

SOLUTION TO THE TRAVELLING SALESPERSON PROBLEM USING SIMULATED ANNEALING ALGORITHM

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ABSTRACT. In this manuscript, we consider a travelling salesperson in Tanzania who needs to travel from Arusha city to all the other cities in Tanzania exactly once to sell his or her products and come back to Arusha city. The salesperson wants to find the shortest road by which the total distance or total time will be minimized. The big challenge of this problem is how can the salesperson manage to do that. This manuscript helps to give solutions to that problem by using the Simulated Annealing (SA) algorithm. We reported the implementation of simulated annealing to solve the Travelling Salesperson Problem (TSP) by using PYTHON 2.7.10 programming language. The data used for solving this problem consist of the latitudes and the longitudes of thirty cities in Tanzania. The best tours obtained from the PYTHON program solution is given in the table 1.

1. INTRODUCTION

Optimization problems can be found in different disciplines, for example, they may be found in sciences, engineering design, manufacturing systems and economics among other disciplines. In order to examine the workable utility of optimization problems, there is a demand for efficient and robust computational algorithms which can be used to solve optimization problems arising in various fields of application. For more detail see [6].

The Travelling Salesperson Problem (TSP) is one of the well-known NP-hard combinatorial optimization class. Many notable researchers have applied several methods of optimization to solve the Travelling Salesperson Problem (TSP) among them are [22], [25], [2], [23], [24], [19] and [11] to mention but a few. Let us assume that we are given a list of towns or cities and the distances between each pair of cities. We need to determine the shortest tour that visits each city once and only once. For example, suppose there is a salesperson in Tanzania with the mission of travelling from Arusha city to several other cities in Tanzania once and only once to sell his or her products and return to the origin city. The main goal of the salesperson is to find the shortest possible road by which the total distance and total time will be minimized [4], [7]. The major setback of this problem is how the salesperson will be able to accomplish his or her mission effectively and efficiently. We have used Simulated Annealing (SA) algorithm to give solution to this problem.

2. SIMULATED ANNEALING (SA)

The Simulated Annealing (SA) method is one of the generic probabilistic meta-heuristic approaches which is extremely important methods for solving NP-hard class problems and it is also used to determine an approximate solution to most of global optimization problems, such as the Quadratic Assignment Problem (QAP) and the Travelling Salesperson Problem (TSP). It was first proposed by [12] and the approach has been used to solved several layout problems for example [17], [9] and others

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researchers. Different intellectuals such as: [26], [20], [1], and [3] applied Simulated Annealing (SA) to solve Travelling Salesperson Problem (TSP). These authors confirmed that Simulated Annealing (SA) is one of the best algorithms to give solutions to the Travelling Salesperson Problem (TSP) and the Quadratic Assignment Problem (QAP).

It is highly encouraged by annealing in metallurgy which is a methodology of slowly cooling of the material to decrease the defects. The simulated annealing in metal-work can be described as follows:

The temperature of a metal is extended up to the time it melts in a heat bath, which means that we put the metal in the heat bath and we increase the temperature so much until the metal melts, that is, it becomes liquid ice or liquid. After that, the metal in the liquid ice or liquid state is then cooled down and instead of increasing the temperature, we reduce the temperature gradually and carefully until the particles are back to an original condition in the ground state of the solid.

The Simulated Annealing (SA) approach begins with a random solution and every iteration forms a random closely solution. If the closely solution is an over top answer, it would substitute by the present solution, but if it is a worsen answer, it may be chosen to substitute the present solution with a probability that will be dependent on the temperature variable (parameter). As the approach continues, the temperature variable (parameter) reduces, thus giving worse answer a lower chance of substituting the present answer. Allowing worse answers assists to stay away from converging to a local minimum rather than the global minimum.

In this manuscript, we shall use the simulated annealing (SA) approach to solve the TSP. Since the main aim of solving the TSP is to minimize distance, so the distance must be our cost function. The parameters that affect the outcome of this approach include, initial temperature, the value at which point the temperature reduced and the stopping condition of the approach. The starting temperature for this work is $1.0e + 10$, the end temperature is 0.1, cooling factor is 0.9500, and the number of iterations is 1. The Simulated Annealing (SA) approach can be practically useful to solve the TSP by using the following steps.

Step One: We need to make the opening list of cities by casting the input list (that is make the order of visit unsystematic).

Step Two: At all iterations, two cities are exchanged in the list. The cost value is the distance travelled by the Salesperson for all the tours.

Step Three: If the new length (distance) calculated after the modification, is smaller than the current length (distance), it is preserved.

Step Four: If the new length is longer than the current length, it is preserved with a positive probability.

Step Five: We need to bring up-to-date the temperature at every iteration by gradually cooling it down.

In addition to that, there are two major optimizations that can be used to raise the calculation of the distances:

1. Instead of recalculating the distance among two cities every time it is needed, the distances among every pair of cities can be recomputed in a table and used later. Really, a three-sided matrix is enough as the distance among cities P and Q is identical as the distance among Q and P .
2. While a change in parameters consists in exchanging two cities, it is unusable to recalculate the total distance for the all tours. Certainly, only the distances modified by the swap should be recalculated [10].

The Simulated annealing (SA) algorithm has also been used by many researchers to solve the Quadratic Assignment Problem (QAP) (see [18], [21], [16] and [14]). However, this implementation has used the good concept from earlier implementations.

We have assumed that the following analogy are equivalent:

1. Feasible solutions of the combinatorial optimization problems correspond to states in a physical system.
2. The values of the objective function correspond to the energy of a states of the physical system.
3. Optimal solutions of the combinatorial optimization problems correspond to ground state.

3. IMPLEMENTATION OF THE SIMULATED ANNEALING METHOD TO THE TSP IN PYTHON

In this section, we will use the simulated annealing approach to solve a Travelling Salesperson Problem (TSP). The aim is to minimize the total distance travelled by a salesperson during his or her visits across all the regions in Tanzania.

Problem:

A travelling salesperson in Tanzania who needs to travel from Arusha city to all the other cities in Tanzania once and only once to sell his or her products and return to the starting city. the salesperson wants to find the shortest possible road by which the total distance and total time taken will be minimized. The big challenge of this problem is that how can the salesperson manage to do that?

We start the algorithm with a very high temperature, and gradually cools it down to a very low temperature. The cooling down is determined by using a loop on a temperature variable, and by reproducing this variable by a random number between zero and one at each iteration:

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Temperature = temperature begins
While temperature > the temperature at the end: Then
    Temperature = temperature × cooling factor.

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We calculated probability based on the distinction between the current and the earlier cost values, and on the temperature.

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Current cost = cost function
Difference = current cost - earlier cost
If difference < 0 or  $\exp\left(\frac{-\text{difference}}{\text{Temperature}}\right) > \text{random}(0,1)$ 
    Earlier cost = current cost
    Temperature = temperature × cooling factor.

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We have implemented simulated annealing to solve the Travelling Salesperson Problem (TSP) stated above by using the PYTHON 2.7.10 programming language. The data used for this problem is the longitude and latitude of 30 Tanzania cities [8] and the data is given as follow:

TABLE 1. Latitude and Longitude of 30 cities in Tanzania

Latitude	Longitude	Name of Regions	Description
3.3333	36.6667	Arusha	Arusha, Arusha Region
6.1333	35.7500	Dodoma	Dodoma, Dodoma Region
2.8000	32.2000	Geita	Geita, Geita Region
1.5000	34.5333	Musoma	Musoma, Mara Region
8.9000	33.4833	Mbeya	Mbeya, Mbeya Region
5.8333	29.2500	Mkokotoni	Mkokotoni, Zanzibar North Region
6.8333	37.6667	Morogoro	Morogoro, Morogoro region
7.8000	35.4300	Iringa	Iringa, Iringa Region
2.0000	31.5000	Bukoba	Bukoba, Kagera Region
6.8333	31.1667	Mpanda	Mpanda, Katavi Region
5.0000	30.0000	Kigoma	Kigoma, Kigoma Region
3.1167	37.3333	Moshi	Moshi Kilimanjaro Region
10.3333	40.3333	Mtwara	Mtwara, Mtwara Region
2.5000	32.9667	Mwanza	Mwanza, Mwanza Region
9.3333	34.8333	Njombe	Mwanza, Mwanza Region
6.8333	39.2000	Dares-Salaam	Dares-Salaam, Dares-Salaam Region
5.0000	39.6167	Chake Chake	Chake, Chake Region
7.0000	39.0000	Kibaha	Kibaha, Pwani Region
9.9667	39.6333	Lindi	Lindi, Lindi Region
6.3333	30.0000	Koami	Koami, Zanzibar South Region
7.9667	32.4833	Shinyanga	Shinyanga, Shinyanga Region
4.1667	34.2500	Babati	Babati, Manyara Region
7.0000	31.5000	Sumbawanga	Sumbawaga, Rukwa Region
10.3333	36.0000	Songea	Songea, Ruvuma Region
5.7500	34.9833	Zanzibar	Zanzibar, Urban West Region
2.1667	37.6000	Bariadi	Bariadi, Simiyu Region
6.0000	34.5000	Singida	Singida, Singida Region
5.0333	32.8333	Tabora	Tabora, Tabora Region
5.0833	39.0333	Tanga	Tanga, Tanga Region
5.0400	39.4300	Wete	Wete, Pemba North Region

The PYTHON solution obtained for this problem is showed in the figures below:

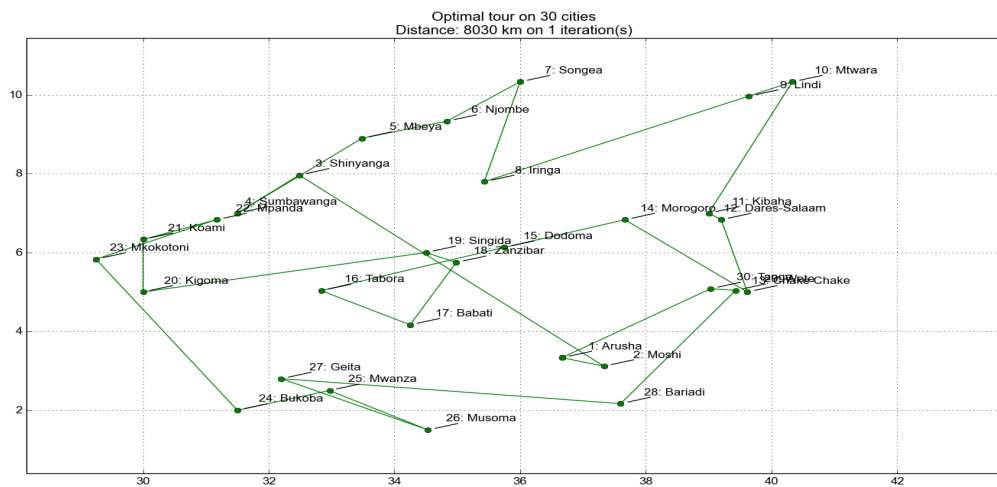


FIGURE 1. Optimal tour of 30 cities in PYTHON using the simulated annealing algorithm.

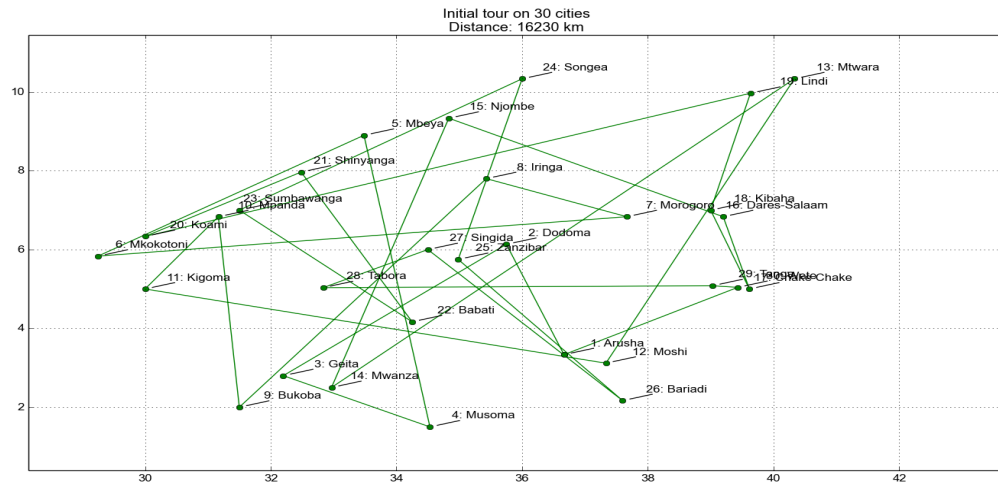


FIGURE 2. Initial tour of 30 cities in PYTHON using the simulated annealing algorithm.

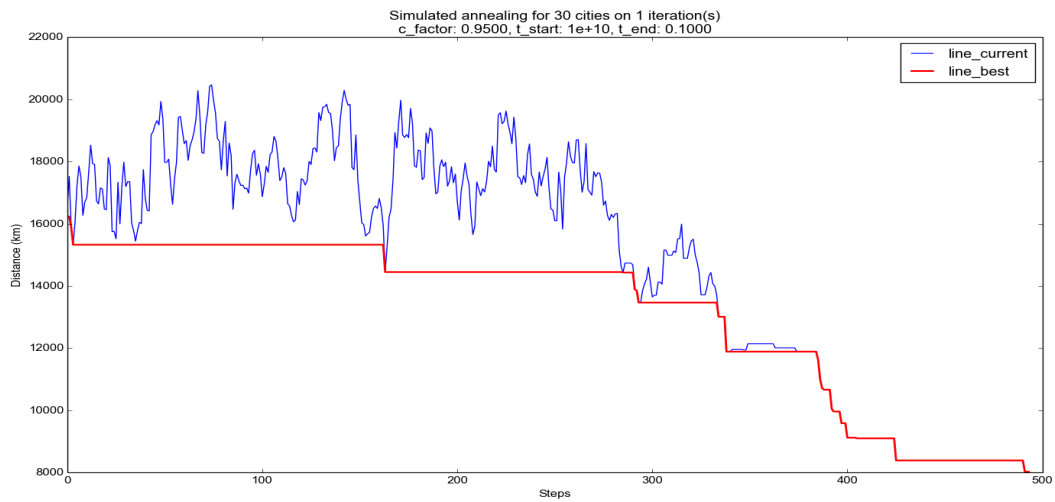


FIGURE 3. Graph of simulated annealing for 30 cities in Tanzania.

The best tours obtained from the PYTHON program solution is given in the table below:

TABLE 2. The table that shows the best tour of distance across all the regions in Tanzania.

The best tour	Distances (km)
Arusha to Moshi	77.9197258116
Moshi to Shinyanga	761.700776371
Shinyanga to Sumbawanga	152.848442809
Sumbawanga to Mbeya	304.233761364
Mbeya to Njombe	156.039234765
Njombe to Songea	169.624739199
Songea to Iringa	288.907303771
Iringa to Lindi	521.451768039
Lindi to Mtwara	86.8922566273
Mtwara to Kibaha	399.045139724
Kibaha to Dares-Salaam	28.8617405569
Dares-Salaam to Chake	209.250776503
Chake to Morogoro	297.121012406
Morogoro to Dodoma	225.887415737
Dodoma to Tabora	345.593309197
Tabora to Babati	184.453765208
Babati to Zanzibar	194.125767958
Zanzibar to Singida	60.3268524483
Singida to Kigoma	510.944040208
Kigoma to Koami	148.435378352
Koami to Mpanda	140.525994171
Mpanda to Mkokotoni	239.524566620
Mkokotoni to Bukoba	494.522270682
Bukoba to Mwanza	172.394419222
Mwanza to Musoma	206.820500354
Musoma to Geita	297.196966423
Geita to Bariadi	604.734303293
Bariadi to Wete	379.023864399
Wete to Tanga	44.2554375660
Tanga to Arusha	327.101576775

The distance between each tour sum together to give the total shortest distance, which is 8030 km.

Figure 2 shows the approximate optimal tour of 30 cities in Tanzania obtained by using the simulated annealing algorithm. We observed that 51% of improvement from the initial tour solution to the approximate optimal tour solution, as the solution obtained for the total distance reduces from 16230 km to 8030 km. Figure 3 shows the initial tour solution of 30 cities in Tanzania generated by an unsystematic visit order. Figure 4 explains graphically how the approximate optimal solution gets the best answer over the course of the simulated annealing.

4. CONCLUSION AND SUGGESTION FOR THE FUTURE WORK

4.1. Conclusion. The objective of this paper is to provide meta-heuristics solutions to the Travelling Salesperson Problem (TSP). We have implemented the Simulated Annealing (SA) method to solve the travelling salesperson problem in Tanzania who needs to travel from Arusha city to all the other cities in Tanzania exactly once to sell his or her products and return to the starting city (Arusha). We have predicted the optimal and initial tours of thirty cities as it can be seen very clearly in the figures 2 and 3. Table 1 shows the best tour of distance across all the regions in Tanzania. We conclude that Simulated Annealing (SA) algorithm gives meta-heuristics solutions to the Travelling Salesperson Problem (TSP).

4.2. Future work. For the future work, we propose to generalize to multi-objective problems which includes the following:

1. The derivation of new Algorithm for solving Travelling Salesperson Problem (TSP).
2. The implementation of the Branch and Bound algorithm for solving the Travelling Salesperson Problem (TSP) in PYTHON programme language.
3. The implementation of the Hungarian algorithm for solving the Travelling Salesperson Problem (TSP) in PYTHON programme language.

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REFERENCES

- [1] B. Cakir, F. Altiparmak, and B. Dengiz, "Multi-objective optimization of a stochastic assembly line balancing: A hybrid simulated annealing algorithm", *Computers and Industrial Engineering* 60, 376-384, 2011.
- [2] M. Dorigo and L. Gambardella, "Ant-Q: A reinforcement learning approach to the traveling salesman problem in Proceedings of ML-95", *Twelfth Intern. Conf. on Machine Learning*, 2014.
- [3] G. Dueck and T. Scheuer, "Threshold accepting: a general purpose optimization algorithm appearing superior to simulated annealing", *Journal of computational physics* 90, 161-175, 1990.
- [4] M. M. Flood, "The Traveling Salesman Problem", *Operations Research*, 1956.
- [5] X. Geng, Z. Chen, W. Yang, D. Shi and K. Zhao, "Solving the traveling salesman problem based on an adaptive simulated annealing algorithm with greedy search", *Applied Soft Computing* 11, 3680-3689, 2011.
- [6] G. Reinelt, "The Traveling Salesman: Computational Solutions for TSP Applications", Springer-Verlag, 1994.
- [7] G. Gutin, "Traveling salesman problems", *Handbook of Graph theory*, Boca Raton, U.S.A, CRC Press, 2003.
- [8] GPS, "Tanzania latitude and longitude maps", Retrieved from <http://www.mapsofworld.com/lat-long/tanzania-lat-long.html>, Accessed in May, 2016.
- [9] Z. Jingwei, R. Ting, F. Husheng, Z. Jinlin, and L. Ming. Simulated annealing ant colony algorithm for qap. In *Natural Computation (ICNC), 2012 Eighth International Conference on*, pages 789 -793. IEEE, 2012.
- [10] P. Karlsson, "Simulated annealing applied to the traveling salesman problem", 2002.
- [11] F. H. Khan, N. Khan, S. Inyatullah, and S. T. Nizami, "Solving TSP Problem by using Genetic Algorithm", *International Journal of Basic and Applied Sciences*, 9(10), 79-88, 2009.
- [12] S. Kirkpatrick, C. D. Gelatt, M. P. Vecchi, et al. Optimization by simulated annealing. *science*, 220 (4598):671-680, 1983.
- [13] S. W. Lin, V. F. and S. Y. Chou, "Solving the truck and trailer routing problem based on a simulated annealing heuristic", *Computers and Operations Research* 36, 1683-1692, 2009.
- [14] A. Misevicius, "A modified simulated annealing algorithm for the quadratic assignment problem", *Informatica*, 14(4), 497-514, 2003.
- [15] M. Malek, M. Guruswamy, M. Pandya, H. Owens, "Serial and parallel simulated annealing and tabu search algorithms for the traveling salesman problem", *Annals of Operations Research* 21, 59-84, 1989.
- [16] G. Paul, "Comparative performance of tabu search and simulated annealing heuristics for the quadratic assignment problem", *Operations Research Letters*, 38(6), 577-581, 2010.

- [17] T. Peng, W. Huanchen, and Z. Dongme. "Simulated annealing for the quadratic assignment problem: A further study". *Computers and industrial engineering*, 31(3):925-928, 1996.
- [18] P. C. Percy, Yip and Yoh-Han Pao, "Combinatorial a guided evolutionary simulated annealing approach to the quadratic assignment problem", *IEEE transactions on systems, man, and cybernetics*, vol. 24, September 1994.
- [19] S. S. Ray, S. Bandyopadhyay and S. K. Pal, "New Genetic Operators for Solving TSP: Application to Microarray Gene Ordering", *Pattern recognition and machine intelligence (1st int. conference)* 37 (76), 605-610 2005.
- [20] R. Sahin, "A Simulated Annealing Algorithm for Solving the Bi-Objective Facility Layout Problem", *Expert Systems with Applications* 38(4), 4460-4465, 2011.
- [21] M. Saifullah Hussin and T. Stutzle, "Tabu search vs. simulated annealing as a function of the size of quadratic assignment problem instances", *Computers and Operations Research*, 43, 286-291, 2014.
- [22] I. Stojanovska and A. Dika, "Impact of the Number of Generations on the Fitness value and the Time Required to Find the Optimal Solution in Standard GA Applications", *Proc of the 2010 IEEE*, pp. 978-1-4244-5540-9 2010.
- [23] T. Stutzle and H. Hoos, "MAX-MIN ant system and local search for the traveling salesman problem", in *Evolutionary Computation, 1997.*, IEEE International Conference on. 1997. IEEE.
- [24] T. Stutzle and M. Dorigo, "ACO Algorithms for the Travelling Salesman Problems", *Evolutionary Algorithms in Engineering and Computer Science*, John-Wiley and Sons 1999, .
- [25] R. Thamilselvan and P. Balasubramanie, "A Genetic Algorithm with a Tabu Search (GTA) for Traveling Salesman Problem", *International Journal of Recent Trends in Engineering*, Vol. 1, No. 1, May 2009 ACEEE.
- [26] R. Zhang and C. Wu, "A hybrid immune simulated annealing algorithm for the job shop scheduling problem", *Applied Soft Computing* 10, 79-89, 2010.

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