Electronic Journal of Mathematical Analysis and Applications
Vol. 6(1) Jan. 2018, pp. 198-203.
ISSN: 2090-729X(online)
http://fcag-egypt.com/Journals/EJMAA/

ON TRIPLE SEQUENCE SPACE OF BERNSTEIN OPERATOR OF χ^3 OF ROUGH $\lambda-$STATISTICAL CONVERGENCE IN PROBABILITY DEFINED BY MUSIELAK-ORLICZ FUNCTION OF $p-$ METRIC

N. SUBRAMANIAN AND A. ESI

Abstract. We introduced the triple sequence space of Bernstein polynomials χ^3 of rough $\lambda-$ statistical convergence in probability and discuss general properties of among these sequence spaces with respect of Musielak-Orlicz function.

1. Introduction

The idea of rough convergence was introduced by Phu [12], who also introduced the concepts of rough limit points and roughness degree. The idea of rough convergence occurs very naturally in numerical analysis and has interesting applications. Aytar [1] extended the idea of rough convergence into rough statistical convergence using the notion of natural density just as usual convergence was extended to statistical convergence. Pal et al. [11] extended the notion of rough convergence using the concept of ideals which automatically extends the earlier notions of rough convergence and rough statistical convergence.

The Bernstein operator of order (r, s, t) is given by

$B_{rst}(f,x) = \sum_{m=0}^{r}\sum_{n=0}^{s}\sum_{k=0}^{t} f\left(\frac{mnk}{rst}\right)\binom{r}{m}\binom{s}{n}\binom{t}{k} x^{m+n+k}(1-x)^{(m-r)+(n-s)+(k-t)}$

where f is a continuous (real or complex valued) function defined on $[0,1]$.

Throughout the paper, $\mathbb{R}$ denotes the real of three dimensional space with metric (X,d). Consider a triple sequence of Bernstein polynomials $(B_{mnk}(f,x))$ such that $(B_{mnk}(f,x)) \in \mathbb{R}, m,n,k \in \mathbb{N}$.

Let f be a continuous function defined on the closed interval $[0,1]$. A triple sequence of Bernstein polynomials $(B_{rst}(f,x))$ is said to be statistically convergent to $0 \in \mathbb{R}$, written as $st - lim\ x = 0$, provided that the set

$$K_\epsilon := \left\{(m,n,k) \in \mathbb{N}^3 : |B_{mnk}(f,x) - f(x)| \geq \epsilon\right\}$$

has natural density zero for any $\epsilon > 0$. In this case, 0 is called the statistical limit of the triple sequence of Bernstein polynomials. i.e., $\delta(K_\epsilon) = 0$. That is,

---

2010 *Mathematics Subject Classification.* 40A35, 40G15, 60B10.
Key words and phrases. rough $\lambda-$ statistical convergence, triple sequences, $\chi-$ sequence .
Submitted March 19, 2016.

$$lim_{rst\to\infty}\frac{1}{rst}\left|\{(m,n,k)\leq(r,s,t):|B_{mnk}(f,x)-(f,x)|\geq\epsilon\}\right|=0.$$

In this case, we write $\delta - limB_{mnk}(f,x)=f(x)$ or $B_{mnk}(f,x)\to^{S_B} f(x)$.

A triple sequence (real or complex) can be defined as a function $x:\mathbb{N}\times\mathbb{N}\times\mathbb{N}\to\mathbb{R}(\mathbb{C})$, where $\mathbb{N},\mathbb{R}$ and $\mathbb{C}$ denote the set of natural numbers, real numbers and complex numbers respectively. The different types of notions of triple sequence was introduced and investigated at the initial by *Sahiner et al. [13,14]*, *Esi et al. [2-5]*, *Datta et al. [6]*,*Subramanian et al. [15]*, *Debnath et al. [7]* and many others.
A triple sequence $x=(x_{mnk})$ is said to be triple analytic if

$$sup_{m,n,k}|x_{mnk}|^{\frac{1}{m+n+k}}<\infty.$$

The space of all triple analytic sequences are usually denoted by Λ^3. A triple sequence $x=(x_{mnk})$ is called triple gai sequence if

$$((m+n+k)!\,|x_{mnk}|)^{\frac{1}{m+n+k}}\to 0 \text{ as } m,n,k\to\infty.$$

The space of all triple gai sequences are usually denoted by χ^3.

2. Definitions and Preliminaries

2.1. **Definition.** An Orlicz function ([see [8]) is a function $M:[0,\infty)\to[0,\infty)$ which is continuous, non-decreasing and convex with $M(0)=0$, $M(x)>0$, for $x>0$ and $M(x)\to\infty$ as $x\to\infty$. If convexity of Orlicz function M is replaced by $M(x+y)\leq M(x)+M(y)$, then this function is called modulus function.

Lindenstrauss and Tzafriri ([9]) used the idea of Orlicz function to construct Orlicz sequence space.

A sequence $g=(g_{mn})$ defined by

$$g_{mn}(v)=sup\{|v|u-(f_{mnk})(u):u\geq 0\}, m,n,k=1,2,\cdots$$

is called the complementary function of a Musielak-Orlicz function f. For a given Musielak-Orlicz function f, [see [10]] the Musielak-Orlicz sequence space t_f is defined as follows

$$t_f=\left\{x\in w^3: I_f(|x_{mnk}|)^{1/m+n+k}\to 0\, as\, m,n,k\to\infty\right\},$$

where I_f is a convex modular defined by

$$I_f(x)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}f_{mnk}(|x_{mnk}|)^{1/m+n+k}, x=(x_{mnk})\in t_f.$$

We consider t_f equipped with the Luxemburg metric

$$d(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}f_{mnk}\left(\frac{|x_{mnk}|^{1/m+n+k}}{mnk}\right)$$

is an exteneded real number.

2.2. **Definition.** Let X,Y be a real vector space of dimension w, where $n\leq m$. A real valued function $d_p(x_1,\dots,x_n)=\|(d_1(x_1,0),\dots,d_n(x_n,0))\|_p$ on X satisfying the following four conditions:
(i) $\|(d_1(x_1,0),\dots,d_n(x_n,0))\|_p=0$ if and only if $d_1(x_1,0),\dots,d_n(x_n,0)$ are linearly dependent,
(ii) $\|(d_1(x_1,0),\dots,d_n(x_n,0))\|_p$ is invariant under permutation,
(iii) $\|(\alpha d_1(x_1,0),\dots,d_n(x_n,0))\|_p=|\alpha|\,\|(d_1(x_1,0),\dots,d_n(x_n,0))\|_p, \alpha\in\mathbb{R}$
(iv) $d_p((x_1,y_1),(x_2,y_2)\cdots(x_n,y_n))=(d_X(x_1,x_2,\cdots x_n)^p+d_Y(y_1,y_2,\cdots y_n)^p)^{1/p}$ $for 1\leq p<\infty$; (or)
(v) $d((x_1,y_1),(x_2,y_2),\cdots(x_n,y_n)):=\sup\{d_X(x_1,x_2,\cdots x_n),d_Y(y_1,y_2,\cdots y_n)\}$,
for $x_1,x_2,\cdots x_n\in X, y_1,y_2,\cdots y_n\in Y$ is called the p product metric of the Cartesian product of n metric spaces (see [16]) .

2.3. **Definition.** Let $\eta = (\lambda_{abc})$ be a non-decreasing sequence of positive real numbers tending to infinity and $\lambda_{111} = 1$ and $\lambda_{a+b+c+3} \leq \lambda_{a+b+c+3}+1$, for all $a, b, c \in \mathbb{N}$. The collection of all such triple sequences λ is denoted by $\Im$.

The generalized de la Vallèe-Poussin means are defined by
$t_{abc}(x) = \lambda_{abc}^{-1} \sum_{m,n,k \in I_{abc}} B_{mnk}(f,x)$, where $I_{abc} = [abc - \lambda_{abc} + 1, abc]$. Let f be a continuous function defined on the closed interval $[0,1]$. A triple sequence of Bernstein polynomials $(B_{mnk}(f,x))$ of real numbers of random variables is said to $(V,\lambda)-$ summable to a number $f(x)$ if $t_{abc}(B_{mnk}(f,x)) \to f(x)$, as $abc \to \infty$.

2.4. **Definition.** Let f be a continuous function defined on the closed interval $[0,1]$. A triple sequence of Bernstein polynomials $(B_{mnk}(f,x))$ of real numbers of random variables is said to strong (V,λ) summable (or shortly : $[V,\lambda]-$ convergent to $f(x)$ if
$lim_{abc\to\infty} \frac{1}{\lambda_{abc}} \sum_{m \in I_a} \sum_{n \in I_b} \sum_{k \in I_c} |B_{mnk}(f,x), f(x)| = 0$. In this case write $B_{mnk}(f,x) \to^{[V,\lambda]} f(x)$.

2.5. **Definition.** Let f be a continuous function defined on the closed interval $[0,1]$. A triple sequence of Bernstein polynomials $(B_{mnk}(f,x))$ of real numbers of random variables is said to be $\lambda-$ statistically convergent (or shortly: $S_\lambda-$ convergent) to $f(x)$ if for any $\epsilon > 0$,
$lim_{abc\to\infty} \frac{1}{\lambda_{abc}} |\{(m,nk) \in I_{abc} : |B_{mnk}(f,x), f(x)| \geq \epsilon\}| = 0$. In this case we write $S_\lambda - limB_{mnk}(f,x) = f(x)$ or by $B_{mnk}(f,x) \to^{S_\lambda} f(x)$. Now we introduce the following main definition:

2.6. **Definition.** Let f be a continuous function defined on the closed interval $[0,1]$. A triple sequence of Bernstein polynomials $(B_{mnk}(f,x))$ of real numbers and α be non negative real number of random variables is said to be rough $[V,\lambda]-$ summable in probability to $X : W \times W \times W \to \mathbb{R} \times \mathbb{R} \times \mathbb{R}$ with respect to the roughness of degree α (or shortly: $\alpha - [V,\lambda]-$ summable in probability to $f(x)$ if for any $\epsilon > 0$,
$lim_{abc\to\infty} \frac{1}{\lambda_{abc}} \sum_{m \in I_a} \sum_{n \in I_b} \sum_{k \in I_c} P(|B_{mnk}(f,x), f(x)| \geq \alpha + \epsilon) = 0$. In this case we write $B_{mnk}(f,x) \to_\alpha^{[V,\lambda]^P} f(x)$. The class of all rough $[V,\lambda]-$ summable triple sequence space of Bernstein polynomials of random variables in probability will be denoted simply by $\alpha [V,\lambda]^P$.

2.7. **Definition.** Let f be a continuous function defined on the closed interval $[0,1]$. A triple sequence of Bernstein polynomials $(B_{mnk}(f,x))$ of real numbers and α be non negative real number of random variables is said to be rough $\lambda-$ statistically convergent in probability to $X : W \times W \times W \to \mathbb{R} \times \mathbb{R} \times \mathbb{R}$ with respect to the roughness of degree α (or shortly: $\alpha - \lambda-$ statistically convergent in probability to $f(x)$ if for any $\epsilon, \delta > 0$, $lim_{abc\to\infty} \frac{1}{\lambda_{abc}} |\{(m,nk) \in I_{abc} : P(|B_{mnk}(f,x), f(x)| \geq \alpha + \epsilon) \geq \delta\}| = 0$. In this case we write $B_{mnk}(f,x) \to_\alpha^{S_\lambda^P} f(x)$. The class of all $\alpha - \lambda-$ statistically convergent triple sequence space of Bernstein polynomials of random variables in probability will be denoted simply by αS_λ^P.

2.8. **Note.** Let f be a continuous function defined on the closed interval $[0,1]$. A triple sequence of Bernstein polynomials $(B_{mnk}(f,x))$ of real numbers and M be an Musielak-Orlicz function is defined by
$\left\|\chi_M^3, (d(x_1), d(x_2), \cdots, d(x_{n-1}))\right\|_p =$

$\left[M_{mnk}\left(\|\mu_{mnk}\left(B_{mnk}\left(f,x\right)\right),\left(d\left(x_1\right),d\left(x_2\right),\cdots,d\left(x_{n-1}\right)\right)\|_p\right)\right],$
where $\mu_{mnk}\left(X\right)=\left(\left((m+n+k)!B_{mnk}\left(f,x\right)\right)^{1/m+n+k},f\left(x\right)\right).$

3. Main Results

3.1. **Theorem.** Let f be a continuous function defined on the closed interval $[0,1]$. A triple sequence of Bernstein polynomials $\left(B_{mnk}\left(f,x\right)\right)$ of real numbers of random variables and M be an Musielak-Orlicz function are equivalent:
(i) $\left\|\chi^3_M\left(B_{mnk}\left(f,x\right)\right),\left(d\left(x_1\right),d\left(x_2\right),\cdots,d\left(x_{n-1}\right)\right)\right\|_p$ is $\alpha-[V,\lambda]-$ summable in probability to $f\left(x\right)$.
(ii) $\left\|\chi^3_M,\left(d\left(x_1\right),d\left(x_2\right),\cdots,d\left(x_{n-1}\right)\right)\right\|_p$ is $\alpha-\lambda-$ statistically convergent in probability to $f\left(x\right)$.
Proof: Similar to the proof of Theorem (3.1) in (see [18]).

3.2. **Theorem.** If $\left\|\chi^3_M\left(B_{mnk}\left(f,x\right)\right),\left(d\left(x_1\right),d\left(x_2\right),\cdots,d\left(x_{n-1}\right)\right)\right\|_p\rightarrow^{S^P}_{\alpha}\ f\left(x\right)$ and
$\left\|\chi^3_M\left(B_{mnk}\left(f,y\right)\right),\left(d\left(x_1\right),d\left(x_2\right),\cdots,d\left(x_{n-1}\right)\right)\right\|_p\rightarrow^{S^P}_{\beta}\ f\left(y\right)$ then
$P\left(\left|\left[M_{mnk}\left(\|\mu_{mnk}\left(X\right),\left(d\left(x_1\right),d\left(x_2\right),\cdots,d\left(x_{n-1}\right)\right)\|_p\right)\right]\right|\geq\alpha+\epsilon\right)=0.$
Proof: Similar to the proof of Theorem (3.1) in (see [17]).

3.3. **Theorem.** Let f be a continuous function defined on the closed interval $[0,1]$. A triple sequence of Bernstein polynomials $\left(B_{mnk}\left(f,x\right)\right)$ of real numbers of random variables and M be an Musielak-Orlicz function. If $\lambda\in\Im$ is such that $\frac{\lambda_{abc}}{(abc)}=1$ then $\alpha S^P_\lambda\subset\alpha S^P$.
Proof: Let $0<\eta<1$ be given. Since $lim_{abc\rightarrow\infty}\frac{\lambda_{abc}}{(abc)}=1$, we can choose $(u,v,w)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}$ such that $\left|\frac{\lambda_{abc}}{(abc)}-1\right|<\frac{\eta}{2}$ for all $(a,b,c)>(u,v,w)$. Now, for $\epsilon,\delta>0$

$$\frac{1}{abc}\left|\left\{(mnk)\leq(abc):P\left(\left|\left[f_{mnk}\left(\|\mu_{mnk}\left(X\right),\left(d\left(x_1\right),d\left(x_2\right),\cdots,d\left(x_{n-1}\right)\right)\|_p\right)\right]\right|\geq\alpha+\epsilon\right)\geq\delta\right\}\right|=$$
$$\frac{1}{abc}\left|\left\{(mnk)\leq(abc)-\lambda_{abc}:P\left(\left|\left[f_{mnk}\left(\|\mu_{mnk}\left(X\right),\left(d\left(x_1\right),d\left(x_2\right),\cdots,d\left(x_{n-1}\right)\right)\|_p\right)\right]\right|\geq\alpha+\epsilon\right)\geq\delta\right\}\right|$$
$$=\frac{1}{abc}\left|\left\{(mnk)\in I_{abc}:P\left(\left|\left[f_{mnk}\left(\|\mu_{mnk}\left(X\right),\left(d\left(x_1\right),d\left(x_2\right),\cdots,d\left(x_{n-1}\right)\right)\|_p\right)\right]\right|\geq\alpha+\epsilon\right)\geq\delta\right\}\right|$$
$$\leq\frac{(abc)-\lambda_{abc}}{(abc)}+$$
$$\frac{1}{abc}\left|\left\{(mnk)\in I_{abc}:P\left(\left|\left[f_{mnk}\left(\|\mu_{mnk}\left(X\right),\left(d\left(x_1\right),d\left(x_2\right),\cdots,d\left(x_{n-1}\right)\right)\|_p\right)\right]\right|\geq\alpha+\epsilon\right)\geq\delta\right\}\right|$$
$$\leq1-\left(1-\frac{\eta}{2}\right)+$$
$$\frac{1}{abc}\left|\left\{(mnk)\in I_{abc}:P\left(\left|\left[f_{mnk}\left(\|\mu_{mnk}\left(X\right),\left(d\left(x_1\right),d\left(x_2\right),\cdots,d\left(x_{n-1}\right)\right)\|_p\right)\right]\right|\geq\alpha+\epsilon\right)\geq\delta\right\}\right|$$
$$=\frac{\eta}{2}+$$
$$\frac{\lambda_{abc}}{abc}\frac{1}{\lambda_{abc}}\left|\left\{(mnk)\in I_{abc}:P\left(\left|\left[f_{mnk}\left(\|\mu_{mnk}\left(X\right),\left(d\left(x_1\right),d\left(x_2\right),\cdots,d\left(x_{n-1}\right)\right)\|_p\right)\right]\right|\geq\alpha+\epsilon\right)\geq\delta\right\}\right|$$
$$<\frac{\eta}{2}+$$
$$\frac{1}{\lambda_{abc}}\left|\left\{(mnk)\in I_{abc}:P\left(\left|\left[f_{mnk}\left(\|\mu_{mnk}\left(X\right),\left(d\left(x_1\right),d\left(x_2\right),\cdots,d\left(x_{n-1}\right)\right)\|_p\right)\right]\right|\geq\alpha+\epsilon\right)\geq\delta\right\}\right|$$

holds for all $(a,b,c)\geq(u,v,w)$.

3.4. **Theorem.** Let f be a continuous function defined on the closed interval $[0,1]$. A triple sequence of Bernstein polynomials $(B_{mnk}(f,x))$ of real numbers of random variables and M be an Musielak-Orlicz function and $\alpha S^P \subset \alpha S^P_\lambda$ if and only if $lim_{abc\to\infty} \frac{\lambda_{abc}}{(abc)} > 0$.

Proof: Let $lim_{abc\to\infty} \frac{\lambda_{abc}}{(abc)} > 0$. Then for $\epsilon, \delta > 0$, we have

$$\frac{1}{abc}\left|\left\{(mnk) \leq (abc) : P\left(\left|\left[f_{mnk}\left(\left\|\mu_{mnk}(X),(d(x_1),d(x_2),\cdots,d(x_{n-1}))\right\|_p\right)\right]\right| \geq \alpha+\epsilon\right) \geq \delta\right\}\right|$$
$$\geq \frac{1}{abc}\left|\left\{(mnk) \in I_{abc} : P\left(\left|\left[f_{mnk}\left(\left\|\mu_{mnk}(X),(d(x_1),d(x_2),\cdots,d(x_{n-1}))\right\|_p\right)\right]\right| \geq \alpha+\epsilon\right) \geq \delta\right\}\right|$$
$$= \frac{\lambda_{abc}}{abc}$$
$$\frac{1}{\lambda_{abc}}\left|\left\{(mnk) \in I_{abc} : P\left(\left|\left[f_{mnk}\left(\left\|\mu_{mnk}(X),(d(x_1),d(x_2),\cdots,d(x_{n-1}))\right\|_p\right)\right]\right| \geq \alpha+\epsilon\right) \geq \delta\right\}\right|.$$

Taking limit $abc \to \infty$ we get $\left\|\chi^3_M(B_{mnk}(f,x)),(d(x_1),d(x_2),\cdots,d(x_{n-1}))\right\|_p \to^{S^P}_\alpha$ $f(x) \Longrightarrow$

$\left\|\chi^3_M(B_{mnk}(f,x)),(d(x_1),d(x_2),\cdots,d(x_{n-1}))\right\|_p \to^{S^P_\lambda}_\alpha f(x)$.

Conversely, let $lim_{abc\to\infty} \frac{\lambda_{abc}}{(abc)} = 0$ then we can choose a subsequence $(a_u, b_v, c_w)_{u,v,w\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}}$ such that $\frac{\lambda_{a_u,b_v,c_w}}{a_u,b_v,c_w} < \frac{1}{uvw}$ for all $u,v,w \in \mathbb{N}$. Define a triple sequence space of Bernstein polynomials of $B_{mnk}(f,x)$ of random numbers of random variables whose probability density function is

$$\mu_{abc}(X) = \begin{cases} 1, & \text{if } 0 < X < 1 \\ 0, & \text{otherwise }, where\,(abc) \in I_{abc} for some\,(uvw) \in \mathbb{N} \end{cases}$$

Let $0 < \epsilon, \delta < 1$. Then

$$P\left(\left|\left[M_{mnk}\left(\left\|\mu_{mnk}(X),(d(x_1),d(x_2),\cdots,d(x_{n-1}))\right\|_p\right)\right]\right| \geq 1+\epsilon\right) = \begin{cases} 1, & \text{if } (abc) \in I_{abc} for some\,(uvw) \in \mathbb{N} \\ \left(1-\frac{\epsilon}{2}\right)^n, & \text{otherwise }. \end{cases}$$

We have

$$\frac{1}{\lambda_{abc}}\left|\left\{(mnk) \in I_{abc} : P\left(\left|\left[M_{mnk}\left(\left\|\mu_{mnk}(X),(d(x_1),d(x_2),\cdots,d(x_{n-1}))\right\|_p\right)\right]\right| \geq 1+\epsilon\right) \geq \delta\right\}\right| =$$
$$\begin{cases} 1, & \text{if } (abc) \in I_{abc} for some\,(uvw) \in \mathbb{N} \\ 0, & \text{otherwise }. \end{cases}$$

Hence $\left\|\chi^3_M(B_{mnk}(f,x)),(d(x_1),d(x_2),\cdots,d(x_{n-1}))\right\|_p \notin^{S^P_\lambda}_\alpha$.

4. Conclusions and Future Work

We introduced triple sequence space of Bernstein polynomials of random numbers of random variables of rough $\lambda-$ statistical convergence in probability with respect sequence of Musielak-Orlicz function. For the reference sections, consider the following introduction described the main results are motivating the research.

5. Competing Interests

The authors declare that there is not any conflict of interests regarding the publication of this manuscript.

References

[1] S. Aytar 2008. Rough statistical Convergence, *Numerical Functional Analysis Optimization*, **29(3)**, 291-303.

[2] A. Esi , On some triple almost lacunary sequence spaces defined by Orlicz functions, *Research and Reviews:Discrete Mathematical Structures*, **1(2)**, (2014), 16-25.

[3] A. Esi and M. Necdet Catalbas,Almost convergence of triple sequences, *Global Journal of Mathematical Analysis*, **2(1)**, (2014), 6-10.

[4] A. Esi and E. Savas, On lacunary statistically convergent triple sequences in probabilistic normed space,*Appl.Math.and Inf.Sci.*, **9 (5)** , (2015), 2529-2534.

[5] A. Esi, S. Araci and M. Acikgoz, Statistical Convergence of Bernstein Operators,*Appl.Math.and Inf.Sci.*, **10 (6)** , (2016), 2083-2086.

[6] A. J. Datta A. Esi and B.C. Tripathy,Statistically convergent triple sequence spaces defined by Orlicz function , *Journal of Mathematical Analysis*, **4(2)**, (2013), 16-22.

[7] S. Debnath, B. Sarma and B.C. Das ,Some generalized triple sequence spaces of real numbers , *Journal of nonlinear analysis and optimization*, **Vol. 6, No. 1** (2015), 71-79.

[8] P.K. Kamthan and M. Gupta, Sequence spaces and series, Lecture notes, Pure and Applied Mathematics, *65 Marcel Dekker, In c. New York* , **1**981.

[9] J. Lindenstrauss and L. Tzafriri, On Orlicz sequence spaces, *Israel J. Math.*, **10** (1971), 379-390.

[10] J. Musielak, Orlicz Spaces,Lectures Notes in Math.,1034, *Springer-Verlag*, **1983**.

[11] S.K. Pal, D. Chandra and S. Dutta 2013. Rough ideal Convergence, *Hacee. Jounral Mathematics and Statistics*, **42(6)**, 633-640.

[12] H.X. Phu 2001. Rough convergence in normed linear spaces, *Numerical Functional Analysis Optimization*, **22**, 201-224.

[13] A. Sahiner, M. Gurdal and F.K. Duden, Triple sequences and their statistical convergence, *Selcuk J. Appl. Math.* , **8 No. (2)**(2007), 49-55.

[14] A. Sahiner, B.C. Tripathy , Some I related properties of triple sequences, *Selcuk J. Appl. Math.* , **9 No. (2)**(2008), 9-18.

[15] N. Subramanian and A. Esi, The generalized tripled difference of χ^3 sequence spaces, *Global Journal of Mathematical Analysis*, **3 (2)** (2015), 54-60.

[16] N. Subramanian and C. Murugesan, The entire sequence over Musielak $p-$ metric space, *Journal of the Egyptian Mathematical Society*, **Vol. 24 (2)** (2016), 233-238.

[17] N. Subramanian , On triple sequence spaces of χ^3 of random variable in probability of rough statistical convergence of $p-$ metric defined by Musielak-Orlicz function, *Mathematical Sciences.*, **communicated** (2016).

[18] N. Subramanian , On triple sequence spaces of χ^3 of rough lacunary statistical convergence in probability of $p-$ metric defined by Musielak-Orlicz function, *Acta arithmetica.*, **communicated** (2016).

N. Subramanian

Department of Mathematics, SASTRA University, Thanjavur-613 401, India

E-mail address: nsmaths@yahoo.com

A. Esi

Department of Mathematics, Adiyaman University, 02040,Adiyaman, Turkey

E-mail address: aesi23hotmail.com