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ON THE ROUGH CONVERGENCE OF A SEQUENCE OF SETS

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Abstract. In the present paper, we give an alternative expression of rough Wijsman convergence of a sequence of sets. Defining the concept of rough Wijsman limit set of a sequence, we prove some inclusion relations related to this limit set. Finally, we examine the relation between rough Wijsman convergence and rough Hausdorff convergence.

1. Introduction

Set convergence theory was pioneered by Painleve in 1902. In fact, this theory was originally developed by Kuratowski [8]. Therefore, this convergence is called the Kuratowski convergence by many authors. On the other hand, Hausdorff defined the concept of "Hausdorff convergence" which corresponds to the uniform convergence of a sequence of distance functions. Wijsman ([16], [17]) introduced the concept of "Wijsman convergence", by using the pointwise convergence of distance functions. In the 2000's, Nuray and Rhoades [9] applied the statistical convergence theory to a sequence of sets. They also gave the definitions of boundedness and Wijsman Cauchy for a sequence of sets. Kişi and Nuray [7] extended the concept of Wijsman convergence to Wijsman I-convergence and Wijsman I^*-convergence. Hazarika and Esi [6] defined the idea of asymptotically equivalent sequences of sets in the sense of ideal Wijsman convergence. Recently, Dündar and Pancaroğlu [4] have introduced the concepts of Wijsman lacunary invariant convergence and Wijsman lacunary invariant statistical convergence.

Rough convergence grew out of Phu [11] investigations of a sequence which is not convergent in the usual sense in a finite dimensional normed space. He introduced this sequence might be convergent to a point with a certain degree of roughness. He gave some basic properties of the rough limit set. Then Aytar [2] obtained the relation between core and rough limit set of a sequence.

Recently, theory of rough convergence has begun to be applied to sequences of sets. In this context, the concept of rough Wijsman convergence was first defined by Ölmez and Aytar [10]. They explored the effect of the asymptotic cone of the

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limit set of a sequence that is rough Wijsman convergent. Subramanian and Esi [14, 15] introduced the notions of rough Wijsman convergence and rough Wijsman statistical convergence for a triple sequence of sets. Defining rough Wijsman limit set for triple sequences of sets, they obtained some convergence criteria. Later, Esi et al. [5] studied on rough Wijsman statistical convergence. They stated the analogous definitions and results for rough Wijsman statistical convergence of a triple sequence.

In this article, an equivalent definitions of rough Wijsman convergence and of rough Hausdorff convergence are given (see Propositions 2.2 and 3.1). Defining the concept of the set of rough Wijsman limit, some inclusion relations related to this limit set are proved (see Propositions 2.1, 2.3 and 2.4). Finally, the relation between rough Wijsman convergence and rough Hausdorff convergence is examined (see Theorem 3.1).

2. Rough Wijsman Convergence

Throughout this paper, we assume that X is a nonempty set and ρ_X is a metric on X and A, A_n are nonempty closed subsets of X for each $n \in \mathbb{N}$.

Let $\{x_n\}$ be a sequence in the metric space X, and r be a nonnegative real number. The sequence $\{x_n\}$ is said to be rough convergent to x with the roughness degree r, denoted by $x_n \xrightarrow{r} x$, if for each $\varepsilon > 0$ there exists an $n(\varepsilon) \in \mathbb{N}$ such that $\rho_X(x_n, x) < r + \varepsilon$ for each $n \geq n(\varepsilon)$ [11].

The *distance function* $d(\cdot, A) : X \to [0, \infty)$ is defined by the formula

$$d(x, A) = \inf\{\rho_X(x, y) : y \in A\} \text{ [1, 16].}$$

We say that the sequence $\{A_n\}$ is *Wijsman convergent* to the set A if

$$\lim_{n\to\infty} d(x, A_n) = d(x, A) \text{ for all } x \in X.$$

In this case, we write $A_n \xrightarrow{W} A$, as $n \to \infty$ [16].

The set A is called *Wijsman cluster point* of the sequence $\{A_n\}$ provided that there is a subsequence that Wijsman converges to A. In this case $L_{\{A_n\}}$ denotes the set of all cluster points of the sequence $\{A_n\}$.

Throughout the paper we suppose that $r \geq 0$.

In 2016, the concept of rough Wijsman convergence was first defined by Ölmez and Aytar [10] as follows:

Definition 2.1. *A sequence $\{A_n\}$ is said to be $r-$Wijsman convergent to the set A if for every $\varepsilon > 0$ and each $x \in X$ there is an $N(\varepsilon, x) \in \mathbb{N}$ such that*

$$|d(x, A_n) - d(x, A)| < r + \varepsilon \text{ for all } n \geq N(\varepsilon, x)$$

and we write $d(x, A_n) \xrightarrow{r} d(x, A)$ or $A_n \xrightarrow{r-W} A$ as $n \to \infty$ [10].

Now define

$$1 - LIM^r A_n = \{A \subset X : A_n \xrightarrow{r-W} A\}. \tag{2.1}$$

If a sequence is Wijsman convergent, then this sequence $r-$Wijsman converges to the same set for each r. However, there are some sequences of sets which are $r-$Wijsman convergent, but not Wijsman convergent as can be seen in the following

Example 2.1 ([1])**.** *Let $X = \mathbb{R}^2$ and define a sequence $\{A_n\}$ as follows:*

$$A_n =: \begin{cases} \left\{\frac{1}{n}\right\} \times [-1,0] & \text{, if } n \text{ is an odd integer} \\ \left\{\frac{1}{n}\right\} \times [0,1] & \text{, if } n \text{ is an even integer} \end{cases}.$$

This sequence is not Wijsman convergent to the set $A = \{(0,0)\}$. But, this sequence is $r-$Wijsman convergent to the set A for $r \geq 1$.
If we take $x = (0,1)$, then we have $d((0,1), A) = 1$ and

$$d((0,1), A_n) = \begin{cases} \sqrt{1 + \frac{1}{n^2}} & \text{, if } n \text{ is an odd integer} \\ \frac{1}{n} & \text{, if } n \text{ is an even integer} \end{cases}.$$

Since

$$\limsup_{n\to\infty} d((0,1), A_n) = 1 \text{ and } \liminf_{n\to\infty} d((0,1), A_n) = 0,$$

we have that $\lim_{n\to\infty} d(x, A_n)$ does not exist. Therefore, this sequence is not Wijsman convergent to the set A, but this sequence is $r-$Wijsman convergent to the set A for $r \geq 1$.

Remark 2.1. *In the Example 2.1, if we take $A \in 1 - LIM^r A_n$, then we have*

$$A \subseteq \overline{B}\left((0,0), r\right) \tag{2.2}$$

where $\overline{B}\left((0,0), r\right) = \left\{x \in \mathbb{R}^2 : \rho((0,0), x) \leq r\right\}$. Now let us prove the inclusion (2.2). On the contrary, suppose that $A \nsubseteq \overline{B}\left((0,0), r\right)$. For the sake of generality let us choose $x = (x_1, x_2) \in \mathbb{R}^2$ such that $x_1 > 0$ and $x_2 > 1$. Then, there exists an $x = (x_1, x_2) \in A$ such that $\sqrt{x_1^2 + x_2^2} > r$. Since $d(x, A) = 0$, we have

$$d(x, A_n) = \begin{cases} \sqrt{\left(x_1 - \frac{1}{n}\right)^2 + x_2^2} & \text{, if } n \text{ is an odd integer} \\ \sqrt{\left(x_1 - \frac{1}{n}\right)^2 + (x_2 - 1)^2} & \text{, if } n \text{ is an even integer} \end{cases}.$$

As $n \to \infty$, we get

$$\begin{aligned} |d(x, A_n) - d(x, A)| &= |d(x, A_n) - 0| \\ &= \begin{cases} \sqrt{x_1^2 + x_2^2} & \text{, if } n \text{ is an odd integer} \\ \sqrt{x_1^2 + (x_2 - 1)^2} & \text{, if } n \text{ is an even integer} \end{cases}. \end{aligned}$$

If n is an odd integer, then we have $|d(x, A_n) - d(x, A)| > r$. Thus we have $A \notin 1 - LIM^r A_n$. This contradiction completes the proof.
If the origin is an element of the set A and A is contained in the $r-$closed ball, then we have $A \in 1 - LIM^r A_n$.

Remark 2.2. *In the Example 2.1, if we take $A = \left\{\left(\frac{1}{2}, 0\right)\right\}$ then the sequence $\{A_n\}$ is not $r-$Wijsman convergent to the set A for $r = 1$. Since $r = \sup_{c \in L_{\{A_n\}}} \left\|\left(\frac{1}{2}, 0\right) - c\right\| = \frac{\sqrt{5}}{2}$, the sequence $\{A_n\}$ is $r-$Wijsman convergent to the set $A = \left\{\left(\frac{1}{2}, 0\right)\right\}$ for $r = \frac{\sqrt{5}}{2}$. Moreover this sequence is not $r-$Wijsman convergent to the set A for all $r < \frac{\sqrt{5}}{2}$.*

Now define

$$2 - LIM^r A_n = \bigcap_{c \in L_{\{A_n\}}} \overline{B}(c, r) = \{A \in X : L_{\{A_n\}} \subseteq \overline{B}(A, r)\}. \tag{2.3}$$

Then, for the usual rough convergence of a sequence of elements in any metric space, the sets defined by the equalities (2.1) and (2.3) coincide with each other. But these sets are not equal for an arbitrary sequence of sets. The following example compares the sets $1-LIM^rA_n$ and $2-LIM^rA_n$.

Example 2.2. *Let $X=\mathbb{R}^2$ and define a sequence $\{A_n\}$ as follows:*

$$A_n := \begin{cases} \left\{-\frac{1}{n}\right\} \times \left[0, 2+\frac{1}{n}\right] & , \textit{if } n \textit{ is an odd integer} \\ \left\{\frac{1}{n}\right\} \times \left[-2-\frac{1}{n}, 0\right] & , \textit{if } n \textit{ is an even integer} \end{cases} .$$

This sequence is not Wijsman convergent to the set $A=\{(0,0)\}\times[-2,2]$. If we take $x=(9,10)$, then we have $d(x,A)=\sqrt{145}$ and

$$d(x,A_n) = \begin{cases} \sqrt{145} & , \textit{if } n \textit{ is an odd integer} \\ \sqrt{181} & , \textit{if } n \textit{ is an even integer} \end{cases} .$$

Since

$$|d(x,A_n)-d(x,A)| = \begin{cases} 0 & , \textit{if } n \textit{ is an odd integer} \\ \sqrt{181}-\sqrt{145} & , \textit{if } n \textit{ is an even integer} \end{cases} ,$$

this sequence is not Wijsman convergent to the set A. But, this sequence is $r-$Wijsman convergent to the set A for $r\geq 2$. Moreover, we have $A\in 1-LIM^rA_n$ and

$$2-LIM^rA_n = \overline{B}\left(\{(0,0)\}\times[0,2],2\right) \cap \overline{B}\left(\{(0,0)\}\times[-2,0],2\right) = \overline{B}\left((0,0),2\right)$$

for $r=2$. Hence, the definition of $1-LIM^rA_n$ does not coincide with that of $2-LIM^rA_n$.

We are ready to give following inclusion.

Proposition 2.1. *If $A\in 1-LIM^rA_n$, then $A\subseteq 2-LIM^rA_n$.*

Proof. Let $A\in 1-LIM^rA_n$. On the contrary, assume that $A\nsubseteq 2-LIM^rA_n$. Then we have there exists $y\in A$ such that $y\notin 2-LIM^rA_n$. Thus we get

$$\begin{aligned} y\notin 2-LIM^rA_n \quad &\Rightarrow y\notin \bigcap_{c\in L_{\{A_n\}}} \overline{B}(c,r) \\ &\Rightarrow \exists c\in L_{\{A_n\}} \therefore y\notin \overline{B}(c,r) \\ &\Rightarrow \rho(y,c)>r \\ &\Rightarrow \inf_{y\in A}\rho(y,c)>r \\ &\Rightarrow d(c,A)\geq r. \end{aligned}$$

Since $A\in 1-LIM^rA_n$, for every $\varepsilon>0$ and each $x\in X$ there exists an $N(\varepsilon,x)\in\mathbb{N}$ such that $|d(x,A_n)-d(x,A)|<r+\varepsilon$ for all $n\geq N(\varepsilon,x)$. If we take $x=c$ which is a cluster point, then $d(c,A_n)<\varepsilon$ for all $n\geq N(\varepsilon,x)$. Because of $d(c,A)\geq r$ and $d(c,A_n)<\varepsilon$, we have $|d(c,A_n)-d(c,A)|\geq r$ as $n\to\infty$. This inequality contradicts to the fact that $A\in 1-LIM^rA_n$. Thus we obtain $A\subseteq 2-LIM^rA_n$. □

Now we will consider a sequence which is $r-$Wijsman convergent and has infinitely many cluster points.

Example 2.3. *Let $X=\mathbb{R}$ and define a sequence $\{A_n\}$ as follows:*

$$A_n := \left(\{0\},\{0\},\{1\},\{0\},\left\{\frac{1}{2}\right\},\{1\},\{0\},\left\{\frac{1}{3}\right\},\left\{\frac{2}{3}\right\},\{1\},\ldots\right).$$

It is clear that, for each $c \in [0,1]$, the singleton $\{c\}$ is a cluster point of $\{A_n\}$. Thus we have

$$2 - LIM^r A_n = \bigcap_{c\in[0,1]} \overline{B}(c,1) = \bigcap_{c\in[0,1]} [c-1, c+1] = [0,1].$$

Moreover, if we take $A = [0,1]$, then we have $A \in 1 - LIM^r A_n$ for $r = 1$.

The following proposition characterizes $r-$Wijsman convergence by means of the upper limit.

Proposition 2.2. *For every $\varepsilon > 0$ and each $x \in X$ there exists an $N(\varepsilon, x) \in \mathbb{N}$ such that*

$$|d(x, A_n) - d(x, A)| < r + \varepsilon \text{ for all } n \geq N(\varepsilon, x)$$

if and only if

$$\limsup_{n\to\infty} |d(x, A_n) - d(x, A)| \leq r.$$

Proof. *(Necessity)* Suppose that $d(x, A_n) \xrightarrow{r} d(x, A)$. Then, for every $\varepsilon > 0$ and each $x \in X$ there exists an $N(\varepsilon, x) \in \mathbb{N}$ such that $f_n(x) := |d(x, A_n) - d(x, A)| < r + \varepsilon$ for all $n \geq N(\varepsilon, x)$. On the contrary, suppose that

$$\limsup_{n\to\infty} f_n(x) > r.$$

Take

$$\delta(x) = \frac{\limsup\limits_{n\to\infty} f_n(x) - r}{3}.$$

By definition of lim sup, we have

$$f_n(x) > \limsup_{n\to\infty} f_n(x) - \delta(x) \text{ for infinitely many } n.$$

Take $\varepsilon = \delta(x)$, then there exists an $N(\varepsilon, x) \in \mathbb{N}$ such that

$$f_n(x) < r + \delta(x) \text{ for all } n \geq N(\varepsilon, x).$$

This inequality contradicts to the definition of lim sup. Thus, we obtain $\limsup\limits_{n\to\infty} f_n(x) \leq r$ for all $x \in X$.

(Sufficiency) Let $\limsup\limits_{n\to\infty} f_n(x) \leq r$ for every $x \in X$. Assume on the contrary that there exists an $\widetilde{\varepsilon} > 0$ and an $x \in X$ and infinitely many n such that $f_n(x) \geq r + \widetilde{\varepsilon}$. Now, by the assumption, there exists an $N(\widetilde{\varepsilon}, x) \in \mathbb{N}$ such that

$$f_n(x) \leq \left[\limsup_{n\to\infty} f_n(x)\right] + \widetilde{\varepsilon} \text{ for all } n \geq N(\widetilde{\varepsilon}, x).$$

This inequality contradicts to the fact that $f_n(x) \geq r + \widetilde{\varepsilon}$, hence the proof is complete. □

Proposition 2.3. *If $\{A_{k_n}\}$ is a subsequence of $\{A_n\}$, then*

$$1 - LIM^r A_n \subseteq 1 - LIM^r A_{k_n}.$$

Proof. Let $A \in 1 - LIM^r A_n$. That is, for every $\varepsilon > 0$ and each $x \in X$ there exists an $N(\varepsilon, x) \in \mathbb{N}$ such that

$$|d(x, A_n) - d(x, A)| < r + \varepsilon \text{ for all } n \geq N(\varepsilon, x).$$

Since $\{A_{k_n}\}$ is a subsequence of $\{A_n\}$, we have

$$|d(x, A_{k_n}) - d(x, A)| < r + \varepsilon \text{ for all } k_n \geq N(\varepsilon, x).$$

Thus we have $A \in 1 - LIM^r A_{k_n}$. □

Proposition 2.4. *If $r_1 \leq r_2$, then we have*

$$1 - LIM^{r_1} A_n \subseteq 1 - LIM^{r_2} A_n \text{ and } 2 - LIM^{r_1} A_n \subseteq 2 - LIM^{r_2} A_n.$$

Proof. Let $r_1 \leq r_2$ and $A \in 1 - LIM^{r_1} A_n$. Hence, for every $\varepsilon > 0$ and each $x \in X$ there exists an $N(\varepsilon, x) \in \mathbb{N}$ such that

$$\begin{aligned} |d(x, A_n) - d(x, A)| &< r_1 + \varepsilon \\ &\leq r_2 + \varepsilon \text{ for all } n \geq N(\varepsilon, x). \end{aligned}$$

Thus we have $A \in 1 - LIM^{r_2} A_n$.

If $r_1 \leq r_2$ and $c \in L_{\{A_n\}}$ then

$$\begin{aligned} \overline{B}(c, r_1) &\subseteq \overline{B}(c, r_2) \\ \bigcap_{c \in L_{\{A_n\}}} \overline{B}(c, r_1) &\subseteq \bigcap_{c \in L_{\{A_n\}}} \overline{B}(c, r_2). \end{aligned}$$

Thus we get $2 - LIM^{r_1} A_n \subseteq 2 - LIM^{r_2} A_n$. □

3. Rough Hausdorff Convergence

The *Hausdorff distance* between A and B, denoted by $H(A, B)$, is defined as

$$H(A, B) := \sup_{x \in X} |d(x, A) - d(x, B)| \text{ [12]}.$$

We say that a sequence $\{A_n\}$ is *Hausdorff convergent* to the set A if $\lim_{n \to \infty} H(A_n, A) = 0$, and we denote $A_n \overset{H}{\to} A$, as $n \to \infty$. That is, we have $A_n \overset{H}{\to} A$ provided that, for every $\varepsilon > 0$ there exists an $N(\varepsilon) \in \mathbb{N}$ such that

$$H(A_n, A) = \sup_{x \in X} |d(x, A_n) - d(x, A)| < \varepsilon \text{ for all } n \geq N(\varepsilon) \text{ [12]}.$$

Example 3.1 ([13])**.** *Define a sequence $\{A_n\}$ by*

$$A_n := \left(-\infty, -1 - \frac{1}{n}\right] \cup \left[2 + \frac{1}{n}, \infty\right).$$

This sequence is Hausdorff convergent to the set $A = (-\infty, -1] \cup [2, \infty)$.

Now we will define a new type of Hausdorff convergence for a sequence of closed sets. The concept of rough convergence in a metric space was first introduced by Debnath and Rakshit [3]. Since the set of all closed sets is a metric space with the Hausdorff metric, following definition is a special case of Debnath and Rakshit's [3] definition.

Definition 3.1. *The sequence $\{A_n\}$ is said to be $r-$Hausdorff convergent to the set A if for every $\varepsilon > 0$ there exists an $N(\varepsilon) \in \mathbb{N}$ such that*

$$H(A_n, A) = \sup_{x \in X} |d(x, A_n) - d(x, A)| < r + \varepsilon \text{ for all } n \geq N(\varepsilon).$$

In this case, we write $A_n \overset{r-H}{\to} A$ as $n \to \infty$.

Now let us define

$$H-LIM^r A_n = \left\{ A \subset X : A_n \overset{r-H}{\to} A \right\}.$$

If a sequence is Hausdorff convergent, then this sequence is $r-$Hausdorff convergent to the same set for each r. However, the converse of this claim does not hold in general, as can be seen following

Example 3.2. *Let us consider the sequence $\{A_n\}$ defined in Example 3.1. This sequence is $r-$Hausdorff convergent to the set $\mathbb{R}$ for each $r \geq \frac{3}{2}$.*

The following proposition characterizes the concept of $r-$Hausdorff convergence by means of the upper limit. Its proof is similar to that of Proposition 2.1, hence we omit it.

Proposition 3.1. *For every $\varepsilon > 0$ there exists an $N(\varepsilon) \in \mathbb{N}$ such that*

$$H(A_n, A) = \sup_{x \in X} |d(x, A_n) - d(x, A)| < r + \varepsilon \text{ for all } n \geq N(\varepsilon)$$

if and only if

$$\limsup_{n \to \infty} \sup_{x \in X} |d(x, A_n) - d(x, A)| \leq r.$$

Finally, giving the relation between the concepts of rough Wijsman convergence and rough Hausdorff convergence, we will end our work.

Theorem 3.1. *If the sequence $\{A_n\}$ is $r-$Hausdorff convergent to the set A, then it is $r-$ Wijsman convergent to the same set.*

Proof. Suppose that the sequence $\{A_n\}$ is $r-$Hausdorff convergent to the set A. Then, for every $\varepsilon > 0$ there exists an $N(\varepsilon) \in \mathbb{N}$ such that

$$H(A_n, A) = \sup_{x \in X} |d(x, A_n) - d(x, A)| < r + \varepsilon \text{ for all } n \geq N(\varepsilon).$$

Hence we have

$$|d(x_*, A_n) - d(x_*, A)| < \sup_{x \in X} |d(x, A_n) - d(x, A)| < r + \varepsilon$$

for all $n \geq N(\varepsilon)$. Since x_* is an arbitrary point, we say that the sequence $\{A_n\}$ is $r-$Wijsman convergent to the set A. □

The converse of the above theorem does not hold in general as can be seen in the following

Example 3.3. *Let $X = \mathbb{R}^2$ and define a sequence $\{A_n\}$ as follows:*

$$A_n := \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 + 2ny = 0\}.$$

This sequence is $r-$ Wijsman convergent to the set $A = \mathbb{R} \times [0, 1]$, but it is not $r-$Hausdorff convergent to the set A for each r.

4. Conclusion

This paper is an application of the rough convergence theory to the sequences of sets in the sense of Wijsman convergence and Hausdorff convergence. We proved some inclusion relations related to rough limit set of a sequence of sets. Similarly these inclusion relations can be extended to the double or multiple sequences of sets. Moreover the relation between rough Wijsman convergence and rough Hausdorff convergence of these types of sequences can be examined.

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