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OPTIMAL ASYNCHRONOUS DETECTION OF RAYLEIGH FADING MFSK SIGNALS

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ABSTRACT

In this paper, we develop and analyze optimum asynchronous detector for M-ary Frequency Shift Keying (MFSK) signals traveled over Rayleigh fading channel. Developing this detector comprises two steps: (1) deriving a synchronous optimal detector based on applying the statistical decision theory to the Rayleigh fading MFSK signal, (2) modifying the synchronous structure of the MFSK detector by averaging over the unknown signal arrival time. The performance of the asynchronous detector is measured in terms of the probability of mis-detection and is compared with the performance of the synchronous one. The asynchronous detector operates satisfactory under lack of knowledge of the arrival time and its performance enhances by increasing the quantization level of epoch.

KEY WORDS

Fading, Synchronization, and Signal Detection

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I. Introduction

The development of communication systems using the radio channel resulted in increasing interest in the study of signal detection in the presence of fading. The most realistic problem that arises in communication applications is when the receiver is not knowledgeable of the time at which the signal arrives. Research documents have focused on the detection of signals when immersed in additive white Gaussian noise (AWGN) and contain a profusion of decision algorithms. These algorithms may be categorized essentially into a structured type that includes those derived from a likelihood-ratio formulation [1], [2], and an ad hoc type which uses the autocorrelation operator [4],[5]. These documents assume available perfect knowledge of signal arrival time or epoch. Indeed the detector is able neither to observe its received input at the correct arrival time nor to synchronously sample the matched filters at the appropriate time instants. Therefore, attention should be devoted to develop asynchronous structure to remedy the situation of unavailable signal arrival time or epoch as described in the research documents [3],[6-10]. These documents are trying to estimate the epoch parameter based on the principles of maximum likelihood estimation. The drawback of this method is that, the performance of the detector is depending on the degree of accuracy of estimation. All the above mentioned research documents focus only on detection of non-fading signals and also none of them was concerned with the asynchronous detection of fading signals.

In this paper, we develop *asynchronous* detector of *fading* MFSK signals. First, a synchronous decision rule is derived and then extended to the asynchronous one by averaging over the unknown epoch instead of spend an effort in trying to measure it. The paper is organized as follows. The problem statement and assumptions are presented in section II. Section III describes the mathematical formulation of the asynchronous MFSK detector. The derivation of the decision rule is presented in this section. In section IV, numerical studies and discussions are presented to demonstrate the performance of the asynchronous detection scheme. Finally, the summary and conclusions are presented in section V.

II. Problem Statement Assumptions

The receiver input *y(t)* is observed over the interval of time *T* which contains under the hypothesis H_i an M_i FSK signal where i = 0,1,2... Given several choices, the receiver must detect which type of modulation format is actually employed. We will consider a binary hypotheses testing problem, extension to more hypotheses testing is straightforward. The complex envelope of the observations, $\tilde{y}(t)$, received through fading channel is given by:

$$\tilde{y}(t) = A\, e^{j\tau}\, \tilde{x}(t) + \tilde{w}(t) \tag{1}$$

where $\tilde{x}(t)$ is the complex envelope of the received M_i FSK signal; *i*=0,1, $\tilde{w}(t)$ is the complex envelope of the white Gaussian noise process with two sided height of spectral density of $N_o/2$. In addition, *A* and τ are random parameters, due to the fading phenomena. These parameters are assumed to be statistically independent of each other. The parameter *A* has a Rayleigh distribution with a power spectral density (psd) given by [11, p. 529]:

$$P_A\ (v)=\frac{2v}{b}\,e^{-v^2/b}\ ; \qquad v,b\ge 0 \tag{2}$$

The parameter τ has a uniform distribution given by:

$$P_\tau=\frac{1}{2\pi}; \qquad 0\le\tau\le 2\pi \tag{3}$$

The receiver announces H_1 (or M_1 FSK) when a threshold is exceeded and H_o (or M_o FSK) otherwise. It is assumed that the hypotheses are equally likely and that $M_o \langle M_1$ with no loss in generality. Generally, the complex envelope of MFSK signal is mathematically expressed as [1], [3]:

$$\tilde{s}(t)=\sqrt{E}\sum_n \exp\{j(2\pi f^{(n)}t+\theta^{(n)}\}\times p(t-nT_s-\varepsilon T_s) \tag{4}$$

where *E* is the signal energy, $f^{(n)}$ is a set of independent identically distributed (i.i.d) discrete random variables (r.v's), the elements of which are uniformly distribution on {*-B/2, B/2*], $\theta^{(n)}$ is a set of random variables that is uniformly distributed over the interval $[-\pi,\pi]$, *n* is the symbol number, and *p(t)* is the standard unit pulse of duration T_s. In addition, the parameter ε is the normalized epoch parameter which accounts for any timing offset that exists between the transmitter's clock and that of the receiver. The parameter ε is considered to be continuous r.v. uniformly distributed on the interval (0,1).

III. Mathematical Formulation of the Detector

Consider the received waveform $\tilde{y}(t)$; $0\le t\le T_s$ which consists of the *per-symbol* complex envelope of the desired signal plus the complex envelope of the additive white Gaussian noise (AWGN) as

$$\tilde{y}(t)=A\ e^{j\tau}\ \tilde{x}(t)+\tilde{w}(t) \tag{5}$$

where $\tilde{x}(t)$ is *the per-symbol* complex envelope of the desired signal which is given by

$$\tilde{x}(t) = \sqrt{E}e^{j(2\pi f t+\theta)} \tag{6}$$

According to Grenander's theorem [8, p. 377], the likelihood function (LF) of $\tilde{y}(t)$ with respect to the random parameters $\theta, \tau, A, f, \varepsilon$ is given by:

$$\Delta[\tilde{y}(t);\theta,\tau,A,f,\varepsilon] = \exp\left\{\frac{1}{N_o}\left(-\left|Ae^{j\tau}\int_0^{\varepsilon T_s}\tilde{x}(t)dt\right|^2 + 2\,\mathrm{Re}\left(Ae^{j\tau}\int_0^{\varepsilon T_s}\tilde{y}(t)\tilde{x}^*(t)\,dt\right)\right)\right\} \tag{7}$$

The relevant part of the LF, given by (7), can be written as

$$\Delta_1[\tilde{y}(t);\theta,\tau,A,f,\varepsilon] = \exp\left\{\frac{2\sqrt{E}}{N_o}A|Y(f,\varepsilon)|\cos(\theta+\tau)\right\} \tag{8}$$

where $Y(f,\varepsilon) = \int_0^{\varepsilon T_s}\tilde{y}(t)e^{-j2\pi f t}\,dt$ is the fourier transform of $\tilde{y}(t)$ within the symbol duration εT_s. For *N* independent and identically distributed (i.i.d) symbols, $\Delta_1[\tilde{y}(t);\theta,\tau,A,f,\varepsilon]$ can be written as

$$\Delta_1[\tilde{y}(t);\theta,\tau,A,f,\varepsilon] = \exp\left\{\sum_{n=1}^{N}\ln E_{\theta,\tau,A,f,\varepsilon}\left[\exp\left(\frac{2\sqrt{E}}{N_o}A|Y^{(n)}(f,\varepsilon)|\cos(\theta+\tau)\right)\right]\right\} \tag{9}$$

where $E_{\theta,\tau,A,f,\varepsilon}$ denotes the expectation with respect to the random variables $\theta, \tau, A, f, \varepsilon$. Expanding the inner exponential using Taylor's series representation; truncated to the third term, taking the expectation with respect to θ and τ and using linear approximation of ln(1+x), the log-likelihood function (LLF) can be written as

$$\Delta_2[\tilde{y}(t);A,f,\varepsilon] \approx \sum_{n=1}^{N}E_{A,f,\varepsilon}\left[\frac{A^2}{4}\left(\frac{2\sqrt{E}}{N_o}\right)^2|Y^{(n)}(f,\varepsilon)|^2\right] \tag{10}$$

Taking the average over the Rayleigh random variable *A* we have

$$\Delta_2[\tilde{y}(t);f,\varepsilon] \approx \frac{1}{4}\left(\frac{2\sqrt{E}}{N_o}\right)^2\sum_{n=1}^{N}E_{f,\varepsilon}\left(\int_0^{\infty}\upsilon^2|Y^{(n)}(f,\varepsilon)|^2\frac{2\upsilon}{b}e^{-\upsilon^2/b}d\upsilon\right) \tag{11}$$

Discretizing the bandwidth B to M values with separation of $\frac{1}{2T_s}$, the minimum orthogonal spacing, then $\{f\}$ is a set of i.i.d discrete random variables whose elements are orthogonal and uniformly distributed on $\left\{\pm\frac{1}{2T_s},\pm\frac{2}{2T_s},...,\pm\frac{M/2}{2T_s}\right\}$. Averaging with respect to f, the log-likelihood function becomes

$$\Delta_2[\tilde{y}(t);\varepsilon]=\frac{b}{4M}\left(\frac{2\sqrt{E}}{N_o}\right)^2\sum_{n=1}^{N}E_{\varepsilon}\left(\sum_{m=\frac{-M}{2}}^{M/2}\left|Y_m^{(n)}(\varepsilon)\right|^2\right) \tag{12}$$

Where $Y_m^{(n)}(\varepsilon)=\int_{(n+\varepsilon)T_s}^{(n+\varepsilon+1)T_s}\tilde{y}(t)e^{-j2\pi\frac{m}{2T_s}t}dt$ is the Fourier transform of the received observation at the m-th frequency location evaluated at the n-th symbol duration. Note that the synchronous LLF can be obtained by substituting $\varepsilon=0$ in (12) and the synchronous decision rule is derived from the resulting equation. It is clear that the synchronous structure is transformed to the asynchronous one by averaging it over the unknown epoch. This is performed by discretizing the epoch parameter ε to K equals intervals and taking the expectation over the discrete random variable ε. The resulting LLF becomes:

$$\Delta_2[\tilde{y}(t)]=\frac{b}{4MK}\sum_{n=1}^{N}\sum_{k=1}^{K}\left(\sum_{m=\frac{-M}{2}}^{M/2}\left|Y_m^{(n)}(\varepsilon_k)\right|^2\right) \tag{13}$$

where

$$Y_m^{(n)}(\varepsilon_k)=\int_{(n+\varepsilon_k)T_s}^{(n+\varepsilon_k+1)T_s}\tilde{y}(t)e^{-j2\pi\frac{m}{2T_s}t}dt \tag{14}$$

and $\varepsilon_k=\frac{k}{K};\ k=1,2,...,K$. Note that the number of levels K, to which the epoch uncertainty parameter is quantized, is designer-chosen and is directly proportional to the involved complexity load; it will be kept small for practical considerations. The binary hypothesis testing [6] states that the optimal detector is the one that compares to a threshold, the likelihood function associated with detection of the signal under H_1 to that of the signal under H_o. Then, from (13), the optimal decision rule to decide between M_0FSK and M_1FSK is given by

$$\sum_{n=1}^{N}\sum_{k=1}^{K}\left(\frac{1}{M_1}\sum_{m=\frac{-M_1}{2}}^{M_1/2}\left|Y_{m,1}^{(n)}(\varepsilon_k)\right|^2\right)-\sum_{n=1}^{N}\sum_{l=1}^{I}\left(\frac{1}{M_o}\sum_{m=\frac{-M_o}{2}}^{M_o/2}\left|Y_{m,0}^{(n)}(\varepsilon_k)\right|\right)\underset{H_0}{\overset{H_1}{\gtrless}}\ln\eta \qquad (15)$$

where

$$Y_{m,l}^{(n)}(\varepsilon_k)=\int_{(n+\varepsilon_k)T_{s,l}}^{(n+\varepsilon_k+1)T_{s,l}}\tilde{r}(t)e^{-j2\pi\frac{m}{2T_{s,l}}t}dt;\quad l=0,1 \qquad (16)$$

$T_{s,l}$; (l=0,1) is the symbol duration of the signal under hypothesis l and η is the threshold level. The receiver announces H_1 (or M_1FSK) when the threshold is exceeded and H_o (or M_0 FSK) otherwise. It is clear that the statistical decision theory provides a simple way of transforming the synchronous structure into asynchronous one. The optimal decision rule actually belongs to the reduced uncertainty model in which the epoch is taken from the discrete set ε_k, $k=1,2,...,K$. It can provide an upper approximation to the truly random case. The accuracy of this approximation depends on the quantization level *K*.

IV. Numerical Studies and Discussions

Experimental evaluation of the performance of the detector is performed. The two hypotheses that are considered are BFSK (M_oFSK) and QFSK (M_1FSK). The complex envelope of a generated QFSK signal is evaluated and added to the complex envelope of a generated white Gaussian noise to form the observation. The decision rule is applied to detect the QFSK signal. For comparison purposes, we start with evaluation of the performance of the decision rule in the synchronous case (when ε=0). The performance of the synchronous detector is evaluated for different values of the threshold γ ($\gamma=\ln\eta$). The probability of misdetection versus the signal to noise ratio (SNR) evaluated at γ= 0, .25, 1 is shown in Fig. 1. This figure shows that as the signal to noise ratio increases the probability of misdetection decreases for any value of the threshold γ. The figure also shows that the case of γ= 0, has the best performance i.e. the lowest probability of misdetection that can be achieved compared with the probability of misdetection for the other values of γ. This is because γ = 0 corresponds to the minimum probability of error criterion.

The performance of the asynchronous detector is evaluated for γ= 0 (the best performance) and for different values of the epoch quantization level *K*. The probability of misdetection versus the signal to noise ratio (SNR) evaluated at *K*= 2, 3, 4 and 5 is shown in Fig. 2. This figure shows that as the signal to noise ratio increases the probability of misdetection decreases. The figure also shows that the probability of misdetection decreased as the epoch quantization level *K* increased. This means that the performance of the asynchronous detector enhances as the epoch quantization level increases. The enhancement in performance after K = 4 is insignificant, so we can use K = 4 in the asynchronous decision rule to provide satisfactory performance. As a comparison between the performance of the synchronous and asynchronous decision rules we provide Fig. 3 which collects the results of both of them at threshold $\gamma = 0$. This figure shows that increasing the uncertainty quantization level K minimizes the gap in performance between the asynchronous and the synchronous detectors.

V. Summary and Conclusions

We developed asynchronous optimum detector of MFSK signals received through fading channel and contaminated with AWGN. The method is based on the principles of the average-likelihood ratio theory. Modifying the synchronous structure of the MFSK detector by averaging over the unknown epoch develops the asynchronous detector. The detector operates satisfactory under lack of knowledge of the arrival time of epoch. The performance of the asynchronous detector depends on the epoch quantization level. Increasing this level enhances the detector performance and decreasing the gap in performance between the synchronous and the asynchronous cases.

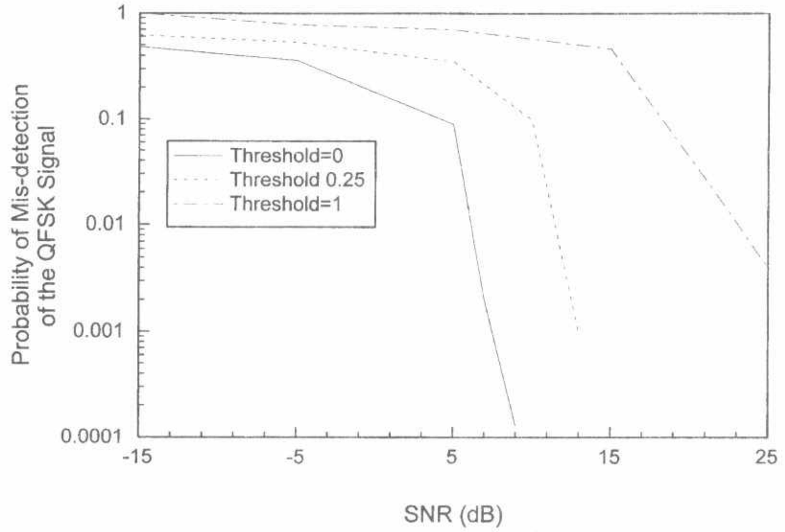

Fig. 1 : The Probability of misdetection of the QFSK signal versus the signal to noise ratio for different values of the threshold

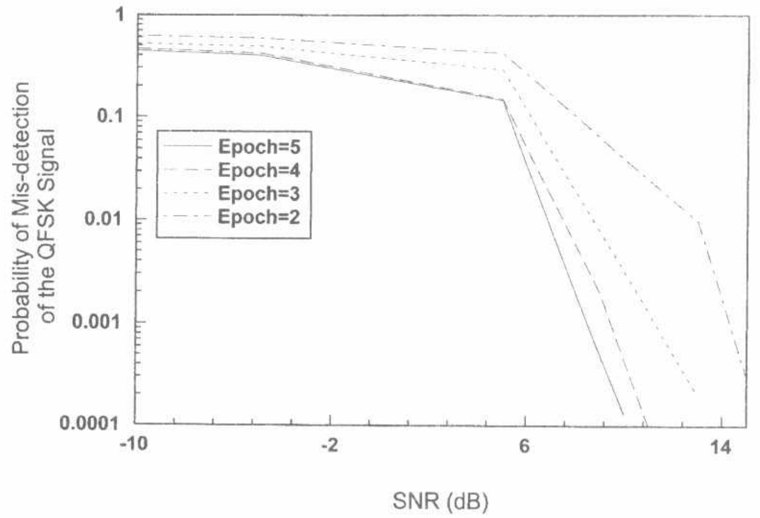

Fig. 2. The Probability of misdetection of the QFSK signal versus the signal to noise ratio for the asynchronous detector and for epoch quantization Levels 2, 3,4, 5.

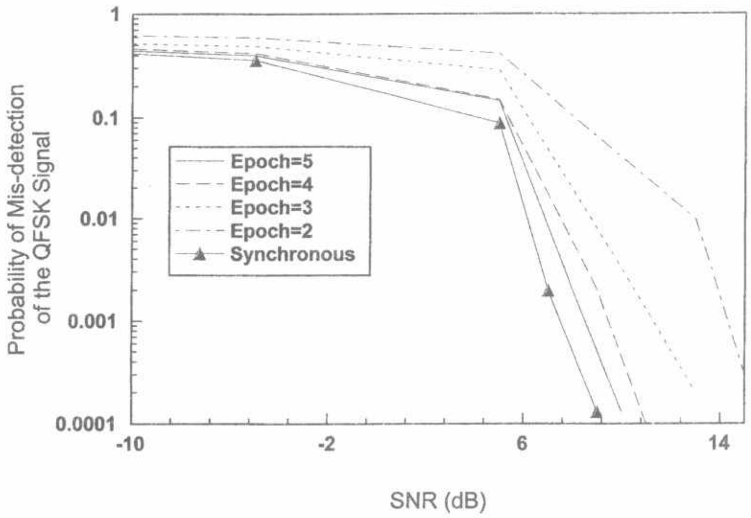

Fig. 3. The Probability of misdetection of the QFSK signal versus the signal to noise ratio for synchronous and asynchronous detectors.

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