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TRIPLED COINCIDENCE POINT THEOREM FOR COMPATIBLE MAPS IN FUZZY METRIC SPACES

P. P. MURTHY, RASHMI AND VISHNU NARAYAN MISHRA*

Abstract. In this paper, we are going to introduce the concept of compatible maps in Fuzzy metric spaces to establish a triple coincidence point Theorem. Our result extends the result of Hu [10]. We are providing an example to support our main result.

1. Introduction

The concept of fuzzy set theory introduced by Zadeh [24]. There are many viewpoints of the notion of the metric space in fuzzy topology. We can divide them into following two groups:
First group involves those results in which a fuzzy metric on a set X is treated as a map d: X x $X \to \mathbb{R}^+$, where X represents the totality of all fuzzy points of a set and satisfy some axioms which are analogous to the ordinary metric axioms. In such an approach numerical distances are set up between fuzzy objects.

Second group focussed on those results in which the distance between objects is fuzzy and the objects themselves may or may not be fuzzy. Erceg [4], Kaleva and Seikkala [12] and Kramosil and Michalek [13] discussed in length about fuzzy metric spaces. Grabiec's [7] proved a fixed point theorem in fuzzy metric space. Subramanyam [23] generalized Grabiec's result for a pair of commuting maps in the setting of Jungck 1976 [11]. George and Veermani [6] modified the concept of fuzzy metric spaces and defined a Hausdorff topology on this fuzzy metric space which a few applications in quantum particle physics particularly in connection with both string and E - infinity theory.

In 2004 the concept of partially ordered metric space which was introduced by Ran and Reurings [21], Guo and Lakshmikantham [8] studied the concept of coupled

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*Corresponding author
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fixed points in partially ordered metric spaces. Bhaskar and Lakshmikantham [3] introduced the monotone property in partially ordered metric spaces and given an application to the existence of periodic boundary value problem. Nieto and Lopez ([19] - [20]) rediscovered the partially ordered metric spaces and applied their problems to periodic boundary value problems. Berinde and Borcut [1] introduced the concept of tripled fixed point for nonlinear contractive mappings $F : X^3 \to X$, in partially ordered complete metric spaces (see also [2]). On the other hand, Sedghi, Altun and Shobe [22] gave a coupled fixed point theorem for contractions in fuzzy metric spaces and Fang [5] gave some common fixed point theorems under ϕ - contractions for compatible and weakly compatible mappings in probabilistic menger metric spaces. Many authors have proved fixed point theorems in (intuitionistic) fuzzy metric space or probabilistic metric spaces.
Hu [10], inspired us to obtain a triple coincidence point theorem under weaker conditions by providing an example to show that the result is a genuine generalization of the corresponding result of Hu [10]. In this aspect several results and applications are established in ([15, 16, 17, 18] and references therein).

2. Preliminaries

We recall a few definitions and symbols which are useful in proving our theorem.

Definition 1. [6] *A binary operation* $* : [0,1] \times [0,1] \to [0,1]$ *is a continuous t-norm if it satisfies the following conditions:*

1. $*$ *is associative and commutative;*
2. $*$ *is continuous;*
3. $a * 1 = a$ *for every* $a \in [0,1]$*;*
4. $a * b \le c * d$ *if* $a \le c$ *and* $b \le d$ *for all* $a, b, c, d \in [0,1]$.

Definition 2. [9] *A special class of t-norms* $\triangle$*(called a Hadžić-type t-norm) is introduced as follows:*
Let $\triangle$ *be a t-norm and* $sup_{0<t<1}\triangle(t,t) = 1$. *A t - norm* $\triangle$ *is said to be of H - type if the family of functions* $\{\triangle^m(t)\}_{m=1}^{\infty}$ *is equicontinuous at* $t = 1$, *where*

$$\triangle^1(t) = t\triangle t, \quad \triangle^{m+1}(t) = t\triangle(\triangle^m(t)), \quad m = 1, 2, ..., t \in [0,1]. \tag{2.1}$$

The t - norm $\triangle_M = min$ *is an example of t - norm of H - type, but there are some other t - norms* $\triangle$ *of H - type* [9].
Obviously, $\triangle$ *is a H - type t norm if and only if for any* $\lambda \in (0,1)$, *there exists* $\delta(\lambda) \in (0,1)$ *such that* $\triangle^m(t) > 1 - \lambda$ *for all* $m \in N$, *when* $t > 1 - \delta$.

Definition 3. [6] *A triplet* $(X, M, *)$ *is said to be a fuzzy metric space, if* X *is an arbitrary set,* $*$ *is a continuous t-norm and* M *is a fuzzy set on* $X^2 \times (0,\infty)$ *satisfying the following conditions for all* $x, y, z \in X$ *and* $t, s > 0$*:*

- (M_1) $M(x,y,t) > 0$,
- (M_2) $M(x,y,t) = 1$ *if and only if* $x = y$,
- (M_3) $M(x,y,t) = M(y,x,t)$,
- (M_4) $M(x,y,t) * M(y,z,s) \le M(x,z,t+s)$,
- (M_5) $M(x,y,\cdot) : (0,\infty) \to [0,1]$ *is continuous.*

In view of (M_1) and (M_2), it is worth pointing out that $0 < M(x,y,t) < 1$ for all $t > 0$, provided $x \ne y$. In view of Definition (3), George and Veermani [21] introduced the concept of Hausdorff topology on fuzzy metric spaces and showed

that every metric space induces a fuzzy metric space.
In fact, we can fuzzify metric spaces into fuzzy metric spaces in a natural way as is shown by the following example. In other words, every metric induces a fuzzy metric.

Example 4. *Let (X, d) be a metric space and define $a * b = ab$ for all $a, b \in [0, 1]$. Also define $M(x, y, t) = \frac{t}{t+d(x,y)}$ for all $x, y \in X$ and $t > 0$. Then $(X, M, *)$ is a fuzzy metric space, called standard fuzzy metric space induced by (X, d).*

Definition 5. [6] *Let $(X, M, *)$ be a fuzzy metric space, then*

(i) *a sequence $\{x_n\}$ in X is said to be convergent and converges to say x if*
$$lim_{n\to\infty} M(x_n, x, t) = 1,$$
for all $t > 0$;

(ii) *a sequence $\{x_n\}$ in X is said to be a Cauchy sequence if for a given $\epsilon > 0$, there exists $n_0 \in \mathbb{N}$, such that*
$$M(x_n, x_m, t) > 1 - \epsilon,$$
for all $t > 0$ and $n, m \geq n_0$;

(iii) *a fuzzy metric space $(X, M, *)$ is said to be complete if and only if every Cauchy sequence in X is a convergent sequence.*

Remark 6. *(see [7]).*

(a) *For all $x, y \in X, M(x, y, .)$ is non - decreasing.*

(b) *It is easy to prove that if $x_n \to x, y_n \to y, t_n \to t$, then*
$$lim_{n\to\infty} M(x_n, y_n, t_n) = M(x, y, t). \tag{2.2}$$

(c) *In a fuzzy metric space $(X, M, *)$, whenever $M(x, y, t) > 1 - r$ for $x, y \in X, t > 0, 0 < r < 1$, we can find a t_0, $0 < t_0 < t$ such that $M(x, y, t_0) > 1 - r$.*

(d) *For any $r_1 > r_2$, we can find r_3 such that $r_1 * r_3 \geq r_2$ and for any r_4 we can find r_5 such that $r_3 * r_5 \geq r_4$,*
where $r_1, r_2, r_3, r_4, r_5 \in (0, 1)$.

Definition 7. [22] *Let $(X, M, *)$ be a fuzzy metric space. M is said to satisfy the n - property on $X \times X \times (0, \infty)$ if*
$$lim_{n\to\infty} [M(x, y, k^n t)]^{n^p} = 1, \tag{2.3}$$
whenever $x, y \in X, k > 1$ and $p > 0$.

Lemma 8. *Let $(X, M, *)$ be a fuzzy metric space and M satisfies the n - property; then*
$$lim_{t\to+\infty} M(x, y, t) = 1, \tag{2.4}$$
for all $x, y \in X$.

Proof. If not, since $M(x, y, .)$ is non - decreasing and $0 \leq M(x, y, .) \leq 1$, there exists $x_0, y_0 \in X$ such that $lim_{t\to+\infty} M(x_0, y_0, t) = \lambda < 1$, then for $k > 1, k^n t \to +\infty$ as $n \to \infty$ and $t > 0$. We get $lim_{n\to\infty}[(M(x_0, y_0, k^n t))^{n^p}] = 0$, which is a contraction. □

Remark 9. *Condition (2.4) cannot guaranteed for the n - property in fuzzy metric space. To see this we have the following example given in [10].*

Example 10. *Let (X, d) be an ordinary metric space, $a * b \leq ab$ for all $a, b \in [0, 1]$, and ψ be define as following:*

$$\psi(t) = \begin{cases} \alpha\sqrt{t}, & \text{if} \quad 0 < t \leq 4, \\ 1 - \frac{1}{Int}, & \text{if} \quad t > 4, \end{cases} \tag{2.5}$$

where $\alpha = (\frac{1}{2})(1 - \frac{1}{In4})$. Then $\psi(t)$ is continuous and increasing in $(0, \infty)$, $\psi(t) \in (0, 1)$ and $\lim_{t\to+\infty} \psi(t) = 1$. Let

$$M(x, y, t) = [\psi(t)]^{d(x,y)}, \qquad \forall x, y \in X, t > 0, \tag{2.6}$$

then $(X, M, *)$ is a fuzzy metric space and

$$\lim_{t\to+\infty} M(x, y, t) = \lim_{t\to+\infty} [\psi(t)]^{d(x,y)} = 1, \qquad \forall \quad x, y \in X, t > 0, \tag{2.7}$$

But for any $x \neq y, p = 1, k > 1, t > 0$,

$$\lim_{n\to\infty} [M(x, y, k^n t)]^{n^p} = \lim_{n\to\infty} [(\psi(k^n t))]^{d(x,y).n^p} = \lim_{n\to\infty} [1 - \frac{1}{In(k^n t)}]^{n.d(x,y)} = e^{\frac{-d(x,y)}{Ink}} \neq 1. \tag{2.8}$$

Now, we recall the following:
$\Phi = \{\phi : R^+ \to R^+\}$, where $R^+ = [0, \infty)$ and each $\phi \in \Phi$ satisfies the following conditions:

(ϕ_1) ϕ is non decreasing;

(ϕ_2) ϕ is upper semi - continuous from the right;

(ϕ_3) $\Sigma_{n=0}^{\infty} \phi^n(t) < +\infty$ for all $t > 0$ where $\phi^{n+1}(t) = \phi(\phi^n(t)), n \in N$.

It is easy to prove that, if $\phi \in \Phi$, then $\phi(t) < t$ for all $t > 0$.

Lemma 11. [5] *Let $(X, M, *)$ be a fuzzy metric space, where $*$ is a continuous t - norm of H - type. If there exists $\phi \in \Phi$ such that if*

$$M(x, y, \phi(t)) \geq M(x, y, t), \tag{2.9}$$

for all $t > 0$, then $x = y$.

In this context, we mildly modify the concept of *tripled fixed point* introduced by Berinde and Borcut [1] in partially ordered metric space into fuzzy metric space. Also we introduce the following notion of tripled coincidence point and tripled fixed point.

Definition 12. *Let $(X, M, *)$ be a nonempty fuzzy metric space. An element $(x, y, z) \in X \times X \times X$ is said to be tripled coincidence point of the maps $F : X \times X \times X \to X$ and $g : X \to X$ if $F(x, y, z) = g(x)$, $F(y, x, z) = g(y)$ and $F(z, y, x) = g(z)$.*

Definition 13. *Let $(X, M, *)$ be a nonempty fuzzy metric space. An element $(x, y, z) \in X \times X \times X$ is said to be tripled fixed point of the map $F : X \times X \times X \to X$ if $F(x, y, z) = x$, $F(y, x, z) = y$ and $F(z, y, x) = z$.*

Now we define the concept of compatibility for a pair of maps $F : X \times X \times X \to X$ and $g : X \to X$.
The notion introduce by us is analogous to the notion of Hu [10] in the setting of fuzzy metric spaces.

Definition 14. *The maps $F : X \times X \times X \to X$ and $g : X \to X$ are said to be compatible pair of maps if*

$$\lim_{n\to\infty} M(g(F(x_n, y_n, z_n)), F(g(x_n), g(y_n), g(z_n)), t) = 1,$$

$$\lim_{n\to\infty} M(g(F(y_n, x_n, z_n)), F(g(y_n), g(x_n), g(z_n)), t) = 1,$$

and

$$\lim_{n\to\infty} M(g(F(z_n, y_n, x_n)), F(g(z_n), g(y_n), g(x_n)), t) = 1,$$

whenever $\{x_n\}, \{y_n\}$ and $\{z_n\}$ are sequences in X, such that

$$\lim_{n\to\infty} F(x_n, y_n, z_n) = \lim_{n\to\infty} g(x_n) = x, \lim_{n\to\infty} F(y_n, x_n, z_n) = \lim_{n\to\infty} g(y_n) = y$$

$$\lim_{n\to\infty} F(z_n, y_n, x_n) = \lim_{n\to\infty} g(z_n) = z,$$

for all $x, y, z \in X$ and $t > 0$.

Definition 15. *The mappings $F : X \times X \times X \to X$ and $g : X \to X$ are said to be commutative if*

$$g(F(x, y, z)) = F(gx, gy, gz),$$

for all $x, y \in X$.

Remark 16. *If F and g are commutative pair of maps, then they are compatible pair of maps.*

3. Main results

Now, we prove tripled coincidence point theorem under weaker conditions, which generalizes the result of Hu [10].

Throughout this section:

$$[M(x, y, t)]^n = \underbrace{M(x, y, t) * M(x, y, t) * ... * M(x, y, t)}_{n}, \tag{3.1}$$

for all $n \in \mathbb{N}$.

Theorem 17. *Let $(X, M, *)$ be a complete FM - Space, where $*$ is a continuous t - norm of H - type satisfying (2.4). Let $F : X \times X \times X \to X$ and $g : X \to X$ be two mappings such that there exists $\phi \in \Phi$ satisfying*

$$M(F(x, y, z), F(u.v, w), \phi(t)) \geq M(g(x), g(u), t) * M(g(y), g(v), t) * M(g(z), g(w), t), \tag{3.2}$$

for all $x, y, z, u, v, w \in X, t > 0$. Suppose that

- (a) $F(X \times X \times X) \subseteq g(X)$;
- (b) *g is continuous;*
- (c) *F and g are compatible.*

Then there exits $x, y, z \in X$ such that $g(x) = F(x, y, z), g(y) = F(y, x, z), g(z) = F(z, y, x)$ that is, F and g have triple coincidence point in X.

Proof. Let $x_0, y_0, z_0 \in X$ be arbitrary points in X. Since $F(X \times X \times X) \subseteq g(X)$, then there exists $x_1, y_1, z_1 \in X$ such that
$g(x_1) = F(x_0, y_0, z_0), g(y_1) = F(y_0, x_0, z_0)$, and $g(z_1) = F(z_0, y_0, x_0)$.
Continuing in this way, we can construct sequences $\{x_n\}, \{y_n\}$ and $\{z_n\}$ in X such that

$$g(x_{n+1}) = F(x_n, y_n, z_n), g(y_{n+1}) = F(y_n, x_n, z_n) \quad \text{and} \quad g(z_{n+1}) = F(z_n, y_n, x_n), \tag{3.3}$$

for all n ≥ 0.
We shall divided the proof in two steps:
Step I: First we show that $\{gx_n\}, \{gy_n\}$ and $\{gz_n\}$ are Cauchy sequences. Since $*$ is a t - norm of H - type, then for any $\lambda > 0$, there exists a $\mu > 0$ such that

$$\underbrace{(1-\mu) * (1-\mu) * ... * (1-\mu)}_{k-times} \geq 1 - \lambda, \tag{3.4}$$

for all $k \in N$.
Since $M(x, y, .)$ is continuous and $lim_{t \to +\infty} M(x, y, t) = 1$ for all $x, y \in X$ there exists $t_0 > 0$ such that

$$\begin{cases} M(gx_0, gx_1, t_0) \geq 1 - \mu, \\ M(gy_0, gy_1, t_0) \geq 1 - \mu, \\ M(gz_0, gz_1, t_0) \geq 1 - \mu. \end{cases} \tag{3.5}$$

On the other hand, since $\phi \in \Phi$, by condition (ϕ_3) we have $\sum_{n=1}^{\infty} \phi^n(t_0) < \infty$. Then for any $t > 0$, there exists $n_0 \in \mathbb{N}$ such that

$$t > \sum_{k=n_0}^{\infty} \phi^k(t_0), \tag{3.6}$$

From condition (3.2), we have

$$\begin{aligned} M(gx_1, gx_2, \phi(t_0)) &= M(F(x_0, y_0, z_0), F(x_1, y_1, z_1), \phi(t_0)) \\ &\geq M(gx_0, gx_1, t_0) * M(gy_0, gy_1, t_0) * M(gz_0, gz_1, t_0). \\ M(gy_1, gy_2, \phi(t_0)) &= M(F(y_0, x_0, z_0), F(y_1, x_1, z_1), \phi(t_0)) \\ &\geq M(gy_0, gy_1, t_0) * M(gx_0, gx_1, t_0) * M(gz_0, gz_1, t_0). \end{aligned}$$

and

$$\begin{aligned} M(gz_1, gz_2, \phi(t_0)) &= M(F(z_0, y_0, x_0), F(z_1, y_1, x_1), \phi(t_0)) \\ &\geq M(gz_0, gz_1, t_0) * M(gy_0, gy_1, t_0) * M(gx_0, gx_1, t_0) \end{aligned}$$

Similarly, we can also get

$$\begin{aligned} M(gx_2, gx_3, \phi^2(t_0)) &= M(F(x_1, y_1, z_1), F(x_2, y_2, z_2), \phi^2(t_0)) \\ &\geq M(gx_1, gx_2, \phi(t_0)) * M(gy_1, gy_2, \phi(t_0)) * M(gz_1, gz_2, \phi(t_0)) \\ &= M(F(x_0, y_0, z_0), F(x_1, y_1, z_1), \phi(t_0)) \\ &\quad * M(F(y_0, x_0, z_0), F(y_1, x_1, z_1), \phi(t_0)) \\ &\quad * M(F(z_0, y_0, x_0), F(z_1, y_1, x_1), \phi(t_0)) \\ &\geq M(gx_0, gx_1, t_0) * M(gy_0, gy_1, t_0) * M(gz_0, gz_1, t_0) \\ &\quad * M(gy_0, gy_1, t_0) * M(gx_0, gx_1, t_0) * M(gz_0, gz_1, t_0) \\ &\quad * M(gz_0, gz_1, t_0) * M(gy_0, gy_1, t_0) * M(gx_0, gx_1, t_0) \end{aligned}$$

$$
\begin{aligned}
&= [M(gx_0, gx_1, t_0)]^3 * [M(gy_0, gy_1, t_0)]^3 * [M(gz_0, gz_1, t_0)]^3. \\
M(gy_2, gy_3, \phi^2(t_0)) &= M(F(y_1, x_1, z_1), F(y_2, x_2, z_2), \phi^2(t_0)) \\
&\geq M(gy_1, gy_2, \phi(t_0)) * M(gx_1, gx_2, \phi(t_0)) * M(gz_1, gz_2, \phi(t_0)) \\
&= M(F(y_0, x_0, z_0), F(y_1, x_1, z_1), \phi(t_0)) \\
&\quad * M(F(x_0, y_0, z_0), F(x_1, y_1, z_1), \phi(t_0)) \\
&\quad * M(F(z_0, y_0, x_0), F(z_1, y_1, x_1), \phi(t_0)) \\
&\geq M(gy_0, gy_1, t_0) * M(gx_0, gx_1, t_0) * M(gz_0, gz_1, t_0) \\
&\quad * M(gx_0, gx_1, t_0) * M(gy_0, gy_1, t_0) * M(gz_0, gz_1, t_0) \\
&\quad * M(gz_0, gz_1, t_0) * M(gy_0, gy_1, t_0) * M(gx_0, gx_1, t_0) \\
&= [M(gy_0, gy_1, t_0)]^3 * [M(gx_0, gx_1, t_0)]^3 * [M(gz_0, gz_1, t_0)]^3. \\
M(gz_2, gz_3, \phi^2(t_0)) &\geq [M(gz_0, gz_1, t_0)]^3 * [M(gy_0, gy_1, t_0)]^3 * [M(gx_0, gx_1, t_0)]^3.
\end{aligned}
$$

Continuing in the same way we get

$$
\begin{aligned}
M(gx_n, gx_{n+1}, \phi^n(t_0)) &\geq [M(gx_0, gx_1, t_0)]^{3^{n-1}} * [M(gy_0, gy_1, t_0)]^{3^{n-1}} * [M(gz_0, gz_1, t_0)]^{3^{n-1}}, \\
M(gy_n, gy_{n+1}, \phi^n(t_0)) &\geq [M(gy_0, gy_1, t_0)]^{3^{n-1}} * [M(gx_0, gx_1, t_0)]^{3^{n-1}} * [M(gz_0, gz_1, t_0)]^{3^{n-1}}, \\
M(gz_n, gz_{n+1}, \phi^n(t_0)) &\geq [M(gz_0, gz_1, t_0)]^{3^{n-1}} * [M(gy_0, gy_1, t_0)]^{3^{n-1}} * [M(gx_0, gx_1, t_0)]^{3^{n-1}},
\end{aligned}
$$

So, from (3.5) and (3.6), for $m > n \geq n_0$, we have

$$
\begin{aligned}
M(gx_n, gx_m, t) &\geq M(gx_n, gx_m, \sum_{k=n_0}^{\infty} \phi^k(t_0)) \\
&\geq M(gx_n, gx_m, \sum_{k=n}^{m-1} \phi^k(t_0)) \\
&\geq M(gx_n, gx_{n+1}, \phi^n(t_0)) * M(gx_{n+1}, gx_{n+2}, \phi^{n+1}(t_0)) \\
&\quad * \cdots * M(gx_{m-1}, gx_m, \phi^{m-1}(t_0)) \\
&\geq [M(gy_0, gy_1, t_0)]^{3^{n-1}} * [M(gx_0, gx_1, t_0)]^{3^{n-1}} * [M(gz_0, gz_1, t_0)]^{3^{n-1}} \\
&\quad * [M(gy_0, gy_1, t_0)]^{3^n} * [M(gx_0, gx_1, t_0)]^{3^n} * [M(gz_0, gz_1, t_0)]^{3^n} \\
&\quad * \cdots * [M(gy_0, gy_1, t_0)]^{3^{m-2}} * [M(gx_0, gx_1, t_0)]^{3^{m-2}} * [M(gz_0, gz_1, t_0)]^{3^{m-2}} \\
&= [(M(gy_0, gy_1, t_0))^{\frac{1}{2}}]^{3^{n-1}(3^{m-n}-1)} * [(M(gx_0, gx_1, t_0))^{\frac{1}{2}}]^{3^{n-1}(3^{m-n}-1)} \\
&\quad * [(M(gz_0, gz_1, t_0))^{\frac{1}{2}}]^{3^{n-1}(3^{m-n}-1)} \\
&\geq (1-\mu) * (1-\mu) * ... * (1-\mu) \\
&\geq 1 - \lambda
\end{aligned}
$$

which implies that

$$
M(gx_n, gx_m, t) > 1 - \lambda, \tag{3.7}
$$

for all $m, n \in N$ with $m > n \geq n_0$ and $t > 0$. So $\{g(x_n)\}$ is a cauchy sequence. Similarly, we can get that $\{g(y_n)\}$ and $\{g(z_n)\}$ is also a Cauchy sequence.

Step II: Here we shall prove that g and F have a tripled coincidence point.
Since X is complete, there exists $x, y, z \in X$ such that

$$lim_{n\to\infty} F(x_n, y_n, z_n) = lim_{n\to\infty} g(x_n) = x,$$

$$lim_{n\to\infty} F(y_n, x_n, z_n) = lim_{n\to\infty} g(y_n) = y,$$

$$lim_{n\to\infty} F(z_n, y_n, x_n) = lim_{n\to\infty} g(z_n) = z,$$

Since F and g are compatible, we have by

$$\begin{aligned} lim_{n\to\infty} M(gF(x_n, y_n, z_n), F(gx_n, gy_n, gz_n), t) &= 1, \\ lim_{n\to\infty} M(gF(y_n, x_n, z_n), F(gy_n, gx_n, gz_n), t) &= 1, \\ lim_{n\to\infty} M(gF(z_n, y_n, x_n), F(gz_n, gy_n, gx_n), t) &= 1. \end{aligned}$$

$\forall \quad t > 0.$
Now, we prove that $g(x) = F(x, y, z), g(y) = F(y, x, z)$ and $g(z) = F(z, y, x)$, so by condition (3.2) and (3.3), we have

$$\begin{aligned} M(gx, F(x, y, z), \phi(t)) &\geq M(ggx_{n+1}, F(x, y, z), \phi(k_1 t)) * M(gx, ggx_{n+1}, \phi(t) - \phi(k_1 t)) \\ &= M(gF(x_n, y_n, z_n), F(x, y, z), \phi(k_1 t)) \\ &\quad * M(gx, ggx_{n+1}, \phi(t) - \phi(k_1 t)) \\ &\geq M(gF(x_n, y_n, z_n), F(gx_n, gy_n, gz_n), \phi(k_1 t) - \phi(k_2 t)) \\ &\quad * M(F(gx_n, gy_n, gz_n), F(x, y, z), \phi(k_2 t)) \\ &\quad * M(gx, ggx_{n+1}, \phi(t) - \phi(k_1 t)) \\ &\geq M(gF(x_n, y_n, z_n), F(gx_n, gy_n, gz_n), \phi(k_1 t) - \phi(k_2 t)) \\ &\quad * M(ggx_n, gx, k_2 t) * M(ggy_n, gy, k_2 t) \\ &\quad * M(ggz_n, gz, k_2 t) * M(gx, ggx_{n+1}, \phi(t) - \phi(k_1 t)), \end{aligned}$$

$\forall \quad 0 < k_2 < k_1 < 1.$
Since g is continuous, $\{g, F\}$ is a compatible pair and by letting $n \to \infty$, we have $M(gx, F(x, y, z), \phi(t)) \geq 1, \Rightarrow gx = F(x, y, z)$.
Similarly, we obtain $gy = F(y, x, z)$ and $gz = F(z, y, x)$.
Hence F and g have a have triple coincidence point. □

As an immediate consequence of the above theorem, we have the following corollaries:

Corollary 18. *Let $(X, M, *)$ be a complete fuzzy metric space, where $*$ is a continuous t - norm of H - type satisfying (2.4). Let $F : X \times X \times X \to X$ and there exists $\phi \in \Phi$ such that*

$$M(F(x, y, z), F(u.v, w), \phi(t)) \geq M(x, u, t) * M(y, v, t) * M(z, w, t), \tag{3.8}$$

for all $x, y, z, u, v, w \in X, t > 0$. Then there exits $x, y, z \in X$ such that $x = F(x, y, z), y = F(y, x, z), z = F(z, y, x)$ that is, F have a common tripled fixed point in X.

Corollary 19. [22] *Let $a * b \geq ab$ for all $a, b \in [0,1]$ and $(X, M, *)$ be a complete fuzzy metric space such that M has n - property. Let $F : X \times X \times X \to X$ and $g : X \to X$ be two function such that*

$$M(F(x,y,z), F(u.v,w), kt) \geq M(gx, gu, t) * M(gy, gv, t) * M(gz, gw, t), \quad (3.9)$$

for all $x, y, z, u, v, w \in X$, where $0 < k < 1, F(X \times X \times X) \subset g(X)$ and g is continuous and commutes with F. Then there exists triple coincidence point $(x, y, z) \in X$ such that
$g(x) = F(x, y, z), g(y) = F(y, x, z), g(z) = F(z, y, x)$.

Now we are ready to give an example to validate our main theorem:

Example 20. *Let $X = [-2, 2], a * b = ab$ for all $a, b \in [0, 1]$. ψ is defined as (2.5). Let*

$$M(x, y, t) = [\psi(t)]^{|x-y|},$$

for all $x, y \in [0, 1]$. Then $(X, M, *)$ is a complete FM - space.
Let $\phi(t) = \frac{t}{2}, g(x) = x$ and $F : X \times X \times X \to X$ be defined as

$$F(x, y, z) = \frac{x + y + z}{4}, \qquad \forall x, y \in X.$$

Thus F satisfies all the condition of Theorem(3.1), and there exists point $(0, 0, 0)$ is tripled coincidence point of g and F.
It is easy to see that $F(X \times X \times X) = [\frac{-2}{3}, \frac{2}{3}]$,

$$M(F(x, y, z), F(u, v, w), \phi(t)) = [\psi(\phi(t))]^{\frac{|x-u+y-v+z-w|}{4}}, \quad (3.10)$$

For all $t > 0$ and $x, y \in [-2, 2]$. (3.8) is equivalent to

$$[\psi(\frac{t}{2})]^{\frac{|x-u+y-v+z-w|}{4}} \geq [\psi(t)]^{|x-u|} \cdot [\psi(t)]^{|y-v|} \cdot [\psi(t)]^{|z-w|} \quad (3.11)$$

Since $\psi \in (0, 1)$, we can get

$$[\psi(\frac{t}{2})]^{\frac{|x-u+y-v+z-w|}{4}} \geq [\psi(\frac{t}{2})]^{\frac{|x-u|}{4}} \cdot [\psi(\frac{t}{2})]^{\frac{|y-v|}{4}} \cdot [\psi(\frac{t}{2})]^{\frac{|z-w|}{4}} \quad (3.12)$$

From (3.11), we only need to verify the following:

$$[\psi(\frac{t}{2})]^{\frac{|x-u|}{4}} \geq [\psi(t)]^{|x-u|} \quad (3.13)$$

that is,

$$\psi(\frac{t}{2}) \geq [\psi(t)]^4 \forall t > 0. \quad (3.14)$$

We consider the following cases.

Case 1: $(0 < t \leq 4)$. *Then (3.14) is equivalent to*

$$\alpha\sqrt{\frac{t}{2}} \geq (\alpha\sqrt{(t)})^4, \quad (3.15)$$

it is easy to verified.

Case 2: $(t \geq 8)$. *Then (3.14) is equivalent to*

$$1 - \frac{1}{In\frac{t}{2}} \geq (1 - \frac{1}{Int})^4 \quad (3.16)$$

which is

$$4In^3t \cdot In\frac{t}{2} + 4Int \cdot In\frac{t}{2} \geq In^4t + 6In^2t \cdot In\frac{t}{2} + In\frac{t}{2} \tag{3.17}$$

since

$$In^4t - 4In^3t \cdot In\frac{t}{2} - 4Int \cdot In\frac{t}{2} + 6In^2t \cdot In\frac{t}{2} + In^4\frac{t}{2} - In^4\frac{t}{2} + In\frac{t}{2} \leq 0, \tag{3.18}$$

hods for all $t \geq 8$. *So (3.14) holds for* $t \geq 8$.

Case 3: $(4 < t < 8)$. *Then (3.14) is equivalent to*

$$\alpha\sqrt{\frac{t}{2}} \geq (1 - \frac{1}{Int})^4 \tag{3.19}$$

Let $t = e^x$, *we only need to verify*

$$\frac{\alpha}{\sqrt{2}}e^{\frac{x}{2}} - (1 - \frac{1}{x})^4 \geq 0, \tag{3.20}$$

for all x *that* $2In2 < x < 3In2$.
Hence it is verified that the functions F, g, ϕ *satisfies all the conditions given in the Theorem(3.1) and* $(x, y, z) = (0, 0, 0)$ *is a triple coincidence point of* F *and* g *in* X.

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P. P. Murthy,
Department of Pure and Applied Mathematics, Guru Ghasidas Vishwavidyalaya(A Central University), Koni, Bilaspur(Chhattisgarh), 495 009, India
E-mail address: ppmurthy@gmail.com

Rashmi,
Department of Pure and Applied Mathematics, Guru Ghasidas Vishwavidyalaya (A Central University), Koni, Bilaspur-495009 (Chhattisgarh), India
E-mail address: rashmikwt@gmail.com

Vishnu Narayan Mishra (Corresponding author),
Department of Applied Mathematics and Humanities, Sardar Vallabhbhai National Institute of Technology, Ichchhanath Mahadev Dumas Road, Surat -395 007 (Gujarat), India.
L. 1627 Awadh Puri Colony Beniganj, Phase -III, Opposite - Industrial Training Institute (I.T.I.), Ayodhya Main Road, Faizabad-224 001, (Uttar Pradesh), India
E-mail address: vishnunarayanmishra@gmail.com, vishnu_narayanmishra@yahoo.co.in