Electronic Journal of Mathematical Analysis and Applications
Vol. 7(1) Jan. 2019, pp. 65-73.
ISSN: 2090-729X(online)
http://fcag-egypt.com/Journals/EJMAA/

DYNAMICS AND CONTROL MEASURES FOR MALARIA USING A MATHEMATICAL EPIDEMIOLOGICAL MODEL

S. I. ONAH, O. C. COLLINS, C. OKOYE, G. C. E. MBAH

Abstract. Malaria is one of the most prevalent illness globally especially in the tropic and sub-tropic regions of the world. This work investigates the transmission dynamics of malaria disease and the different ways the disease can be controlled by formulating appropriate mathematical epidemiological model. To evaluate the impacts of control measures, we determine the important mathematical features of the model such as the basic reproduction number and analyze then accordingly. The disease free equilibrium and endemic equilibrium point of the model were derived and its stability investigated. For instance, our analysis showed that the disease free equilibrium point is stable when $R_0 < 1$. Stability analyses of the endemic equilibrium is investigated using the centre manifold theorem. Numerical simulations were carried out using realistic parameter values to support our analytical predictions.

1. Introduction

Malaria is an infectious disease caused by the plasmodium parasite and transmitted between humans through the bite of the female Anopheles mosquito. Malaria can also be spread by other medium such as organ transplants, blood transfusions and sharing of needles by intravenous drug (IV drugs) users. There are several species of plasmodium parasites in different parts of the world but there are only four species of parasite that can cause infections in humans, and they are Plasmodium Falciparum, Plasmodium Vivax, Plasmodium Malariae, Plasmodium [1]. Malaria infection in human begins with an inoculums of plasmodium parasites from an infectious Anopheles Mosquito. Malaria shares many characteristics with other Protozoan parasites, which cause disease such as African Trypanosomiasis and Visceral Leishmaniasis. However, malaria is most prevalent of these diseases among humans. In 2002, it was estimated that 2.2 billion people were exposed to the threat of the most dangerous species, Plasmodium Falciparum. People who live in the poor areas of the world are more prone to the risk of malaria. These people constitute above 40 percent of the worlds population. The malaria disease according to the WHO 2010 report, is prevalent in the tropic and sub-tropic regions of

2010 *Mathematics Subject Classification.* 92D30; 49M05.
Key words and phrases. Mathematical modeling, malaria, dynamical system, infectious disease dynamics, stability analyses, basic reproduction.
Submitted March 27, 2018. Revised April 2, 2018.

the world [2]. Malaria constitutes the cause of death in different parts of this area which is about one million annually. Given the human and economic costs of this disease, there is a great need for eradication through a better understanding on how to effectively control the spreads of the disease just like some other diseases such as Ebola, chiken-pox etc that are almost wiped out by effective control intervention ([3], [4]). Therefore in this work, we considered the incidence of malaria, some of the factors influencing the spread of the disease and how to control it and then formulate a mathematical model describing the dynamics of malaria.

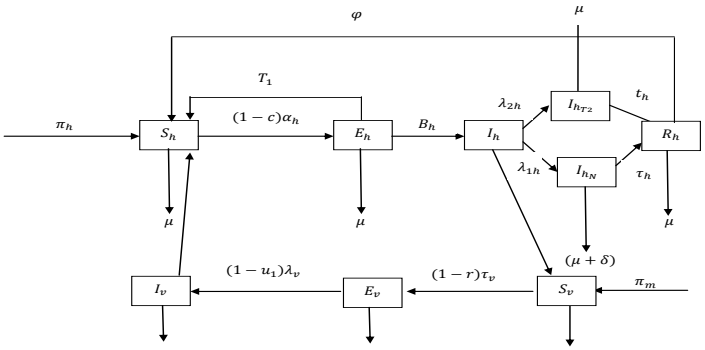
2. Model Equation

Many approaches have been used in studying the dynamics of malaria disease ([5], [6], [7], [8], [9]). For this paper we consider a mathematical model for malaria transmission in the form of model by ([10],[11],[12],[13]) and extended the model by introducing different control measures such as reduction of breeding sites of mosquito, use of ITNs, prophylactic drugs and treatment.

Table 1.0.

Symbols	Meaning
N_h	Total human population
π_h	Recruited humans
φ	Rate of loss of immunity for humans
S_h	Susceptible humans
I_h	Infected humans
E_h	Exposed humans
$I_{h\tau_2}$	Infected humans treated
I_{h_N}	Infected humans not treated
R_h	Recovered humans
α_h	Rate of progression from susceptible human to exposed human
β_h	Rate of progression from exposed human to infected human
λ_{2h}	Rate at which humans are treated
λ_{1h}	Rate at which humans are not treated
δ	Disease induced death of humans
ι_h	Rate of progression from treated human to recovered humans
τ_h	Rate of progression from untreated human to recovered humans
c	Campaign strategy for malaria control
μ	Natural death rate of humans
T_1	Treatment in form of Prophylactic for humans
T_2	Treatment of infected humans with drugs
N_m	Total mosquito population
π_m	Recruited mosquito
S_v	Susceptible class of mosquitoes
ϵ_l	Death of mosquito caused by natural death, quest to suck blood and insecticides
r	Reduction of breeding sites for mosquitoes
τ_v	Rate of progression from susceptible class to exposed class
E_v	Exposed class of mosquitoes
I_v	Class of mosquitoes that are infective
u_1	Adherence to awareness program on malaria control practices
λ_v	Rate of progression of mosquitoes from exposed class to infective class
θ_{mh}	Probability of malaria transmission from an infectious mosquito to a susceptible human, if there is a bite.
θ_{hm}	Probability of malaria transmission from an infectious human to a susceptible mosquito, if there is a bite.
Φ	Biting rate of mosquitoes.

2.1 FLOW DIAGRAM FOR THE MODEL



Considering the assumptions, definitions of state variables and parameters above and the flow diagram in Fig 2.1, we obtain the system of non-linear differential equations.

$\frac{dS_h}{dt} = \pi_h + \varphi R_h - (1-c)\alpha_h S_h + T_1 E_h - \mu S_h$
$\frac{dE_h}{dt} = (1-c)\alpha S_h - \beta_h E_h - \mu E_h - T_1 E_h$
$\frac{dI_h}{dt} = \beta_h E_h - \lambda_{1h} I_h - \lambda_h$
$\frac{dI_{hN}}{dt} = \lambda_{1h} I_h - (\mu+\delta) I_{hN} - \tau_h I_{hN}$
$\frac{dR_h}{dt} = t_h I_{hT2} + \tau_h I_{hN} - \mu R_h - \varphi R_h$
$\frac{dS_v}{dt} = \pi_m - \epsilon_v S_v - (1-r)\tau_v S_v$
$\frac{dE_v}{dt} = (1-r)\tau_v S_v - \epsilon_v E_v - (1-u_1)\lambda_v E_v$
$\frac{dI_v}{dt} = (1-u_1)\lambda_v E_v - \epsilon_v I_v$

where

$\alpha_h = \frac{\theta_{mh}\phi I_v}{N_h}$ and $\tau_v = \frac{\theta_{hm}\phi I_h}{N_h}$

The meaning of variables and parameters are given in Table 1.0. The initial conditions are assumed as follows:

$S_h(0) = S_{h0}, E_h(0) = E_{h0}, I_h(0) = I_{h0}, I_{hN}(0) = I_{hN0}, I_{hT2}(0) = I_{hT20}, R_h(0) = R_{h0}$

3. Results

In this section, we present the results of the mathematical analyses of the malaria model.

Theorem 3.1 The solutions of system (2.1) are positive and meaningful for all $t > 0$ if they enter the invariant region
$\Omega = \Omega_h \times \Omega_m$ where
$\Omega_h = (S_h, E_h, I_h, I_{hN}, I_{hT2}, R_h)$ and
$\Omega_m = (S_v, E_v, I_v)$
$\Omega = \{(S_h, E_h, I_h, I_{hN}, I_{hT2}, R_h, S_v, E_v, I_v) \in {R_+}^9; (S_h, S_v) > 0, (E_h, I_h, R_h, E_v, I_v) \geq 0, N_h \leq \frac{\pi_h}{\mu}; N_m \leq \frac{\pi_m}{\epsilon_v}\}$
Therefore, the region Ω is positively invariant (ie. solution remain positive for all times, t) . So, the model (2.1) is biologically meaningful and mathematically well posed in the domain Ω

3.1. **Disease Free Equilibrium.** The disease free equilibrium point of the malaria model (2.1) is given by, $E_0 = (S_h^{t_0}, E_h^{t_0}, I_h^{t_0}, I_{hN}^{t_0}, I_{hT2}^{t_0}, R_h^{t_0}, S_v^{t_0}, E_v^{t_0}, I_v^{t_0})$
$= (\frac{\pi_h}{\mu}, 0, 0, 0, 0, 0, \frac{\pi_m}{\epsilon_v}, 0, 0)$ 3.1.1

3.2. **Basic Reproduction Number (R_0).** Diekmann et al. [14] defined the basic reproduction number, R_0, as the number of secondary infections that one infectious individual will create over the duration of the infectious period, provided that everyone else is susceptible. If the basic reproduction number $R_0 < 1$ it implies that an infected individual produces an average less than one infected person, and by that calculation it means that with time the disease will die out from the population. On the other hand, if $R_0 > 1$, it means that an infected person produces more than one infected person in the population. For this case, the disease will remain in the population. Therefore for the disease to die out of the population $R?_0$must be less than one. The basic reproduction number R_0 of the malaria model is the dominant eigenvalue of the next generation matrix FV^{-1}, where V^{-1} is the inverse of V and is given by
$R_0 = \sqrt{R_{0h} R_{0m}}$ 3.2.3
where
$R_{0h} = \frac{(1-c)\phi\beta_h\theta_{mh}\mu}{\pi_h(\beta_h+\mu+T_1)(\lambda_{1h}+\lambda_{2h})}$
$R_{0m} = \frac{(1-r)((1-u_1)\lambda_v)\theta_{hm}\phi\pi_m}{\epsilon_v^2(\epsilon_v+(1-u_1)\lambda_v)}$
The value of R_0 here is the product of two parameters, since the model is between two interactions ie. mosquito and human. Therefore, R_0 is divided into R_{0h} and R_{0m}, where R_{0h} is the number of humans, a mosquito bites and transmits malaria in its lifetime, while R_{0m} signifies the number of susceptible mosquitoes an infected human infects malaria parasite during its infection period. This is true for all humans and mosquitoes been susceptible. The bite of mosquito is what determines the transmission of human to mosquito and vice versa, that is why the quantity F appeared in the both expressions above. The basic reproduction number can be useful in investigating the stability of the disease free equilibrium point.

3.3. **Local Stability of the Disease Free Equilibrium.** The local stability of the disease-free equilibrium is summarized in the Theorem below.

Theorem 3.2

The disease free equilibrium point for system (2.1) is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$

We can show that the determinants of the Hurwitz matrices are positive, then by implication all eigenvalues of the Jacobian matrices have negative real part when $R_0 < 1$, [15]. Thus, disease-free equilibrium is stable provided $R_0 < 1$. The epidemiological implication of this is that the disease can be eradicated from the population in the presence of control measures provided $R_0 < 1$. On the other hand, if $R_0 > 1$, we obtain that all the roots of the polynomial cannot have negative real parts. Therefore, the disease-free equilibrium point is unstable if $R_0 > 1$. In this case the epidemiological implication is that the disease will persist in the population. Consequently, it will be difficult to completely eradicate the disease from the population when $R_0 > 1$ since the average transmission is greater than one.

3.4. **Endemic Equilibrium Point.** The endemic equilibrium (EEP) of the model is given by

$EEP = (S_h^e, E_h^e, I_h^e, I_{hN}^e, I_{hT2}^e, R_h^e, S_v^e, E_v^e, I_v^e)$.....3.2.4

where

$I_v^e = \frac{(1-u_1)\lambda_v E_v^e}{\epsilon_v}$, $E_v^e = \frac{(1-r)\theta_{hm}\phi I_h^e E_v^e}{N_h(\epsilon_v+(1-u_1)\lambda_v)}$, follows that $I_v^e = \frac{((1-u_1)\lambda_v)(1-r)\theta_{hm}\phi I_h^e S_v^e}{\epsilon_v N_h(\epsilon_v+(1-u_1)\lambda_v)}$ but $S_v^e = \frac{\pi_m N_h}{(1-r)\theta_{hm}\phi I_h^e+\epsilon_v N_h}$ therefore $I_v^e = \frac{\epsilon_v((1-u_1)\lambda_v)(1-r)\theta_{hm}\phi I_h^e\pi_m}{\epsilon_v^2(\epsilon_v+(1-u_1)\lambda_v)((1-r)\theta_{hm}\phi I_h+\epsilon_v N_h)} = \frac{R_{0m}\epsilon_v I_h^e}{(1-r)\theta_{hm}\phi I_h+\epsilon_v N_h}$ since $R_{0m} = \frac{(1-r)((1-u_1)\lambda_v)\theta_{hm}\phi\pi_m}{\epsilon_v^2(\epsilon_v+(1-u_1)\lambda_v)}$

Also $E_h^e = \frac{(\lambda_{1h}+\lambda_{2h})I_h^e}{\beta_h}$, $S_h^e = \frac{\mu(1-r)\theta_{hm}\phi I_h^e+\epsilon_v\pi_h}{R_0^2\epsilon_v\mu}$, $R_h^e = \frac{t_h I_{hT2}^e+\tau_h I_{hN}^e}{\mu+\varphi}$ where $I_{hN}^e + I_{hT2}^e = I_h^e$, then $R_h^e = \frac{(t_h+\tau_h)I_h^e}{\mu+\varphi}$, it implies that $I_h^e = \frac{-B+\sqrt{B^2-4AC}}{2A} = \frac{\sqrt{B^2-4AC}-B}{2A} = \phi$, $I_h^e \geq 0$

Substituting accordingly, we have the following:

Theorem 3.3

The malaria model (2.1) has,

(1) Precisely one unique endemic equilibria if $C < 0 \iff R_0 > 1$.
(2) Precisely one unique endemic equilibria if $B < 0 and C = 0 or B^2 - 4AC = 0$.
(3) Precisely two unique endemic equilibria if $C > 0, B < 0 and B^2 - 4AC > 0$.
(4) No endemic equilibria otherwise.

3.5. **Local Stability of the Endemic Equilibrium.** There are many methods of investigating the stability of a system, such as the geometric approach Li and Muldowney [16] and the use of Lyapunov function ([17],[18],[19]). In this work we consider the Centre Manifold Theorem as described in Castillo Chavez and Song [20] and the results of our stability analyses is summarized in the Theorem below.

Theorem 3.4

The model (4.2.1) has a unique endemic equilibrium which is locally asymptotically stable when $R_0 > 1$ and unstable when $R_0 < 1$.

4. Numerical Simulation

This section we consider numerical simulations to investigate the dynamics of the malaria disease in both the human and mosquito population. The effect of the control measures on human and mosquito at different phases in the transmission process is also investigated numerically. Parameter values used in the simulations

and their reference are given in Table 4.2.

Table 2.1:

PARAMETER	VALUE	REFERENCES
π_h	0.03914	Silva & Torres [12]
φ	0.0146	Chitnis et al. [10]
μ	0.00003914	Silva & Torres [12]
β_h	0.08333	Chitnis et al. [11]
t_h	0.25	Silva & Torres [12]
τ_h	0.003704	Chitnis et al. [10]
c	0.50	Estimated
T_1	0.02	Estimated
λ_{1h}	0.0723	Estimated
λ_{2h}	0.0533	Estimated
δ	0.001	Silva & Torres [12]
π_m	0.4	Chitnis et al. [10]
τ_v	0.09091	Chitnis et al. [10]
λ_v	0.1	Chitnis et al. [10]
ϵ_v	0.04762	Silva & Torres [12]
r	0.15	Estimated
u_1	0.75	Estimated
θ_{hm}	0.020	Chitnis et al. [10]
θ_{mh}	0.0833	Chitnis et al. [10]
Φ	0.4	Chitnis et al. [10]

The basic reproduction number R_0 obtained using this parameter values is less than unity. This means that malaria can be eradicated from the population with time for this parameter value since $R_0 < 1$.

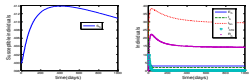


Figure 2.2: Plot illustrating the long term dynamics of the state variables of the malaria model with time.

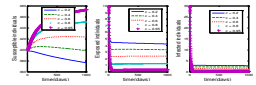


Figure 2.3: Plot illustrating the impact of control strategy on the $S_h(t)$, $E_h(t)$ and $I_h(t)$, for various values of c.

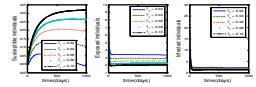


Figure 2.4: : Plot illustrating the impact of prophylactic treatment on the $S_h(t)$, $E_h(t)$ and $I_h(t)$, for various values of T_1

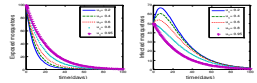


Figure 2.5: Plot showing impact of use of insecticides on the $E_v(t)$ and $I_v(t)$ for various values of u_1.

5. Conclusion

We have investigated the transmission dynamics and control of malaria infection using a mathematical epidemiological model. We computed the basic reproduction number R_0 for the model and showed that if $R_0 < 1$, the disease cannot persist in the population and when $R_0 > 1$ the disease persists. Our analyses revealed that the disease free equilibrium of the model is stable when $R_0 < 1$. Epidemiologically, this results implies that malaria disease can be eradicated from the population using control measures provided that $R_0 < 1$. On the other hand, when $R_0 > 1$, we discovered using center manifold theorem that the endemic equilibrium is stable.

Next, we considered numerical simulation to gain deeper understanding on the dynamics and control of malaria disease. The results of our simulations revealed the following: (i) Fig. 4.1 shows the long term dynamics of the malaria model with time. From this figure we discovered that each of the disease state variables decreases for the basic reproduction number less than unity. This suggests that the control measure have some influence in reducing the spread of the disease. (ii) Fig. 4.2 illustrates the impact of control strategy, on the susceptible, exposed and infectious humans. From the plot it can be seen that an increase in c leads to a decrease in exposed and infected population and an increase in susceptible population. This implies that control strategy reduces the rate at which humans are exposed to malaria and the rate at which humans become infectious. (iii) Fig. 4.3 illustrates the impact of treatment in form of prophylactic drugs (T_1) on the susceptible, exposed and infectious human. From the figure we discovered that an increase in T_1 results to a decrease in exposed and infected population with an increase in susceptible population. Thus, this shows that prophylactic drugs reduce the rate at which human population becomes exposed or infectious. (iv)Fig. 4.4 illustrates the impact of adherence to mosquito control strategies (u_1) such as sleeping under treated nets, use of insecticides etc. For this figure, we discovered that an increase in u_1 leads to a decrease in exposed and infectious mosquitoes population respectively. It is also important to highlights some effects of pesticides on our environment and on our body, Pesticides can contaminate soil, water, turf, and other vegetation. In addition to killing insects or weeds, pesticides can be toxic to a host of other organisms including birds, fish, beneficial insects, and non-target plants. Its Oral exposure can contaminate the foods or water containing we drink. Its exposure can cause irritation or burns. In more serious cases, your skin can absorb the pesticide into the body, causing other health effects. But if well handled, the pesticide could be of great advantage to curbing the malaria disease. Thus, increasing the reduction of mosquito breeding sites, malaria control strategy in form of spraying of insecticides reduces infectious mosquitoes and exposed mosquitoes. It means that if the breeding sites of mosquitoes are reduced and insecticides are well sprayed, it will go a long way to eradicate the vector in the population. All our findings using the malaria model agree with intuitive expectation. Thus, our model can be used to study and make predictions of future dynamics of malaria in any community where the disease is endemic.

References

[1] Ishikawa, H., Ishii, A., Nagai, N., Ohmae, H., Harada, M., Suguri, S., and Leafasia, J. (2003). A mathematical model for the transmission of Plasmodium vivax malaria. Parasitology International, 52(1), 81-93.

[2] WHO. (2010). World Health Malaria Report.

[3] Area, I., Batarfi, H., Losada, J., Nieto, J. J., Shammakh, W., and Torres, . (2015). On a fractional order Ebola epidemic model. Advances in Difference Equations, 2015 (1), 1.

[4] Area, I., Losada, J., Ndarou, F., Nieto, J. J., and Tcheutia, D. D. (2015). Mathematical modeling of 2014 Ebola outbreak. Mathematical Methods in the Applied Sciences.

[5] Yakob, L. (2016). Endectocide-treated cattle for malaria control: A coupled entomological-pidemiological model. Parasite Epidemiology and Control, 1(1), 2-9.

[6] Ngwa, G. A., and Shu, W. S. (2000). A mathematical model for endemic malaria with variable human and mosquito populations. Mathematical and Computer Modelling, 32(7), 747-763.

[7] Niger, A. M., and Gumel, A. B. (2008). Mathematical analysis of the role of repeated exposure on malaria transmission dynamics. Differential Equations and Dynamical Systems, 16(3), 251-287.

[8] Teboh-Ewungkem, M. I., Podder, C. N., and Gumel, A. B. (2010). Mathematical study of the role of gametocytes and an imperfect vaccine on malaria transmission dynamics. Bulletin of mathematical biology, 72(1), 63-93.
[9] Tumwiine, J., Mugisha, J. Y. T., and Luboobi, L. S. (2007). On oscillatory pattern of malaria dynamics in a population with temporary immunity. Computational and Mathematical Methods in Medicine, 8(3), 191-203.
[10] Chitnis, N, Cushing J.M. and Hyman , J. M (2006). Bifurcation Analysis of a Mathematical Model for Malaria Transmission. SIAM J. Appl. Math., 67(1), 2445.
[11] Chitnis, N., Hyman, J. M., and Cushing, J. M. (2008). Determining important parameters in the spread of malaria through the sensitivity analysis of a mathematical model. Bulletin of mathematical biology 70(5), 1272 - 1296.
[12] Silva, C. J., and Torres, D. F. (2013). An optimal control approach to malaria prevention via insecticide-treated nets. In Conference Papers in Science (Vol. 2013). Hindawi Publishing Corporation.
[13] Azu-Tungmah G.T. (2012). A Mathematical Model to Control the Spread of Malaria in Ghana. A thesis submitted to College of Science Nkwameh Nkrumah University of Science and Technology.
[14] Diekmann., O. J. A. Heesterbeek, and Metz. J.A.J. (1990). On the Definition and Computation of Basic Reproduction Ratio R0 in the model for infection disease in a heterogeneous population, Journal of Mathematical Biology 28, 365-382.
[15] DeJesus, E. X., and Kaufman, C. (1987). Routh-Hurwitz criterion in the examination of eigenvalues of a system of nonlinear ordinary differential equations. Physical Review A, 35(12), 5288.
[16] Li, M. Y., and Muldowney, J. S. (1996). A geometric approach to global-stability problems. SIAM Journal on Mathematical Analysis, 27(4), 1070-1083.
[17] Korobeinikov, A., and Wake, G. C. (2002). Lyapunov functions and global stability for SIR, SIRS, and SIS epidemiological models. Applied Mathematics Letters, 15(8), 955-960.
[18] Korobeinikov, A., and Maini, P. K. (2004). A Lyapunov function and global properties for SIR and SEIR epidemiological models with nonlinear incidence. Mathematical Biosciences and Engineering, 1(1), 57-60.
[19] Garba, S. M., Gumel, A. B., and Bakar, M. A. (2008). Backward bifurcations in dengue transmission dynamics. Mathematical biosciences, 215(1), 11-25.
[20] Castillo Chavez, C. and Song, B. (2004). Applications Centre Manifold Theorem, Springer-Verg, New York. 1(2):361-404..

S. I. Onah
Department of Mathematics, University of Nigeria, Nsukka, Nigeria
E-mail address: sunday.ifeanyi@unn.edu.ng

O. C. Collins
Department of Mathematics, University of Nigeria, Nsukka, Nigeria
E-mail address: obiora.collins@unn.edu.ng

C. Okoye
Department of Mathematics, University of Nigeria, Nsukka, Nigeria
E-mail address: chukwudi.okoye@unn.edu.ng

G. C. E. Mbah
Department of Mathematics, University of Nigeria, Nsukka, Nigeria
E-mail address: godwin.mbah@unn.edu.ng