



Original article

Entropy inside the atoms

Ahmed M. M. Elsheekh*

High Institute for Engineering & Technology – Obour, Km 21 Cairo/Belbeis Rd. Egypt.

ARTICLE INFO

Received 28/05/2022
Revised 05/08/2022
Accepted 03/10/2022

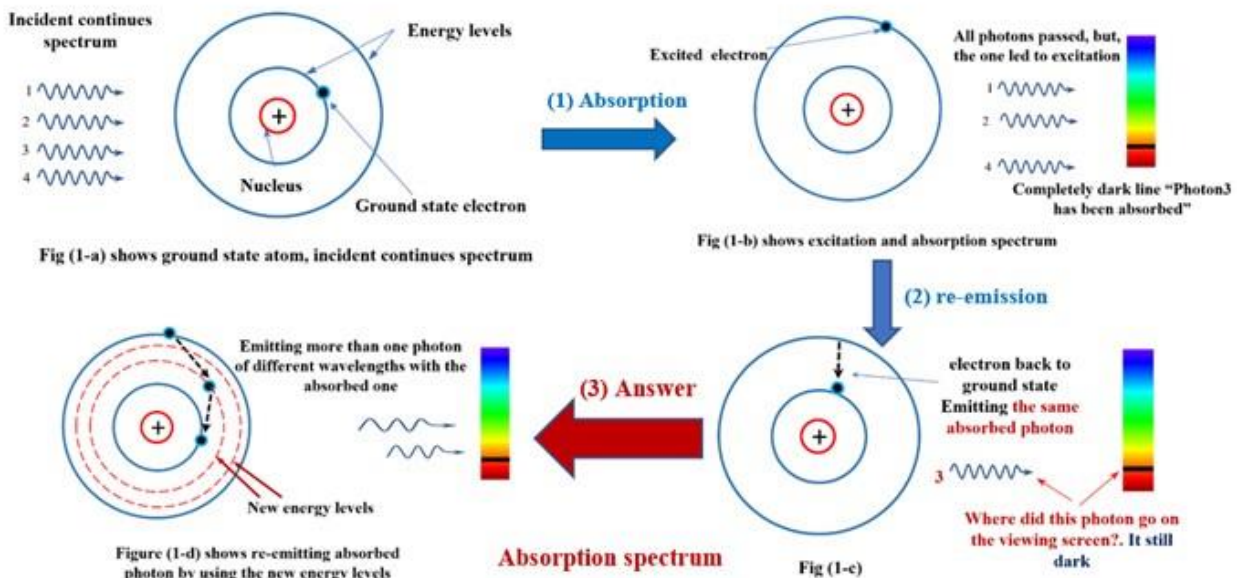
Keywords

Entropy
Theoretical Physics
Spectroscopy
Atomic physics
Quantum Physics

ABSTRACT

All experimental data in atomic physics proved that, emission and absorption processes are completely reverses to each other. Emitted photon is the same as absorbed; excited atom returns to its ground state without any residual tells us there is an action has been done. While studying and interpreting the process of absorption spectra, there is a question that remains unanswered till now, which is; when the electron returns to the ground state, it re-emits the same absorbed photon, but where did this photon go? If it was re-emitted, then the observed absorbed spectrum should appear pale, not completely dark line which mean it has been disappeared. In order to be able to say that the electron lost its energy while returning to its ground state without re-emitting the same absorbed photon or by emitting more than one photon, we must introduce theoretically new energy levels to be attributable to the answer to the question. In this article, we present a mathematical proposal that leads to the inference of new quantized energy levels, where an electron can move among them thermally or spectroscopically. All elements' ionization energy has been proved by using this new mathematical proposal led to results, which are in agreement with the National Institute of Standards and Technology (NIST) data. Not only entropy inside the atom has been achieved but also, the disappeared absorbed photon has been explained. At last, our mathematical proposal connected between modern physics and classical physics.

Graphical abstract



* Corresponding author

E-mail address: elsheekh_physics@yahoo.com

DOI: 10.21608/IJTAR.2023.141132.1008

1. Introduction

Niels Bohr assumed in his great paper angular momentum of electron = $h/2\pi$ [1-5].

$$2\pi m_e v r = n h \quad (1)$$

Where, (m_e) is the mass of the electron ($m_e=9.10938291 \times 10^{-31}$ kg) [9], (v) is the velocity of the electron, and (r) is the radius of the electron, (h) is Planck's constant ($h=6.62606957 \times 10^{-34}$ J.s) [9], (π) is constant ($\pi=3.14159265$) [9] and (n) is an integer number. Although this assumption of "angular momentum" is a physical quantity, the other side of the equation is just a half integer repetition of Planck's constant divided by (π), at this time of the 20th century this assumption couldn't be accepted, unless it proved experimental data. This assumption gave birth to Bohr's theory, which is considered a physical revolution in the records of humanity history because it was able to explain the emission spectra of the hydrogen atom. Although Bohr's theory was true only and only for the hydrogen atom, and completely failed to explain the spectra of the rest of the elements of the periodic table, it has been accepted because of this principle "If you reject the current theory, then, please, introduce another theory or equation, more acceptable and accurate". Quantum physics and quantum mechanics have been created to complete Bohr work.

2. Overview

According to Bohr assumption, the square velocity of electron is given by the following equation

$$v^2 = \left(\frac{nh}{2\pi m_e r} \right)^2 \quad (2)$$

Where, (r) is radius of electron and (n) is constant. Balance between coulomb force and central fugal force has been given by the following equation,

$$\frac{m_e v^2}{r} = \frac{Ze^2}{4\pi\epsilon_0 r^2} \quad (3)$$

Where, (Z) is atomic number Hydrogen atom, (e) is charge of electron ($e = 1.602176565 \times 10^{-19}$ C) [9] and (ϵ_0) is dielectric constant ($\epsilon_0 = 8.854187817 \times 10^{-12}$ F/m) [9]

$$v^2 = \frac{e^2}{4 m_e \pi \epsilon_0 r} \quad (4)$$

General common universal equation of electron's total energy [5], is given by,

$$E = \frac{1}{2} m_e v^2 - \frac{Ze^2}{4\pi\epsilon_0 r} \quad (5)$$

Where, (E) is total energy of electron (kinetic energy and potential energy). By substitution from equation (2) into equation (5)

$$E_{hv} = \frac{n^2 h^2}{8m\pi^2} \cdot \frac{1}{r^2} - \frac{Ze^2}{4\pi\epsilon_0} \cdot \frac{1}{r} \quad (6)$$

By substitution from equation (4) into equation (5)

$$E_p = \frac{-e}{8\pi\epsilon_0} \cdot \frac{1}{r} \quad (7)$$

By using reused mass (m).

$$m = \frac{m_e \times Z m_p}{m_e + Z m_p} \quad (8)$$

Where (m_p) is mass of protons (Proton rest mass $m_p = 1.672621777 \times 10^{-27}$ kg) [9].

By solving equation s (1,2 and 3), Bohr concluded electron energy for Hydrogen atom as given in equation s (10, 11),

$$r = \frac{n^2 h^2 \epsilon_0}{m Z e^2 \pi} \quad (9)$$

$$E = \frac{-Z e^2}{8\pi\epsilon_0} \cdot \frac{1}{r} \quad (10)$$

Can we conclude equation (11) by using another mathematical dealing?

3. Conflict factor of differential

In Can we say velocity of electron as a wave = velocity of electron as a particle? In other words, is equation (2) = equation (4)? All previous work proved that equation (2) = equation (4). We can prove previous work by first order of differential energy equation as shown in equation (12),

$$\frac{dE}{dr} = \left(\frac{-n^2 h^2}{4m\pi^2} \cdot \frac{1}{r^3} \right) + \left(\frac{e^2}{4\pi\epsilon_0} \cdot \frac{1}{r^2} \right) = 0 \quad (12)$$

Equation (12) gives us allowed radius where, $\left(\frac{-n^2 h^2}{4m\pi^2} \cdot \frac{1}{r^3} \right) = \left(\frac{e^2}{4\pi\epsilon_0} \cdot \frac{1}{r^2} \right)$, this leads to calculating value of energy level as given in equation (11) "as same as Bohr theory". But, sequence differential of equation (6) "D order of differential" generated allowed radius for electron "New Energy Levels of the Atom" [6], as shown in equation (13)

$$r = D \frac{n^2 h^2 \epsilon_0}{m Z e^2 \pi} \quad (13)$$

$$, D = \left(1, \frac{3}{2}, 2, \frac{5}{2}, \dots, \frac{d+1}{2} \right)$$

Where (D) is order of differential, which means number of orbitals. At last, by substituting from equation (13) in equation (7) leads to equation (14) [6].

$$E_e = \frac{1}{D} \cdot \frac{m Z^2 e^4}{8n^2 h^2 \epsilon_0^2} \quad (14)$$

$$, D = \left(1, \frac{3}{2}, 2, \frac{5}{2}, \dots, \frac{d+1}{2} \right)$$

Where, (E_e) energy of electron as a particle.

By substitution from equation (13) into equation (6) gives us other radiuses as shown in next relation

$$E_{hv} = \frac{1 - 2D}{D^2} \cdot \frac{m Z^2 e^4}{8n^2 h^2 \epsilon_0^2} \quad (15)$$

$$, D = \left(1, \frac{3}{2}, 2, \frac{5}{2}, \dots, \frac{d+1}{2} \right)$$

Where, (E_{hv}) energy of electron as a wave.

We call equation (14) energy of electron as a particle, where the initial radius and velocity of the electron has been concluded from Coulomb's law, but equation (15) is called energy of electron as a wave where its radius and velocity has been concluded from Bohr postulation. This conflict factor is given by equation (16)

$$M_f = \pm \left[\frac{1 - 3D}{D^2} \right] \cdot \frac{mZ^2e^4}{8n^2h^2\epsilon_0^2} \quad (16)$$

$$, D = \left(1, \frac{3}{2}, 2, \frac{5}{2}, \dots, \frac{d+1}{2} \right)$$

We call here equation (14) energy of electron as a particle, where the initial radius and velocity of the electron has been concluded from Coulomb's law or equation (4), but equation (15) energy of combining wave or “electron as wave behavior”.

4. Theoretical ionization energy

Sequence differential of energy equation generated New Energy Levels in the Atom and proved and satisfied all theoretical ionization energy of all elements that was declared by National Institute of Standards and Technology (NIST) [7,8]. Previous work did that for the electron as a particle only[6].

Table 1. Ionization energy of elements by Elsheekh Theory comparing with experimental data of (NIST).

El	Wave behavior		Particle behavior		(NIST)
	D _{Wave}	E _{hν}	D _{Pricle}	E _e	
H	1	13.5983	1	13.5983	13.5984
He	4	23.8035	2	27.204	24.5874
Li	45	5.3808	22.5	5.4413	5.3917
Be	46	9.3607	23.5	9.2622	9.3227
B	81.5	8.2949	41	8.2953	8.298
C	86.5	11.2585	43.5	11.2589	11.2603
N	91	14.5706	45.5	14.6511	14.5341
O	127.5	13.6046	64	13.6048	13.6181
F	126	17.4226	63.5	17.3542	17.4228
Ne	125.5	21.5948	63	21.5952	21.5645
Na	640	5.1404	320.5	5.1364	5.1391
Mg	512	7.6454	256	7.6529	7.6462
Al	767.5	5.9877	384	5.9877	5.9858
Si	653.5	8.1548	327	8.1548	8.1517
P	583.5	10.4835	292	10.4835	10.4867
S	672	10.3582	336	10.3659	10.36
Cl	606	12.9659	303	12.9766	12.9676
Ar	559	15.7573	279.5	15.7714	15.7596
K	2262.5	4.3407	1131.5	4.3407	4.3407
Ca	1780	6.1130	890	6.1148	6.1132
Sc	1828.5	6.5609	914.5	6.5609	6.5615
Ti	1928.5	6.8274	964.5	6.8274	6.8281
V	2133	6.7469	1067	6.7453	6.7462
Cr	2316	6.7660	1158	6.7674	6.7665
Mn	2287	7.4346	1144	7.433	7.434
Fe	2327	7.9031	1164	7.9014	7.9024
Co	2516.5	7.8811	1258.5	7.8811	7.881
Ni	2792	7.6395	1396	7.6409	7.6399
Cu	2961.5	7.7260	1481	7.726	7.7264
Zn	2606.5	9.3939	1303.5	9.3939	9.3942
Ga	4358.5	5.9990	2179.5	5.999	5.9993
Ge	3527	7.8991	1763.5	7.9002	7.8994
As	3027	9.7878	1513.5	9.7895	9.7886
Se	3225	9.7522	1612.5	9.7538	9.7524
Br	2821	11.8141	1408.5	11.833	11.8318
Kr	2518.5	13.9998	1259.5	13.9998	13.9996
Rb	8917.5	4.1772	4459	4.1772	4.1771
Sr	6899	5.6950	3450	5.6946	5.6949
Y	6656.5	6.2172	3328.5	6.2172	6.2173
Zr	6562.5	6.6338	3281.5	6.6338	6.6339
Nb	6767	6.7590	3384	6.7585	6.7589
Mo	6767.5	7.0922	3384	7.0922	7.0924
Tc	6910.5	7.2802	3455.5	7.2802	7.28

El	Wave behavior		Particle behavior		(NIST)
	D _{Wave}	E _{hν}	D _{Pricle}	E _e	
Te	8166	9.0098	4083.5	9.0093	9.0096
I	7313	10.4514	3657	10.4506	10.4513
Xe	6541	12.1299	3271	12.129	12.1298
Cs	21138.5	3.8939	10569.5	3.8939	3.8939
Ba	16373	5.2117	8187	5.2116	5.2117
La	15852	5.5770	7926.5	5.5768	5.5769
Ce	16526.5	5.5387	8263.5	5.5387	5.5387
Pr	17335	5.4640	8653.5	5.473	5.473
Nd	17730	5.5249	8865	5.5251	5.525
Pm	18145	5.5800	9069.5	5.582	5.582
Sm	18533.5	5.6437	9267	5.6437	5.6437
Eu	19046	5.6704	9523	5.6705	5.6704
Gd	18123	6.1499	9062	6.1497	6.1498
Tb	19606	5.8637	9803	5.8639	5.8638
Dy	19958	5.9389	9979.5	5.9388	5.9389
Ho	20285	6.0216	10143	6.0214	6.0215
Er	20600.5	6.1077	10300.5	6.1077	6.1077
Tm	20948	6.1843	10474.5	6.1842	6.1843
Yb	21318.5	6.2543	10659.5	6.2543	6.2542
Lu	25280.5	5.4259	12640.5	5.4259	5.4259
Hf	20667.5	6.8252	10334	6.8252	6.8251
Ta	19207	7.5496	9603.5	7.5498	7.5496
W	18947.5	7.8641	9474	7.8641	7.864
Re	19539	7.8335	9770	7.8333	7.8335
Os	18625.5	8.4383	9313	8.4383	8.4382
Ir	17991.5	8.9670	8996	8.967	8.967
Pt	18479	8.9587	9239.5	8.959	8.9588
Au	18407.5	9.2256	9204	9.2256	9.2255
Hg	16684.5	10.4376	8342.5	10.4376	10.4375
Tl	29228	6.1082	14614	6.1083	6.1082
Pb	24669	7.4168	12335	7.4166	7.4167
Bi	25730	7.2854	12865	7.2856	7.2855
Po	22811.5	8.4167	11409.5	8.4141	8.414
Rn	18723.5	10.7485	9362	10.7485	10.7485
Fr	50571	4.0727	25285.5	4.0727	4.0727
Ra	39921.5	5.2784	19961	5.2784	5.2784
Ac	41690	5.1700	20029	5.3807	5.3807
Th	34948	6.3067	17474	6.3068	6.3067
Pa	38257	5.8900	19128.5	5.8901	5.89
U	37182.5	6.1941	18592	6.1939	6.1939
Np	37561	6.2657	18781	6.2656	6.2657
Pu	39898.5	6.0262	19950	6.026	6.026
Am	41109	5.9738	20555	5.9738	5.9738

Table 1 continued

Ru	7156.5	7.3607	3578.5	7.3607	7.3605
Rh	7387	7.4589	3693.5	7.4594	7.4589
Pd	6906	8.3369	3453	8.3375	8.3369
Ag	7933.5	7.5762	3967	7.5762	7.5762
Cd	6970.5	8.9936	3485.5	8.9936	8.9938
In	11290.5	5.7864	5645.5	5.7864	5.7864
Sn	9262.5	7.3440	4631.5	7.344	7.3439
Sb	8221	8.6087	4111	8.6081	8.6084

Cm	41855	5.9915	20928	5.9915	5.9914
Bk	41308.5	6.1979	20654.5	6.1979	6.1979
Cf	41602.5	6.2817	20801.5	6.2817	6.2817
Es	41541	6.4200	20941.5	6.3677	6.3676
Fm	41863	6.5000	20931.5	6.5001	6.5
Md	42185	6.5800	21093	6.58	6.58
No	42572	6.6500	21286	6.65	6.65

5. New Energy Levels in Hydrogen

The most important results of our dealing here are hydrogen energy levels, we concluded all previous work addition to 90 new energy level as shown in table

(2). All great Bohr energy levels of Hydrogen atom are marked by (*) and bold font, it has been done in previous work[6]

Table 2. New energy levels for hydrogen atom.

D	E (eV)	D	E (eV)	D	E (eV)	D	E (eV)	D	E (eV)	D	E (eV)
1	13.59829*	9.5	1.43140	18	0.75546	26.5	0.51314	35	0.38852	43.5	0.31260
1.5	9.06552	10	1.35983	18.5	0.73504	27	0.50364	35.5	0.38305	44	0.30905
2	6.79914	10.5	1.29507	19	0.71570	27.5	0.49448	36	0.37773*	44.5	0.30558
2.5	5.43931	11	1.23621	19.5	0.69735	28	0.48565	36.5	0.37256	45	0.30218
3	4.53276	11.5	1.18246	20	0.67991	28.5	0.47713	37	0.36752	45.5	0.29886
3.5	3.88522	12	1.13319	20.5	0.66333	29	0.46891	37.5	0.36262	46	0.29561
4	3.39957*	12.5	1.08786	21	0.64754	29.5	0.46096	38	0.35785	46.5	0.29244
4.5	3.02184	13	1.04602	21.5	0.63248	30	0.45328	38.5	0.35320	47	0.28933
5	2.71966	13.5	1.00728	22	0.61810	30.5	0.44585	39	0.34867	47.5	0.28628
5.5	2.47242	14	0.97131	22.5	0.60437	31	0.43865	39.5	0.34426	48	0.28330
6	2.26638	14.5	0.93781	23	0.59123	31.5	0.43169	40	0.33996	48.5	0.28038
6.5	2.09204	15	0.90655	23.5	0.57865	32	0.42495	40.5	0.33576	49	0.27752*
7	1.94261	15.5	0.87731	24	0.56660	32.5	0.41841	41	0.33167	49.5	0.27471
7.5	1.81310	16	0.84989*	24.5	0.55503	33	0.41207	41.5	0.32767	50	0.27197
8	1.69979	16.5	0.82414	25	0.54393*	33.5	0.40592	42	0.32377	50.5	0.26927
8.5	1.59980	17	0.79990	25.5	0.53327	34	0.39995	42.5	0.31996	51	0.26663
9	1.51092*	17.5	0.77704	26	0.52301	34.5	0.39415	43	0.31624	51.5	0.26404

6. Ratio of Circumference to Wavelength

There are unexpected data where, by changing radius of electron energy change, the energy of individual level change of course, but " ratio between the circumference of each level and the wavelength of this level is still constant

$$\frac{2\pi r}{\lambda} = n \quad (17)$$

Where, (λ) is emitted wavelength, (r) is radius of the energy level and (n) is constant, so

$$2\pi r = n \lambda$$

$$2\pi r (mv) = n \lambda (mv) \quad (18)$$

Where, (m) is mass of electron and (v) is velocity of electron, so de Broglie principle ($P = h/\lambda$) or ($mv = h/\lambda$) can be considered as a conclusion from Bohr assumption. This leads to confirmation of our new mathematical proposal. Ratio between circumference and emitted wavelength (CirWav) are shown in table (3)

Table 3. Ratio between the circumference and wavelength of each element.

El.	CirWav	El.	CirWav	El.	CirWav
H	0.003646152109436600	Br	0.127615323830281000	Tm	0.251584495551126000
He	0.007292304218873210	Kr	0.131261475939718000	Yb	0.255230647660562000
Li	0.010938456328309800	Rb	0.134907628049154000	Lu	0.258876799769999000
Be	0.014584608437746400	Sr	0.138553780158591000	Hf	0.262522951879435000
B	0.018230760547183000	Y	0.142199932268028000	Ta	0.266169103988872000
C	0.021876912656619600	Zr	0.145846084377464000	W	0.269815256098309000
N	0.025523064766056200	Nb	0.149492236486901000	Re	0.273461408207745000
O	0.029169216875492800	Mo	0.153138388596337000	Os	0.277107560317182000
F	0.032815368984929400	Tc	0.156784540705774000	Ir	0.280753712426619000

Table 3 continued

Ne	0.036461521094366100	Ru	0.160430692815211000	Pt	0.284399864536055000
Na	0.040107673203802600	Rh	0.164076844924647000	Au	0.288046016645492000
Mg	0.043753825313239300	Pd	0.167722997034084000	Hg	0.291692168754928000
Al	0.047399977422675900	Ag	0.171369149143520000	Tl	0.295338320864365000
Si	0.051046129532112500	Cd	0.175015301252957000	Pb	0.298984472973802000
P	0.054692281641549100	In	0.178661453362394000	Bi	0.302630625083238000
S	0.058338433750985700	Sn	0.182307605471830000	Po	0.306276777192675000
Cl	0.061984585860422300	Sb	0.185953757581267000	Rn	0.313569081411548000
Ar	0.065630737969858900	Te	0.189599909690703000	Fr	0.317215233520985000
K	0.069276890079295500	I	0.193246061800140000	Ra	0.320861385630421000
Ca	0.072923042188732100	Xe	0.196892213909577000	Ac	0.324507537739858000
Sc	0.076569194298168700	Cs	0.200538366019013000	Th	0.328153689849294000
Ti	0.080215346407605300	Ba	0.204184518128450000	Pa	0.331799841958731000
V	0.083861498517041900	La	0.207830670237887000	U	0.335445994068168000
Cr	0.087507650626478500	Ce	0.211476822347323000	Np	0.339092146177604000
Mn	0.091153802735915100	Pr	0.215122974456760000	Pu	0.342738298287041000
Fe	0.094799954845351700	Nd	0.218769126566196000	Am	0.346384450396477000
Co	0.098446106954788300	Pm	0.222415278675633000	Cm	0.350030602505914000
Ni	0.102092259064225000	Sm	0.226061430785069000	Bk	0.353676754615351000
Cu	0.105738411173662000	Eu	0.229707582894506000	Cf	0.357322906724787000
Zn	0.109384563283098000	Gd	0.233353735003943000	Es	0.360969058834224000
Ga	0.113030715392535000	Tb	0.236999887113379000	Fm	0.364615210943660000
Ge	0.116676867501971000	Dy	0.240646039222816000	Md	0.368261363053097000
As	0.120323019611408000	Ho	0.244292191332253000	No	0.371907515162534000
Se	0.123969171720845000	Er	0.247938343441689000		

7. Entropy of electron inside the atom

There is no completely reversible process. Emission and absorption processes are completely reversible processes for each other where, all emission lines coincidence exactly with absorption lines. Any electron absorbs photon will transfer to high energy level, then it will re-emit this same photon itself with same frequency and back to its original energy level, this inconsistent with meaning of entropy. By using these new energy levels, we can say that if electron absorbed photon, it will back to its original energy level by emitting many photons due to many transitions. On the other hand, electron will be back to its original energy level by many spectroscopic transitions or by thermal transition among these new energy levels “means absorption process”, which means increasing intensity of short frequencies – high energies are dissipated into low energies “universe is going to be cold”. But, if the electron is excited by electric energy or any other energy except electromagnetic waves then, it will jump or transfers from its ground state to higher energy level(s) till it arrives to photonic energy level “energy level allows electron to emit photon” where it can come back to its energy level by one single transition, which means emission process. This means that, emission and absorption processes are not completely reversible

processes for each other and entropy is satisfied, which means increasing the chaos.

8. New experiments of spectroscopy

Emission-Absorption Sequence Experiment (EASE). We must do this new experiment to have a clear and strong evidence about reality of absorption process. Figure (1) shows that, hydrogen lamb is connected with power supply for excitation (emission process). Emitted spectrum pass through a long length hole, then it passes through an excellent thermal insulator closed tube of glass is filled with hydrogen gas (the same gas that is used to emit spectrum), there is two terminals one of digital thermometer and the other of digital manometer to measure temperature and pressure of gas which will do absorption process, side surface area of this tube must be in black, this is absorption process. The detectors, sensors and viewing screen are used to find out any spectra, these instruments of long wide range from sound wave to UV, this to detect any spectrum.

According to quantum physics, spectroscopy and all our current theories, we will never detect any spectrum. If there are any spectrum, then, that means we found out the absorbed photon, but it has been re-emitted through two transitions not through one transition, this will change our old explanation and prove experimentally new energy levels inside the atom and entropy inside

the atom. Figure (2) shows Emission-Double Absorption Sequence Experiment (EDASE). There are

two tubes of hydrogen gas for absorption process, so this experiment is very important for atomic physics.

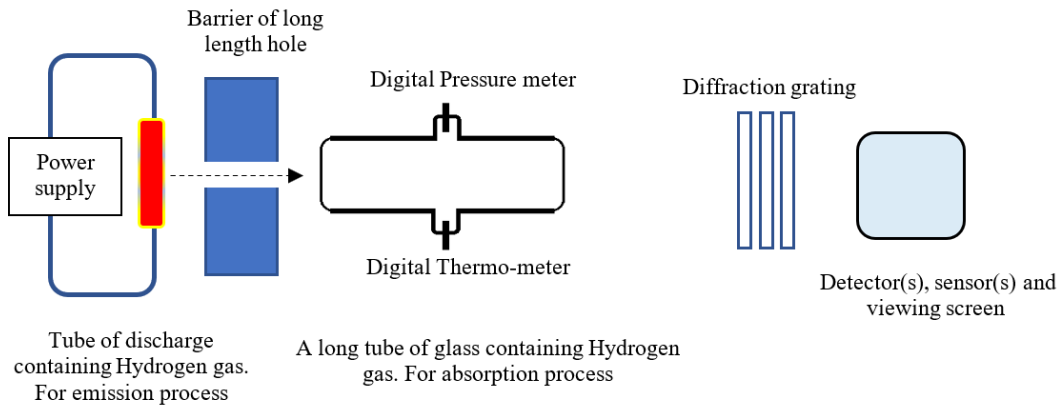


Fig.1 Indicating emission-absorption sequence experiment.

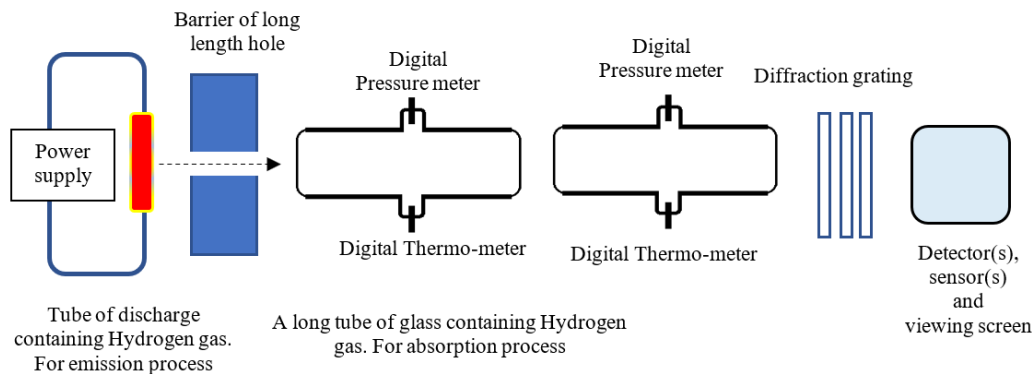


Fig.2 Indicating emission-absorption sequence experiment.

9. New experiments of spectroscopy

It is clear that, Elsheekh Theory approved all experimental ionization energies for all elements either the electron was a particle or a wave. It generated new energy levels in addition to approve all previous energy levels. Classical vision of energy level as a sphere around nucleus still stood. We found out the conflict between wave behavior and particle behavior, this conflict inside the base equation of electron velocity and its radius. We can classify energy levels as photonic energy levels and thermal energy levels. Emission process occurs where, we excited the electron(s) by any energy source except electromagnetic waves, the excited electron transfers among thermal energy levels till it arrive to high photonic energy level then, emits a photon(s) by transition among photonic energy levels only. Absorption process occurs where, we excited the electron(s) by electromagnetic waves only, the excited electron transfers among photonic levels only, but it back to origin energy level due to only two predict explanations, first if the electron returned to its ground state thermally then we will not observe any spectrum in Emission-Absorption Sequence Experiment (EASE). Second, if the electron returned to its ground state by emitting more than one photon then we will observe spectrum. We could not observe these spectra before

where, these lines were added to the continuous spectrum of incident light that were used to do absorption spectra experiment. Classical physics generated quantized energy levels satisfied and proved experimental results of ionization energy for all elements.

Physical constants have been used [9].

Dielectric constant $\epsilon_0 = 8.854187817 \times 10^{-12}$ F/m

Electron rest mass $m_e = 9.10938291 \times 10^{-31}$ kg

Proton rest mass $m_p = 1.672621777 \times 10^{-27}$ kg

Elementary charge $e = 1.602176565 \times 10^{-19}$ C

Planck's constant $h = 6.62606957 \times 10^{-34}$ J.s

Constant (Pi) $\pi = 3.14159265$

References

1. N. Bohr, Philos. Mag. 26, 1 (1913).
2. Max Born, The Mechanics of The Atom, University of Gottingen. Printed in Great Britain (1927).
3. Michel Ghins, Model-Based Reasoning in Science and Technology. Institute Supérieur de Philosophie. Belgium (2013).

-
4. P. Weinberger, Niels Bohr and the dawn of quantum theory. *Philosophical Magazine* (2014).
 5. Arthur Beiser, Concepts of Modern Physics, 6th Ed., ISBN: 0-07-244848-2. (2003).
 6. Ahmed M. M. Elsheekh. New Theory and New Energy Levels in The Atom. *International Journal of Theoretical and Mathematical Physics*, 3(4): 117-122, (2013).
 7. National Institute of Standards and Technology. (NIST).
(2021).<https://physics.nist.gov/PhysRefData/ASD/ionEnergy.html>
 8. National Institute of Standards and Technology. (NIST). (2021).<https://physics.nist.gov/cgi-bin/ASD/ie.pl>
 9. National Institute of Standards and Technology. (NIST). (2021).
<https://physics.nist.gov/cuu/Constants/>