag{9}$$

setting $F(u) = \ln u$, then (9) takes the form

$$\ln u_i^{j+1} = \ln u_i^j + \frac{k}{u_i^j}((u_i^j)_{xx}^2 + pu_i^j - q(u_i^j)^2) \tag{10}$$

so,

$$u_i^{j+1} = \exp(\frac{k}{u_i^j}((u_i^j)_{xx}^2 + pu_i^j - q(u_i^j)^2). \tag{11}$$

Applying the difference formulas (2)-(4) to (11), we have the following recurrence relations:

$$u_i^{j+1} = (u_i^j)\exp(\frac{k}{u_i^j}\frac{2(u_i^j)^2 - 5(u_{i+1}^j)^2 + 4(u_{i+2}^j)^2 - (u_{i+3}^j)^2}{h^2} + p(u_i^j) - q(u_i^j)^2), i = 1, 2 \tag{12}$$

$$u_i^{j+1} = (u_i^j)\exp(\frac{k}{u_i^j}\frac{(u_{i+1}^j)^2 - 2(u_i^j)^2 + (u_{i-1}^j)^2}{2h^2} + p(u_i^j) - q*(u_i^j)^2), i = 3 : N-2 \tag{13}$$

$$u_i^{j+1} = (u_i^j)\exp(\frac{k}{u_i^j}\frac{2(u_i^j)^2 - 5(u_{i-1}^j)^2 + 4(u_{i-2}^j)^2 - (u_{i-3}^j)^2}{h^2} + p(u_i^j) - q*(u_i^j)^2), i = N-1, N \tag{14}$$

3.2. **Logarithmic finite difference method applied to the nonlinear reaction-diffusion equation.** In (Log. FDM) we assume $F(u) = \exp u$, equation (9) transformed to

$$\exp(u_i^{j+1}) = \exp(u_i^j) + k\exp(u_i^j)((u_i^j)^2_{xx} + pu_i^j - q(u_i^j)^2) \tag{15}$$

we have,

$$u_i^{j+1} = u_i^j + \ln(1 + k((u_i^j)^2_{xx} + pu_i^j - q(u_i^j)^2). \tag{16}$$

Similarly applying the difference formulas (2)-(4) to (16), we have the following recurrence relations

$$u_i^{j+1} = u_i^j + \ln(1+k(\frac{(2(u_i^j)^2 - 5(u_{i+1}^j)^2 + 4(u_{i+2}^j)^2 - (u_{i+3}^j)^2)}{h^2} + pu_i^j - q(u_i^j)^2)), i = 1, 2 \tag{17}$$

$$u_i^{j+1} = u_i^j + \ln(1+k(\frac{(u_{i+1}^j)^2 - 2(u_i^j)^2 + (u_{i-1}^j)^2}{2h^2} + pu_i^j - q(u_i^j)^2)), i = 3 : N-2 \tag{18}$$

$$u_i^{j+1} = u_i^j + \ln(1+k(\frac{(2(u_i^j)^2 - 5(u_{i-1}^j)^2 + 4(u_{i-2}^j)^2 - (u_{i-3}^j)^2)}{h^2} + pu_i^j - q(u_i^j)^2)), i = N-1, N \tag{19}$$

4. Fitzhugh–Nagumo equation

The classical Fitzhugh–Nagumo equation [16]-[17],is given by

$$u_t = u_{xx} + u(1-u)(p-u) \tag{20}$$

where $0 \preceq p \preceq 1$ and $u(x,t)$ is the unknown function depending on the temporal variable t and the spatial variable x. This equation combines diffusion, and nonlinearity which is controlled by the term $u(1-u)(p-u)$. When $p = 1$, (20) reduces to the real Newell–Whitehead equation. To apply CFDS, we reset (20) as follows

$$\frac{F(u_i^{j+1}) - F(u_i^j)}{k} = \frac{\partial F}{\partial u}(u_{xx} + u(1-u)(p-u)) \tag{21}$$

ie.

$$F(u_i^{j+1}) = F(u_i^j) + k\frac{\partial F}{\partial u}(u_{xx} + u(1-u)(p-u)) \tag{22}$$

4.1. **Exp. FDM method applied to FN equation.** Replacing the derivatives in (22) by the difference formulas (2)-(4), we obtain

$$u_i^{j+1} = (u_i^j * \exp\frac{k}{u_i^j}(\frac{2u_i^j - 5u_{i+1}^j + 4u_{i+2}^j - u_{i+3}^j}{h^2} + u_i^j(1-u_i^j)(p-u_i^j)), i = 1, 2 \tag{23}$$

$$u_i^{j+1} = (u_i^j * \exp\frac{k}{u_i^j}(\frac{u_{i+1}^j - 2u_i^j + u_{i-1}^j}{2h^2} + u_i^j(1-u_i^j)(p-u_i^j)), i = 3, N-2 \tag{24}$$

and

$$u_i^{j+1} = (u_i^j * \exp\frac{k}{u_i^j}(\frac{2u_i^j - 5u_{i-1}^j + 4u_{i-2}^j - u_{i-3}^j}{h^2} + u_i^j(1-u_i^j)(p-u_i^j)), i = N-1, N \tag{25}$$

4.2. **Log. FDM method applied to FN equation.** For the Log. FDM, we have the following iterative formulas

$$u_i^{j+1} = (u_i^j + \ln(1 + k(\frac{2u_i^j - 5u_{i+1}^j + 4u_{i+2}^j - u_{i+3}^j}{h^2} + u_i^j(1 - u_i^j)(p - u_i^j))), i = 1, 2 \tag{26}$$

$$u_i^{j+1} = (u_i^j + \ln(1 + k(\frac{u_{i+1}^j - 2u_i^j + u_{i-1}^j}{2h^2} + u_i^j(1 - u_i^j)(p - u_i^j))), i = 3, N-2 \tag{27}$$

and and for $i = N - 1, N$ we have

$$u_i^{j+1} = u_i^j + \ln(1 + k(\frac{2u_i^j - 5u_{i-1}^j + 4u_{i-2}^j - u_{i-3}^j}{h^2} + u_i^j(1 - u_i^j)(p - u_i^j)) \tag{28}$$

5. Application of CFDS to the modified KdV equation

Consider the modified Korteweg–de Vries equation (mKdV), which takes the form

$$u_t + 6u^2 u_x + u_{xxx} = 0 \tag{29}$$

Similarly, we obtain

$$F(u_i^{j+1}) = F(u_i^j) - k(6(u_i^j)^2(u_i^j)_x + (u_i^j)_{xxx}) \tag{30}$$

5.1. **Exp. FDM method applied to mKdv equation.** The recurrence relations of Exp. FDM are

$$u_i^{j+1} = (u_i^j * \exp\frac{-k}{u_i^j}(6(u_i^j)^2\frac{-3u_i^j + 4u_{i+1}^j - u_{i+2}^j}{2h} + \frac{-5u_i^j + 18u_{i+1}^j - 24u_{i+2}^j + 14u_{i+3}^j - 3u_{i+4}^j}{2h^3}), i = 1, 2 \tag{31}$$

$$u_i^{j+1} = (u_i^j * \exp\frac{-k}{u_i^j}(6(u_i^j)^2\frac{u_{i+1}^j - u_{i-1}^j}{2h} + \frac{u_{i+2}^j - 2u_{i+1}^j + 2u_{i-1}^j - u_{i-2}^j}{2h^3}), i = 3, N-2 \tag{32}$$

and

$$u_i^{j+1} = (u_i^j * \exp\frac{-k}{u_i^j}(6(u_i^j)^2\frac{3u_i^j - 4u_{i-1}^j + u_{i-2}^j}{2h} + \frac{5u_i^j - 18u_{i-1}^j + 24u_{i-2}^j - 14u_{i-3}^j + 3u_{i-4}^j}{2h^3}), i = N-1, N \tag{33}$$

5.2. **Log. FDM method applied to mKdV equation.** In case of Log. FDM equation (30) transformed to

$$u_i^{j+1} = u_i^j + \ln(1 - k(6(u_i^j)^2(u_i^j)_x + (u_i^j)_{xxx})). \tag{34}$$

Similarly applying the difference formulas (2)-(4) to (34), we have the following recurrence relations:

$$u_i^{j+1} = u_i^j + \ln(1 - k(6(u_i^j)^2\frac{-3u_i^j + 4u_{i+1}^j - u_{i+2}^j}{2h} + \frac{-5u_i^j + 18u_{i+1}^j - 24u_{i+2}^j + 14u_{i+3}^j - 3u_{i+4}^j}{2h^3})), i = 1, 2 \tag{35}$$

$$u_i^{j+1} = u_i^j + \ln(1 - k(6(u_i^j)^2\frac{u_{i+1}^j - u_{i-1}^j}{2h} + \frac{u_{i+2}^j - 2u_{i+1}^j + 2u_{i-1}^j - u_{i-2}^j}{2h^3})), i = 3, N-2 \tag{36}$$

Table 1. Numerical results of solving NRD equation at different times,absolute errors for CFDS

x	$t = 0.01$	$t = 0.1$	$t = 0.5$
2	0.0001351	0.0011230	0.0304806
3	0.0000716	0.0008931	0.0159891
4	0.0000363	0.0002172	0.0016654
5	0.0000695	0.0006197	0.0016458
6	0.0000416	0.0003921	0.0014175
7	0.0000249	0.0002358	0.0009075
8	0.0000150	0.0001420	0.0005527
9	0.0000091	0.0000857	0.0003337
10	0.0000055	0.0000516	0.0001966
11	0.0000028	0.0000258	0.0000844
12	0.0000017	0.0000184	0.0001753

Table 2. Numerical results of solving mKdV equation at different times

x	$t = 0.01$	$t = 0.1$	$t = 0.5$
10	$4.7 * 10^{-7}$	$5.2 * 10^{-6}$	0.00009056
11	$1.7 * 10^{-7}$	$1.6 * 10^{-6}$	0.00001313
12	$6.4 * 10^{-8}$	$4.9 * 10^{-7}$	0.00001414
13	$1.2 * 10^{-8}$	$1.3 * 10^{-7}$	$2.8 * 10^{-6}$
14	$4.4 * 10^{-9}$	$2.5 * 10^{-8}$	$1.1 * 10^{-6}$
15	$1.7 * 10^{-9}$	$2.0 * 10^{-8}$	$3.9 * 10^{-7}$
16	$6.3 * 10^{-10}$	$6.5 * 10^{-9}$	$5.6 * 10^{-8}$
17	$2.3 * 10^{-10}$	$2.8 * 10^{-9}$	$5.3 * 10^{-8}$
18	$8.1 * 10^{-11}$	$5.8 * 10^{-10}$	$1.6 * 10^{-8}$
19	$1.0 * 10^{-9}$	$9.8 * 10^{-9}$	$1.8 * 10^{-7}$
20	$4.7 * 10^{-10}$	$7.5 * 10^{-9}$	$1.3 * 10^{-7}$

and

$$u_i^{j+1} = u_i^j + \ln(1 - k(6(u_i^j)^2 \frac{3u_i^j - 4u_{i-1}^j + u_{i-2}^j}{2h} + \frac{5u_i^j - 18u_{i-1}^j + 24u_{i-2}^j - 14u_{i-3}^j + 3u_{i-4}^j}{2h^3})), i = N-1, N \tag{37}$$

6. Numerical examples

In this section, we apply preceding algorithm to three numerical examples associated with the appropriate initial conditions **Example. 1** consider the reaction-diffusions equation (1), when $2 \preceq x \preceq 12$ in case of $p = 1, q = 1$, at $h = 1$and $k = 0.00001$, we start with the initial approximation, $u(x, 0) = \frac{1}{3}(3 + e^{\frac{-x}{2}})$. The exact solution is $u(x, t) = \frac{1}{3}(3 + e^{\frac{x+t}{2}})$. **Example. 2** consider the mKdV equation (29), when $10 \preceq x \preceq 20$ in case of $h = 1$and $k = 0.00001$, we start with the initial approximation, $u(x, 0) = sech(x)$. The exact solution is $u(x, t) = sech(x - t)$. **Example. 3** consider the Fitzhugh–Nagumo equation (20), when $0 \preceq x \preceq 10$ in case of $h = 1$and $k = 0.00001$, we start with the initial approximation, $u(x, 0) = \frac{1}{2}(1 + \tanh \frac{x}{2\sqrt{2}})$.

Table 3. Numerical results of solving FN equation at different times

x	$t = 0.01$	$t = 0.1$	$t = 0.5$
0	0.0002393	0.0022600	0.0045646
1	0.0001758	0.0018887	0.0126318
2	0.0000301	0.0003588	0.0032443
3	0.0000025	0.0000086	0.0004057
4	0.0000034	0.0000312	0.0001697
5	0.0000036	0.0000350	0.0001613
6	0.0000024	0.0000237	0.0001158
7	0.0000013	0.0000133	0.0000565
8	0.0000007	0.0000027	0.0000676
9	0.0000086	0.0000931	0.0006180
10	0.0000042	0.0000238	0.0005093

The exact solution is $u(x,t) = \frac{1}{2}(1 + \tanh(\frac{x}{2\sqrt{2}} - \frac{2p-1}{4})), p = 0.75$. Tables (1- 3) illustrate the numerical results of solving RD equation, mKdV equation and FN equation using CFDS at different times.

7. Conclusion

The CFDS is effective for solving linear and nonlinear partial differential equations especially for small time intervals. The numerical results show that the solution using CFDS give high accuracy and no more conditions or restrictions are needed.

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