Electronic Journal of Mathematical Analysis and Applications
Vol. 5(2) July 2017, pp. 271- 288.
ISSN: 2090-729X(online)
http://fcag-egypt.com/Journals/EJMAA/

NEW EXACT SOLUTIONS FOR A SEMICONDUCTOR NONLINEAR REACTION-DIFFUSION EQUATION:THE COMBINATION OF THE FACTORIZATION METHOD TO THE PROJECTIVE RICCATI EQUATION METHOD

MIBAILE JUSTIN, GASISSOU MARCEL, GAMBO BETCHEWE, SERGE Y. DOKA AND KOFANE TIMOLEON CREPIN

Abstract. The projective Riccati equation method is applied to the first equation of the semiconductor one level impact ionization nonlinear reaction-diffusion equation. The obtained solutions, combined to the factorization method leads to the solutions of the second one equation. Such a use of the combination of these two methods yields new solutions which are classified as, periodic, and soliton. As a result, some of that obtained solutions behave like: two different fasten anti-kink soliton, and peakon soliton. The obtained solutions are found to be very interesting since they could be helpful in semiconductor electronics device engineering and many other domains.

1. Introduction

Nowadays, many nonlinear evolution equations are involved in various domains such as in economics, in astronomy, in physics, in mechanics, in chemistry, in biology, in engineering, and in mathematics. Among all these evolution equations, partial differential equations are very used to model nonlinear wave phenomena; which are observed in fluid dynamics, plasma, semiconductors, optical fibers, and so on [1].
Moreover, exact solutions of nonlinear evolution equations could be an important tool for the understanding of the phenomena that they describe. By means of these equations scientists and engineers could have an idea of the physical behavior of the above phenomena. Specially, a deep insight on their stability could be obtained. Therefore, it is very important to investigate explicit solutions of nonlinear evolution equations.
Then, researchers have established many powerful methods for such purpose. Among all the existing methods, we can list: the sub- ODE. method [2], the ansatz method

2010 *Mathematics Subject Classification.* 82D37, 35-XX, 83C15, 74J35, 35C08.
Key words and phrases. The projective Riccati equation method, the factorization method, semiconductor nonlinear reaction-diffusion equation, periodic, and soliton solutions.
Submitted March 3, 2016.

[3, 4], the Hirota bilinear method [5], various tanh methods [6]-[9], the homogeneous balance method [10], the projective Riccati equation method [11], the Weierstrass elliptic function method [12], the Sine- Cosine method [13],[14], the $(G^{'}/G)$ -expansion method [15],[16], the factorization method[17], just to name a few.
Among all the solutions obtained by the above methods, the solitary and the periodic are the most famous since they are the most sought through literature, and they are those who have many applications. Moreover, such solutions could help engineers to introduce new devices; for example new classes of solitary and periodic current filaments formation in semiconductor could be helpful for introducing new electronic devices. In addition, wave form of the electric field generated by a semiconductor could also give birth to new kind of wireless communication.
In the framework of this work, a particular type of nonlinear evolution equations concerns us: the nonlinear reaction diffusion equation (NRDE). This type of equations are generally used to describe the evolution of concentrations of one or many spatially distributed substances, which are submitted to two processes: a local chemical reaction where substances are transformed and the diffusion process which spreads spatial the substances. Such a description let us think that NRDEs are only involve in chemistry. Whoever, NRDEs are widely used to model other dynamic phenomena in nature.
Then, in this paper, we focus on the following semiconductor NRDE,

$$\frac{\partial n}{\partial t} = f_{gr} + D_n \frac{\partial^2 n}{\partial x^2}, \tag{1a}$$

$$\frac{\partial E}{\partial t} = f_E + D_E \frac{\partial^2 E}{\partial x^2}, \tag{1b}$$

where

$$f_{gr} = n(N_D - N_A - n)A_I + (N_D - N_A - n)A_T - n(n + N_A)B_T, \tag{2a}$$

$$f_E = -\gamma(E - E_0 + enCE\mu_r). \tag{2b}$$

These equations model the one level impact ionization phenomenon [18],[19]; which is the nonequilibrium phase transitions phenomena stemming from the self-organization processes of generation and recombination. The function f_{gr} describes the generation recombination processes. The parameters n, E, E_0 and C are, the electron density, the electric field, the external applied electric field, the slope of load line, respectively. In the same way, the quantities D_n, D_E and μ_r are, the electron diffusion constant, the diffusion constant which is related to the electric field and the electron mobility, respectively. The parameter γ is a constant. The constants N_D and N_A stand for the concentration of donors and acceptors; moreover, A_I, A_T and B_T are the impact ionization rate, the thermal ionization rate and the thermal recombination rate, respectively.
A study on this equation have recently be done [19] and as solutions according to the electrons density only tanh and coth have been found as solutions; and the electric field has been expressed as series expansion function. Periodic and combination of other hyperbolic solutions of this equation has not be found yet. This constitute the scope of this work. Our aim is to solve the Eq(1a) by means of the projective Riccati equation method, and then, use its obtained solutions combined to the factorization method to solve the Eq(1b). In other to fulfill such propose, we outline our paper as follows: in section 2 we present briefly the projective Riccati

equation method and the factorization method, the section 3 is devoted to their discussion to the Eq(1), and then, the section 4 is allowed to concluding remarks.

2. Description of the methods

2.1. **The the projective Riccati equation method.** This method is well described in [20]. Consider the following partial differential equation:

$$P(u, u_x, u_t, u_{xx}, u_{xt}, u_{tt}, ...) = 0. \tag{3}$$

By setting $\xi = wx - ct$ where c is the propagation speed, and w a constant, we obtain,

$$Q(u, u_z, u_z, u_{zz}, ...) = 0, \tag{4}$$

which is an ordinary differential equation.
We consider that the solution of Eq (4) is as follows

$$u(\xi) = \sum_{i=1}^{N} (\sigma(\xi))^{i-1}[A_i\sigma(\xi) + B_i\tau(\xi)] + A_0, \tag{5}$$

Where σ and τ satisfy,

$$\sigma^{'}(\xi) = e\sigma(\xi)\tau(\xi), \tau^{'}(\xi) = e\tau(\xi)^2 - \mu\sigma(\xi) + r. \tag{6}$$

The parameters $r \neq 0$, μ, A_i, B_i, $e = \pm 1$ and N are constants to be determined. The determination of N for example is achieved by balancing the highest order derivatives and nonlinear terms appearing in Eq (4). Also, it is easy to realize that Eq (6) possesses a first integral of the following form

$$\tau^2(\xi) = e[r + \frac{\mu^2 - 1}{r}\sigma^2(\xi) - 2\mu\sigma(\xi)]. \tag{7}$$

Solutions of Eq (6) are generally given as [20]:
Case 1 when $e = -1$ and $r \neq 0$ solutions are

$$\sigma_1(\xi) = \frac{\sqrt{r} sech(\sqrt{r}\xi)}{\mu sech(\sqrt{r}\xi) + 1}, \tau_1(\xi) = \frac{\sqrt{r}\tanh(\sqrt{r}\xi)}{\mu\tanh(\sqrt{r}\xi) + 1}, \tag{8a}$$

$$\sigma_2(\xi) = \frac{\sqrt{r} csch(\sqrt{r}\xi)}{\mu csch(\sqrt{r}\xi) + 1}, \tau_2(\xi) = \frac{\sqrt{r}\coth(\sqrt{r}\xi)}{\mu\coth(\sqrt{r}\xi) + 1}. \tag{8b}$$

Case 2 when $e = 1$ and $r \neq 0$ solutions are

$$\sigma_3(\xi) = \frac{\sqrt{r}\sec(\sqrt{r}\xi)}{\mu\sec(\sqrt{r}\xi) + 1}, \tau_3(\xi) = \frac{\sqrt{r}\tan(\sqrt{r}\xi)}{\mu\tan(\sqrt{r}\xi) + 1}, \tag{9a}$$

$$\sigma_4(\xi) = \frac{\sqrt{r}\csc(\sqrt{r}\xi)}{\mu\csc(\sqrt{r}\xi) + 1}, \tau_4(\xi) = \frac{\sqrt{r}\cot(\sqrt{r}\xi)}{\mu\cot(\sqrt{r}\xi) + 1}. \tag{9b}$$

Case 3 when $r = \mu = 0$ solutions are

$$\sigma_5(\xi) = \lambda\xi, \tau_5(\xi) = \frac{1}{e(\xi)}. \tag{10}$$

2.2. **The factorization method.** This method is based on the factorization technics of differential equations [17]. Let us consider the following equation,

$$u^{''} + g(u)u^{'} + f(u) = 0. \tag{11}$$

The symbol $^{'}$ represents $D = \frac{\partial}{\partial z}, g(u), f(u)$, are polynomial in u. We can write the Eq (11) as follows:

$$(D^2 + g(u)D + \frac{f(u)}{u})u = 0. \tag{12}$$

For $f(u) = uh(u)$, equation (11) can be factorized as follows

$$(D - \psi_2(u))(D - \psi_1(u))u = 0. \tag{13}$$

The expansion of equation (13) leads to

$$u^{''} - (\psi_2 + \psi_1 + \frac{\partial \psi_1}{\partial u}u)u^{'} + u\psi_1\psi_2 = 0. \tag{14}$$

Identifying each member of equation (11) to those of (14) yields:

$$g(u) = -(\psi_2 + \psi_1 + \tfrac{\partial \psi_1}{\partial u}u), \tag{15a}$$

$$f(u) = u\psi_2\psi_1. \tag{15b}$$

The function g is supposed to have the same order with ψ_2 and ψ_1 for f polynomial. The relation (15) will be used for finding constant parameters. The equation (13) can be converted to two systems of first order differential equations,

$$u^{'} - \psi_1(u)u = 0, \tag{16a}$$

$$u^{'} - \psi_2(u)u = 0. \tag{16b}$$

Then, by choosing an appropriate ψ_i a particular solution of Eq.11 can be found.

3. Discussion of the methods to a Semiconductor nonlinear reaction-diffusion Equation

3.1. **Discussion of the projective Riccati equation method to Eq(1a).** By setting $\xi = wx - ct$, we write Eq(1a) as follows:

$$w^2 D_n n^{''} + cn^{'} + f_{gr} = 0. \tag{17}$$

Balancing the order of $n^{''}$ to n^2 leads to $N = 2$; then we express n as:

$$n = A_1\sigma + B_1\tau + A_0 + A_2\sigma^2 + B_2\sigma\,\tau, \tag{18}$$

Substituting Eq(18) into Eq(17), using Eq (6) along with Eq(7) and collecting all the terms with the same power of $\sigma^i\tau^j$, ($i = 0, 1, 2, 3, 4$ and $j = 0, 1$) together, equating each term to zero, leads to the following set of algebraic equations:

$-6\,\frac{D_n B_2 e^3 \mu^2}{r} - 2\,A_I A_2 B_2 + 6\,\frac{D_n B_2 e^3}{r} - 2\,B_T A_2 B_2 = 0,$

$-B_T {A_2}^2 + \frac{A_I {B_2}^2 e\mu^2}{r} + \frac{B_T {B_2}^2 e\mu^2}{r} - \frac{A_I {B_2}^2 e}{r} - 6\,\frac{D_n A_2 e^3 \mu^2}{r} - A_I {A_2}^2 - \frac{B_T {B_2}^2 e}{r} + 6\,\frac{D_n A_2 e^3}{r} = 0,$

$A_I B_1 N_D - 2\,A_I B_1 A_0 - 2\,B_T B_1 A_0 - 2\,D_n B_1 e^3 r + 2\,D_n B_1 e r - B_T B_1 N_A - A_T B_1 - A_I B_1 N_A = 0,$

$12\,D_n B_2 e^3 \mu - 2\,B_T B_1 A_2 - 2\,\frac{D_1 B_1 e^3 \mu^2}{r} + 2\,\frac{D_n B_1 e^3}{r} - 2\,A_I B_1 A_2 - 2\,A_I A_1 B_2 - 6\,D_n B_2 e\mu - 2\,B_T A_1 B_2 + 2\,cA_2 e = 0,$

$A_I {B_1}^2 er - cB_1 e^2 r + cB_1 r - A_I {A_0}^2 - B_T A_0 N_A - A_T N_A + A_I A_0 N_D - A_I A_0 N_A + A_T N_D - A_T A_0 + B_T {B_1}^2 er - B_T {A_0}^2 = 0,$

$-6\,D_n B_2 e^3 r - A_I B_2 N_A + cA_1 e - 2\,B_T A_1 B_1 + 4\,D_n B_1 e^3 \mu - 2\,B_T A_0 B_2 - 3\,D_n B_1 \mu\, e -$

$2\,A_IA_0B_2 - A_TB_2 - 2\,A_iA_1B_1 - B_TB_2N_A + A_IB_2N_D + 5\,D_nB_2er = 0,$
$-2\,A_I{B_2}^2e\mu - 2\,B_TA_1A_2 - 2\,\frac{cB_2e^2\mu^2}{r} - 2\,\frac{A_IB_1eB_2}{r} - 2\,D_1A_2e\mu + 2\,\frac{D_nA_1e^3}{r} - 2\,B_T{B_2}^2e\mu - 2\,A_IA_1A_2 + 2\,\frac{B_TB_1eB_2\mu^2}{r} + 12\,D_nA_2e^3\mu - 2\,\frac{B_TB_1eB_2}{r} + 2\,\frac{cB_2e^2}{r} + 2\,\frac{A_IB_1eB_2\mu^2}{r} - 2\,\frac{D_nA_1e^3\mu^2}{r} = 0,$
$-A_IA_1N_A + 2\,A_IB_1eB_2r - 2\,D_nA_1e^3r + 2\,B_TB_1eB_2r + A_IA_1N_D - 2\,B_T{B_1}^2e\mu + 2\,cB_1e^2\mu - 2\,B_TA_1A_0 + D_nA_1er - B_TA_1N_A - 2\,A_IA_1A_0 - cB_1\mu + cB_2r - 2\,cB_2e^2r - A_TA_1 - 2\,A_I{B_1}^2e\mu = 0,$
$-\frac{cB_1e^2\mu^2}{r} + A_IA_2N_D + \frac{A_i{B_1}^2e\mu^2}{r} + \frac{B_T{B_1}^2e\mu^2}{r} - 2\,B_TA_0A_2 - 4\,A_IB_1eB_2\mu + A_I{B_2}^2er - 4\,B_TB_1eB_2\mu + 4\,D_nA_1e^3\mu - A_I{A_1}^2 - A_IA_2N_A - A_TA_2 - B_T{A_1}^2 - D_nA_1e\mu - B_TA_2N_A + B_T{B_2}^2er - cB_2\mu + 2\,D_nA_2er - \frac{B_T{B_1}^2e}{r} - 6\,D_nA_2e^3r - \frac{A_I{B_1}^2e}{r} - 2\,A_iA_0A_2 + \frac{cB_1e^2}{r} + 4\,cB_2e^2\mu = 0.$

The resolution of this set of equations by means of Maple leads to two kinds of solutions viz the solitary and the periodic ones:

3.1.1. *Periodic solutions.* They all correspond to the solutions of the above equation for the case $e = 1$

Case 1

$e = 1,$
$\mu = \frac{\sqrt{\alpha+\beta+24\,{A_I}^2{B_T}^2{A_1}^2{N_A}^2+8\,A_IB_T{A_1}^2{A_T}^2+16\,A_I{B_T}^3{A_1}^2{N_A}^2+c^4}}{c^2}$
$\alpha = -24\,{A_I}^3B_T{A_1}^2N_DN_A - 24\,{A_I}^2{B_T}^2{A_1}^2N_AN_D + 32\,{A_i}^2B_T{A_1}^2A_TN_D + 40\,A_I{B_T}^2{A_1}^2A_TN_D - 8\,A_I{B_T}^3{A_1}^2N_AN_D - 24\,A_I{B_T}^2{A_1}^2A_TN_A,$
$\beta = 4\,{B_T}^2{A_1}^2{A_T}^2 + 4\,{B_T}^4{A_1}^2{N_A}^2 + 4\,{A_I}^4{A_1}^2{N_A}^2 + 4\,{A_I}^4{A_1}^2{N_D}^2 + 4\,{A_I}^2{A_1}^2{A_T}^2 - 8\,{B_T}^3{A_1}^2A_TN_A + 16\,{B_T}^3{A_1}^2A_TN_D - 24\,{A_I}^2B_T{A_1}^2A_TN_A - 8\,{A_i}^4{A_1}^2N_DN_A + 16\,{A_I}^3B_T{A_1}^2{N_A}^2 - 8\,{A_I}^3{A_1}^2A_TN_A + 8\,{A_i}^3{A_1}^2A_TN_D + 8\,{A_I}^3B_T{A_1}^2{N_D}^2 + 4\,{A_I}^2{B_T}^2{A_1}^2{N_D}^2,$
$r = -\frac{\theta - 2\,B_TN_AA_IN_D + 2\,B_T{N_A}^2A_I + {A_I}^2{N_D}^2 - 2\,{A_I}^2N_DN_A + {A_I}^2{N_A}^2 + 4\,A_TN_DB_T}{c^2},$
$\theta = {A_T}^2 - 2\,A_TB_TN_A + 2\,A_TA_IN_D - 2\,A_TA_IN_A + {B_T}^2{N_A}^2,$
$A_0 = 1/2\,\frac{-A_T - B_TN_A + A_IN_D - A_IN_A}{B_T + A_I},$
$A_1 = A_1, A_2 = 0, B_1 = 1/2\,(B_T + A_I)^{-1}, B_2 = 0, w = 0.$

This leads to the following solutions:

$$n_1 = \frac{A_1\sqrt{r}\sec(\sqrt{r}\xi)}{\mu\,\sec(\sqrt{r}\xi) + 1} + \frac{B_1\sqrt{r}\tan(\sqrt{r}\xi)}{\mu\,\tan(\sqrt{r}\xi) + 1} + A_0, \tag{19a}$$

$$n_2 = \frac{A_1\sqrt{r}\csc(\sqrt{r}\xi)}{\mu\,\csc(\sqrt{r}\xi) + 1} + \frac{B_1\sqrt{r}\cot(\sqrt{r}\xi)}{\mu\,\cot(\sqrt{r}\xi) + 1} + A_0. \tag{19b}$$

Case 2

$e = 1, \mu = 1,$
$r = -\frac{\delta + 2\,B_T{N_A}^2A_I + {A_I}^2{N_D}^2 - 2\,{A_I}^2N_DN_A + {A_I}^2{N_A}^2 + 4\,A_TN_DB_T}{c^2}$
$\delta = {A_T}^2 - 2\,A_TB_TN_A + 2\,A_TA_IN_D - 2\,A_TA_IN_A + {B_T}^2{N_A}^2 - 2\,B_TN_AA_IN_D,$
$A_0 = 1/2\,\frac{-A_T - B_TN_A + A_IN_D - A_IN_A}{B_T + A_I}, A_1 = 0, A_2 = 0, B_1 = 1/2\,\frac{c}{B_T + A_I}, B_2 = 0, w = 0$

This also leads to the following solutions:

$$n_3 = \frac{B_1\sqrt{r}\tan(\sqrt{r}\xi)}{\tan(\sqrt{r}\xi) + 1} + A_0, \tag{20a}$$

$$n_4 = \frac{B_1\sqrt{r}\cot(\sqrt{r}\xi)}{\cot(\sqrt{r}\xi) + 1} + A_0 \tag{20b}$$

Case 3
$e=1, \mu=-1, r=-\frac{\epsilon+2\,B_TN_A{}^2A_I+A_I{}^2N_D{}^2-2\,A_I{}^2N_DN_A+A_I{}^2N_A{}^2+4\,A_TN_DB_T}{c^2}$,
$\epsilon=A_T{}^2-2\,A_TB_TN_A+2\,A_TA_IN_D-2\,A_TA_IN_A+B_T{}^2N_A{}^2-2\,B_TN_AA_IN_D$,
$A_0=1/2\,\frac{-A_T-B_TN_A+A_IN_D-A_IN_A}{B_T+A_I}, A_1=0, A_2=0$,
$B_1=1/2\,\frac{c}{B_T+A_I}, B_2=0, w=0$
Then, the associated solutions are:

$$n_5=\frac{B_1\sqrt{r}\tan(\sqrt{r}\xi)}{-\tan(\sqrt{r}\xi)+1}+A_0, \tag{21a}$$

$$n_6=\frac{B_1\sqrt{r}\cot(\sqrt{r}\xi)}{-\cot(\sqrt{r}\xi)+1}+A_0. \tag{21b}$$

Case 4
$e=1, \mu=1, A_0=1/2\,\frac{-A_T-B_TN_A+A_IN_D-A_IN_A}{B_T+A_I}$,
$A_1=\frac{D_n18\sqrt{(\eta-1250\,D_n{}^2A_I{}^2N_DN_A-1250\,D_n{}^2A_TB_TN_A-1250\,D_n{}^2B_TN_AA_IN_D+1250\,D_n{}^2B_TN_A{}^2A_I)^{-1}}c^2}{B_T+A_i}$,
$\eta=-1250\,D_n{}^2A_TA_IN_A+625\,D_n{}^2B_T{}^2N_A{}^2+625\,D_n{}^2A_T{}^2+1250\,D_n{}^2A_TA_IN_D+2500\,D_n{}^2A_TN_DB_T+625\,D_n{}^2A_I{}^2N_D{}^2+625\,D_n{}^2A_I{}^2N_A{}^2, A_2=0, B_1=3/5\,\frac{c}{B_T+A_I}, B_2=0, r=-\frac{25}{36}\,\frac{\zeta+2\,B_TN_A{}^2A_I+A_I{}^2N_D{}^2-2\,A_I{}^2N_DN_A+A_I{}^2N_A{}^2+4\,A_TN_DB_T}{c^2}, \zeta=A_T{}^2-2\,A_TB_TN_A+2\,A_TA_IN_D-2\,A_TA_IN_A+B_T{}^2N_A{}^2-2\,B_TN_AA_IN_D$,
$w=\sqrt[4]{36}\sqrt[4]{\left(\vartheta-1250\,D_n{}^2A_TB_TN_A-1250\,D_n{}^2B_TN_AA_IN_D+1250\,D_n{}^2B_TN_A{}^2A_I\right)^{-1}}c$
$\vartheta=-1250\,D_n{}^2A_TA_IN_A+625\,D_n{}^2B_T{}^2N_A{}^2+625\,D_n{}^2A_T{}^2+1250\,D_n{}^2A_TA_IN_D+2500\,D_n{}^2A_TN_DB_T+625\,D_n{}^2A_I{}^2N_D{}^2+625\,D_n{}^2A_I{}^2N_A{}^2-1250\,D_n{}^2A_i{}^2N_DN_A$.
The corresponding solutions are the following:

$$n_7=\frac{A_1\sqrt{r}\sec(\sqrt{r}\xi)}{\sec(\sqrt{r}\xi)+1}+\frac{B_1\sqrt{r}\tan(\sqrt{r}\xi)}{\tan(\sqrt{r}\xi)+1}+A_0, \tag{22a}$$

$$n_8=\frac{A_1\sqrt{r}\csc(\sqrt{r}\xi)}{\csc(\sqrt{r}\xi)+1}+\frac{B_1\sqrt{r}\cot(\sqrt{r}\xi)}{\cot(\sqrt{r}\xi)+1}+A_0. \tag{22b}$$

Case 5
$e=1, \mu=-1, r=-\frac{25}{36}\,\frac{\varpi-2\,A_I{}^2N_DN_A+A_I{}^2N_A{}^2+4\,A_TN_DB_T}{c^2}$,
$\varpi=A_T{}^2-2\,A_TB_TN_A+2\,A_TA_IN_D-2\,A_TA_IN_A+B_T{}^2N_A{}^2-2\,B_TN_AA_IN_D+2\,B_TN_A{}^2A_I+A_I{}^2N_D{}^2, A_0=1/2\,\frac{-A_T-B_TN_A+A_IN_D-A_IN_A}{B_T+A_I}$,
$A_1=-18\,\frac{D_n{}^2\sqrt{(\psi-1250\,D_n{}^2A_I{}^2N_DN_A-1250\,D_n{}^2A_TB_TN_A-1250\,D_n{}^2B_TN_AA_IN_D+1250\,D_n{}^2B_TN_A{}^2A_I)^{-1}}c^2}{B_T+A_I}$,
$\psi=-1250\,D_1{}^2A_TA_IN_A+625\,D_n{}^2B_T{}^2N_A{}^2+625\,D_n{}^2A_T{}^2+1250\,D_n{}^2A_TA_IN_D+2500\,D_n{}^2A_TN_DB_T+625\,D_n{}^2A_i{}^2N_D{}^2+625\,D_n{}^2A_I{}^2N_A{}^2$,
$A_2=0, B_1=3/5\,\frac{c}{B_T+A_I}, B_2=0$,
$w=\sqrt[4]{36}\sqrt[4]{\left(\varrho-1250\,D_n{}^2A_TB_TN_A-1250\,D_n{}^2B_TN_AA_IN_D+1250\,D_n{}^2B_TN_A{}^2A_I\right)^{-1}}c$,
$\varrho=-1250\,D_n{}^2A_TA_IN_A+625\,D_n{}^2B_T{}^2N_A{}^2+625\,D_n{}^2A_T{}^2+1250\,D_n{}^2A_TA_IN_D+2500\,D_n{}^2A_TN_DB_T+625\,D_n{}^2A_I{}^2N_D{}^2+625\,D_n{}^2A_I{}^2N_A{}^2-1250\,D_n{}^2A_i{}^2N_DN_A$,
Then, the obtained solutions are:

$$n_9=\frac{A_1\sqrt{r}\sec(\sqrt{r}\xi)}{-\sec(\sqrt{r}\xi)+1}+\frac{B_1\sqrt{r}\tan(\sqrt{r}\xi)}{-\tan(\sqrt{r}\xi)+1}+A_0, \tag{23a}$$

$$n_10=\frac{A_1\sqrt{r}\csc(\sqrt{r}\xi)}{-\csc(\sqrt{r}\xi)+1}+\frac{B_1\sqrt{r}\cot(\sqrt{r}\xi)}{-\cot(\sqrt{r}\xi)+1}+A_0. \tag{23b}$$

Case 6

$e=1, \mu=\mu, r=-\frac{25}{36}\,\frac{\chi+2\,B_TN_A{}^2A_I+A_I{}^2N_D{}^2-2\,A_I{}^2N_DN_A+A_I{}^2N_A{}^2+4\,A_TN_DB_T}{c^2}$,
$\chi=A_T{}^2-2\,A_TB_TN_A+2\,A_TA_IN_D-2\,A_TA_IN_A+B_T{}^2N_A{}^2-2\,B_TN_AA_IN_D$,
$A_0=1/2\,\frac{-A_T-B_TN_A+A_IN_D-A_IN_A}{B_T+A_I}$,
$A_1=3\,\frac{\sqrt{36}\sqrt{(\rho-1250\,D_n{}^2B_TN_AA_IN_D+1250\,D_n{}^2B_TN_A{}^2A_I)^{-1}}D_n\left(\mu^2+\sqrt{\mu^2-1}\mu-1\right)c^2}{\sqrt{\mu^2-1}(B_T+A_I)}$,
$\rho=-1250\,D_n{}^2A_TA_IN_A+625\,D_n{}^2B_T{}^2N_A{}^2+625\,D_1{}^2A_T{}^2+1250\,D_n{}^2A_TA_IN_D+2500\,D_n{}^2A_TN_DB_T+625\,D_n{}^2A_I{}^2N_D{}^2+625\,D_n{}^2A_I{}^2N_A{}^2-1250\,D_n{}^2A_I{}^2N_DN_A-1250\,D_n{}^2A_TB_TN_A$,
$A_2=\frac{108}{25}\,\frac{\sqrt{36}\sqrt{\Psi}c^4D_1\left(\mu^2-1\right)}{\Upsilon(B_T+A_I)}$,
$\Psi=(-1250\,D_1{}^2A_TA_IN_A+625\,D_n{}^2B_T{}^2N_A{}^2+625\,D_n{}^2A_T{}^2+1250\,D_n{}^2A_TA_iN_D+2500\,D_n{}^2A_TN_DB_T+625\,D_n{}^2A_I{}^2N_D{}^2+625\,D_n{}^2A_I{}^2N_A{}^2-1250\,D_n{}^2A_I{}^2N_DN_A-1250\,D_n{}^2A_TB_TN_A-1250\,D_n{}^2B_TN_AA_IN_D+1250\,D_n{}^2B_TN_A{}^2A_I)^{-1}$,
$\Upsilon=(A_T{}^2-2\,A_TB_TN_A+2\,A_TA_IN_D-2\,A_TA_IN_A+B_T{}^2N_A{}^2-2\,B_TN_AA_IN_D+2\,B_TN_A{}^2A_I+A_I{}^2N_D{}^2-2\,A_I{}^2N_DN_A+A_I{}^2N_A{}^2+4\,A_TN_DB_T)$,
$B_1=3/5\,(B_T+A_I)^{-1}, B_2=\frac{108}{125}\,\frac{\sqrt{\mu^2-1}c^3}{\Omega(B_T+A_I)}, \Omega=(A_T{}^2-2\,A_TB_TN_A+2\,A_TA_IN_D-2\,A_TA_IN_A+B_T{}^2N_A{}^2-2\,B_TN_AA_IN_D+2\,B_TN_A{}^2A_I+A_I{}^2N_D{}^2-2\,A_I{}^2N_DN_A+A_I{}^2N_A{}^2+4\,A_TN_DB_T)$,
$w=\sqrt[4]{36}\sqrt[4]{\Gamma^{-1}}c$,
$\Gamma=-1250\,D_n{}^2A_TA_IN_A+625\,D_n{}^2B_T{}^2N_A{}^2+625\,D_n{}^2A_T{}^2+1250\,D_n{}^2A_TA_IN_D+2500\,D_n{}^2A_TN_DB_T+625\,D_n{}^2A_I{}^2N_D{}^2+625\,D_n{}^2A_I{}^2N_A{}^2-1250\,D_n{}^2A_i{}^2N_DN_A-1250\,D_n{}^2A_TB_TN_A-1250\,D_n{}^2B_TN_AA_IN_D+1250\,D_n{}^2B_TN_A{}^2A_I$.
The associated solutions are

$$n_{11}=\frac{A_1\sqrt{r}\sec(\sqrt{r}\xi)}{\mu\,\sec(\sqrt{r}\xi)+1}+\frac{B_1\sqrt{r}\tan(\sqrt{r}\xi)}{\mu\,\tan(\sqrt{r}\xi)+1}+\frac{A_2r\left(\sec(\sqrt{r}\xi)\right)^2}{\left(\mu\,\sec(\sqrt{r}\xi)+1\right)^2}+R(\xi), \tag{24a}$$

$$R(\xi)=\frac{B_2r\sec(\sqrt{r}\xi)\tan(\sqrt{r}\xi)}{(\mu\,\sec(\sqrt{r}\xi)+1)(\mu\,\tan(\sqrt{r}\xi)+1)}+A_0, \tag{24b}$$

$$n_{12}=\frac{A_1\sqrt{r}\csc(\sqrt{r}\xi)}{\mu\,\csc(\sqrt{r}\xi)+1}+\frac{B_1\sqrt{r}\cot(\sqrt{r}\xi)}{\mu\,\cot(\sqrt{r}\xi)+1}+\frac{A_2r\left(\csc(\sqrt{r}\xi)\right)^2}{\left(\mu\,\csc(\sqrt{r}\xi)+1\right)^2}+Q(\xi), \tag{24c}$$

$$Q(\xi)=\frac{B_2r\csc(\sqrt{r}\xi)\cot(\sqrt{r}\xi)}{(\mu\,\csc(\sqrt{r}\xi)+1)(\mu\,\cot(\sqrt{r}\xi)+1)}+A_0, \tag{24d}$$

Case 7 $e=1, \mu=0, r=-1/4\,\frac{M}{c^2}$,
$M=A_T{}^2-2\,A_TB_TN_A+2\,A_TA_iN_D-2\,A_TA_IN_A+B_T{}^2N_A{}^2-2\,B_TN_AA_IN_D+2\,B_TN_A{}^2A_I+A_I{}^2N_D{}^2-2\,A_I{}^2N_DN_A+A_I{}^2N_A{}^2+4\,A_TN_DB_T$,
$A_0=1/2\,\frac{-A_T-B_TN_A+A_IN_D-A_IN_A}{B_T+A_I}, A_1=0, A_2=0, B_1=\frac{c}{B_T+A_I}, B_2=0, w=0$.
Then, solutions related to these coefficients are,

$$n_{13}=B_1\sqrt{r}\tan\left(\sqrt{r}\xi\right)+A_0, \tag{25a}$$

$$n_{14}=B_1\sqrt{r}\cot\left(\sqrt{r}\xi\right)+A_0. \tag{25b}$$

Case 8

$e=1, \mu=0, r=-\frac{25}{144}\,\frac{X}{c^2}$,
$X=A_T{}^2-2\,A_TB_TN_A+2\,A_TA_IN_D-2\,A_TA_IN_A+B_T{}^2N_A{}^2-2\,B_TN_AA_IN_D+2\,B_TN_A{}^2A_I+A_I{}^2N_D{}^2-2\,A_I{}^2N_DN_A+A_I{}^2N_A{}^2+4\,A_TN_DB_T$,
$A_0=1/2\,\frac{-A_T-B_TN_A+A_IN_D-A_IN_A}{B_T+A_I}, A_1=0, A_2=-\frac{864}{25}\,\frac{c^4D_n\sqrt{36}\sqrt{P^{-1}}}{(B_T+A_I)U}$,

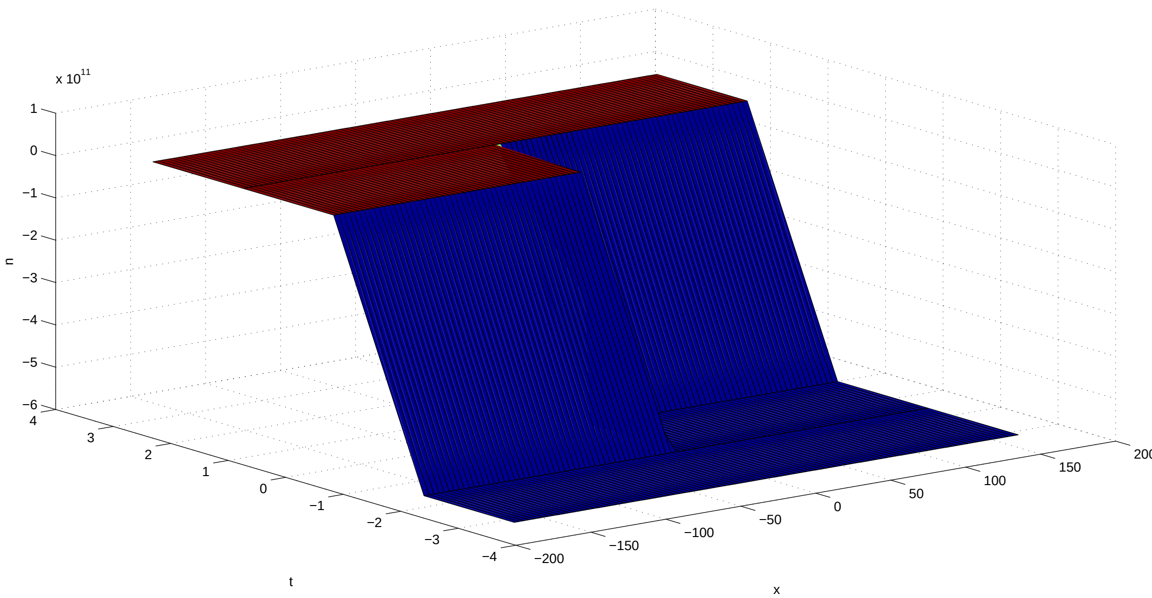

Figure 1. Two different fasten anti-kink: solution for equation (26a) obtained for sb-doped n-Ge at temperature of T=7.14k, with parameters values as follows:$\mu_r = 1.2 \times 10^6 cm/Vs, N_D = 2.5 \times 10^{13}, N_A = .6 \times 10^{12}, A_I = 5 \times 10^{-8}, A_T = 1.6 \times 10^{-2}, B_T = 8.4 \times 10^{-4}$; This solitary wave representation of a periodic solution is obtained because the parameter r is negative for the above semiconductor parameters.

$P = (-1250\, {D_n}^2 A_T A_I N_A + 625\, {D_n}^2 {B_T}^2 {N_A}^2 + 625\, {D_n}^2 {A_T}^2 + 1250\, {D_1}^2 A_T A_I N_D + 2500\, {D_n}^2 A_T N_D B_T + 625\, {D_n}^2 {A_I}^2 {N_D}^2 + 625\, {D_n}^2 {A_i}^2 {N_A}^2 - 1250\, {D_n}^2 {A_I}^2 N_D N_A - 1250\, {D_n}^2 A_T B_T N_A - 1250\, {D_n}^2 B_T N_A A_I N_D + 1250\, {D_n}^2 B_T {N_A}^2 A_I)$,
$U = ({A_T}^2 - 2\, A_T B_T N_A + 2\, A_T A_I N_D - 2\, A_T A_I N_A + {B_T}^2 {N_A}^2 - 2\, B_T N_A A_I N_D + 2\, B_T {N_A}^2 A_I + {A_I}^2 {N_D}^2 - 2\, {A_I}^2 N_D N_A + {A_I}^2 {N_A}^2 + 4\, A_T N_D B_T)$,
$, B_1 = 6/5\, \frac{c}{B_T + A_I}, B_2 = 0,$
$w = \sqrt[4]{36}\sqrt[4]{L^{-1}}c, L = (-1250\, {D_n}^2 A_T A_I N_A + 625\, {D_n}^2 {B_T}^2 {N_A}^2 + 625\, {D_n}^2 {A_T}^2 + 1250\, {D_n}^2 A_T A_I N_D + 2500\, {D_n}^2 A_T N_D B_T + 625\, {D_n}^2 {A_I}^2 {N_D}^2 + 625\, {D_n}^2 {A_I}^2 {N_A}^2 - 1250\, {D_n}^2 {A_i}^2 N_D N_A - 1250\, {D_n}^2 A_T B_T N_A - 1250\, {D_n}^2 B_T N_A A_I N_D + 1250\, {D_n}^2 B_T {N_A}^2 A_I)$.
Then, solutions related to these coefficients are,

$$n_{15} = B_1 \sqrt{r} \tan\left(\sqrt{r}\xi\right) + A_0 + A_2 r \left(\sec\left(\sqrt{r}\xi\right)\right)^2, \tag{26a}$$

$$n_{16} = B_1 \sqrt{r} \cot\left(\sqrt{r}\xi\right) + A_0 + A_2 r \left(\csc\left(\sqrt{r}\xi\right)\right)^2. \tag{26b}$$

The graphical representation of solution (26a) is given by Fig. 1.

3.1.2. *Soliton solutions.* These solutions are derived from the resolution of the set of equations giving by the coefficients for $e = -1$

Case 1

$e=-1, \mu=\frac{\sqrt{S}}{c^2}$,
$S=4\,{B_T}^4{A_1}^2{N_A}^2+4\,{A_I}^2{A_1}^2{A_T}^2+4\,{A_I}^4{A_1}^2{N_D}^2+4\,{B_T}^2{A_1}^2{A_T}^2+4\,{A_I}^4{A_1}^2{N_A}^2-24\,{A_I}^3B_T{A_1}^2N_DN_A-24\,{A_I}^2{B_T}^2{A_1}^2N_DN_A-24\,{A_i}^2B_T{A_1}^2A_TN_A+32\,{A_I}^2B_T{A_1}^2N_DA_T-8\,A_I{B_T}^3{A_1}^2N_DN_A-24\,A_I{B_T}^2{A_1}^2A_TN_A+40\,A_I{B_T}^2{A_1}^2N_DA_T+8\,{A_I}^3{A_1}^2N_DA_T+16\,{B_T}^3{A_1}^2N_DA_T-8\,{B_T}^3{A_1}^2A_TN_A-8\,{A_I}^4{A_1}^2N_DN_A-8\,{A_I}^3{A_1}^2A_TN_A+8\,{A_I}^3B_T{A_1}^2{N_D}^2+16\,{A_I}^3B_T{A_1}^2{N_A}^2+24\,{A_I}^2{B_T}^2{A_1}^2{N_A}^2+4\,{A_I}^2{B_T}^2{A_1}^2{N_D}^2+16\,A_I{B_T}^3{A_1}^2{N_A}^2+8\,A_IB_T{A_1}^2{A_T}^2+c^4$,
$r=\frac{F}{c^2}, F={A_I}^2{N_D}^2-2\,A_IN_DB_TN_A+2\,A_IN_DA_T-2\,{A_I}^2N_DN_A+{B_T}^2{N_A}^2-2\,B_TN_AA_T+2\,B_T{N_A}^2A_I+{A_T}^2-2\,A_TA_IN_A+{A_I}^2{N_A}^2+4\,A_TN_DB_T$,
$A_0=1/2\,\frac{A_IN_D-B_TN_A-A_T-A_IN_A}{B_T+A_I}, A_1=A_1, A_2=0, B_1=-1/2\,\frac{c}{B_T+A_I}, B_2=0, w=0.$
solutions are:

$$n_{17}=\frac{A_1\sqrt{r}sech\left(\sqrt{r}\xi\right)}{\mu\,sech\left(\sqrt{r}\xi\right)+1}+\frac{B_1\sqrt{r}\tanh\left(\sqrt{r}\xi\right)}{\mu\,\tanh\left(\sqrt{r}\xi\right)+1}+A_0, \tag{27a}$$

$$n_{18}=\frac{A_1\sqrt{r}csch\left(\sqrt{r}\xi\right)}{\mu\,csch\left(\sqrt{r}\xi\right)+1}+\frac{B_1\sqrt{r}\coth\left(\sqrt{r}\xi\right)}{\mu\,\coth\left(\sqrt{r}\xi\right)+1}+A_0. \tag{27b}$$

Case 2

$e=-1, \mu=1, r=1/4\,\frac{\Lambda}{c^2}, \Lambda={A_I}^2{N_D}^2-2\,A_IN_DB_TN_A+2\,A_IN_DA_T-2\,{A_I}^2N_DN_A+{B_T}^2{N_A}^2-2\,B_TN_AA_T+2\,B_T{N_A}^2A_i+{A_T}^2-2\,A_TA_IN_A+{A_I}^2{N_A}^2+4\,A_TN_DB_T$,
$A_0=1/2\,\frac{A_IN_D-B_TN_A-A_T-A_IN_A}{B_T+A_I}, A_1=0, A_2=0, B_1=-\frac{c}{B_T+A_I}, B_2=0, w=0.$
This leads us to:

$$n_{19}=B_1\sqrt{r}\tanh\left(\sqrt{r}\xi\right)+A_0, \tag{28a}$$

$$n_{20}=B_1\sqrt{r}\coth\left(\sqrt{r}\xi\right)+A_0. \tag{28b}$$

Case 3

$e=-1, \mu=0, r=\frac{G}{c^2}, G={A_I}^2{N_D}^2-2\,A_IN_DB_TN_A+2\,A_IN_DA_T-2\,{A_I}^2N_DN_A+{B_T}^2{N_A}^2-2\,B_TN_AA_T+2\,B_T{N_A}^2A_I+{A_T}^2-2\,A_TA_IN_A+{A_I}^2{N_A}^2+4\,A_TN_DB_T, A_0=1/2\,\frac{A_IN_D-B_TN_A-A_T-A_IN_A}{B_T+A_I}, A_1=0, A_2=0, B_1=-1/2\,\frac{c}{B_T+A_I}, B_2=0, w=0.$
Correspondingly, solutions are:

$$n_{21}=\frac{B_1\sqrt{r}\tanh\left(\sqrt{r}\xi\right)}{\tanh\left(\sqrt{r}\xi\right)+1}+A_0, \tag{29a}$$

$$n_{22}=\frac{B_1\sqrt{r}\coth\left(\sqrt{r}\xi\right)}{\coth\left(\sqrt{r}\xi\right)+1}+A_0. \tag{29b}$$

Case 4

$e=-1, \mu=-1r=\frac{J}{c^2}, J={A_I}^2{N_D}^2-2\,A_IN_DB_TN_A+2\,A_IN_DA_T-2\,{A_I}^2N_DN_A+{B_T}^2{N_A}^2-2\,B_TN_AA_T+2\,B_T{N_A}^2A_I+{A_T}^2-2\,A_TA_IN_A+{A_I}^2{N_A}^2+4\,A_TN_DB_T, A_0=1/2\,\frac{A_IN_D-B_TN_A-A_T-A_IN_A}{B_T+A_I}, A_1=0, A_2=0, B_1=-1/2\,\frac{c}{B_T+A_I}, B_2=0, w=0.$

$$n_{23}=\frac{B_1\sqrt{r}\tanh\left(\sqrt{r}\xi\right)}{\mu\,\tanh\left(\sqrt{r}\xi\right)+1}+A_0, \tag{30a}$$

$$n_{24}=\frac{B_1\sqrt{r}\coth\left(\sqrt{r}\xi\right)}{\mu\,\coth\left(\sqrt{r}\xi\right)+1}+A_0. \tag{30b}$$

Case 5

$e=-1, \mu=-1, r=\frac{25}{144}\,\frac{H}{c^2}, H={A_I}^2{N_D}^2-2\,A_IN_DB_TN_A+2\,A_IN_DA_T-2\,{A_I}^2N_DN_A+{B_T}^2{N_A}^2-2\,B_TN_AA_T+2\,B_T{N_A}^2A_I+{A_T}^2-2\,A_TA_IN_A+{A_I}^2{N_A}^2+4\,A_TN_DB_T$,

$A_0 = 1/2\,\frac{A_I N_D - B_T N_A - A_T - A_I N_A}{B_T + A_I}, A_1 = 0,$
$A_2 = -\frac{864}{25}\,\frac{c^4 D_n \sqrt{36}\sqrt{\Theta^{-1}}}{(B_T+A_I)\Xi},$
$\Theta = (-1250\,{D_n}^2 B_T N_A A_T + 2500\,{D_n}^2 A_T N_D B_T + 625\,{D_n}^2 {A_I}^2 {N_D}^2 - 1250\,{D_n}^2 {A_I}^2 N_D N_A + 1250\,{D_n}^2 A_I N_D A_T + 1250\,{D_n}^2 B_T {N_A}^2 A_I + 625\,{D_1}^2 {B_T}^2 {N_A}^2 + 625\,{D_n}^2 {A_I}^2 {N_A}^2 - 1250\,{D_n}^2 A_I N_D B_T N_A + 625\,{D_1}^2 {A_T}^2 - 1250\,{D_n}^2 A_T A_I N_A),$
$\Xi = ({A_I}^2 {N_D}^2 - 2\,A_I N_D B_T N_A + 2\,A_I N_D A_T - 2\,{A_i}^2 N_D N_A + {B_T}^2 {N_A}^2 - 2\,B_T N_A A_T + 2\,B_T {N_A}^2 A_I + {A_T}^2 - 2\,A_T A_I N_A + {A_I}^2 {N_A}^2 + 4\,A_T N_D B_T),$
$B_1 = -6/5\,\frac{c}{B_T+A_I}, B_2 = 0,$
$w = \sqrt[4]{36}\sqrt[4]{F^{-1}}c, F = (-1250\,{D_n}^2 B_T N_A A_T + 2500\,{D_n}^2 A_T N_D B_T + 625\,{D_n}^2 {A_i}^2 {N_D}^2 - 1250\,{D_n}^2 {A_I}^2 N_D N_A + 1250\,{D_n}^2 A_I N_D A_T + 1250\,{D_n}^2 B_T {N_A}^2 A_I + 625\,{D_n}^2 {B_T}^2 {N_A}^2 + 625\,{D_1}^2 {A_I}^2 {N_A}^2 - 1250\,{D_n}^2 A_I N_D B_T N_A + 625\,{D_n}^2 {A_T}^2 - 1250\,{D_n}^2 A_T A_I N_A)$
solutions are:

$$n_{25} = B_1\sqrt{r}\tanh\left(\sqrt{r}\xi\right) + A_0 + A_2 r\left(sech\left(\sqrt{r}\xi\right)\right)^2, \tag{31a}$$

$$n_{26} = B_1\sqrt{r}\coth\left(\sqrt{r}\xi\right) + A_0 + A_2 r\left(csch\left(\sqrt{r}\xi\right)\right)^2. \tag{31b}$$

Case 6
$e = -1, \mu = 1, r = \frac{25}{36}\,\frac{Y}{c^2}, Y = {A_I}^2 {N_D}^2 - 2\,A_I N_D B_T N_A + 2\,A_I N_D A_T - 2\,{A_I}^2 N_D N_A + {B_T}^2 {N_A}^2 - 2\,B_T N_A A_T + 2\,B_T {N_A}^2 A_I + {A_T}^2 - 2\,A_T A_I N_A + {A_I}^2 {N_A}^2 + 4\,A_T N_D B_T, A_0 = 1/2\,\frac{A_I N_D - B_T N_A - A_T - A_I N_A}{B_T + A_I},$
$A_1 = -3\,\frac{D_n \sqrt{36}\sqrt{\Pi^{-1}}c^2}{B_T + A_i}, \Pi = (-1250\,{D_n}^2 B_T N_A A_T + 2500\,{D_n}^2 A_T N_D B_T + 625\,{D_n}^2 {A_I}^2 {N_D}^2 - 1250\,{D_n}^2 {A_I}^2 N_D N_A + 1250\,{D_n}^2 A_I N_D A_T + 1250\,{D_1}^2 B_T {N_A}^2 A_I + 625\,{D_n}^2 {B_T}^2 {N_A}^2 + 625\,{D_n}^2 {A_I}^2 {N_A}^2 - 1250\,{D_n}^2 A_I N_D B_T N_A + 625\,{D_n}^2 {A_T}^2 - 1250\,{D_n}^2 A_T A_I N_A),$
$B_1 = -3/5\,\frac{c}{B_T+A_I}, B_2 = 0, A_2 = 0,$
$w = \sqrt[4]{36}\sqrt[4]{m^{-1}}c, m = (-1250\,{D_n}^2 B_T N_A A_T + 2500\,{D_n}^2 A_T N_D B_T + 625\,{D_n}^2 {A_i}^2 {N_D}^2 - 1250\,{D_n}^2 {A_I}^2 N_D N_A + 1250\,{D_n}^2 A_I N_D A_T + 1250\,{D_n}^2 B_T {N_A}^2 A_I + 625\,{D_n}^2 {B_T}^2 {N_A}^2 + 625\,{D_1}^2 {A_I}^2 {N_A}^2 - 1250\,{D_n}^2 A_I N_D B_T N_A + 625\,{D_n}^2 {A_T}^2 - 1250\,{D_n}^2 A_T A_I N_A),$
Then, we get:

$$n_{27} = \frac{A_1\sqrt{r}\,sech\left(\sqrt{r}\xi\right)}{sech\left(\sqrt{r}\xi\right)+1} + \frac{B_1\sqrt{r}\tanh\left(\sqrt{r}\xi\right)}{\tanh\left(\sqrt{r}\xi\right)+1} + A_0, \tag{32a}$$

$$n_{28} = \frac{A_1\sqrt{r}\,csch\left(\sqrt{r}\xi\right)}{csch\left(\sqrt{r}\xi\right)+1} + \frac{B_1\sqrt{r}\coth\left(\sqrt{r}\xi\right)}{\coth\left(\sqrt{r}\xi\right)+1} + A_0. \tag{32b}$$

Case 7
$e = -1, \mu = -1, r = \frac{25}{36}\,\frac{Z}{c^2}, Z = {A_I}^2 {N_D}^2 - 2\,A_I N_D B_T N_A + 2\,A_I N_D A_T - 2\,{A_I}^2 N_D N_A + {B_T}^2 {N_A}^2 - 2\,B_T N_A A_T + 2\,B_T {N_A}^2 A_I + {A_T}^2 - 2\,A_T A_I N_A + {A_I}^2 {N_A}^2 + 4\,A_T N_D B_T, A_0 = 1/2\,\frac{A_I N_D - B_T N_A - A_T - A_I N_A}{B_T + A_I}, A_1 = 3\,\frac{D_n \sqrt{36}\sqrt{s^{-1}}c^2}{B_T + A_i}, s = (-1250\,{D_n}^2 B_T N_A A_T + 2500\,{D_n}^2 A_T N_D B_T + 625\,{D_n}^2 {A_I}^2 {N_D}^2 - 1250\,{D_n}^2 {A_I}^2 N_D N_A + 1250\,{D_n}^2 A_I N_D A_T + 1250\,{D_1}^2 B_T {N_A}^2 A_I + 625\,{D_n}^2 {B_T}^2 {N_A}^2 + 625\,{D_n}^2 {A_I}^2 {N_A}^2 - 1250\,{D_n}^2 A_I N_D B_T N_A + 625\,{D_n}^2 {A_T}^2 - 1250\,{D_n}^2 A_T A_I N_A), A_1 = 0, B_1 = -3/5\,\frac{c}{B_T+A_I}, B_2 = 0,$
$w = \sqrt[4]{36}\sqrt[4]{d^{-1}}c, d = (-1250\,{D_n}^2 B_T N_A A_T + 2500\,{D_n}^2 A_T N_D B_T + 625\,{D_n}^2 {A_i}^2 {N_D}^2 - 1250\,{D_n}^2 {A_I}^2 N_D N_A + 1250\,{D_n}^2 A_I N_D A_T + 1250\,{D_n}^2 B_T {N_A}^2 A_I + 625\,{D_n}^2 {B_T}^2 {N_A}^2 + 625\,{D_1}^2 {A_I}^2 {N_A}^2 - 1250\,{D_n}^2 A_I N_D B_T N_A + 625\,{D_n}^2 {A_T}^2 - 1250\,{D_n}^2 A_T A_I N_A).$
Hence, solutions related to these coefficients are:

$$n_{29} = \frac{A_1\sqrt{r}\,sech\,(\sqrt{r}\xi)}{-sech\,(\sqrt{r}\xi)+1} + \frac{B_1\sqrt{r}\tanh\,(\sqrt{r}\xi)}{-\tanh\,(\sqrt{r}\xi)+1} + A_0, \tag{33a}$$

$$n_{30} = \frac{A_1\sqrt{r}\,csch\,(\sqrt{r}\xi)}{-csch\,(\sqrt{r}\xi)+1} + \frac{B_1\sqrt{r}\coth\,(\sqrt{r}\xi)}{-\coth\,(\sqrt{r}\xi)+1} + A_0. \tag{33b}$$

3.2. **Discussion of the factorization method to Eq(1b).** This equation has been already handled by this method in[19]. We are just going to present the main results. By setting $\xi = wx - ct$, and following all the instructions of the paragraph (2.2), we write Eq(1b) as [19]:

$$E' + (\frac{-\gamma n C\mu_r}{c})E = \quad 0, \tag{34a}$$

$$E' - (c/D_E W^2 C\mu_r)(\frac{E-E_0}{En} + eC\mu)E = \quad 0. \tag{34b}$$

A particular solution of the Eq(1b) can be obtained by solving Eq(34a) ie:

$$E = K\exp(-\gamma e C\mu_r \int ndz/c). \tag{35}$$

This leads to two classes of solutions ie depending on the kind of solutions of Eq(1a).

3.2.1. *Solutions deriving from periodic solutions of Eq(1a).* ***Case 1***

$$E_1 \quad = K\left(1+(\tan(\sqrt{r}\xi))^2\right)^{1/2\,\frac{B_1\gamma\,e\mu_r C}{(1+\mu^2)c}}(\mu\,\tan(\sqrt{r}\xi)+1)^{-\frac{B_1\gamma\,e\mu_r C}{(1+\mu^2)c}}\,\mathrm{e}^{\Psi(\xi)}, \tag{36a}$$

$$\Psi(\xi) \quad = \gamma\,e\mu_r C(2\,A_1\arctan(\frac{(\mu-1)\tan(1/2\,\sqrt{r}\xi)}{\sqrt{(\mu+1)(\mu-1)}})\frac{1}{\sqrt{(\mu+1)(\mu-1)}} + \Psi_1(\xi))c^{-1}, \tag{36b}$$

$$\Psi_1(\xi) \quad = \frac{B_1\mu\,\arctan\left(\tan\left(\sqrt{r}\xi\right)\right)}{1+\mu^2} + A_0\xi, \tag{36c}$$

$$E_2 \quad = K\left((\cot(\sqrt{r}\xi))^2+1\right)^{-1/2\,\frac{\gamma\,e\mu_r CB_1}{c(1+\mu^2)}}(\mu\,\cot(\sqrt{r}\xi)+1)^{\frac{\gamma\,e\mu_r CB_1}{(1+\mu^2)c}}\,\mathrm{e}^{\Omega(\xi)}, \tag{36d}$$

$$\Omega(\xi) = \gamma\,e\mu_r C\left(2\,A_1\arctan\left(1/2\,\frac{2\,\mu\,\tan\left(1/2\,\sqrt{r}\xi\right)+2}{\sqrt{-1+\mu^2}}\right)\frac{1}{\sqrt{-1+\mu^2}}\Omega_1(\xi)\right)c^{-1}, \tag{36e}$$

$$\Omega_1(\xi) \quad = -1/2\,\frac{B_1\mu\,\pi}{1+\mu^2} + \frac{B_1\mu\,arccot\left(\cot\left(\sqrt{r}\xi\right)\right)}{1+\mu^2} + A_0\xi. \tag{36f}$$

All the coefficients of this equation are the same with those of Eq(19).
Case 2
These solutions derive from Eq(20) hence, they possess the same coefficients

$$E_3 \quad = K\,(\tan(\sqrt{r}\xi)+1)^{-1/2\,\frac{\gamma\,e\mu_r CB_1}{c}}\left(1+(\tan(\sqrt{r}\xi))^2\right)^{1/4\,\frac{\gamma\,e\mu_r CB_1}{c}}\,\mathrm{e}^{\Theta(\xi)}, \tag{37a}$$

$$\Theta(\xi) \quad = \frac{\gamma\,e\mu_r C\left(1/2\,B_1\arctan\left(\tan\left(\sqrt{r}\xi\right)\right)+A_0\xi\right)}{c}, \tag{37b}$$

$$E_4 \quad = K\,(\cot(\sqrt{r}\xi)+1)^{1/2\,\frac{\gamma\,e\mu_r CB_1}{c}}\left(1+(\cot(\sqrt{r}\xi))^2\right)^{-1/4\,\frac{\gamma\,e\mu_r CB_1}{c}}\,\Delta(\xi), \tag{37c}$$

$$\Delta(\xi) \quad = \mathrm{e}^{\frac{\gamma\,e\mu_r C\left(-1/4\,B_1\pi+1/2\,B_1\,arccot\left(\cot\left(\sqrt{r}\xi\right)\right)+A_0\xi\right)}{c}} \tag{37d}$$

Case 3

$$E_5 = K\left(\tan\left(\sqrt{r}\xi\right)-1\right)^{-1/2\,\frac{\gamma\,e\mu_r C B_1}{c}}\left(1+\left(\tan\left(\sqrt{r}\xi\right)\right)^2\right)^{1/4\,\frac{\gamma\,e\mu_r C B_1}{c}}\mathrm{e}^{\eta(\xi)}, \tag{38a}$$

$$\eta(\xi) = \frac{\gamma\,e\mu_r C\left(-1/2\,B_1\arctan\left(\tan\left(\sqrt{r}\xi\right)\right)+A_0\xi\right)}{c}, \tag{38b}$$

$$E_6 = K\left(\cot\left(\sqrt{r}\xi\right)-1\right)^{1/2\,\frac{\gamma\,e\mu_r C B_1}{c}}\left(1+\left(\cot\left(\sqrt{r}\xi\right)\right)^2\right)^{-1/4\,\frac{\gamma\,e\mu_r C B_1}{c}}\mathrm{e}^{\beta(\xi)}, \tag{38c}$$

$$\beta(\xi) = \frac{\gamma\,\mu_r e\,C\left(1/4\,B_1\pi-1/2\,B_1\,arccot\left(\cot\left(\sqrt{r}\xi\right)\right)+A_0\xi\right)}{c}. \tag{38d}$$

$$\tag{38e}$$

These solutions possess the same parameters like those of Eq(21)

Case 4

These solutions derived from those of Eq(22), they then share the same coefficients,

$$E_7 = K\left(1+\left(\tan\left(\sqrt{r}\xi\right)\right)^2\right)^{-1/4\,\frac{\gamma\,e\mu_r C B_1}{c}}\left(\tan\left(\sqrt{r}\xi\right)+1\right)^{1/2\,\frac{\gamma\,e\mu_r C B_1}{c}}\mathrm{e}^{\theta(\xi)}, \tag{39a}$$

$$\theta(\xi) = \frac{\gamma\,e\mu_r C\left(A_1\tan\left(1/2\,\sqrt{r}\xi\right)+1/2\,B_1\arctan\left(\tan\left(\sqrt{r}\xi\right)\right)+A_0\xi\right)}{c}, \tag{39b}$$

$$E_8 = K\left(\cot\left(\sqrt{r}\xi\right)+1\right)^{1/2\,\frac{\gamma\,\mu\_r\,e C B_1}{c}}\left(\left(\cot\left(\sqrt{r}\xi\right)\right)^2+1\right)^{-1/4\,\frac{\gamma\,\mu\_r\,e C B_1}{c}}\mathrm{e}^{\zeta(\xi)}, \tag{39c}$$

$$\zeta(\xi) = \gamma\,\mu\_r\,eC\left(-2\,\frac{A_1}{\tan\left(1/2\,\sqrt{r}\xi\right)+1}-1/4\,B_1\pi+1/2\,B_1\,arccot\left(\cot\left(\sqrt{r}\xi\right)\right)+A_0\xi\right)c^{-1}, \tag{39d}$$

Case 5

This equation shares the same parameters with the Eq(23) since it derives from it.

$$E_9 = K\left(1+\left(\tan\left(\sqrt{r}\xi\right)\right)^2\right)^{-1/4\,\frac{\gamma\,e\mu_r C B_1}{c}}\left(\tan\left(\sqrt{r}\xi\right)-1\right)^{1/2\,\frac{\gamma\,e\mu_r C B_1}{c}}\mathrm{e}^{\varrho(\xi)}, \tag{40a}$$

$$\varrho(\xi) = -\gamma\,e\mu_r C\left(\frac{A_1}{\tan\left(1/2\,\sqrt{r}\xi\right)}-1/2\,B_1\arctan\left(\tan\left(\sqrt{r}\xi\right)\right)+A_0\xi\right)c^{-1}, \tag{40b}$$

$$E_{10} = K\left(\left(\cot\left(\sqrt{r}\xi\right)\right)^2+1\right)^{\gamma\,1/4\,e\,\mu r\,B_1}\left(\cot\left(\sqrt{r}\xi\right)-1\right)^{-\gamma\,1/2\,e\,\mu r\,B_1}\mathrm{e}^{\upsilon(\xi)}, \tag{40c}$$

$$\upsilon(\xi) = -\gamma\,e\,\mu r\left(2\,\frac{A_1}{\tan\left(1/2\,\sqrt{r}\xi\right)-1}+1/4\,B_1\pi-1/2\,B_1\,arccot\left(\cot\left(\sqrt{r}\xi\right)\right)+A_0\xi\right) \tag{40d}$$

Case 6

The parameters of these solutions are the same with those of Eq(24).

$$E_{11} = KA(\xi)^{\frac{\gamma\,e\mu_r C B_2\sqrt{r}}{c\mu^3}}B(\xi)^{-\frac{\gamma\,e\mu_r C B_2\sqrt{r}}{c\mu^3}}\left(\mu\,\cot\left(\sqrt{r}\xi\right)+1\right)^{\frac{\gamma\,e\mu_r C B_1}{c(\mu^2+1)}}P(\xi), \tag{41a}$$

$$A(\xi) = \left(\mu\,\left(\tan\left(1/2\,\sqrt{r}\xi\right)\right)^2-\mu-2\,\tan\left(1/2\,\sqrt{r}\xi\right)\right), \tag{41b}$$

$$B(\xi) = \left(\mu\,\left(\tan\left(1/2\,\sqrt{r}\xi\right)\right)^2+\mu+2\,\tan\left(1/2\,\sqrt{r}\xi\right)\right), \tag{41c}$$

$$P(\xi) = \left(\left(\cot\left(\sqrt{r}\xi\right)\right)^2+1\right)^{-1/2\,\frac{\gamma\,e\mu_r C B_1}{c(\mu^2+1)}}\mathrm{e}^{\varsigma(\xi)}, \tag{41d}$$

$$\varsigma(\xi) = \gamma\,e\mu_r C\left(\varsigma(\xi)_1+\Gamma(\xi)+\nu(\xi)+\psi(\xi)+\Upsilon(\xi)+\delta(\xi)+A_0\xi\right)c^{-1}, \tag{41e}$$

$$\varsigma_1(\xi) = 2\,A_1\arctan\left(1/2\,\frac{2\,\mu\,\tan\left(1/2\,\sqrt{r}\xi\right)+2}{\sqrt{\mu^2-1}}\right)\frac{1}{\sqrt{\mu^2-1}}-1/2\,\frac{B_1\mu\,\pi}{\mu^2+1}+\frac{B_1\mu\,arccot\left(\cot\left(\sqrt{r}\xi\right)\right)}{\mu^2+1}, \tag{41f}$$

$$\Gamma(\xi) = 2\,A_2\sqrt{r}\arctan\left(1/2\,\frac{2\,\mu\,\tan\left(1/2\,\sqrt{r}\xi\right)+2}{\sqrt{\mu^2-1}}\right)\mu^{-1}\frac{1}{\sqrt{\mu^2-1}}, \tag{41g}$$

and

$$\nu(\xi) = 8\,\frac{A_2\sqrt{r}\tan\left(1/2\,\sqrt{r}\xi\right)}{\mu\,(4\,\mu^2-4)\left(\mu\left(\tan\left(1/2\,\sqrt{r}\xi\right)\right)^2+\mu+2\,\tan\left(1/2\,\sqrt{r}\xi\right)\right)}, \tag{42a}$$

$$\psi(\xi) = 8\,\frac{A_2\sqrt{r}}{(4\,\mu^2-4)\left(\mu\left(\tan\left(1/2\,\sqrt{r}\xi\right)\right)^2+\mu+2\,\tan\left(1/2\,\sqrt{r}\xi\right)\right)}, \tag{42b}$$

$$\Upsilon(\xi) = 8\,A_2\sqrt{r}\arctan\left(1/2\,\frac{2\,\mu\,\tan\left(1/2\,\sqrt{r}\xi\right)+2}{\sqrt{\mu^2-1}}\right)\mu^{-1}\left(4\,\mu^2-4\right)^{-1}\frac{1}{\sqrt{\mu^2-1}}, \tag{42c}$$

$$\delta(\xi) = 2\,B_2\sqrt{r}\,arctanh\left(1/2\,\frac{2\,\mu\,\tan\left(1/2\,\sqrt{r}\xi\right)-2}{\sqrt{\mu^2+1}}\right)\mu^{-3}\frac{1}{\sqrt{\mu^2+1}}+q(\xi), \tag{42d}$$

$$q(\xi) = 2\,B_2\sqrt{r}\sqrt{\mu^2-1}\arctan\left(1/2\,\frac{2\,\mu\,\tan\left(1/2\,\sqrt{r}\xi\right)+2}{\sqrt{\mu^2-1}}\right)\mu^{-3}, \tag{42e}$$

$$E_{12} = KT(\xi)L(\xi)X(\xi)a(\xi)b(\xi)\mathrm{e}^{H(\xi)}, \tag{42f}$$

$$T(\xi) = \left(1+\left(\tan\left(\sqrt{r}\xi\right)\right)^2\right)^{-1/2\,\frac{\gamma\,e\mu_r CB_1}{c(\mu^2+1)}}, \tag{42g}$$

$$L(\xi) = \left(\mu\,\tan\left(\sqrt{r}\xi\right)+1\right)^{-\frac{\gamma\,e\mu_r CB_1}{\mu^2+1}}, \tag{42h}$$

$$X(\xi) = \left(\mu\,\left(\tan\left(1/2\,\sqrt{r}\xi\right)\right)^2+\mu-\left(\tan\left(1/2\,\sqrt{r}\xi\right)\right)^2+1\right)^{\frac{\gamma\,e\mu_r C\sqrt{r}}{\mu^2(\mu-1)}}, \tag{42i}$$

$$a(\xi) = \left(\mu\,\left(\tan\left(1/2\,\sqrt{r}\xi\right)\right)^2+\mu-\left(\tan\left(1/2\,\sqrt{r}\xi\right)\right)^2+1\right)^{\frac{\gamma\,e\mu_r CB_2\sqrt{r}}{c\mu^3(\mu-1)}}, \tag{42j}$$

$$b(\xi) = \left(-2\,\mu\,\tan\left(1/2\,\sqrt{r}\xi\right)+\left(\tan\left(1/2\,\sqrt{r}\xi\right)\right)^2-1\right)^{\frac{\gamma\,e\mu_r CB_2\sqrt{r}}{c\mu^3}}, \tag{42k}$$

$$H(\xi) = -\gamma\,e\mu_r C\left(k(\xi)+z(\xi)-h(\xi)-g(\xi)+u(\xi)+v(\xi)+p(\xi)-\lambda(\xi)\right)c^{-1}, \tag{42l}$$

$$k(\xi) = 2\,A_1\arctan\left(\frac{(\mu-1)\tan\left(1/2\,\sqrt{r}\xi\right)}{\sqrt{(\mu-1)(\mu+1)}}\right)\frac{1}{\sqrt{(\mu-1)(\mu+1)}}, \tag{42m}$$

$$z(\xi) = \frac{B_1\mu\,\arctan\left(\tan\left(\sqrt{r}\xi\right)\right)}{\mu^2+1}+A_0\xi, \tag{42n}$$

$$h(\xi) = 2\,\frac{A_2\sqrt{r}\tan\left(1/2\,\sqrt{r}\xi\right)}{(\mu-1)(\mu+1)\left(\mu\left(\tan\left(1/2\,\sqrt{r}\xi\right)\right)^2+\mu-\left(\tan\left(1/2\,\sqrt{r}\xi\right)\right)^2+1\right)}, \tag{42o}$$

$$g(\xi) = 2\,A_2\sqrt{r}\arctan\left(\frac{(\mu-1)\tan\left(1/2\,\sqrt{r}\xi\right)}{\sqrt{(\mu-1)(\mu+1)}}\right)(\mu-1)^{-1}(\mu+1)^{-1}\frac{1}{\sqrt{(\mu-1)(\mu+1)}}, \tag{42p}$$

$$u(\xi) = 2\,A_2\sqrt{r}\arctan\left(\frac{(\mu-1)\tan\left(1/2\,\sqrt{r}\xi\right)}{\sqrt{(\mu-1)(\mu+1)}}\right)(\mu-1)^{-1}\frac{1}{\sqrt{(\mu-1)(\mu+1)}}, \tag{42q}$$

$$v(\xi) = 2\,B_2\sqrt{r}\,arctanh\left(1/2\,\frac{-2\,\mu+2\,\tan\left(1/2\,\sqrt{r}\xi\right)}{\sqrt{\mu^2+1}}\right)\mu^{-3}\frac{1}{\sqrt{\mu^2+1}}, \tag{42r}$$

$$p(\xi) = 2\,B_2\sqrt{r}\arctan\left(\frac{(\mu-1)\tan\left(1/2\,\sqrt{r}\xi\right)}{\sqrt{(\mu-1)(\mu+1)}}\right)\mu^{-1}\frac{1}{\sqrt{(\mu-1)(\mu+1)}}, \tag{42s}$$

$$\lambda(\xi) = 2\,B_2\sqrt{r}\arctan\left(\frac{(\mu-1)\tan\left(1/2\,\sqrt{r}\xi\right)}{\sqrt{(\mu-1)(\mu+1)}}\right)\mu^{-3}\frac{1}{\sqrt{(\mu-1)(\mu+1)}}, \tag{42t}$$

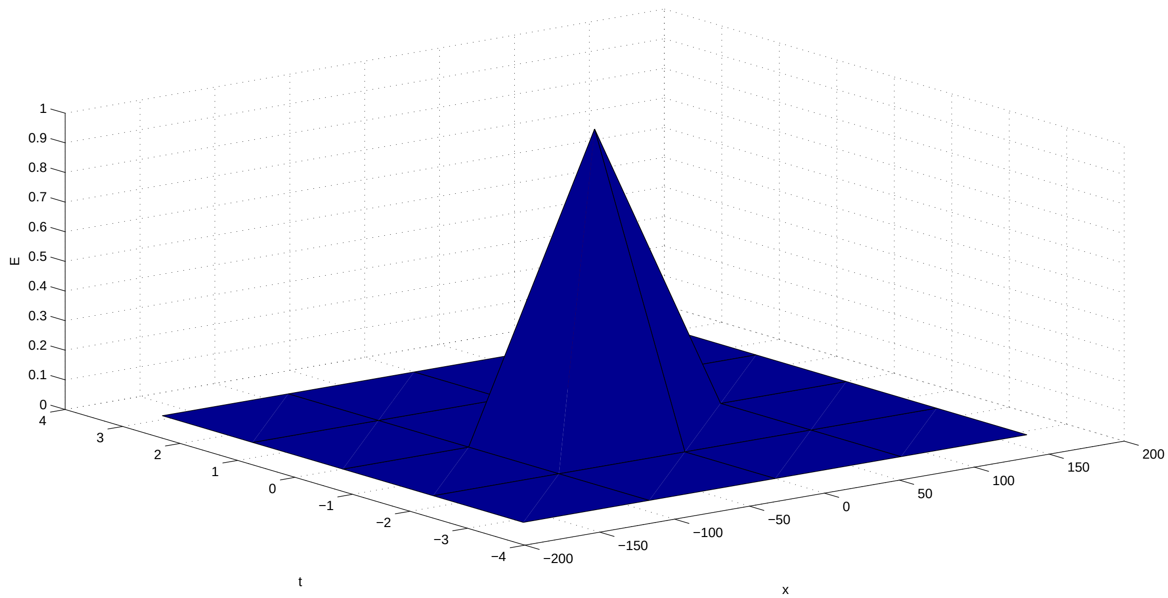

FIGURE 2. Peakon soliton associated to equation (44a).The parameters are those of fig(1)

Case 7

These solutions share their coefficients with those of Eq(25).

$$E_{13} = K\left(1 + \left(\tan\left(\sqrt{r}\xi\right)\right)^2\right)^{1/2\,\frac{\gamma\, e\mu_r C B_1}{c}} \mathrm{e}^{\frac{\gamma\, e1\; \mu e1\; C A_0 \xi}{c}}, \tag{43a}$$

$$E_{14} = K\left(1 + \left(\cot\left(\sqrt{r}\xi\right)\right)^2\right)^{1/2\,\frac{\gamma\, e\mu_r C B_1}{c}} \mathrm{e}^{\frac{\gamma\, e\mu_r C A_0 \xi}{c}} \tag{43b}$$

Case 8

$$E_{15} = K\left(1 + \left(\tan\left(\sqrt{r}\xi\right)\right)^2\right)^{1/2\,\frac{\gamma\, e\mu_r C B_1}{c}} \mathrm{e}^{\gamma\, e\mu_r C\left(A_0\xi + \frac{A_2\sqrt{r}\sin\left(\sqrt{r}\xi\right)}{\cos\left(\sqrt{r}\xi\right)}\right)c^{-1}}, \tag{44a}$$

$$E_{16} = K\left(\left(\cot\left(\sqrt{r}\xi\right)\right)^2 + 1\right)^{-1/2\,\frac{\gamma\, e\mu_r C B_1}{c}} \mathrm{e}^{\gamma\, e\mu_r C\left(A_0\xi - \frac{A_2\sqrt{r}\cos\left(\sqrt{r}\xi\right)}{\sin\left(\sqrt{r}\xi\right)}\right)c^{-1}}. \tag{44b}$$

.

These solutions possess the same parameters with Eq(26). It graphical representation is given at fig(2)

3.2.2. *Solutions deriving from soliton solutions of Eq(1a).* .

Case 1

These solutions have the same coefficients with Eq(27).

$$E_{17} = KR(\xi)\left(\tanh\left(\sqrt{r}\xi\right)-1\right)^{-\frac{\gamma e\mu_r CB_1}{c(2\mu+2)}}\left(\tanh\left(\sqrt{r}\xi\right)+1\right)^{\frac{\gamma e\mu_r C}{cB_1(2\mu-2)}}, \tag{45a}$$

$$R(\xi) = \left(\mu\tanh\left(\sqrt{r}\xi\right)+1\right)^{-\frac{\gamma e\mu_r CB_1}{c(\mu+1)(\mu-1)}} e^{\gamma e\mu_r C\left(2A_1 \mathit{arctanh}\left(\frac{(\mu-1)\tanh\left(1/2\sqrt{r}\xi\right)}{\sqrt{(\mu+1)(\mu-1)}}\right)\frac{1}{\sqrt{(\mu+1)(\mu-1)}}+A_0\xi\right)c^{-1}}, \tag{45b}$$

$$E_{18} = \rho(\xi)\left(\coth\left(\sqrt{r}\xi\right)+1\right)^{\frac{\gamma e\mu_r CB_1}{c(2\mu-2)}} e^{\gamma e\mu_r C\left(2A_1 \mathit{arctanh}\left(1/2\,\frac{2\mu\tanh\left(1/2\sqrt{r}\xi\right)-2}{\sqrt{\mu^2+1}}\right)\frac{1}{\sqrt{\mu^2+1}}+A_0\xi\right)c^{-1}}, \tag{45c}$$

$$\rho(\xi) = K\left(\mu\coth\left(\sqrt{r}\xi\right)+1\right)^{-\frac{\gamma e\mu_r CB_1}{c(\mu+1)(\mu-1)}}\left(\coth\left(\sqrt{r}\xi\right)-1\right)^{-\frac{\gamma e\mu_r CB_1}{c(2\mu+2)}}, \tag{45d}$$

Case 2

$$E_{19} = K\left(\tanh\left(\sqrt{r}\xi\right)-1\right)^{-1/2\,\frac{\gamma e\mu_r CB_1}{c}}\left(\tanh\left(\sqrt{r}\xi\right)+1\right)^{-1/2\,\frac{\gamma e\mu_r CB_1}{c}} q(\xi), \tag{46a}$$

$$E_{20} = K\left(\coth\left(\sqrt{r}\xi\right)-1\right)^{-1/2\,\frac{\gamma e\mu_r CB_1}{c}}\left(\coth\left(\sqrt{r}\xi\right)+1\right)^{-1/2\,\frac{\gamma e\mu_r CB_1}{c}} q(\xi), \tag{46b}$$

$$q(\xi) = e^{\frac{\gamma e\mu_r CA_0\xi}{c^1}}. \tag{46c}$$

The coefficients of These solutions are those of Eq(28).

Case 3

These solutions use the same coefficients with Eq(29).

$$E_{21} = K\left(\tanh\left(\sqrt{r}\xi\right)+1\right)^{1/4\,\frac{\gamma e\mu_r CB_1}{c}}\left(\tanh\left(\sqrt{r}\xi\right)-1\right)^{-1/4\,\frac{\gamma e\mu_r CB_1}{c}}\varphi(\xi), \tag{47a}$$

$$\varphi(\xi) = e^{\gamma e\mu_r C\left(1/2\,\frac{B_1}{\tanh\left(\sqrt{r}\xi\right)+1}+A_0\xi\right)c^{-1}}, \tag{47b}$$

$$E_{22} = K\left(\coth\left(\sqrt{r}\xi\right)+1\right)^{1/4\,\frac{\gamma e\mu_r CB_1}{c}}\left(\coth\left(\sqrt{r}\xi\right)-1\right)^{-1/4\,\frac{\gamma e\mu_r CB_1}{c}}\theta(\xi), \tag{47c}$$

$$\theta(\xi) = e^{\gamma e\mu_r C\left(1/2\,\frac{B_1}{\coth\left(\sqrt{r}\xi\right)+1}+A_0\xi\right)c^{-1}}. \tag{47d}$$

Case 4

$$E_{23} = K\left(\tanh\left(\sqrt{r}\xi\right)+1\right)^{\frac{\gamma e\mu_r CB_1}{c(2\mu-2)}}\left(\tanh\left(\sqrt{r}\xi\right)-1\right)^{-\frac{\gamma e\mu_r CB_1}{c(2\mu+2)}} s(\xi), \tag{48a}$$

$$s(\xi) = \left(\mu\tanh\left(\sqrt{r}\xi\right)+1\right)^{-\frac{\gamma e\mu_r CB_1(\mu-1)}{c(\mu+1)}} e^{\frac{\gamma e\mu_r CA_0\xi}{c}}, \tag{48b}$$

$$E_{24} = K\left(\coth\left(\sqrt{r}\xi\right)+1\right)^{\frac{\gamma e\mu_1 CB_1}{c(2\mu-2)}}\left(\coth\left(\sqrt{r}\xi\right)-1\right)^{-\frac{\gamma e\mu_r CB_1}{c(2\mu+2)}}\varpi(\xi), \tag{48c}$$

$$\varpi(\xi) = \left(\mu\coth\left(\sqrt{r}\xi\right)+1\right)^{-\frac{\gamma e\mu_r CB_1(\mu-1)}{c(\mu+1)}} e^{\frac{\gamma e\mu_r CA_0\xi}{c}}, \tag{48d}$$

These solutions are associated to those of Eq(30), they then possess the same parameters.

Case 5

The following solutions derived from Eq(31), then, both share the same parameters,

$$E_{25} = \left(\tanh\left(\sqrt{r}\xi\right) - 1\right)^{-1/2\,\frac{\gamma\, e\mu_r C B_1}{c}} \left(\tanh\left(\sqrt{r}\xi\right) + 1\right)^{-1/2\,\frac{\gamma\, e\mu_r C B_1}{c}} \eta(\xi), \tag{49a}$$

$$\eta(\xi) = K e^{\gamma\, e\mu_r C \left(A_0\xi + \frac{A_2\sqrt{r}\sinh(\sqrt{r}\xi)}{\cosh(\sqrt{r}\xi)}\right) c^{-1}}, \tag{49b}$$

$$E_{26} = \left(\coth\left(\sqrt{r}\xi\right) - 1\right)^{-1/2\,\frac{\gamma\, e\mu_r C B_1}{c}} \left(\coth\left(\sqrt{r}\xi\right) + 1\right)^{-1/2\,\frac{\gamma\, e\mu_r C B_1}{c}} \phi(\xi), \tag{49c}$$

$$\phi(\xi) = K e^{\gamma\, e\mu_r C \left(A_0\xi - \frac{A_2\sqrt{r}\cosh(\sqrt{r}\xi)}{\sinh(\sqrt{r}\xi)}\right) c^{-1}}, \tag{49d}$$

Case 6

$$E_{27} = K\left(\tanh\left(\sqrt{r}\xi\right) - 1\right)^{-1/4\,\frac{\gamma\, e\mu_r C B_1}{c}} \left(\tanh\left(\sqrt{r}\xi\right) + 1\right)^{1/4\,\frac{\gamma\, e\mu_r C B_1}{c}} V(\xi), \tag{50a}$$

$$V(\xi) = e^{\gamma\, e\mu_r C \left(A_1 \tanh\left(1/2\sqrt{r}\xi\right) + 1/2\,\frac{B_1}{\tanh(\sqrt{r}\xi)+1} + A_0\xi\right) c^{-1}}, \tag{50b}$$

$$E_{28} = K\left(\coth\left(\sqrt{r}\xi\right) - 1\right)^{-1/4\,\frac{\gamma\, e\mu_r C B_1}{c}} \left(\coth\left(\sqrt{r}\xi\right) + 1\right)^{1/4\,\frac{\gamma\, e\mu_r C B_1}{c}} \psi(\xi), \tag{50c}$$

$$\psi(\xi) = e^{\gamma\, e\mu_r C \left(A_1\sqrt{2}\, arctanh\left(1/4\left(2\,\tanh\left(1/2\sqrt{r}\xi\right) - 2\right)\sqrt{2}\right) + 1/2\,\frac{B_1}{\coth(\sqrt{r}\xi)+1} + A_0\xi\right) c^{-1}}, \tag{50d}$$

All the constants of this equation are the same with those of Eq(32).

Case 7

$$E_{29} = K\left(\tanh\left(\sqrt{r}\xi\right) + 1\right)^{-1/4\,\frac{\gamma\, e\mu_r C B_1}{c}} \left(\tanh\left(\sqrt{r}\xi\right) - 1\right)^{1/4\,\frac{\gamma\, e\mu_r C B_1}{c}} \Psi(\xi), \tag{51a}$$

$$\Psi(\xi) = e^{\gamma\, e\mu_r C \left(-\frac{A_1}{\tanh(1/2\sqrt{r}\xi)} - 1/2\,\frac{B_1}{\tanh(\sqrt{r}\xi)-1} + A_0\xi\right) c^{-1}}, \tag{51b}$$

$$E_{30} = K\left(\coth\left(\sqrt{r}\xi\right) + 1\right)^{-1/4\,\frac{\gamma\, e\mu_r C B_1}{c}} \left(\coth\left(\sqrt{r}\xi\right) - 1\right)^{1/4\,\frac{\gamma\, e\mu_r C B_1}{c}} \Phi(\xi), \tag{51c}$$

$$\Phi(\xi) = e^{\gamma\, e\mu_r C \left(-A_1\sqrt{2}\, arctanh\left(1/4\left(2\,\tanh\left(1/2\sqrt{r}\xi\right) + 2\right)\sqrt{2}\right) - 1/2\,\frac{B_1}{\coth(\sqrt{r}\xi)-1} + A_0\xi\right) c^{-1}}, \tag{51d}$$

These solutions share their constant parameters with those of Eq(32).

4. Conclusion

This paper focused on the semiconductors one level reaction-diffusion equation system. This equation has been subject to many investigations such as chaos [18], current filament [18], Turing bifurcation [21], and solitons [19]. According to our knowledge a study on which two methods are combined to solve a given system of equation has not been done yet. Except the Eq(28) all the obtained solutions are new: there are classified as, periodic and hyperbolic ie according to electron density. These solutions may find their applications in many domains where semiconductor devices are used: in nonlinear electronics for the introduction of new switching devices. Moreover, solitary carriers transport in semiconductors could be used to reduce losses in solar cells like in [22]. In addition the solitary form of the electric field (fig(2)) could be helpful for developing lossless wireless data transport devices needed in telecommunication.

References

[1] Khaled A. Gepreel, Exact solutions for nonlinear partial Differential equations and its applications, International Journal of Pure and Applied Mathematics vol. 86, 241-255, 2013.

[2] Li XZ, Wang ML, A sub-ODE method for finding exact solutions of a generalized KdV-mKdV equation with higher order nonlinear terms, Phys. Lett. A, vol. 361, 115-118, 2007.

[3] Hu JL, Explicit solutions to three nonlinear physical models, Phys. Lett. A., vol. 287, 81-89, 2001.

[4] Hu JL, A new method for finding exact traveling wave solutions to nonlinear partial differential equations, Phys. Lett. A, vol. 286, 175-179, 2001.

[5] Hirota R, The Direct Method in Soliton Theory, Cambridge University Press, 2004.

[6] Parkes EJ and Duffy BR, Travelling solitary wave solutions to a compound KdV-Burgers equation, Phys. Lett. A, vol 229 217-220, 1997.

[7] Fan E., Extended tanh-function method and its applications to nonlinear equations, Phys. Lett. A, vol 277, 212-218 2000.

[8] Yan ZY, New explicit travelling wave solutions for two new integrable coupled nonlinear evolution equations, Phys. Lett. A, vol. 292, 100-106, 2001.

[9] Chen Y and Zheng Y, Generalized extended tanh-function method to construct new explicit exact solutions for the approximate equations for long water waves, Int. J. Mod. Phys. C., vol. 14, 601-611, 2003.

[10] Wang ML, Application of a homogeneous balance method to exact solutions of nonlinear equations in mathematical physics, Phys. Lett. A, vol., 216, 67-75, 1996.

[11] Lu DC and Hong BJ, New exact solutions for the (2+1)-dimensional Generalized Broer-Kaup system, Appl. Math. Comput., vol., 199, 572-580, 2008.

[12] Porubov AV, Periodical solution to the nonlinear dissipative equation for surface waves in a convecting liquid, Phys. Lett. A, vol., 221, 391-394, 1996.

[13] Wazwaz M, he tanh and sine- cosine method for compact and noncompact solutions of nonlinear Klein Gordon equation, Appl. Math. Comput., vol., 167, 1179-1195, 2005.

[14] Yan ZY and Zhang HQ, New explicit solitary wave solutions and periodic wave solutions for WhithamBroerKaup equation in shallow water, Phys. Lett. A, vol., 285, 355-362, 2001.

[15] Zayed EME and Gepreel KA, The $(G^{'}/G)$- expansion method for finding traveling wave solutions of nonlinear PDEs in mathematical physics, J. Math. Phys., vol. 50, 013502-013514, 2009.

[16] Gepreel KA, Exact Solutions for Nonlinear PDEs with the Variable Coefficients in Mathematical Physics, J. Information and Computing Science, vol., 6, 3-14, 2011.

[17] E. S. Fahmy and H. A. Abdusalam, Exact Solutions for Some Reaction Diffusion Systems with Nonlinear Reaction Polynomial Terms, Applied Mathematical Sciences, 533-540, 2009.

[18] Kazunori Aoki, Nonlinear Dynamics and Chaos in Semiconductors, IOP Publishing Ltd, London, 2001.

[19] Mibaile Justin, Gambo Betchewe, Serge Y. Doka, Kofane Timoleon Crepin, Exact solutions of a semiconductor nonlinear reaction-diffusion equation through factorization method, Applied Mathematics and computation vol., 219, 2917-2922, 2012.

[20] Jian-qin Mei, Hong-qing Zhang, Dong-mei Jiang, New Exact Solutions for a Reaction-diffusion Equation and a Quasi-Camassa Holm Equation , Applied Mathematics E-notes, vol., 4, 85-91, 2004.

[21] K. Aoki, Turing bifurcation during the nucleation process of a current density filament in n-GaAs, Solid State Communications, vol., 114, 203-207, 2000.

[22] Zieba Falama R, Justin M, Djongyang N and Yamigno Doka S, A Novel Technique for improving the efficiency of a photovoltaic silicon solar cell based on the band-trap impact ionization phenomenon. J. Fundam. Appl. Sci., vol., 7, 375-393, 2015.

Mibaile Justin

Higher Teachers' Training College of Maroua, The university of Maroua, P.O. Box. 46, Cameroon

E-mail address: thejust7@yahoo.fr

Gasissou Marcel
Higher Teachers' Training College of Maroua, The university of Maroua, P.O. Box. 46, Cameroon
E-mail address: marcelgasissou@ymail.com

Gambo Betchewe
Department of Physics, Faculty of Science, The university of Maroua, P.O. Box. 46, Cameroon
E-mail address: gambobetch@yahoo.fr

Serge Y. Doka
Department of Physics, Faculty of Science, University of Ngaoundere, P.O. Box 454, Cameroon
E-mail address: numami@gmail.com

Kofane Timoleon Crepin
Department of Physics, Faculty of Science, The university of Yaounde I, P.O. Box. 812, Cameroon
E-mail address: tckofane@yahoo.com