Electronic Journal of Mathematical Analysis and Applications
Vol. 7(1) Jan. 2019, pp. 5-13.
ISSN: 2090-729X(online)
http://fcag-egypt.com/Journals/EJMAA/

DEGREE SEQUENCES OF JOIN AND CORONA PRODUCTS OF GRAPHS

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Abstract. Topological indices are very important in Mathematics and have a lot of applications in Molecular Chemistry and many other areas. About the half of them are defined in terms of vertex degrees. The degree sequence of a graph is the set of vertex degrees. It gives many information about the properties of the graph and also the real life situations that the graph corresponds.

Graph operations are used to obtain larger graphs from given graphs and vice versa. Two of the most important and applicable graph operations, the join and corona product, are studied and the degree sequences of the graphs obtained by these two operations from two simple connected graphs were determined by the authors recently. Also the algebraic structure of this two graph operations were determined. In this paper, we generalize these results to the join and corona products of any number of graphs and obtain several formulae giving the degree sequence of the resulting graphs.

1. Introduction

Throughout this paper, we take $G = (V(G), E(G))$ as a simple and connected undirected graph with $\mid V(G) \mid= n$ vertices and $\mid E(G) \mid= m$ edges. For disconnected graphs, similar calculations can be made. Usually we use the abbreviated notations V and E instead of $V(G)$ and $E(G)$, respectively. Here, by the word "simple", we understand that the graphs we consider neither have loops nor multiple edges. Similar studies can be done for non-simple graphs as well with some notational and computational differences. This problem has not yet been considered for directed and oriented graphs, but one can study the same problem for these more complicated graph types but naturally, there will be many possibilities to consider.

For a vertex $v \in V$, we denote the degree of v by $deg_G(v)$, $d_G(v)$ or briefly $d(v)$, which is defined as the number of edges of G meeting at vertex v. A vertex with degree one is usually called a pendant vertex.

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2010 *Mathematics Subject Classification.* 05C07, 05C76.
Key words and phrases. degree sequence, join, corona product, graph operation.
Submitted Mar. 26, 2018.

The notion of degree of a graph provides us an area to study various structural properties of graphs and hence attracts the attention of many graph theorists. If $d_i,\ 1 \leq i \leq n$, are the degrees of the vertices v_i of a graph G in any order, then the degree sequence (DS) of G is the sequence $\{d_1,\ d_2,\ \cdots,\ d_n\}$. In some places, it is also denoted by $\{d_1\ d_2\ \cdots d_n\}$, but we prefer the former notation. Also, in many papers, the DS is taken to be a non-decreasing sequence, whenever possible. Conversely, given a non-negative sequence $\{d_1,\ d_2,\ \cdots,\ d_n\}$, this sequence will be called realizable if it is the DS of any graph. It is well-known that for a realizable DS, there is at least one graph having this DS. See the graphs in Figure 1 which have the same DS.

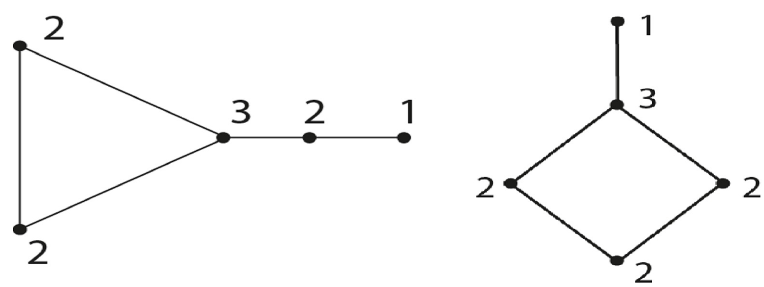

Fig. 1 Graphs with the same DS

For convenience and brevity, we shall denote the DS having repeated degrees with a shorter DS. For example, if the degree d_i of the vertex v_i appears z_i times in the DS of a graph G, then we use $\{d_1^{z_1},\ d_2^{z_2},\ \cdots,\ d_l^{z_l}\}$ instead of $\{d_1,\ d_2,\ \cdots,\ d_n\}$ where $l \leq n$. Here the members z_i are called the frequencies of the degrees. When $l = n$, that is, when all degrees are different, the DS is called perfect.

It is an open problem to determine that which DSs are realizable and there are several algorithms to determine that. As many graph theoretical aspects are widely used in some areas including Chemistry and Pharmacology, the determination of DSs of more complex graphs is becoming more and more important.

As usual, we denote by $P_n,\ C_n,\ S_n,\ K_n,\ K_{r,s}$ and $T_{r,s}$ the path, cycle, star, complete, complete bipartite and tadpole graphs, respectively, which are the most used graph examples in literature, see Fig. 2.

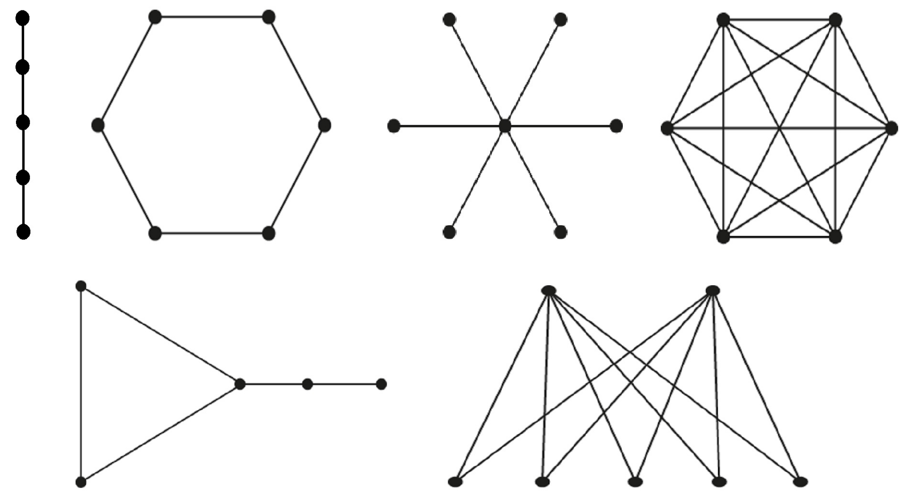

Fig. 2 $P_5,\ C_6,\ S_7,\ K_6,\ T_{3,2},\ K_{2,5}$

In Graph Theory, graph operations are used to obtain larger graphs from given graphs and vice versa. In that way, the properties of the larger graph can be obtained by means of the same properties of smaller graphs. These include several combinatorial properties of graphs such as the number of loops, pendant edges, chords, etc. and some other properties such as chromatic number, chromatic polynomial, M-polynomila etc. Two of the most important graph operations, the join and corona product, are studied and the DSs of the graphs obtained by these two operations from two simple connected graphs were determined by the authors in [4] for some widely used graph classes. Also the algebraic structure of this two graph operations were determined in that paper. In this paper, we generalize these results to the join and corona products of any number of graphs and obtain several formulae giving the DS of the resulting graphs.

Another important reason to study the DS of graphs is topological indices. A topological index (or a graph invariant) is a fixed number invariant for two isomorphic graphs and gives some information about the graph under consideration. These indices are especially useful in the study of molecular graphs. A large number of the topological indices are defined by means of the vertex degrees: first and second Zagreb indices, first and second multiplicative Zagreb indices, atom-bond connectivity index, Narumi-Katayama index, geometric-arithmetic index, harmonic index and sum-connectivity index etc. There are hundreds of papers on degree based topological indices, see e.g. [2, 3, 5, 6, 9, 11, 15] and the references therein. Therefore to know about the DS of the graph will help to obtain information about, e.g., the chemical properties of the graph. Fnally, to move between graphs corresponds to moving between the corresponding DSs. All these reasons show the importance and the need for the studies with DSs.

The modern study of DSs started in 1981 by Bollobas, [1]. The same year, Tyshkevich et.al. established a correspondence between DS of a graph and some structural properties of this graph, [13]. In 1987, Tychkevich et.al. written a survey on the same correspondence, [14]. In [16], the authors gave a new version of the

Erdös-Gallai theorem on the realizability of a given DS. In 2008, a new criterion on the same problem is given by Triphati and Tyagi, [12]. The same year, Kim et al gave a necessary and sufficient condition for the same problem, [8]. Ivanyi et.al, [7], gave an enumeration of DSs of simple graphs. Miller, [10], also gave some criteria for the realizability of given DSs.

There are several graph operations used in calculating some chemical invariants of graphs. Amongst these the join, cartesian, corona product, union, disjunction, and symmetric difference are well-known. The authors studied several graph operations by means of DSs in [4]. In this paper, after recalling two of these operations, join and corona product, we shall determine the DS of the join and corona product of any number of graphs.

Let G_1 and G_2 be two graphs with n_1 and n_2 vertices and m_1 and m_2 edges, respectively. The join $G_1 \vee G_2$ of graphs G_1 and G_2 with disjoint vertex sets $V(G_1)$ and $V(G_2)$ and edge sets $E(G_1)$ and $E(G_2)$ is the graph union $G_1 \cup G_2$ together with all the edges joining $V(G_1)$ and $V(G_2)$. We have $\mid V(G_1 \vee G_2) \mid= n_1 + n_2$ and $\mid E(G_1 \vee G_2) \mid= m_1 + m_2 + n_1 n_2$.

The corona product $G_1 \circ G_2$ of two graphs G_1 and G_2 is defined to be the graph obtained by taking one copy of G_1 (which has n_1 vertices) and n_1 copies of G_2, and then joining the i-th vertex of G_1 to every vertex in the i-th copy of G_2, for $i = 1,\ 2,\ \cdots,\ n_1$.

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs such that $V(G_1) = \{u_1,\ u_2,\ \cdots, u_{n_1}\}$, $\mid E(G_1) \mid= m_1$ and $V(G_2) = \{v_1,\ v_2,\ \cdots,\ v_{n_2}\}$, $\mid E(G_2) \mid= m_2$. Then it follows from the definition of the corona product that $G_1 \circ G_2$ has $n_1(1+n_2)$ vertices and $m_1 + n_1 m_2 + n_1 n_2$ edges, where

$$V(G_1 \circ G_2) = \{(u_i, v_j),\ i = 1,\ 2,\ \cdots, n_1;\ j = 0,\ 1,\ 2,\ \cdots,\ n_2\}$$

and

$$\begin{aligned} E(G_1 \circ G_2) = &\{((u_i, v_0), (u_k, v_0)), (u_i, u_k) \in E(G_1)\} \\ &\cup \{((u_i, v_j), (u_i, v_\ell)), (v_j, v_\ell) \in E(G_2),\ i = 1, 2, ..., n_1\} \\ &\cup \{((u_i, v_0), (u_i, v_\ell)), \ell = 1, 2, ..., n_2,\ i = 1, 2, ..., n_1\}. \end{aligned}$$

It is clear that if G_1 is connected, then $G_1 \circ G_2$ is connected, and in general, $G_1 \circ G_2$ is not isomorphic to $G_2 \circ G_1$.

2. General Formulae for the DSs of the join operation

In this section, we obtain the DS of the join of any given number of simple connected graphs. First we start with two graphs G_1, G_2 and obtain the DS of $G_1 \vee G_2$, and using mathematical induction, we obtain the general formula for $G_1 \vee G_2 \vee \cdots \vee G_l$ in terms of the number of vertices of G_i's. We give an alternative formula for the DS of $G_1 \vee G_2 \vee \cdots \vee G_l$ in terms of vertex degrees.

2.1. **Theorem.** Let G_1 and G_2 be two simple connected graphs with DSs

$$DS(G_1) \;=\; \{\alpha_{11}^{\beta_{11}},\;\cdots,\;\alpha_{1k_1}^{\beta_{1k_1}}\}$$

and

$$DS(G_2) \;=\; \{\alpha_{21}^{\beta_{21}},\;\cdots,\;\alpha_{2k_2}^{\beta_{2k_2}}\},$$

respectively. Then the DS of the join of the two graphs G_1 and G_2 is

$$DS(G_1 \vee G_2) \;=\; \{(n_2+\alpha_{11})^{\beta_{11}},\;\cdots,\;(n_2+\alpha_{1k_1})^{\beta_{1k_1}},\;(n_1+\alpha_{21})^{\beta_{21}},\;\cdots,\;(n_2+\alpha_{2k_2})^{\beta_{2k_2}}\}.$$

Note that to obtain $DS(G_1 \vee G_2)$, we add the number n_2 of vertices of G_2 to each of α_{1j} where $1 \le j \le k_1$, without changing the powers β_{1j}, and add the number n_1 of vertices of G_1 to each of α_{2j} where $1 \le j \le k_2$, without changing the powers β_{2j}.

2.2. **Example.** Let us consider $P_r = \{1^2, 2^{r-2}\}$ and $K_{s,t} = \{s^t, t^s\}$. We will find the DS of $P_r \vee K_{s,t}$. Let the number of vertices of P_r be n_1, and the number of vertices of $K_{s,t}$ be n_2.

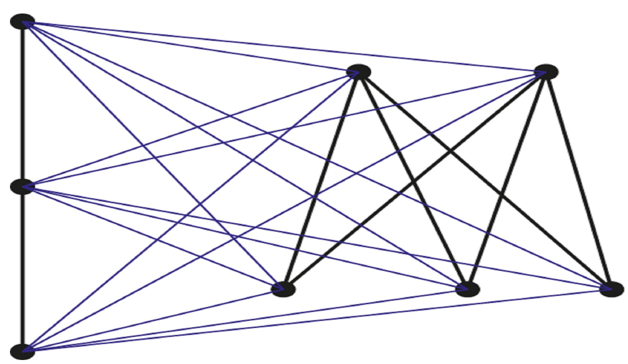

Fig. 3 $P_3 \vee K_{2,3}$

As $\alpha_{11} = 1$, $\beta_{11} = 2$, $\alpha_{12} = 2$, $\beta_{12} = r-2$, $\alpha_{21} = s$, $\beta_{21} = t$, $\alpha_{22} = t$ and $\beta_{22} = s$, by the definition of join operation, we have

$$DS(P_r \vee K_{s,t}) = \{(s+t+1)^2, (s+t+2)^{r-2}, (r+s)^t, (r+t)^s\}.$$

Now we take the join of l simple connected graphs $G_1,\ G_2,\ \cdots,\ G_l$ where $l \ge 2$ is a finite integer. The DS of $G_1 \vee G_2 \vee \cdots \vee G_l$ is given as follows:

2.3. **Theorem.** Let $G_1,\ G_2,\ \cdots,\ G_l$ be l simple connected graphs. Let G_i have n_i vertices for $i = 1,\ 2,\ \cdots,\ l$. Also let the DS of G_i be

$$DS(G_i) = \{\alpha_{i1}^{\beta_{i1}},\;\cdots,\;\alpha_{ik_i}^{\beta_{ik_i}}\}.$$

Then the DS of the join of $G_1,\ G_2,\ \cdots,\ G_l$ is

$$\begin{aligned} DS(G_1 \vee G_2 \vee \cdots \vee G_l) = \{ & (n_2 + n_3 + \ \cdots \ + n_l + \alpha_{11})^{\beta_{11}}, \ \cdots, \\ & (n_2 + n_3 + \ \cdots \ + n_l + \alpha_{1k_1})^{\beta_{1k_1}}, \\ & (n_1 + n_3 + \ \cdots \ + n_l + \alpha_{21})^{\beta_{21}}, \ \cdots, \\ & (n_1 + n_3 + \ \cdots \ + n_l + \alpha_{2k_2})^{\beta_{2k_2}}, \\ & \qquad \cdots, \\ & (n_1 + n_2 + \ \cdots \ + n_{l-1} + \alpha_{l1})^{\beta_{l1}}, \ \cdots, \\ & (n_1 + n_2 + \ \cdots \ + n_{l-1} + \alpha_{lk_l})^{\beta_{lk_l}} \}. \end{aligned}$$

2.4. **Example.** We give an example for the case $l = 3$:

$$\begin{aligned} DS(P_4 \vee C_3 \vee P_3) &= \{1^2, 2^2\} \vee \{2^3\} \vee \{1^2, 2\} \\ &= \{(3+1)^2, (3+2)^2, (4+2)^3\} \vee \{1^2, 2\} \\ &= \{(3+4)^2, (3+5)^2, (3+6)^3, (7+1)^2, (7+2)\} \\ &= \{7^2, 8^2, 9^3, 8^2, 9\} \\ &= \{7^2, 8^4, 9^4\}. \end{aligned}$$

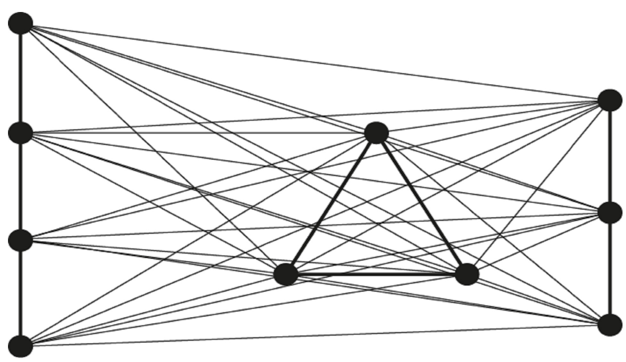

Fig. 4 $P_4 \vee C_3 \vee P_3$

It is possible to give the $DS(G_1 \vee G_2 \vee \cdots \vee G_l)$ in a brief form:

2.5. **Corollary.**

$$\begin{aligned} DS(G_1 \vee G_2 \vee \cdots \vee G_l) = \{ & (A - n_1 + \alpha|11)^{\beta_{11}}, \ (A - n_1 + \alpha_{12})^{\beta_{12}}, \cdots, \ (A - n_1 + \alpha_{1k_1})^{\beta_{1k_1}}, \\ & (A - n_2 + \alpha_{21})^{\beta_{21}}, \ (A - n_2 + \alpha_{22})^{\beta_{22}}, \cdots, \ (A - n_2 + \alpha_{2k_2})^{\beta_{2k_2}}, \\ & \qquad \cdots, \\ & (A - n_l + \alpha_{l1})^{\beta_{l1}}, \ (A - n_l + \alpha_{l2})^{\beta_{l2}}, \cdots, \ (A - n_l + \alpha_{lk_l})^{\beta_{lk_l}} \} \end{aligned}$$

where $A_{r,s} = (A - n_r + \alpha_{rs})^{\beta_{rs}}$ with $r = 1, \ 2, \ \cdots, \ l; \ s = 1, \ 2, \ \cdots, \ k_r$ and $A = \sum_{u=1}^{l} n_u$.

Now we give $DS(G_1 \vee G_2 \vee G_l)$ only in terms of the vertex degrees α_{ij} and their frequencies β_{ij}:

2.6. **Corollary.**

$$\begin{aligned} DS(G_1 \vee G_2 \vee \cdots \vee G_l) = \{ & (B-b_1+\alpha_{11})^{\beta_{11}},\ (B-b_1+\alpha_{12})^{\beta_{12}}, \cdots,\ (B-b_1+\alpha_{1k_1})^{\beta_{1k_1}}, \\ & (B-b_2+\alpha_{21})^{\beta_{21}},\ (B-b_2+\alpha_{22})^{\beta_{22}}, \cdots,\ (B-b_2+\alpha_{2k_2})^{\beta_{2k_2}}, \\ & \cdots, \\ & (B-b_l+\alpha_{l1})^{\beta_{l1}},\ (B-b_l+\alpha_{l2})^{\beta_{l2}}, \cdots,\ (B-b_l+\alpha_{lk_l})^{\beta_{lk_l}} \}. \end{aligned}$$

Finally we obtain a compact formula for $DS(G_1 \vee G_2 \vee \cdots \vee G_l)$ as follows:

2.7. **Corollary.** Let $b_i = \sum_{t=1}^{k_i} \alpha_{it}$ for $i = 1,\ 2,\ \cdots,\ l$ and $B = \sum_{u=1}^{k_l} (\sum_{t=1}^{l} \alpha_{tu})$. Then

$$DS(G_1 \vee G_2 \vee \cdots \vee G_l) = \{(B-b_r+\alpha_{rs})^{\beta_{rs}}\ :\ r = 1,\ 2,\ \cdots,\ l;\ s = 1,\ 2,\ \cdots,\ k_r\}.$$

3. General Formulae for the Degree Sequences of Corona Product Operation

Now, similarly to the join operation, we obtain the DSs of the corona product of any given number of simple connected graphs. Again, we start with two graphs $G_1,\ G_2$ and obtain the DS of $G_1 \circ G_2$, and using mathematical induction, we obtain the general formula for $G_1 \circ G_2 \circ \cdots \circ G_l$ in terms of the number of vertices of G_i's. We give an alternative formula for the DS of $G_1 \circ G_2 \circ \cdots \circ G_l$ in terms of vertex degrees.

3.1. **Theorem.** Let G_1 and G_2 be two simple connected graphs with DSs $DS(G_1) = \{\alpha_{11}^{\beta_{11}},\ \cdots,\ \alpha_{1k_1}^{\beta_{1k_1}}\}$ and $DS(G_2) = \{\alpha_{21}^{\beta_{21}},\ \cdots,\ \alpha_{2k_2}^{\beta_{2k_2}}\}$, respectively. Then the DS of the corona product of the two graphs G_1 and G_2 is

$$DS(G_1 \circ G_2) = \left\{(n_2+\alpha_{11})^{\beta_{11}},\ \cdots,\ (n_2+\alpha_{1k_1})^{\beta_{1k_1}}, (1+\alpha_{21})^{n_1\cdot\beta_{21}},\ \cdots,\ (1+\alpha_{2k_2})^{n_1\cdot\beta_{2k_2}}\right\}.$$

Note that to obtain $DS(G_1 \circ G_2)$, we add the number n_2 of vertices of G_2 to each of α_{1j} where $1 \le j \le k_1$, without changing the powers β_{1j}, and add 1 to each of α_{2j} where $1 \le j \le k_2$ while multiplying each of the powers β_{2j} by the number n_1 of vertices of G_1.

Let us consider the corona product $P_5 \circ P_3$. Now we have $n_1 = 5,\ n_2 = 3$, $\alpha_{11} = 1$, $\beta_{11} = 2$, $\alpha_{12} = 2$, $\beta_{12} = 3$, $\alpha_{21} = 1$, $\beta_{21} = 2$, $\alpha_{22} = 2$ and $\beta_{22} = 1$. By the definition of corona product operation, we have the DS of $P_5 \circ P_3$ as

$$DS(P_5 \circ P_3) = \{1^2, 2^3\} \circ \{1^2, 2\} = \{(3+1)^2, (3+2)^3, (1+1)^{5\cdot 2}, (1+2)^{5\cdot 1}\} = \{4^2, 5^3, 2^{10}, 3^5\}.$$

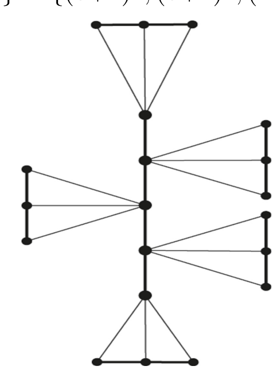

Fig. 5 $P_5 \vee P_3$

Finally we generalize our result for two graphs to obtain a compact formula for $DS(G_1 \circ G_2 \circ \cdots \circ G_l)$ as follows:

3.2. **Corollary.** The DS of l graphs $G_1,\ G_2,\ \cdots,\ G_l$ is given by

$$\begin{aligned} DS(G_1 \circ\ \cdots\ G_l) = \{ & (A_1+\alpha_{1s})^{\beta_{1s}}, && \text{for } s = 1,\ 2,\ \cdots,\ k_1, \\ & (1+A_2+\alpha_{2s})^{n_1 \cdot \beta_{2s}}, && \text{for } s = 1,\ 2,\ \cdots,\ k_2, \\ & (1+A_r+\alpha_{rs})^{n_1 \cdot n_2\ \cdots\cdots\ n_r} && \text{for } s = 1,\ \cdots,\ k_{l-1};\ r = 3,\ \cdots,\ l-1\} \end{aligned}$$

where $A_i = \sum_{k=i+1}^{l} n_k$.

Now we give $DS(G_1 \circ G_2\ \circ\ \cdots\ \circ G_l)$ only in terms of the number of vertices n_i, vertex degrees α_{ij} and their frequencies β_{ij}:

$$\begin{aligned} DS(G_1 \circ G_2\ \circ\ \cdots\ \circ G_l) = \{ & (n_l + n_{l-1} + \cdot + n_3 + n_2 + \alpha_{11})^{\beta_{11}},\ \cdots, \\ & (n_l + n_{l-1} +\ \cdots\ + n_3 + n_2 + \alpha_{1k_1})^{\beta_{1k_1}}, \\ & (1 + n_l + n_{l-1} +\ \cdots\ + n_3 + \alpha_{21})^{n_1 \cdot \beta_{21}},\ \cdots, \\ & (1 + n_l + n_{l-1} +\ \cdots\ + n_3 + \alpha_{2k_2})^{n_1 \cdot \beta_{2k_2}}, \\ & (1 + n_l + n_{l-1} +\ \cdots\ + n_4 + \alpha_{31})^{n_1 \cdot n_2 \cdot n_3},\ \cdots, \\ & (1 + n_l + n_{l-1} +\ \cdots\ + n_4 + \alpha_{3k_3})^{n_1 \cdot n_2 \cdot n_3},\ \cdots, \\ & (1 + n_l + \alpha_{(l-1)1})^{n_1 \cdot n_2 \cdot n_{l-1}},\ \cdots, \\ & (1 + n_l + {}_{(l-1)k_{l-1}})^{n_1 \cdot n_2 \cdots n_{l-1}}\}. \end{aligned}$$

Finally we obtain a compact formula for $DS(G_1 \circ G_2\ \circ\ \cdots\ \circ G_l)$ as follows:

3.3. **Corollary.** Let $A_i = \sum_{k=i+1}^{l} n_k$. Then

$$\begin{aligned} DS(G_1 \circ G_2\ \circ\ \cdots\ \circ G_l) = \{ & (A_1 + \alpha_{11})^{\beta_{11}},\ \cdots,\ (A_1 + \alpha_{1k_1})^{\beta_{1k_1}}, \\ & (1 + A_2 + \alpha_{21})^{n_1 \cdot \beta_{21}},\ \cdots,\ (1 + A_2 + \alpha_{2k_2})^{n_1 \cdot \beta_{2k_2}}, \\ & (1 + A_3 + \alpha_{31})^{n_1 \cdot n_2 \cdot n_3},\ \cdots,\ (1 + A_3 + \alpha_{3k_3})^{n_1 \cdot n_2 \cdot n_3}, \\ & (1 + A_4 + \alpha_{41})^{n_1 \cdot n_2 \cdot n_3 \cdot n_4},\ \cdots,\ (1 + A_4 + \alpha_{4k_4})^{n_1 \cdot n_2 \cdot n_3 \cdot n_4}, \\ & \cdots, \\ & (1 + A_{l-1} + \alpha_{(l-1)1})^{n_1 \cdot n_2 \cdot \cdots \cdot n_{l-1}},\ \cdots, \\ & (1 + A_{l-1} + \alpha_{(l-1)k_{l-1}})^{n_1 \cdot n_2 \cdot \cdots \cdot n_{l-1}}\}. \end{aligned}$$

Competing Interests: The authors declare that there is no conflict of interests regarding the publication of this research paper.

Acknowledgement

The authors are extremely grateful to the anonymous referee(s) for their keen reading, valuable suggestions and constructive comments for the improvement of the manuscript. The third author was supported by the Research Fund of Uludag University project no: F-2015/17.

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