Electronic Journal of Mathematical Analysis and Applications
Vol. 5(1) Jan. 2017, pp. 17-31.
ISSN: 2090-729(online)
http://fcag-egypt.com/Journals/EJMAA/

ON EXPONENTIAL STABILITY OF SOLUTIONS OF NEUTRAL DIFFERENTIAL SYSTEMS WITH MULTIPLE VARIABLE DELAYS

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Abstract. In this paper, the globally exponentially stability of the solutions to a certain neutral delay differential system with nonlinear uncertainties is investigated. A globally exponentially stability criterion is derived for the considered system. Based on the Lyapunov-Krasovskii functional approach, we prove a new result on the topic. Our result includes and improves the results in the literature.

1. Introduction

The dynamical systems with time delays have been considered by many authors during the past few decades (see [3, 6, 8, 19, 23, 24]). In particular, the interest in neutral differential equations has been growing rapidly due to their successful applications in practical fields such as circuit theory [2], bioengineering [17], population dynamics [5], automatic control [4,13] and so on. Current efforts on the problem of stability of time delay systems of neutral type can be divided into two categories; delay independent criteria and delay dependent criteria. A number of sufficient delay independent criteria for the asymptotic stability of neutral delay differential systems have been discussed by various researchers (see, for example [21, 22]). It should be noted that, in general, it is important both theoretically and practically to determine the delay independent and delay dependent criteria for the exponential stability of solutions.

In a recent paper, Syed Ali [18] investigated globally exponentially stability of solutions to the following neutral delay differential system:

$$\begin{cases} \dot{x}(t) &= Ax(t) + Bx(t-h(t)) + C\dot{x}(t-h(t)) + f_1(t,x(t)) + f_2(t,x(t-h(t))) \\ & \quad +f_3(t,\dot{x}(t-h(t))), \\ x(s) &= \phi(s), \quad \dot{x}(s) = \varphi(s), \ s \in [-h,0]. \end{cases}$$

In fact, many researches have also studied the exponential stability analysis for systems with time delays in the literature [1,7,9-12, 14, 15].

2010 *Mathematics Subject Classification.* 34K20.
Key words and phrases. Exponential stability, neutral delay differential system, first order, multiple variable delays.

In this paper, instead of the above system, we consider following neutral differential system multiple variable delays:

$$\begin{cases} \dot{x}(t) &= A(t)x(t) + \sum_{i=1}^{n} B_i(t)x(t-h_i(t)) + \sum_{i=1}^{n} C_i(t)\dot{x}(t-h_i(t)) \\ & +f_1(t,x(t)) + f_2(t,x(t-h_1(t)),...,x(t-h_n(t))) \\ & +f_3(t,\dot{x}(t-h_1(t)),...,\dot{x}(t-h_n(t))), \\ x(s) &= \varphi(s), \quad \dot{x}(s)=\phi(s),\ s\in[-h_i,0],\ (i=1,2,...,n), \end{cases} \tag{1}$$

where $x \in R^n$, $\phi(.)$ and $\varphi(.)$ are continuous vector valued initial functions, $A(t)$, $B_i(t)$, $C_i(t)$, $(i=1,2,...,n)$, are $n\times n$ real symmetric matrix functions, $h_i(t)$, $(i=1,2,...,n)$, are differentiable and denote the time-varying delays such that

$$0 \le h_i(t) \le h_{M_i}, \quad 0 \le \dot{h}_i(t) \le d_i < 1$$

hold, where h_{M_i} and d_i are positive constants, $f_1(t,x(t))$, $f_2(t,x(t-h_1(t)),...,x(t-h_n(t)))$, $f_3(t,\dot{x}(t-h_1(t)),...,\dot{x}(t-h_n(t)))$ are continuous nonlinear uncertainties and satisfy the following assumptions,

$$||f_1(t,x(t))|| \le \alpha_1 ||x(t)||,$$

$$||f_2(t,x(t-h_1(t)),...,x(t-h_n(t)))|| \le \alpha_2 \sum_{i=1}^{n} ||x(t-h_i(t))||,$$

$$||f_3(t,\dot{x}(t-h_1(t)),...,\dot{x}(t-h_n(t)))|| \le \alpha_3 \sum_{i=1}^{n} ||\dot{x}(t-h_i(t))||, t>0,$$

where α_1, α_2, α_3 are certain positive constants.

We can rewrite system (1) as the following descriptor system:

$$\begin{cases} \dot{x}(t) &= y(t), \\ y(t) &= A(t)x(t) + \sum_{i=1}^{n} B_i(t)x(t-h_i(t)) + \sum_{i=1}^{n} C_i(t)y(t-h_i(t)) \\ & +f_1(t,x(t)) + f_2(t,x(t-h_1(t)),...,x(t-h_n(t))) \\ & +f_3(t,y(t-h_1(t)),...,y(t-h_n(t))), \\ x(s) &= \phi(s), \quad y(s)=\varphi(s),\ s\in[-h_i,0],\ (i=1,2,...,n). \end{cases} \tag{2}$$

The motivation of this paper comes from the recent papers of [1, 9, 16, 20, 25]. Our aim is to extend and improve the results obtained in Syed Ali [18] for a more general case, that is, from one delay to multiple delays for the globally exponentially stability of solutions. This case shows the novelty of the paper. By this work, our aim is to do a contribution to the literature.

2. Stability

We need to the following basic definition, lemmas and theorem.

Definition 2.1 ([18]). System (1) is said to be globally exponentially stable with convergence rate α if there exist two positive constants α and λ such that

$$||x(t)|| \le \lambda e^{-\alpha t}, t \ge 0.$$

Lemma 2.1 (Schur Complement [18]). Let M, P and Q be given matrices such that $Q > 0$. Then

$$\begin{bmatrix} P & M^T \\ M & -Q \end{bmatrix} \le 0 \Longleftrightarrow P + M^T Q^{-1} M < 0.$$

Lemma 2.2 ([18]). For any vectors $a, b \in R^n$ and scalar $\varepsilon > 0$, we have

$$2a^T b \le \varepsilon a^T a + \varepsilon^{-1} b^T b.$$

Lemma 2.3 ([18]). For any constant matrix $M \in R^{n\times n}$, $M = M^T > 0$, scalar $\eta > 0$, vector function $w : [0, \eta] \to R^n$ such that the integrations concerned are well defined, then

$$\left[\int_0^\eta w(s)ds\right]^T M \left[\int_0^\eta w(s)ds\right] \le \eta \int_0^\eta w^T(s) M w(s) ds.$$

The main stability result of this paper is the following theorem.

Theorem. Let $P_i > 0$, $(i = 1, 2, 3, 4)$, and N_j, $(j = 1, 2, 3)$, be positive matrices, and ε_i, (i=1,2,...,12), be positive real numbers. If the following LMI condition

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Psi_1 & \Psi_2 & 0 & 0 \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} & 0 & 0 & \Psi_3 & 0 \\ * & * & \Xi_{33} & \Xi_{34} & 0 & 0 & 0 & \Psi_4 \\ * & * & * & \Xi_{44} & 0 & 0 & 0 & 0 \\ * & * & * & * & \Delta_1 & 0 & 0 & 0 \\ * & * & * & * & * & \Delta_2 & 0 & 0 \\ * & * & * & * & * & * & \Delta_3 & 0 \\ * & * & * & * & * & * & * & \Delta_4 \end{bmatrix} < 0,$$

holds, then system (1) is globally exponentially stable, where

$$\begin{aligned}
\Xi_{11} &= P_1 A(t) + A^T(t) P_1 + 2\alpha P_1 + \sum_{i=1}^n P_2 - \sum_{i=1}^n e^{-2\alpha h_i(t)} P_4 \\
&\quad + N_2^T A(t) + A^T(t) N_2 + \varepsilon_a \alpha_1^2, \\
\Xi_{12} &= \sum_{i=1}^n e^{-2\alpha h_i(t)} P_4 + \sum_{i=1}^n N_2^T B_i(t) + \sum_{i=1}^n P_1 B_i(t) + A^T(t) N_3, \\
\Xi_{13} &= -N_2^T + A^T(t) N_1, \\
\Xi_{14} &= \sum_{i=1}^n P_1 C_i(t) + \sum_{i=1}^n N_2^T C_i(t), \\
\Xi_{22} &= -\sum_{i=1}^n (1 - d_i) e^{-2\alpha h_i(t)} P_2 - \sum_{i=1}^n e^{-2\alpha h_i(t)} P_4 + \sum_{i=1}^n N_3^T B_i(t) \\
&\quad + \sum_{i=1}^n B_i^T(t) N_3 + \varepsilon_b \alpha_2^2, \\
\Xi_{23} &= \sum_{i=1}^n B_i^T(t) N_1 - N_3^T, \\
\Xi_{24} &= \sum_{i=1}^n N_3^T C_i(t),
\end{aligned}$$

$$\begin{aligned}
\Xi_{33} &= \sum_{i=1}^{n} P_3 + \sum_{i=1}^{n} h_{M_i}^2 P_4 - N_1^T - N_1, \\
\Xi_{34} &= \sum_{i=1}^{n} N_1^T C_i(t), \\
\Xi_{44} &= -\sum_{i=1}^{n}(1-d_i)e^{-2\alpha h_i(t)}P_3 + \varepsilon_c \alpha_3^2, \\
\Psi_1 &= [P_1^T P_1^T P_1^T],\ \Psi_2 = [N_2^T N_2^T N_2^T],\ \Psi_3 = [N_3^T N_3^T N_3^T],\ \Psi_4 = [N_1^T N_1^T N_1^T], \\
\Delta_1 &= diag\{-\varepsilon_1^{-1} I, \varepsilon_2^{-1} I, \varepsilon_3^{-1} I\},\ \Delta_2 = diag\{-\varepsilon_7^{-1} I, -\varepsilon_8^{-1} I, -\varepsilon_9^{-1} I\}, \\
\Delta_3 &= diag\{-\varepsilon_{10}^{-1} I, \varepsilon_{11}^{-1} I, \varepsilon_{12}^{-1} I\},\ \Delta_4 = diag\{-\varepsilon_4^{-1} I, -\varepsilon_5^{-1} I, -\varepsilon_6^{-1} I\}, \\
\varepsilon_a &= (\varepsilon_1^{-1} + \varepsilon_4^{-1} + \varepsilon_7^{-1} + \varepsilon_{10}^{-1}),\ \varepsilon_b = (\varepsilon_2^{-1} + \varepsilon_5^{-1} + \varepsilon_8^{-1} + \varepsilon_{11}^{-1}), \\
\varepsilon_c &= (\varepsilon_3^{-1} + \varepsilon_6^{-1} + \varepsilon_9^{-1} + \varepsilon_{12}^{-1}).
\end{aligned}$$

Proof. Define the Lyapunov-Krasovskii functional,

$$\begin{aligned}
V(t) &= e^{2\alpha t} x^T(t) P_1 x(t) + \sum_{i=1}^{n} \int_{t-h_i(t)}^{t} e^{2\alpha s} x^T(s) P_2 x(s) ds \\
&\quad + \sum_{i=1}^{n} \int_{t-h_i(t)}^{t} e^{2\alpha s} y^T(s) P_3 y(s) ds \\
&\quad + \sum_{i=1}^{n} h_{Mi} \int_{-h_{Mi}}^{0} \int_{t+\beta}^{t} e^{2\alpha s} \dot{x}^T(s) P_4 \dot{x}(s) ds d\beta.
\end{aligned}$$

The time derivative of $V(t)$ along the trajectories of (2) satisfies

$$\begin{aligned}
\dot{V}(t) &= 2\alpha e^{2\alpha t} x^T(t) P_1 x(t) + e^{2\alpha t} \dot{x}^T(t) P_1 x(t) \\
&\quad + e^{2\alpha t} x^T(t) P_1 \dot{x}(t) + \sum_{i=1}^{n} e^{2\alpha t} x^T(t) P_2 x(t) \\
&\quad - \sum_{i=1}^{n} (1 - \dot{h}_i(t)) e^{2\alpha(t-h_i(t))} x^T(t-h_i(t)) P_2 x(t-h_i(t)) \\
&\quad + \sum_{i=1}^{n} e^{2\alpha t} y^T(t) P_3 y(t) \\
&\quad - \sum_{i=1}^{n} (1 - \dot{h}_i(t)) e^{2\alpha(t-h_i(t))} y^T(t-h_i(t)) P_3 y(t-h_i(t)) \\
&\quad + \sum_{i=1}^{n} h_{Mi} \int_{-h_{Mi}}^{0} \Big[e^{2\alpha t} \dot{x}^T(t) P_4 \dot{x}(t) \\
&\quad - e^{2\alpha(t+\beta)} \dot{x}^T(t+\beta) P_4 \dot{x}(t+\beta) \Big] d\beta.
\end{aligned}$$

From system (1), we have

$$
\begin{aligned}
\dot{V}(t) \quad = \quad & 2\alpha e^{2\alpha t}x^T(t)P_1x(t) + e^{2\alpha t}\Bigg[x^T(t)A^T(t) + \sum_{i=1}^{n} x^T(t-h_i(t))B_i^T(t) \\
& + \sum_{i=1}^{n} \dot{x}^T(t-h_i(t))C_i^T(t) + f_1^T(t,x(t)) \\
& + f_2^T(t, x(t-h_1(t)), ..., x(t-h_n(t))) \\
& + f_3^T(t, \dot{x}(t-h_1(t)), ..., \dot{x}(t-h_n(t)))\Bigg] P_1x(t) + e^{2\alpha t}x^T(t)P_1\Bigg[A(t)x(t) \\
& + \sum_{i=1}^{n} B_i(t)x(t-h_i(t)) + \sum_{i=1}^{n} C_i(t)\dot{x}(t-h_i(t)) \\
& + f_1(t,x(t)) + f_2(t, x(t-h_1(t)), ..., x(t-h_n(t))) \\
& + f_3(t, \dot{x}(t-h_1(t)), ..., \dot{x}(t-h_n(t)))\Bigg] + \sum_{i=1}^{n} e^{2\alpha t}x^T(t)P_2x(t) \\
& - \sum_{i=1}^{n}(1-\dot{h}_i(t))e^{2\alpha(1-h_i(t))}x^T(t-h_i(t))P_2x(t-h_i(t)) \\
& + \sum_{i=1}^{n} e^{2\alpha t}y^T(t)P_3y(t) \\
& - \sum_{i=1}^{n}(1-\dot{h}_i(t))e^{2\alpha(t-h_i(t))}y^T(t-h_i(t))P_3y(t-h_i(t)) \\
& + \sum_{i=1}^{n} h_{Mi}^2 e^{2\alpha t}\dot{x}^T(t)P_4\dot{x}(t) - \sum_{i=1}^{n} h_{Mi}\int_{t-h_{Mi}}^{t} e^{2\alpha s}\dot{x}^T(s)P_4\dot{x}(s)ds.
\end{aligned}
$$

In view of the following estimates

$$
\begin{aligned}
& \sum_{i=1}^{n} x^T(t-h_i(t))B_i^T(t)P_1x(t) + \sum_{i=1}^{n} \dot{x}^T(t-h_i(t))C_i^T(t)P_1x(t) + f_1^T(t,x(t))P_1x(t) \\
& + f_2^T(t, x(t-h_1(t)), ..., x(t-h_n(t)))P_1x(t) \\
& + f_3^T(t, \dot{x}(t-h_1(t)), ..., \dot{x}(t-h_n(t)))P_1x(t) \\
& = \sum_{i=1}^{n} x^T(t)P_1B_i(t)x(t-h_i(t)) \\
& + \sum_{i=1}^{n} x^T(t)P_1C_i(t)\dot{x}(t-h_i(t)) + x^T(t)P_1f_1(t,x(t)) \\
& + x^T(t)P_1f_2(t, x(t-h_1(t)), ..., x(t-h_n(t))) \\
& + x^T(t)P_1f_3(t, \dot{x}(t-h_1(t)), ..., \dot{x}(t-h_n(t))), \\
& - e^{2\alpha t(t-h_i(t))} \le -e^{-2\alpha h_i(t)}, \ (i=1,2,...,n), \ (t \ge 0),
\end{aligned}
$$

and system (2), it follows that

$$\begin{aligned}
\dot{V}(t) \quad \leq \quad & e^{2\alpha t}\Bigg\{x^T(t)\Bigg[P_1A(t)+A^T(t)P_1+2\alpha P_1\Bigg]x(t) \\
& +2\sum_{i=1}^{n}x^T(t)P_1B_i(t)x(t-h_i(t))+2\sum_{i=1}^{n}x^T(t)P_1C_i(t)y(t-h_i(t)) \\
& +2x^T(t)P_1[f_1(t,x(t))+f_2(t,x(t-h_1(t)),...,x(t-h_n(t))) \\
& +f_3(t,y(t-h_1(t)),...,y(t-h_n(t)))]+\sum_{i=1}^{n}x^T(t)P_2x(t) \\
& -\sum_{i=1}^{n}(1-\dot{h}_i(t))e^{2\alpha h_i(t)}x^T(t-h_i(t))P_2x(t-h_i(t)) \\
& +\sum_{i=1}^{n}y^T(t)P_3y(t)-\sum_{i=1}^{n}(1-\dot{h}_i(t))e^{2\alpha h_i(t)}y^T(t-h_i(t))P_3y(t-h_i(t)) \\
& +\sum_{i=1}^{n}h_{Mi}^2e^{2\alpha t}y^T(t)P_4y(t)-\sum_{i=1}^{n}h_{Mi}\int_{t-h_{Mi}}^{t}e^{2\alpha(s-t)}\dot{x}^T(s)P_4\dot{x}(s)ds\Bigg\}.
\end{aligned}$$

Hence

$$\begin{aligned}
\dot{V}(t) \quad \leq \quad & e^{2\alpha t}\Bigg\{x^T(t)\left[P_1A(t)+A^T(t)P_1+2\alpha P_1+\sum_{i=1}^{n}P_2\right]x(t) \\
& +2\sum_{i=1}^{n}x^T(t)P_1B_i(t)x(t-h_i(t))+2\sum_{i=1}^{n}x^T(t)P_1C_i(t)y(t-h_i(t)) \\
& -\sum_{i=1}^{n}(1-\dot{h}_i(t))e^{2\alpha h_i(t)}x^T(t-h_i(t))P_2x(t-h_i(t)) \\
& +\sum_{i=1}^{n}y^TP_3y(t)+\sum_{i=1}^{n}h_{Mi}^2e^{2\alpha t}y^T(t)P_4y(t) \\
& -\sum_{i=1}^{n}(t-\dot{h}_i(t))e^{2\alpha h_i(t)}y^T(t-h_i(t))P_3y(t-h_i(t)) \\
& +2x^T(t)P_1[f_1(t,x(t))+f_2(t,x(t-h_1(t)),...,x(t-h_n(t))) \\
& +f_3(t,y(t-h_1(t)),...,y(t-h_n(t)))] \\
& -\sum_{i=1}^{n}h_{Mi}\int_{t-h_{Mi}}^{t}e^{2\alpha(s-t)}\dot{x}^T(s)P_4\dot{x}(s)ds\Bigg\}.
\end{aligned}$$

From system (2) we have

$$\begin{aligned}
& 2[y^T(t)N_1^T+x^T(t)N_2^T+\sum_{i=1}^{n}x^T(t-h_i(t))N_3^T]\times[-y(t)+A(t)x(t) \\
& +\sum_{i=1}^{n}B_i(t)x(t-h_i(t))+\sum_{i=1}^{n}C_i(t)y(t-h_i(t))+f_1(t,x(t)) \\
& +f_2(t,x(t-h_1(t)),...,x(t-h_n(t)))+f_3(t,y(t-h_1(t)),...,y(t-h_n(t)))]=0
\end{aligned}$$

and

$$
\begin{aligned}
&- 2y^T(t)N_1^T y(t) + 2y^T(t)N_1^T A(t)x(t) + 2y^T(t)N_1^T \sum_{i=1}^{n} B_i x(t-h_i(t)) \\
&+ 2y^T(t)N_1^T \sum_{i=1}^{n} C_i(t)y(t-h_i(t)) - 2x^T(t)N_2^T y(t) + 2x^T(t)N_2^T A(t)x(t) \\
&+ 2x^T(t)N_2^T \sum_{i=1}^{n} B_i(t)x(t-h_i(t)) + 2x^T(t)N_2^T \sum_{i=1}^{n} C_i(t)y(t-h_i(t)) \\
&- 2\sum_{i=1}^{n} x^T(t-h_i(t))N_3^T y(t) + 2\sum_{i=1}^{n} x^T(t-h_i(t))N_3^T A(t)x(t) \\
&+ 2\sum_{i=1}^{n} x^T(t-h_i(t))N_3^T B_i(t)x(t-h_i(t)) + 2\sum_{i=1}^{n} x(t-h_i(t))N_3^T C_i(t)y(t-h_i(t)) \\
&+ [2y^T(t)N_1^T + 2x^T(t)N_2^T + 2\sum_{i=1}^{n} x^T(t-h_i(t))N_3^T] \times [f_1(t,x(t)) \\
&+ f_2(t,x(t-h_1(t)),...,x(t-h_n(t))) + f_3(t,y(t-h_1(t)),...,y(t-h_n(t)))] = 0.
\end{aligned}
$$

Then

$$
\begin{aligned}
\dot{V}(t) \quad \leq \quad & e^{2\alpha t}\{x^T(t)\left[P_1A(t) + A^T(t)P_1 + 2\alpha P_1 + \sum_{i=1}^{n} P_2\right]x(t) \\
& +2\sum_{i=1}^{n} x^T(t)P_1B_i(t)x(t-h_i(t)) + 2\sum_{i=1}^{n} x^T(t)P_1C_i(t)y(t-h_i(t)) \\
& -\sum_{i=1}^{n}(1-\dot{h}_i(t))e^{2\alpha h_i(t)}x^T(t-h_i(t))P_2x(t-h_i(t)) \\
& +\sum_{i=1}^{n} y^T(t)P_3y(t) + \sum_{i=1}^{n} h_{Mi}^2 e^{2\alpha t}y^T(t)P_4y(t) \\
& -\sum_{i=1}^{n}(1-\dot{h}_i(t))e^{2\alpha h_i(t)}y^T(t-h_i(t))P_3y(t-h_i(t)) \\
& -2y^T(t)N_1^T y(t) + 2y^T(t)N_1^T A(t)x(t) + 2y^T(t)N_1^T \sum_{i=1}^{n} B_i(t)x(t-h_i(t)) \\
& +2y^T(t)N_1^T \sum_{i=1}^{n} C_i(t)y(t-h_i(t)) - 2x^T(t)N_2^T y(t) + 2x^T(t)N_2^T A(t)x(t) \\
& +2x^T(t)N_2^T \sum_{i=1}^{n} B_i(t)x(t-h_i(t)) + 2x^T(t)N_2^T \sum_{i=1}^{n} C_i(t)y(t-h_i(t)) \\
& -2\sum_{i=1}^{n} x^T(t-h_i(t))N_3^T y(t) + 2\sum_{i=1}^{n} x^T(t-h_i(t))N_3^T A(t)x(t)
\end{aligned}
$$

$$+ 2\sum_{i=1}^{n} x^T(t-h_i(t))N_3^T B_i(t)x(t-h_i(t)) + 2\sum_{i=1}^{n} x(t-h_i(t))N_3^T C_i(t)y(t-h_i(t))$$
$$+ [2y^T(t)N_1^T + 2x^T(t)N_2^T + 2\sum_{i=1}^{n} x^T(t-h_i(t))N_3^T] \times [f_1(t,x(t))$$
$$+ f_2(t,x(t-h_1(t)),...,x(t-h_n(t))) + f_3(t,y(t-h_1(t)),...,y(t-h_n(t)))]$$
$$- \sum_{i=1}^{n} h_{Mi}\int_{t-h_{Mi}}^{t} e^{2\alpha(s-t)}\dot{x}^T(s)P_4\dot{x}(s)ds\}.$$

If we use the following inequalities

$$||f_1(t,x(t))|| \le \alpha_1||x(t)||,$$

$$||f_2(t,x(t-h_1(t)),...,x(t-h_n(t)))|| \le \alpha_2\sum_{i=1}^{n}||x(t-h_i(t))||,$$

$$||f_3(t,\dot{x}(t-h_1(t)),...,\dot{x}(t-h_n(t)))|| \le \alpha_3\sum_{i=1}^{n}||\dot{x}(t-h_i(t))||, t>0,$$

and Lemma 2.2 , then we get

$$\begin{aligned} 2x^T(t)P_1f_1(t,x(t)) &\le \varepsilon_1x^T(t)P_1P_1^Tx(t) + \varepsilon_1^{-1}f_1^T(t,x(t))f_1(t,x(t)) \\ &\le \varepsilon_1x^T(t)P_1P_1^Tx(t) + \varepsilon_1^{-1}x^T(t)\alpha_1^2x(t), \end{aligned}$$

$$2x^T(t)P_1f_2(t,x(t-h_1(t)),...,x(t-h_n(t))) \le \varepsilon_2x^T(t)P_1P_1^Tx(t)$$
$$+ \varepsilon_2^{-1}f_2^T(t,x(t-h_1(t)),...,x(t-h_n(t)))f_2(t,x(t-h_1(t)),...,x(t-h_n(t)))$$
$$\le \varepsilon_2x^T(t)P_1P_1^Tx(t) + \varepsilon_2^{-1}\sum_{i=1}^{n}x^T(t-h_i(t))\alpha_2^2\sum_{i=1}^{n}x(t-h_i(t)),$$
$$2x^T(t)P_1f_3(t,y(t-h_1(t)),...,y(t-h_n(t))) \le \varepsilon_3x^T(t)P_1P_1^Tx(t)$$
$$+ \varepsilon_3^{-1}f_3^T(t,y(t-h_1(t)),...,y(t-h_n(t)))f_3(t,y(t-h_1(t)),...,y(t-h_n(t)))$$
$$\le \varepsilon_3x^T(t)P_1P_1^Tx(t) + \varepsilon_3^{-1}\sum_{i=1}^{n}y^T(t-h_i(t))\alpha_3^2\sum_{i=1}^{n}y(t-h_i(t)),$$
$$2y^T(t)N_1^Tf_1(t,x(t)) \le \varepsilon_4y^T(t)N_1^TN_1y(t) + \varepsilon_4^{-1}f_1^T(t,x(t))f_1(t,x(t))$$
$$\le \varepsilon_4y^T(t)N_1^TN_1y(t) + \varepsilon_4^{-1}x^T(t)\alpha_1^2x(t),$$
$$2y^T(t)N_1^Tf_2(t,x(t-h_1(t)),...,x(t-h_n(t))) \le \varepsilon_5y^T(t)N_1^TN_1y(t)$$
$$+ \varepsilon_5^{-1}f_2^T(t,x(t-h_1(t)),...,x(t-h_n(t)))f_2(t,x(t-h_1(t)),...,x(t-h_n(t)))$$
$$\le \varepsilon_5y^T(t)_1N_1^TN_1y(t) + \varepsilon_5^{-1}\sum_{i=1}^{n}x^T(t-h_i(t))\alpha_2^2\sum_{i=1}^{n}x(t-h_i(t)),$$
$$2y^T(t)N_1^Tf_3(t,y(t-h_1(t)),...,y(t-h_n(t))) \le \varepsilon_6y^T(t)N_1^TN_1y(t)$$
$$+ \varepsilon_6^{-1}f_3^T(t,y(t-h_1(t)),...,y(t-h_n(t)))f_3(t,y(t-h_1(t)),...,y(t-h_n(t)))$$

$$\leq \varepsilon_6 y^T(t)N_1^T N_1 y(t) + \varepsilon_6^{-1}\sum_{i=1}^{n} y^T(t-h_i(t))\alpha_3^2 \sum_{i=1}^{n} y(t-h_i(t)),$$

$$2x^T(t)N_2^T f_1(t,x(t)) \leq \varepsilon_7 x^T(t)N_2^T N_2 x(t) + \varepsilon_7^{-1} f_1^T(t,x(t))f_1(t,x(t))$$

$$\leq \varepsilon_7 x^T(t)N_2^T N_2 x(t) + \varepsilon_7^{-1} x^T(t)\alpha_1^2 x(t),$$

$$2x^T(t)N_2^T f_2(t,x(t-h_1(t)),...,x(t-h_n(t))) \leq \varepsilon_8 x^T(t)N_2^T N_2 x(t)$$

$$+ \varepsilon_8^{-1} f_2^T(t,x(t-h_1(t)),...,x(t-h_n(t)))f_2(t,x(t-h_1(t)),...,x(t-h_n(t)))$$

$$\leq \varepsilon_8 x^T(t)N_2^T N_2 x(t) + \varepsilon_8^{-1}\sum_{i=1}^{n} x^T(t-h_i(t))\alpha_2^2 \sum_{i=1}^{n} x(t-h_i(t)),$$

$$2x^T(t)N_2^T f_3(t,y(t-h_1(t)),...,y(t-h_n(t))) \leq \varepsilon_9 x^T(t)N_2^T N_2 x(t)$$

$$+ \varepsilon_9^{-1} f_3^T(t,y(t-h_1(t)),...,y(t-h_n(t)))f_3(t,y(t-h_1(t)),...,y(t-h_n(t)))$$

$$\leq \varepsilon_9 x^T(t)N_2^T N_2 x(t) + \varepsilon_9^{-1}\sum_{i=1}^{n} y^T(t-h_i(t))\alpha_3^2 \sum_{i=1}^{n} y(t-h_i(t)),$$

$$2\sum_{i=1}^{n} x^T(t-h_i(t))N_3^T f_1(t,x(t)) \leq \varepsilon_{10}\sum_{i=1}^{n} x^T(t-h_i(t))N_3^T N_3 \sum_{i=1}^{n} x^T(t-h_i(t))$$

$$+ \varepsilon_{10}^{-1} x^T(t)\alpha_1^2 x(t),$$

$$2\sum_{i=1}^{n} x^T(t-h_i(t))N_3^T f_2(t,x(t-h_1(t)),...,x(t-h_n(t)))$$

$$\leq \varepsilon_{11}\sum_{i=1}^{n} x^T(t-h_i(t))N_3^T N_3 \sum_{i=1}^{n} x(t-h_i(t)))$$

$$+ \varepsilon_{11}^{-1} f_2^T(t,x(t-h_1(t)),...,x(t-h_n(t)))f_2(t,x(t-h_1(t)),...,x(t-h_n(t)))$$

$$\leq \varepsilon_{11}\sum_{i=1}^{n} x^T(t-h_i(t))N_3^T N_3 \sum_{i=1}^{n} x(t-h_i(t)))$$

$$+ \varepsilon_{11}^{-1}\sum_{i=1}^{n} x^T(t-h_i(t))\alpha_2^2 \sum_{i=1}^{n} x(t-h_i(t))),$$

$$2\sum_{i=1}^{n} x^T(t-h_i(t))N_3^T f_3(t,y(t-h_1(t)),...,y(t-h_n(t)))$$

$$\leq \varepsilon_{12}\sum_{i=1}^{n} x^T(t-h_i(t))N_3^T N_3 \sum_{i=1}^{n} x(t-h_i(t)))$$

$$+ \varepsilon_{12}^{-1} f_3^T(t,y(t-h_1(t)),...,y(t-h_n(t)))f_3(t,y(t-h_1(t)),...,y(t-h_n(t)))$$

$$\leq \varepsilon_{12}\sum_{i=1}^{n} x^T(t-h_i(t))N_3^T N_3 \sum_{i=1}^{n} x(t-h_i(t))$$

$$+ \varepsilon_{12}^{-1}\sum_{i=1}^{n} y^T(t-h_i(t))\alpha_3^2 \sum_{i=1}^{n} y(t-h_i(t))$$

so that

$$
\begin{aligned}
\dot{V}(t) \quad \leq \quad & e^{2\alpha t}\{x^T(t)\left[P_1A(t)+A^T(t)P_1+2\alpha P_1+\sum_{i=1}^{n}P_2\right]x(t) \\
& +2\sum_{i=1}^{n}x^T(t)P_1B_i(t)x(t-h_i(t))+2\sum_{i=1}^{n}x^T(t)P_1C_i(t)y(t-h_i(t)) \\
& -\sum_{i=1}^{n}(1-\dot{h}_i(t))e^{2\alpha h_i(t)}x^T(t-h_i(t))P_2x(t-h_i(t))+\sum_{i=1}^{n}y^T(t)P_3y(t) \\
& +\sum_{i=1}^{n}h_{Mi}^2e^{2\alpha t}y^T(t)P_4y(t) \\
& -\sum_{i=1}^{n}(t-\dot{h}_i(t))e^{2\alpha h_i(t)}y^T(t-h_i(t))P_3y(t-h_i(t)) \\
& -2y^T(t)N_1^Ty(t)+2y^T(t)N_1^TA(t)x(t)+2y^T(t)N_1^T\sum_{i=1}^{n}B_i(t)x(t-h_i(t)) \\
& +2y^T(t)N_1^T\sum_{i=1}^{n}C_i(t)y(t-h_i(t))-2x^T(t)N_2^Ty(t)+2x^T(t)N_2^TA(t)x(t) \\
& +2x^T(t)N_2^T\sum_{i=1}^{n}B_i(t)x(t-h_i(t))+2x^T(t)N_2^T\sum_{i=1}^{n}C_i(t)y(t-h_i(t)) \\
& -2\sum_{i=1}^{n}x^T(t-h_i(t))N_3^Ty(t)+2\sum_{i=1}^{n}x^T(t-h_i(t))N_3^TA(t)x(t) \\
& +2\sum_{i=1}^{n}x^T(t-h_i(t))N_3^TB_i(t)x(t-h_i(t)) \\
& +2\sum_{i=1}^{n}x(t-h_i(t))N_3^TC_i(t)y(t-h_i(t)) \\
& +(\varepsilon_1+\varepsilon_2+\varepsilon_3)x^T(t)P_1^TP_1x(t)+(\varepsilon_7+\varepsilon_8+\varepsilon_9)x^T(t)N_2^TN_2x(t) \\
& +(\varepsilon_4+\varepsilon_5+\varepsilon_6)y^T(t)N_1^TN_1y(t) \\
& +(\varepsilon_{10}+\varepsilon_{11}+\varepsilon_{12})\sum_{i=1}^{n}x^T(t-h_i(t))N_3^TN_3\sum_{i=1}^{n}x(t-h_i(t)) \\
& +(\varepsilon_1^{-1}+\varepsilon_4^{-1}+\varepsilon_7^{-1}+\varepsilon_{10}^{-1})x^T(t)\alpha_1^2x(t) \\
& +(\varepsilon_2^{-1}+\varepsilon_5^{-1}+\varepsilon_8^{-1}+\varepsilon_{11}^{-1})\sum_{i=1}^{n}x^T(t-h_i(t))\alpha_2^2\sum_{i=1}^{n}x(t-h_i(t)) \\
& +(\varepsilon_3^{-1}+\varepsilon_6^{-1}+\varepsilon_9^{-1}+\varepsilon_{12}^{-1})\sum_{i=1}^{n}y^T(t-h_i(t))\alpha_3^2\sum_{i=1}^{n}y(t-h_i(t)) \\
& -\sum_{i=1}^{n}h_{Mi}\int_{t-h_{Mi}}^{t}e^{2\alpha(s-t)}\dot{x}^T(s)P_4\dot{x}(s)ds\}.
\end{aligned}
$$

By Lemma 2.3, the mean value theorem for the integrals and the Leibniz-Newton formula, we have

$$\begin{aligned}
&-\sum_{i=1}^{n} h_{Mi}\int_{t-h_{Mi}}^{t} e^{2\alpha(s-t)}\dot{x}^T(s)P_4\dot{x}(s)ds \\
&\le -\sum_{i=1}^{n} h_{Mi}e^{-2\alpha h_{Mi}}\int_{t-h_{Mi}}^{t} \dot{x}^T(s)P_4\dot{x}(s)ds \\
&\le -\sum_{i=1}^{n} h_{Mi}e^{-2\alpha h_i(t)}\int_{t-h_i(t)}^{t} \dot{x}^T(s)P_4\dot{x}(s)ds \\
&\le -\sum_{i=1}^{n} e^{-2\alpha h_i(t)}\left(\int_{t-h_i(t)}^{t}\dot{x}(s)ds\right)^T P_4\left(\int_{t-h_i(t)}^{t}\dot{x}(s)ds\right) \\
&= -\sum_{i=1}^{n} e^{-2\alpha h_i(t)}[x(t)-x(t-h_i(t))]^T P_4[x(t)-x(t-h_i(t))] \\
&= -\sum_{i=1}^{n} x^T(t)e^{-2\alpha h_i(t)}P_4x(t) + 2\sum_{i=1}^{n} x^T(t)e^{-2\alpha h_i(t)}P_4x(t-h_i(t)) \\
&\quad -\sum_{i=1}^{n} x^T(t-h_i(t))e^{-2\alpha h_i(t)}P_4x(t-h_i(t)). \qquad (3)
\end{aligned}$$

By (3) and $0 \le \dot{h}_i(t) \le d_i \le 1$, we can obtain

$$\begin{aligned}
\dot{V}(t) \quad \le \quad & e^{2\alpha t}\Bigg\{x^T(t)\left[P_1A(t)+A^T(t)P_1+2\alpha P_1+\sum_{i=1}^{n}P_2\right]x(t) \\
&+2\sum_{i=1}^{n} x^T(t)P_1B_i(t)x(t-h_i(t)) + 2\sum_{i=1}^{n} x^T(t)P_1C_i(t)y(t-h_i(t)) \\
&-\sum_{i=1}^{n}(1-d_i(t))e^{2\alpha h_i(t)}x^T(t-h_i(t))P_2x(t-h_i(t)) \\
&+\sum_{i=1}^{n} y^T(t)P_3y(t) + \sum_{i=1}^{n} h_{Mi}^2 e^{2\alpha t}y^T(t)P_4y(t) \\
&-\sum_{i=1}^{n}(t-d_i(t))e^{2\alpha h_i(t)}y^T(t-h_i(t))P_3y(t-h_i(t)) \\
&-\sum_{i=1}^{n} x^T(t)e^{-2\alpha h_i(t)}P_4x(t) + 2\sum_{i=1}^{n} x^T(t)e^{-2\alpha h_i(t)}P_4x(t-h_i(t)) \\
&-\sum_{i=1}^{n} x^T(t-h_i(t))e^{-2\alpha h_i(t)}P_4x(t-h_i(t)) - 2y^T(t)N_1^Ty(t) \\
&+2y^T(t)N_1^TA(t)x(t) + 2y^T(t)N_1^T\sum_{i=1}^{n}B_i(t)x(t-h_i(t)) \\
&+2y^T(t)N_1^T\sum_{i=1}^{n}C_i(t)y(t-h_i(t)) - 2x^T(t)N_2^Ty(t) + 2x^T(t)N_2^TA(t)x(t)
\end{aligned}$$

$$+ 2x^T(t)N_2^T\sum_{i=1}^{n}B_i(t)x(t-h_i(t)) + 2x^T(t)N_2^T\sum_{i=1}^{n}C_i(t)y(t-h_i(t))$$
$$- 2\sum_{i=1}^{n}x^T(t-h_i(t))N_3^Ty(t) + 2\sum_{i=1}^{n}x^T(t-h_i(t))N_3^TA(t)x(t)$$
$$+ 2\sum_{i=1}^{n}x^T(t-h_i(t))N_3^TB_i(t)x(t-h_i(t)) + 2\sum_{i=1}^{n}x(t-h_i(t))N_3^TC_i(t)y(t-h_i(t))$$
$$+ (\varepsilon_1+\varepsilon_2+\varepsilon_3)x^T(t)P_1^TP_1x(t) + (\varepsilon_7+\varepsilon_8+\varepsilon_9)x^T(t)N_2^TN_2x(t)$$
$$+ (\varepsilon_4+\varepsilon_5+\varepsilon_6)y^T(t)N_1^TN_1y(t)$$
$$+ (\varepsilon_{10}+\varepsilon_{11}+\varepsilon_{12})\sum_{i=1}^{n}x^T(t-h_i(t))N_3^TN_3\sum_{i=1}^{n}x(t-h_i(t))$$
$$+ (\varepsilon_1^{-1}+\varepsilon_4^{-1}+\varepsilon_7^{-1}+\varepsilon_{10}^{-1})x^T(t)\alpha_1^2x(t)$$
$$+ (\varepsilon_2^{-1}+\varepsilon_5^{-1}+\varepsilon_8^{-1}+\varepsilon_{11}^{-1})\sum_{i=1}^{n}x^T(t-h_i(t))\alpha_2^2\sum_{i=1}^{n}x(t-h_i(t))$$
$$+ (\varepsilon_3^{-1}+\varepsilon_6^{-1}+\varepsilon_9^{-1}+\varepsilon_{12}^{-1})\sum_{i=1}^{n}y^T(t-h_i(t))\alpha_3^2\sum_{i=1}^{n}y(t-h_i(t))\Bigg\}.$$

so that

$$\begin{aligned}
\dot{V}(t) \quad \leq \quad & e^{2\alpha t}\Bigg\{x^T(t)\Bigg[P_1A(t) + A^T(t)P_1 + 2\alpha P_1 + \sum_{i=1}^{n}P_2 - \sum_{i=1}^{n}e^{-2\alpha h_i(t)}P_4 \\
& +2N_2^TA(t) + (\varepsilon_1+\varepsilon_2+\varepsilon_3)P_1^TP_1 + (\varepsilon_7+\varepsilon_8+\varepsilon_9)N_2^TN_2 \\
& +(\varepsilon_1^{-1}+\varepsilon_4^{-1}+\varepsilon_7^{-1}+\varepsilon_{10}^{-1})\alpha_1^2\Bigg]x(t) + 2x^T(t)\Bigg[\sum_{i=1}^{n}e^{-2\alpha h_i(t)}P_4 \\
& +\sum_{i=1}^{n}N_2^TB_i(t) + \sum_{i=1}^{n}P_1B_i(t) + A^T(t)N_3\Bigg]\sum_{i=1}^{n}x(t-h_i(t)) \\
& +2x^T(t)[A^T(t)N_1 - N_2^T]y^T(t) \\
& +2x^T(t)\Bigg[\sum_{i=1}^{n}P_1C_i(t) + \sum_{i=1}^{n}N_2^TC_i(t)\Bigg]\sum_{i=1}^{n}y(t-h_i(t)) \\
& +\sum_{i=1}^{n}x^T(t-h_i(t))[2\sum_{i=1}^{n}N_3^TB_i(t) \\
& -\sum_{i=1}^{n}(1-d_i)e^{-2\alpha h_i(t)}P_2 - \sum_{i=1}^{n}e^{-2\alpha h_i(t)}P_4 \\
& +(\varepsilon_{10}+\varepsilon_{11}+\varepsilon_{12})N_3^TN_3 \\
& +(\varepsilon_2^{-1}+\varepsilon_5^{-1}+\varepsilon_8^{-1}+\varepsilon_{11}^{-1})\alpha_2^2]\sum_{i=1}^{n}x(t-h_i(t)) \\
& +2\sum_{i=1}^{n}x^T(t-h_i(t))\Bigg[\sum_{i=1}^{n}B_i^T(t)N_1 - N_3^T\Bigg]y(t)
\end{aligned}$$

$$
\begin{aligned}
&+ 2\sum_{i=1}^{n} x^T(t-h_i(t))N_3^T C_i(t)y(t-h_i(t)) \\
&+ y^T(t)\left[\sum_{i=1}^{n} P_3 + \sum_{i=1}^{n} h_{Mi}^2 P_4 - 2N_1^T + (\varepsilon_4+\varepsilon_5+\varepsilon_6)N_1^T N_1\right] y(t) \\
&+ 2y^T(t)N_1^T \sum_{i=1}^{n} C_i(t)y(t-h_i(t)) \\
&+ \sum_{i=1}^{n} y^T(t-h_i(t))[-\sum_{i=1}^{n}(1-d_i)e^{-2\alpha h_i(t)}P_3 \\
&+ (\varepsilon_3^{-1}+\varepsilon_6^{-1}+\varepsilon_9^{-1}+\varepsilon_{12}^{-1})\alpha_3^2]\sum_{i=1}^{n} y(t-h_i(t))\bigg\}.
\end{aligned}
$$

In this case, we achieve the following result

$$\dot{V}(t) \le e^{2\alpha t}\xi^T \Sigma \xi,$$

where

$$\xi^T = [x^T(t)x^T(t-h(t))y^T(t)y^T(t-h(t))],$$

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} \\ * & \Sigma_{22} & \Sigma_{23} & \Sigma_{24} \\ * & * & \Sigma_{33} & \Sigma_{34} \\ * & * & * & \Sigma_{44} \end{bmatrix},$$

$$
\begin{aligned}
\Sigma_{11} &= P_1A(t) + A^T(t)P_1 + 2\alpha P_1 + \sum_{i=1}^{n} P_2 - \sum_{i=1}^{n} e^{-2\alpha h_i(t)}P_4 + N_2^T A(t) \\
&\quad + A^T(t)N_2 + (\varepsilon_1+\varepsilon_2+\varepsilon_3)P_1^T P_1 + (\varepsilon_7+\varepsilon_8+\varepsilon_9)N_2^T N_2 \\
&\quad + (\varepsilon_1^{-1}+\varepsilon_4^{-1}+\varepsilon_7^{-1}+\varepsilon_{10}^{-1})\alpha_1^2, \\
\Sigma_{12} &= \sum_{i=1}^{n} e^{-2\alpha h_i(t)}P_4 + \sum_{i=1}^{n} N_2^T B_i(t) + \sum_{i=1}^{n} P_1 B_i(t) + A^T(t)N_3, \\
\Sigma_{13} &= -N_2^T + A^T(t)N_1, \\
\Sigma_{14} &= \sum_{i=1}^{n} P_1C_i(t) + \sum_{i=1}^{n} N_2^T C_i(t), \\
\Sigma_{22} &= -\sum_{i=1}^{n}(1-d_i)e^{-2\alpha h_i(t)}P_2 - \sum_{i=1}^{n} e^{-2\alpha h_i(t)}P_4 \\
&\quad + \sum_{i=1}^{n} N_3^T B_i(t) + \sum_{i=1}^{n} B_i^T(t)N_3 \\
&\quad + (\varepsilon_{10}+\varepsilon_{11}+\varepsilon_{12})N_3^T N_3 + (\varepsilon_2^{-1}+\varepsilon_5^{-1}+\varepsilon_8^{-1}+\varepsilon_{11}^{-1})\alpha_2^2, \\
\Sigma_{23} &= \sum_{i=1}^{n} B_i^T(t)N_1 - N_3^T,
\end{aligned}
$$

$$\begin{aligned}
\Sigma_{24} &= \sum_{i=1}^{n} N_3^T C_i(t), \\
\Sigma_{33} &= \sum_{i=1}^{n} P_3 + \sum_{i=1}^{n} h_{Mi}^2 P_4 - N_1^T - N_1 + (\varepsilon_4 + \varepsilon_5 + \varepsilon_6) N_1^T N_1, \\
\Sigma_{34} &= \sum_{i=1}^{n} N_1^T C_i(t), \\
\Sigma_{44} &= -\sum_{i=1}^{n} (1-d_i) e^{-2\alpha h_i(t)} P_3 + (\varepsilon_3^{-1} + \varepsilon_6^{-1} + \varepsilon_9^{-1} + \varepsilon_{12}^{-1}) \alpha_3^2.
\end{aligned}$$

By applying Lemma 2.1 in Σ with some effort, we get $\Xi < 0$. Therefore; we can conclude the following result

$$\lambda_m(P_1) e^{2\alpha t} ||x(t)||^2 \leq V(t) \leq V(0),$$

where

$$\lambda = \lambda_M(P_1) + \lambda_M(P_2) h_M^n + \lambda_M(P_3) h_M^n + \lambda_M(P_4) h_M^{n+2}$$

and

$$\begin{aligned}
V(0) &= x^T(0) P_1(x)(0) + \sum_{i=1}^{n} \int_{-h_i(0)}^{0} e^{2\alpha s} x^T(s) P_2 x(s) ds \\
&\quad + \sum_{i=1}^{n} \int_{-h_i(0)}^{0} e^{2\alpha s} y^T(s) P_3 y(s) ds \\
&\quad + \sum_{i=1}^{n} h_{Mi} \int_{-h_{Mi}}^{0} \int_{\beta}^{0} e^{2\alpha s} \dot{x}^T(s) P_4 \dot{x}(s) P_4 \dot{x}(s) ds d\beta = \lambda |\mu|_h^2.
\end{aligned}$$

Hence

$$||x(t)|| \leq \sqrt{(\lambda_M(P_1) + \lambda_M(P_2) h_M^n + \lambda_M(P_3) h_M^n + \lambda_M(P_4) h_M^{n+2}) / \lambda_M(P_1)} |\mu|_h e^{-\alpha t}$$

for $t \geq 0$.

This implies that system (1) is globally exponentially stable.

3. CONCLUSION

We give certain sufficient conditions, which guarantee globally exponentially stability of the considered system. By the defining a suitable Lyapunov-Krasovskii functional, we prove a result on the topic. Our result includes and improves some resent ones in the literature.

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