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SOME COMMON FIXED POINT THEOREMS FOR WEAKLY SUBSEQUENTIALLY CONTINUOUS MAPPINGS IN MENGER SPACES

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Abstract. The aim of this paper is to prove some common fixed point theorems for two weakly subsequentially continuous and compatible of type (E) pairs of self mappings in Menger spaces, two examples are given to illustrate our results.

1. Introduction

Menger introduced the notion of probabilistic metric spaces(shortly, PM-spaces), which is a generalization of metric spaces. This notion based in idea to use distribution functions instead of non- negative real numbers as values of the metric.The concept of PM-space corresponds to situations when we do not know exactly the distance between two points, but we know probabilities of possible values of this distance. Since the work of Schweizer and Sklar [26], many authors have some results in probabilistic metric spaces due its importance in probabilistic functional analysis. Recently the study of fixed point or common fixed point in PM-spaces has a part by many authors in their researches.

Jungck[16] introduced the notion of compatible maps, the same author Jungck and Rhoades[17] weakened the concept of compatibility to the weak compatibility. Recently Al-Thagafi and Shahzad[2] gave a generalization, which is called the occasional weak compatibility property, this notion is weaker than the weak compatibility due to Jungck and Rhoades[17]. Doric et al.[11] mentioned that the condition of occasionally weak compatibility reduces to weak compatibility, in the case where the two mappings have a unique point of coincidence (or a unique common fixed point). In 2009 Bouhadjera and Godet Tobie[8] introduced the concepts of subcompatibility and subsequential continuity which are more general than the occasional weak compatibility and the reciprocal continuity due to pant[23] respectively, later Imdad et al.[15] improved the results of Bouhadjera and Godet Thobie[8], by using subcompatibility with reciprocal continuity or subsequential continuity with

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compatibility.Many authors proved some results concerning common fixed point in Menger spaces as in papers [1, 4, 5, 6, 14, 12].

Branciari[9] introduced and used the contraction of integral type to generalize Banach contraction and proved a fixed point theorem in metric space. Altun et al.[3] established a common fixed point by using contractive condition od integral type in Menger space, also Chauhan et al.[10]have some results in this way.

2. Preliminaries

Definition 1 A mapping $\triangle : [0,1] \times [0,1] \times [0,1]$ is a t-norm (or a triangular norm) if it satisfies the following conditions:

(1) $\triangle(a,1) = a$, for all $a \in [0,1]$,
(2) $\triangle(a,b) = \triangle(b,a)$,
(3) $\triangle(a,b) \le \triangle(c,d)$ for all $a \le c$ and $b \le d$,
(4) $\triangle(\triangle(a,b),c) = \triangle(a,\triangle(b,c))$.

Example 1 Let (X,d) be a metric space, define $\triangle(a,b) = \min\{a,b\}$ for all $a,b \in [0,1]$, then $\triangle$ is a t-norm.
Also $\triangle(a,b) = ab$ and $\triangle(a,b) = \max\{0, a+b-1\}$ are t-norms.
Definition 2 A real valued mapping $F : \mathbb{R} \to \mathbb{R}_+$ is called a distribution function, if it is non decreasing and left-continuous with:

$$\inf F(x) = 0, \sup_{x\in} F(x) = 1.$$

We denote by $\mathfrak{F}$ set of all distribution functions, and denote by H the Heaviside distribution function defined by:

$$H(t) = \begin{cases} 0, & t \le 0 \\ 1, & t > 0 \end{cases}$$

Definition 3 Let X be a non empty set, an order pair (X,F) is called a probabilistic metric space if F is a mapping from $X \times X$ into $\{f \in \mathfrak{F}, f(0) = 0\}$ and satisfying the following conditions:

(1) $F_{xy} = H$, if and only if $x = y$,
(2) $F_{xy} = F_{yx}$, for all $x,y \in X$,
(3) if $F_{xy}(t) = 1$ and $F_{yz}(s) = 1$, then $F_{xz}(t+s) = 1$ for all $x,y,z \in X$ and $t,s \ge 0$.

If F satisfies only (1) and (2), the pair $X,F)$ is called a probabilistic semi metric space.
Definition 4 A triplet $(X,F,\triangle)$ is called to be a Menger space if (X,F) is a probabilistic metric space and $\triangle$ is a t-norm such for all $x,y \in X$ and $t,s \ge 0$ the following inequality holds:

$$F_{xz}(t+s) \ge \triangle(F_{xy}, F_{yz}).$$

If (X,d) is a metric space, by taking $F_{xy} = H(t - d(x,y)$, it becomes (X,F) probabilistic metric space, so every metric space can be realized as a probabilistic metric space.
Definition 5 Let $(X,F,\triangle)$ be a Menger space with a continuous t-norm

(i) A sequence $\{x_n\}$ in X is said to be convergent to $x \in X$ if and only if for every $\varepsilon > 0$ and $\lambda \in (0,1)$, there exists an integer N such $F_{x_n x}(\varepsilon) > 1 - \lambda$ for all $n \ge N$.

(*ii*) A sequence $\{x_n\}$ in X is called to a Cauchy one, if and only if for every $\varepsilon > 0$ and $\lambda \in (0,1)$, there exists an integer N such $F_{x_n x_m}(\varepsilon) > 1-\lambda$ for all $n, m \geq N$.

(*iii*) A Menger space is called to be complete if every Cauchy sequence in it, is convergent.

Definition 6 A pair (A,S) of self mappings from a Menger space $(X,F,\triangle)$ into itself is compatible if and only if

$$\lim_{n\to\infty} F_{ASx_n,SAx_n} = 1,$$

for all $t \geq 0$, whenever $\{x_n\}$ is a sequence in X such

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z,$$

for some $z \in X$.

Definition 7 Two self mappings A, S of a Menger space $(X,F,\triangle)$ into itself are called to be weakly compatible if and only if they commute at their coincidence points, i.e if $Ax = Sx$ for some $x \in X$, then $ASx = SAx$.

Kumar and Pant[19] generalized the reciprocal continuity concept due to Pant[23] in the setting of Menger space as follows:

Definition 8 Two self mappings A and S of a Menger space $(X,F,\triangle)$ are called reciprocally continuous if $\lim_{n\to\infty} ASx_n = Az$ and $\lim_{n\to\infty} SAx_n) = Sz$, whenever $x_n\}$ is a sequence in X such $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} SAx_n = z$, for some $z \in X$.

Bouhadjera and Ghodet Tobie[8] introduced the concept of subsequential continuity in metric spaces, in the setting of Menger spaces it becomes:

Definition 9 Let $(X,F,\triangle)$ be a Menger space,the pair of self mappings (A,S) is said to be subsequentially continuous, if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$, for some $z \in X$ and $\lim_{n\to\infty} ASx_n = Az$.

Motivated by the above definition, define:

Definition 10 The pair (A,S) is said to be weakly subsequentially continuous (wsc), if there exists a sequence $\{x_n\}$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$, for some $z \in X$ and $\lim_{n\to\infty} ASx_n = Az$, or $\lim_{n\to\infty} SAx_n = Sz$.

The pair (A,S) is said to be A-subsequentially continuous(S-subsequentially continuous), if there exists a sequence $\{x_n\}$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$, $\lim_{n\to\infty} SAx_n = Sz$.

Example 2 Let $X = [0,\infty)$ and let a continuous t-norm: $\triangle(x,y) = \frac{t}{t+|x-y|}$ for all $t > 0$, define A, S as follows:

$$Ax = \begin{cases} 2+x, & 0 \leq x \leq 2 \\ 0, & x > 2 \end{cases}, \quad Sx = \begin{cases} 2-x, & 0 \leq x \leq 2 \\ \frac{x}{2}, & x > 2 \end{cases}$$

Clearly that A and S are discontinuous at 1.

Consider a sequence $\{x_n\}$ such that for each $n \geq 1: x_n = \dfrac{1}{n}$,
it is clear that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = 2$, also we have:

$$\lim_{n\to\infty} ASx_n = \lim_{n\to\infty} A(2-\frac{1}{n}) = A(1) = 1,$$

then (A,S) is A-subsequentially continuous,i.e., it is wsc.

Singh and Mahendra Singh [27, 28] introduced the notion of compatibility of type

(E) in metric spaces, in the setting of the Menger spaces, it becomes:
Definition 11 Self maps A and S of a Menger space $(X, F, \triangle)$ are said to be compatible of type (E), if $\lim_{n\to\infty} S^2x_n = \lim_{n\to\infty} SAx_n = Az$ and $\lim_{n\to\infty} A^2x_n = \lim_{n\to\infty} ASx_n = Sz$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$, for some $z \in X$.
Definition 12 Two self maps A and S of a Menger space $(X, M, \triangle)$ into itself are said to be A-compatible of type (E), if $\lim_{n\to\infty} A^2x_n = \lim_{n\to\infty} ASx_n = Sz$, for some $z \in X$.
The pair $\{A, S\}$ is said to be S-compatible of type (E), if $\lim_{n\to\infty} S^2x_n = \lim_{n\to\infty} SAx_n = Az$, for some $z \in X$.
Notice that if A and S are compatible of type (E), then they are A-compatible and S-compatible of type (E), but the converse is not true.

Example 3 Let $X = [0, \infty)$ with the continuous t-norm $\triangle(x, y) = \frac{t}{t+|x-y|}$ for all $t \geq 0$, define A, S as follows:

$$Ax = \begin{cases} \frac{x+1}{2}, & 0 \leq x \leq 1 \\ \frac{x}{2}, & x > 1 \end{cases} \qquad Sx = \begin{cases} 2-x, & 0 \leq x \leq 1 \\ 2x-1 & x > 1 \end{cases}$$

Consider a sequence $\{x_n\}$ which defined by: $x_n = 1 - \dfrac{1}{n}$, for all $n \geq 1$, we have:

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = 1,$$

$$\lim_{n\to\infty} SAx_n = \lim_{n\to\infty} S(1 - \frac{1}{2n}) = A(1) = 1,$$

$$\lim_{n\to\infty} S^2x_n = \lim_{n\to\infty} S(1 + \frac{1}{n}) = A(1)$$

then the pair (A, S) is S-compatible of type (E), but never compatible of type (E) since:

$$\lim_{n\to\infty} ASx_n = \lim_{n\to\infty} S(\frac{1}{2} + \frac{1}{2(n+1)}) = \frac{1}{2} \neq S(2)$$

The aim of this paper is to prove the existence and the uniqueness of common fixed point for two pairs of self-mappings in Menger metric space, which satisfying implicit relation by using the weak subsequential continuity with compatibility of type (E), to illustrate our results we give two examples.
Lemma 1 [21] Let $(X, F, \triangle)$ be a Menger space. If there exists a constant $k \in (0, 1)$ such that $F_{x,y}(kt) \geq F_{x,y}(t)$, for all $t > 0$ and fixed $x, y \in X$, then $x = y$. ‘
As a generalization to lemma2 Altun et al.[3] gave the following lemma:
Lemma 2 [3] Let $(X, F, \triangle)$ be a Menger space. If there exists a constant $k \in (0, 1)$ such that for all $t > 0$ and fixed $x, y \in X$ we have

$$\int_0^{F_{x,y}(kt)} \varphi(t)dt \geq \int_0^{F_{x,y}(t)} \varphi(t)dt,$$

for all $t > 0$ and fixed $x, y \in X$, where $\varphi : \mathbb{R}_+ \to \mathbb{R}_+$ is a Lebesgue integrable and summable function such for each $\varepsilon > 0$ $\int_0^{\varepsilon} \varphi(t)dt > 0$. Then $x = y$.

3. Main results

Theorem 1 Let $(X, F, \triangle)$ be a Menger space and let A, B, S be four mappings on X. If the two pairs (A, S) and (B, T) are weakly subsequentially continuous (wsc) and compatible of type (E), then (A, S) and (B, T) has a coincidence point. Further if there exists $k \in [0, 1)$ such for all $x, y \in X$ and each $t > 0$, we have:

$$F_{Sx,Ty}(kt) \geq \min\{F_{Ax,By}(t), F_{Ax,Sx}(t), F_{By,Ty}(t), F_{Ax,Ty}(t), F_{By,Sx}(t)\} \geq 0, \quad (1)$$

then A, B, S and T have a unique common fixed point in X.

Proof Since (A, S) is wsc, there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ for some $z \in X$ and $\lim_{n\to\infty} ASx_n = Az$, $\lim_{n\to\infty} SAx_n = Sz$, the compatibility of type (E) of (A, S) implies that

$$\lim_{n\to\infty} ASx_n = \lim_{n\to\infty} A^2 x_n = Sz$$

and

$$\lim_{n\to\infty} SAx_n = \lim_{n\to\infty} S^2 x_n = Az,$$

then $Az = Sz$ and z is a coincidence point for A and S. Similarly for B and T, since (B, T) is wsc (suppose that it is B-subsequentially continuous) there exists a sequence $\{y_n\}$ such

$$\lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = w$$

for some $w \in X$ and

$$\lim_{n\to\infty} BTy_n = Bw,$$

also the pair (B, T) is compatible of type (E) implies that

$$\lim_{n\to\infty} BTy_n = \lim_{n\to\infty} B^2 y_n = Tw$$

$$\lim_{n\to\infty} TBy_n = \lim_{n\to\infty} T^2 y_n = Bw,$$

so we have $Bw = Tw$.

We claim $Az = Bw$, if not by using (1) we get:

$$F_{Sz,Tw}(kt) \geq \min\{F_{Az,Bw}(t), F_{Az,Sz}(t), F_{Bw,Tw}(t), F_{Az,Tw}(t), F_{Bw,Sz}(t)\}$$

since $Az = Sz$ and $Bw = Tw$, we get:

$$F_{Az,Bw}(kt) \geq \min\{F_{Az,Bw}(t), 1, 1, F_{Az,Bw}(t), F_{Az,Bw}(t)\} = F_{Az,Bw}(t),$$

from lemma2, we obtain $Az = Bw$

Now we prove $z = Az$, if not by using(1) we get:

$$F_{Sx_n,Tw}(kt) \geq \min\{F_{Ax_n,Bw}(t), F_{Ax_n(t),Sx_n}(t), F_{Bw,Tw}(t), F_{Ax_n,Tw}(t), F_{Bw,Sx_n}(t)\},$$

letting $n \to \infty$ we get:

$$F_{z,Tw}(kt) \geq \{F_{z,Bw}(t), 1, 1, F_{z,Tw}(t), F_{Bw,z}(t)\},$$

since $Az = Bw = Tw$, we get:

$$F_{z,Az}(kt) \geq \min\{F_{z,Az}(t), 1, 1, F_{z,Az}(t), F_{z,Az}(t)\} = F_{z,Az}(t).$$

Hence $z = Az = Sz$.

Nextly we shall prove $z = t$, if not by using (1) we get:

$$F_{Sx_n,Tyn}(kt) \geq \min\{F_{Ax_n,Byn}(t), F_{Ax_n,Sx_n}(t), F_{Byn,Tyn}(t), F_{Ax_n,Tyn}(t), F_{Byn,Sx_n}(t)\},$$

letting $n \to \infty$ we get:

$$F_{z,w}(kt) \geq \min\{F_{z,w}(t), 1, 1, F_{z,w}(t), F_{w,z}(t)\} = F_{z,w}(t).$$

Hence z is a fixed point for A, B, S and T.
For the uniqueness, if q is another fixed point q, by using (1) we get:

$$F_{Sz,Tq}(kt) \geq \min\{F_{Az,Bq}(t), F_{Az,Sq}(t), F_{(Bq,Tq}(t), F_{Az,Tq}(t), F_{Bq,Sz}(t)) = F_{z,q}(t).$$

hence $z = q$, and z is unique.

If $A = B$, we get the following corollary:
Corollary 1 Let $(X, F, \triangle)$ be a Menger space and let A, S and T be three self mappings on X, if there exists $k \in (0, 1)$ such for all $x, y \in X$ we have:

$$F_{Sx,Ty}(kt) \geq \min\{F_{Ax,Ay}(t), F_{Ax,Sx}(t), F_{Ay,Ty}(t), F_{Ax,Ty}(t), F_{Ay,Sx}(t)\}.$$

Further, if the pair (A, S) is weakly subsequentially continuous and compatible of type (E), then A, B, S and T have a unique common fixed point.
If $A = B$ and $S = T$, we get the following corollary:
Corollary 2 Let $(X, F, \triangle)$ be a Menger space and let A, B, S and T be four self mappings on X.Suppose that there exists $k \in (0, 1)$ such for all $x, y \in X$ we have:

$$F_{Sx,Sy}(kt) \geq \min\{F_{Ax,Ay}(t), F_{Ax,Sx}(t), F_{Ay,Sy}(t), F_{Ax,Sy}(t), F_{Ay,Sx}(t)\},$$

if the pair (A, S) is weakly subsequentially continuous and compatible of type (E), then A, B, S and T have a unique common fixed point.
Theorem 2 Let $(X, F, \triangle)$ be a Menger space and let A, B, S and T be self mappings on X, if

1. the pair (A, S) is weakly subsequentially continuous and compatible of type (E),
2. the pair (B, T) is weakly subsequentially continuous and compatible of type (E).

Hence (A, S) and (B, T) has a coincidence point.
Moreover the maps A, B, S and T have a unique common fixed point provided there exists $k \in (0, 1)$ such for all $x, y \in X$ and $t > 0$ we have:

$$F_{Sx,Ty}(kt) \geq \phi(\min\{F_{Ax,Ay}(t), F_{Ax,Sx}(t), F_{Ay,Sy}(t), F_{Ax,Sy}(t), F_{Ay,Sx}(t)\}), \quad (2)$$

where $\phi : [0, 1] \to [0, 1]$ is a lower semi continuous function such $\phi(t) > t$ for each $t \in (0, 1)$ with $\phi(0) = 0$ and $\phi(1) = 1$.
Proof Since for all $x, y \in X$ and $t > 0$ we have $\phi(t) > t$, then result of Theorem 3 is a consequence of the result of Theorem 3.
Remark 1 Theorem 3 and Theorem 3remain true if we replace the weakly subsequentially continuity and compatibility of type (E) by one of the following conditions:

1. S, T-subsequentially continuity and S, T-compatibility of type (E),
2. subsequentially continuity and A, B-compatibility of type (E),
3. subsequentially continuity and S, T-compatibility of type (E),
4. subsequentially continuity and compatibility of type (E).

Theorem 3 Let $(X, F, \triangle)$ be a Menger space and let A, B, S and T be self mappings on X, if

1. the pair (A, S) is A-subsequentially continuous and A-compatible of type (E),

(2) the pair (B,T) is B-subsequentially continuous and B-compatible of type (E).

Hence (A,S) and (B,T) has a coincidence point.
Moreover the maps A,B,S and T have a unique common fixed point provided the maps satisfy (1) or (2).
Now we prove a common fixed point of integral type in Menger space.
Theorem 4 Let $(X,F,\triangle)$ be a Menger space and let A,B,S and T be self mappings on X, if

(1) the pair (A,S) is weakly subsequentially continuous and compatible of type (E),
(2) the pair (B,T) is weakly subsequentially continuous and compatible of type (E).

Hence (A,S) and (B,T) has a coincidence point.
Moreover the maps A,B,S and T have a unique common fixed point provided there exists $k\in(0,1)$ such for all $x,y\in X$ and $t0$ we have:

$$\int_0^{F_{Sx,Ty}(kt)}\varphi(t)dt\geq\int_0^{m(x,y)}\varphi(t)dt, \tag{3}$$

where $m(x,y)=\min\{F_{Ax,Ay}(t),F_{Ax,Sx}(t),F_{Ay,Sy}(t),F_{Ax,Sy}(t),F_{Ay,Sx}(t)\}$ and $\varphi:\mathbb{R}_+\to\mathbb{R}_+$ is a Lebesgue integrable and summable function such for each $\varepsilon>0$ $\int_0^\varepsilon\varphi(t)dt>0$.
Proof As in proof of Theorem 3, A and S have a coincidence point z(say) Also the pair(B,T) has a coincidence point w(say)
We prove $Az=Bw$, if not by using (3) we get:

$$\int_0^{F_{Sz,Tw}(kt)}\varphi(t)dt\geq\int_0^{\min\{F_{Az,Bw}(t),F_{Az,Sz}(t),F_{Bw,Tw}(t),F_{Az,Tw}(t),F_{Bw,Sz}(t)\}}\varphi(t)dt$$

since $Az=Sz$ and $Bw=Tw$, we get:

$$\int_0^{F_{Az,Bw}(kt)}\varphi(t)dt\geq\int_0^{\min\{F_{Az,Bw}(t),1,1,F_{Az,Bw}(t),F_{Az,Bw}(t)\}}\varphi(t)dt=\int_0^{F_{Az,Bw}(t)}\varphi(t)dt,$$

the lemma2 implies that $Az=Bw$.
Now we prove $z=Az$, if not by using(3) we get:

$$\int_0^{F_{Sx_n,Tw}(kt)}\varphi(t)dt\geq\int_0^{m(x_n,w)}\varphi(t)dt,$$

letting $n\to\infty$ we get:

$$\int_0^{F_{z,Tw}(kt)}\varphi(t)dt\geq\int_0^{F_{z,Bw}(t)}\varphi(s)ds,$$

since $Az=Bw=Tw$, we get:

$$\int_0^{F_{z,Az}(kt)}\varphi(t)dt\geq\int_0^{F_{z,Az}(t)}\varphi(t)dt.$$

Hence $z=Az=Sz$.
Nextly we shall prove $z=t$, if not by using (3) we get:

$$\int_0^{F_{Sx_n,Tyn}(kt)}\varphi(t)dt\geq\int_0^{m(x_n,y_n)}\varphi(t)dt,$$

letting $n \to \infty$ we get:

$$\int_0^{F_{z,w}(kt)} \varphi(t)dt \geq \int_0^{m(z,w)} \varphi(t)dt = \int_0^{F_{z,w}(t)} \varphi(t)dt.$$

Hence z is a fixed point for A, B, S and T.
For the uniqueness, if q is another fixed point q, by using (2) we get:

$$\int_0^{F_{Sz,Tq}(kt)} \varphi(t)dt \geq \int_0^{m(z,q)} \varphi(t)dt = \int_0^{F_{z,q}(t)} \varphi(t)dt.$$

Hence $z = q$, and z is unique.
Remark 2 Theorem3 remain true if we replace the weak subsequential continuity and compatibility of type (E) by A or S-subsequential continuity with A or S compatibility of type (E) respectively and B or T-subsequential continuity with B or T compatibility of type (E) respectively.
Example 4 Let $(X, F, \triangle)$ be a Menger metric space such $X = [0, \infty)$, $\triangle(x, y) = \min(x, y)$ and

$$F_{x,y} = \begin{cases} \frac{t}{t+|x-y|}, & \text{if } t > 0 \\ 0, & t = 0 \end{cases}$$

define mappings A and S as follows:

$$Ax = \begin{cases} x, & 0 \leq x \leq 1 \\ 1, & x > 1 \end{cases} \qquad Sx = \begin{cases} \frac{x+1}{2} & 0 \leq x \leq 1 \\ \frac{1}{4}, & x > 1 \end{cases},$$

We consider a sequence $\{x_n\}$ which defined for each $n \geq 1$ by:
$x_n = 1 - \frac{1}{n}$, clearly that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = 1$, also we have:

$$\lim_{n\to\infty} ASx_n = A(\frac{1}{2}) = S(1) = 1$$

$$\lim_{n\to\infty} A^2 x_n = S(1) = 1,$$

then (A, S) is A-subsequentially continuous and A-compatible of type (E).
For the inequality (1) we have the following cases:

(1) For $x, y \in [0, 1]$, we have

$$d(Sx, Sy) = \frac{1}{4}|x - y| \leq |x - y| = d(Ax, Ay),$$

which implies that for $k = \frac{1}{2}$, we have:

$$F_{Sx,Sy}(\frac{1}{2}t) \geq F_{Sx,Sy}(t) \geq F_{Ax,Ay}(t)$$

(2) For $x \in [0, 1]$ and $1 < y \leq 2$, we have

$$d(Sx, Sy) = \frac{1}{4}|2x - 1| \leq \frac{3}{4} = d(Ay, Sy),$$

so there exists $k = \frac{1}{2}$ such:

$$F_{Sx,Sy}(\frac{1}{2}t) \geq F_{Sx,Sy}(t) \geq F_{Ay,Sy}(t),$$

for any $k \in (0, 1)$.

(3) For $x \in (1, \infty)$ and $y \in [0, 1]$, we have

$$d(Sx, Sy) = \frac{1}{4}|2y - 1| \leq \frac{3}{4} = d(Ax, Sx),$$

which implies that for $k = \frac{1}{2}$ we have:

$$F_{Sx,Sy}(\frac{1}{2}t) \geq F_{Sx,Sy}(t) \geq F_{Ax,Sx}(t)$$

(4) For $x, y \in (1, \infty)$, it is obviously, because $F_{Sx,Ty}(kt) = 1$.

Consequently, all hypotheses of Corollary 3 are satisfied, and the point 1 is the unique common fixed for A and S.

Example 5 Let $(X, F, \triangle)$ be the probabilistic metric space as defined in the above example with $X = \mathbb{R}_+$, define mappings A and S as follows:

$$Ax = \begin{cases} 2x, & 0 \leq x \leq 1 \\ 2x - 1, & x > 1 \end{cases} \qquad Sx = \begin{cases} \frac{x}{4}, & 0 \leq x \leq 1 \\ 0, & x > 1 \end{cases}$$

Consider a sequence $\{x_n\}$ such for each $n \geq 1$ we have:
$x_n = \frac{1}{n}$, clearly that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = 0$, also we have:

$$\lim_{n\to\infty} ASx_n = A(0) = 0$$

$$\lim_{n\to\infty} A^2x_n = S(0) = 0,$$

then (A, S) is A-subsequentially continuous and A-compatible of type (E).
For the inequality (1) we have the following cases:

(1) For $x, y \in [0, 1]$, we have

$$|Sx - Sy| = \frac{1}{4}|x - y| \leq 2|x - y| = |Ax - Ay|,$$

then for $k = \frac{1}{4}$ we have:

$$F_{Sx,Sy}(\frac{1}{4}t) \geq F_{Sx,Sy}(t)) \geq F_{Ax,Ay}(t).$$

(2) For $x \in [0, 1]$ and $y > 1$, we have

$$|Sx - Sy| = \frac{1}{4}x \leq \frac{7}{4}x = |Ax - Sx|,$$

which implies that for $k = \frac{1}{4}$ we have:

$$F_{Sx,Sy}(\frac{1}{4}t) \geq F_{Sx,Sy}(t) \geq F_{Ax,Sx}(t)),$$

(3) For $x \in (1, \infty)$ and $y \in [0, 1]$, we have

$$|Sx - Sy| = \frac{1}{4}y \leq \frac{7}{4}y = |Ay - Sy|,$$

this yield for $k = \frac{1}{4}$, we have:

$$F_{Sx,Sy}(\frac{1}{4}t) \geq F_{Sx,Sy}(t) \geq F_{Ay,Sy}(t)),$$

(4) For $x, y \in (1, \infty)$, we have $|Sx - Sy| = 0$, so it is obviously that the inequality (1) satisfied.

Consequently, all hypotheses of Theorem 3 with $A = B$ and $S = T$ are satisfied, and the point 0 is the unique common fixed for A and S.

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