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COMMON FIXED POINT RESULTS IN DISLOCATED QUASI METRIC SPACES

MUJEEB UR REHMAN AND SAFIULLAH MUSSA

Abstract. The purpose of this paper is to study a common fixed point results in dislocated quasi metric spaces. We will prove common fixed point results for some new types of contractile conditions which generalize, modify and unify some existing well known fixed point results in the literature. Moreover, a particular example is given here to elucidate the usability of our establish results. We have noticed that by using our results some fixed point theorems in the context of dislocated quasi metric spaces can be deduced.

1. INTRODUCTION

Fixed point theory is one of the most dynamic research subject in nonlinear analysis. In this field the first important and superb result was proved by Banach in 1922 for contraction mapping in complete metric space. The well-known Banach contraction theorem may be stated as follows: "Every contraction mapping of a complete metric space X into itself has a unique fixed point" (Bonsall 1962). Dass and Gupta [3] generalize the Banach contraction principle in metric space for some rational type contractive conditions.

The role of topology in logic programming has come to recognized (see [1] completely and the references cited therein). Particularly, topological methods are applied to obtain fixed point semantics for logic programs. The concept of dislocated metric spaces was motivated by these considerations. This idea was not new and has been studied in the context of domain theory in[10] where the dislocated metrics were called metric domains.

Hitzler and Seda[14] investigate the useful applications of dislocated topology in the context of logic programming semantics. In order to obtain a unique supported model for these programmes, they introduce the notation of dislocated metric space and generalizes the Banach contraction principle in such spaces.

Zeyada et al.[7] generalize the results of Hitzler and Seda [14] and introduce the concept of complete dislocated quasi metric space. Aage and Salunke[5] derived some fixed point theorems in dislocated quasi metric space.

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Isufati[15] proved some fixed point results for continuous contractive condition with rational type expression in the context of dislocated quasi metric space. Kohli et al.[22] investigate a fixed point theorem which generalize the result of Isufati. Zoto[15] give some new results in dislocated and dislocated quasi metric spaces. For continuous self-mapping a fixed point theorem in dislocated quasi metric space was investigated by Madhu Shrivastava[12]. In 2013, S. T. Patel and M. Patel[22] construct some fixed point results in dislocated quasi metric space.
Recently, in[12] Dubey et al proved some common fixed point results in the setting of complex valued b-metric space.
In this paper we have established common fixed point results for a pair of continuous self-mappings in the context of dislocated quasi metric spaces, which generalizes, the results of Banach [1], Kannan[10], Chaterjee[9], Reich[5], Hardy and Roger[6] and Dass and Gupta[3]. Moreover many more fixed point results can be deduce from our established results.
Throughout this paper R+ will represent the set of non-negative real numbers.

2. PRELIMINARIES

2.1. **Definition.** Let X be a non-empty set and let $d : X \times X \to R+$ be a function satisfying the conditions,
$d_1)$ $d(x, x) = 0$;
$d_2)$ $d(x, y) = d(y, x) = 0$ implies that $x = y$;
$d_3)$ $d(x, y) = d(y, x)$ for all $x, y, z \in X$;
$d_4)$ $d(x, y) \leq d(x, z) + d(z, y)$ for all $x, y, z \in X$. If d satisfy the conditions from d_1 to d_4 then it is called metric on X, if d satisfy conditions d_2 to d_4 then it is called dislocated metric (d-metric) on X, and if d satisfy conditions d_2 and d_4 only then it is called dislocated quasi metric(dq-metric) on X. Clearly every metric on X is a dislocated metric on X but the converse is not necessarily true as clear form the following example.

2.2. **Example.** Let $X=R+$ define the distance function $d : X \times X \to R+$ by , $d(x, y) = max\{x, y\}$ Clearly the define function is dislocated metric but not a metric on X. From the following example one can say that dislocated quasi metric on X need not to be dislocated metric on X.

2.3. **Example.** Let $X = [0, 1]$ we define the function $d : X \times X \to R+$ as, $d(x, y) = |x - y| + |x|$ for all $x, y \in X$.
In our main work we will use the following definitions which can be found.

2.4. **Example.** A sequence $\{x_n\}$ in dq-metric space is called Cauchy sequence if for $\in > 0$ there exist a positive integer N such that for $m, n \geq N$, we have $d(x_m, x_n) < \in$.

2.5. **Definition.** A sequence $\{x_n\}$ is called dq-convergent in X if for $n \geq N$, we have $d(x_n,x) < s$ where x is called the dq-limit of the sequence $\{x_n\}$.

2.6. **Definition.** A dq-metric space (X, d) is said to be complete if every Cauchy sequence in X converge to a point of X.

2.7. **Definition.** Let (X, d) be a dq-metric space, a mapping $T : X \to X$ is called contraction if there exist $0 \leq \alpha < 1$ such that $d(T x, T y) \leq \alpha d(x, y)$ for all $x, y \in X$ and $\alpha \in [0, 1)$. The following lemma can be found in [1].

2.7.1. *Lemma.* Limit in dq-metric space is unique.

2.8. **Definition.** Let (X, d) be a metric space and let $T : X \to X$ be a self-mapping. Then T is called kannan mapping if, $d(T x, T y) \leq \alpha[d(x, T x) + d(y, T y)]$ *for all* $x, y \in X$ *and* $\alpha \in [0, 1/2)$ (1)

2.9. **Definition.** Let (X, d) be a metric space, a self-mapping $T : X \to X$ is called generalized contraction if and only for all $x, y \in X$, there exist c_1, c_2, c_3, c_4 such that $\sup\{c_1 + c_2 + c_3 + 2c_4\} < 1$ and
$d(T x, T y) \leq c_1.d(x, y) + c_2.d(x, T x) + c_3.d(y, T y) + c_4.[d(x, T y) + d(y, T x)]$ (2)
R. Kannan[10] established a unique fixed point theorem for mapping which satisfy condition (1) in metric spaces . Ciric[2] investigate a unique fixed point theorem for mapping which satisfy condition (2) in the context of metric spaces. In the following theorem Zeyada et al.[7] generalized the Banach contraction principle in dislocated quasi metric spaces.

2.10. **Theorem.** *Let* (X, d) be a complete dq-metric space $T{:}X \to X$be a continuous contraction then T has a unique fixed point in X. Aage and Salunke[5] established the following theorems for single and a pair of continuous mappings in dislocated quasi metric spaces.

2.11. **Theorem.** Let (X, d) be a complete dq-metric space and $T{:}X \to X$ be a continuous self-mapping satisfying the following condition, $d(T x, T y) \leq a.d(x, y) + b.d(x, T x) + c.d(y, T y)$ where $a, b, c \geq 0$ with $a + b + c < 1$, and for all $x, y \in X$, then T has a unique fixed point.

2.12. **Theorem.** Let (X, d) be a complete dq-metric space and $S, T : X \to X$ be continuous self-mappings satisfying the following condition, $d(Sx, T y) \leq a.d(x, y) + b.d(x, Sx) + c.d(y, T y)$ where $a, b, c > 0$ with $a + b + c < 1$, and for all $x, y \in X$, then S and T have a unique common fixed point. The following theorems of C. T. Aage and J. N. Salunke[5] are the generalization of Kannan type contraction and generalized contraction in dislocated quasi metric spaces respectively .

2.13. **Theorem.** Let (X, d) be a complete dq-metric space and $T : X \to X$ be a continuous self-mappings satisfying the following condition
$d(T x, T y) \leq a.[d(x, T x) + d(y, T y)$
where $a \geq 0$ with $a < \frac{1}{\_}$, and for all $x, y \in X$, then T has a unique fixed point.

2.14. **Theorem.** Let (X, d) be a complete dq-metric space and $T : X \to X$ be a continuous self-mappings satisfying the following condition,
$d(T x, T y) \leq a \ \ d(x, y) + b \ \ d(x, T x) + c \ \ d(y, T y) + e \ \ [d(x, T y) + d(y, T x)]$
where $a, b, c, e \geq 0$ with $a + b + c + 2e < 1$, and for all $x, y \in X$, then T has a unique fixed point.
Isufati in[15] derived the following two results, where the first one generalized the result of Dass and Gupta[4] in dislocated quasi metric spaces.

2.15. **Theorem.** Let (X, d) be a complete dq-metric space and $T : X \to X$ be a continuous self-mapping satisfying the following condition,
$d(y, T y)[1 + d(x, T x)]d(T x, T y) \leq a.+ b.d(x, y)$
$1 + d(x, y)$where $a, b > 0$ with $a + b < 1$, and for all $x, y \in X$, then T has a unique fixed point.

2.16. **Theorem.** Let *(X, d)* be a complete dq-metric space and $T : X \rightarrow X$ be a continuous self-mapping satisfying the following condition,

$$d(Tx, Ty) \leq a.d(x, y) + b.d(y, Tx) + c.d(x, Ty)$$

where $a,\ b,\ c > 0$ with sup $a + 2b + 2c < 1$, and for all $x,\ y \in X$, then T has a unique fixed point.
Kohli, Shrivastava and Sharma[22] proved the following theorem in the context of dislocate quasi metric spaces which generalize Theorem 2.16.

2.17. **Theorem.** Let (X, d) be a complete dq-metric space and T:X→X be a continuous self-mapping satisfying the following condition.

$$1 + d(x, Tx)$$

$$d(y, Ty)$$
$$d(Tx, Ty) \leq a.$$

where $a, b, c > 0$ with $a + b + c < 1$ and for all $x, y \in X$ then T has a unique fixed point.
Dubey, Tripathi and Pandey[12] derived the following theorem in the context of complex valued b-metric spaces.

2.18. **Theorem.** Let the a complete complex valued b-metric space with the coefficient s $\geq$ 1 and lft *S,* be mappings satisfying: for alμ , where α, β, γ, δ, λ, μ and are non-negative reals with $\alpha + 2\beta + 2s\ \gamma + \delta + l < 1$. Then and have a unique common fixed point in.
Now we state and prove tre our result fallowing theorem for a peih of self-mappings for a unique common fixed point in dislocated quasi metric space.

3. MAIN RESULTS

Now we state and prove the our result following theorem for a pair of self-mappings for a unique common fixed point in dislocated quasi metric space.

3.1. **Theorem.** Suppose (X, d) be a complete dq-metric space, an$d\ S, T{:}X \rightarrow X$be two continuous self-mappings satisfying the succeeding property:

$$d(Sx, Ty) \leq \alpha d(x, y) + \beta[d(x, Sx) + d(y, Ty)] + \gamma[d(y, Sx) + d(x, Ty)]$$
$$+\delta\frac{d(y, Ty)[1 + d(x, Sx)]}{1 + d(x, y)} + \lambda\frac{d(y, Sx)[1 + d(x, Ty)]}{1 + d(x, Sy)} + \mu d(y, Sx)$$

$\forall x, y \in X$with$\alpha + 2\beta + 2\gamma + \delta + \lambda + \mu < 1$. Then S and Thave a unique common fixed point in$X$.
Proof: Let $\{x_n\}$ be an arbitrary sequence in$X$, we define the sequence $\{x_n\}$ for $n = 0, 1, 2, 3, \ldots$such that

$$x_{2n+1} = Sx_{2n}$$

and

$$x_{2n} = Tx_{2n-1}.$$

Now to show that $\{x_n\}$ is a Cauchy sequence in X.
For this let us consider!

$$d(x_{2n+1}, x_{2n+2}) = d(Sx_{2n}, Tx_{2n+1}).$$

By using the given condition we have

$$d\left(x_{2n+1}, x_{2n+2}\right) \leq \alpha d\left(x_{2n}, x_{2n+1}\right) + \beta\left[d\left(x_{2n}, Sx_{2n}\right) + d\left(x_{2n+1}, Tx_{2n+1}\right)\right]$$

$$+\gamma\left[d\left(x_{2n+1}, Sx_{2n}\right) + d\left(x_{2n}, Tx_{2n+1}\right)\right] + \delta\frac{d\left(x_{2n+1}, Tx_{2n+1}\right)\left[1 + d\left(x_{2n}, Sx_{2n}\right)\right]}{1 + d\left(x_{2n}, x_{2n+1}\right)}$$

$$+\lambda\frac{d\left(x_{2n+1}, Sx_{2n}\right)\left[1 + d\left(x_{2n}, Tx_{2n+1}\right)\right]}{1 + d\left(x_{2n}, Sx_{2n+1}\right)} + \mu d\left(x_{2n+1}, Sx_{2n}\right)$$

$$\leq \alpha d\left(x_{2n}, x_{2n+1}\right) + \beta\left[d\left(x_{2n}, x_{2n+1}\right) + d\left(x_{2n+1}, x_{2n+2}\right)\right]$$

$$+\gamma\left[d\left(x_{2n+1}, x_{2n+1}\right) + d\left(x_{2n}, x_{2n+2}\right)\right] + \delta\frac{d\left(x_{2n+1}, x_{2n+2}\right)\left[1 + d\left(x_{2n}, x_{2n+1}\right)\right]}{1 + d\left(x_{2n}, x_{2n+1}\right)}$$

$$+\lambda\frac{d\left(x_{2n+1}, x_{2n+1}\right)\left[1 + d\left(x_{2n}, x_{2n+2}\right)\right]}{1 + d\left(x_{2n}, x_{2n+2}\right)} + \mu d\left(x_{2n+1}, x_{2n+1}\right)$$

$x_{2n+1}, d(x_{2n+1})x_{2n+1}, d(x_{2n+1}) + \mu.d \leq \alpha d(x_{2n}, x_{2n+1}) + \beta[d(x_{2n}, x_{2n+1}) + d(x_{2n+1}, x_{2n+2})] +$ $\gamma[d(x_{2n+1}, x_{2n+1}) + d(x_{2n}, x_{2n+2})] + \delta d(x_{2n+1}, x_{2n+2}) + \lambda.d\, x_{2n+1}, d\left(x_{2n+1}\right) \leq \alpha d\left(x_{2n}, x_{2n+1}\right) +$ $\beta.d\left(x_{2n}, x_{2n+1}\right) + \beta.d\left(x_{2n+1}, x_{2n+2}\right) + \gamma.d\left(x_{2n+1}, x_{2n+1}\right) + \gamma.d\left(x_{2n}, x_{2n+2}\right)$ $+ \delta d\left(x_{2n+1}, x_{2n+2}\right) + \lambda.d\left(x_{2n+1}, x_{2n+1}\right) + \mu.d$

$$\gamma$$

$$\leq d\left(x_{2n}, x_{2n+1}\right).\left(\alpha + \beta\right) + d\left(x_{2n+1}, x_{2n+2}\right).\left(\beta + \delta\right) + d\left(x_{2n+1}, x_{2n+1}\right). \quad +\lambda + \mu + \gamma.d\left(x_{2n}, x_{2n+2}\right)$$

$$\gamma$$

$$\leq d\left(x_{2n}, x_{2n+1}\right).\left(\alpha + \beta\right) + d\left(x_{2n+1}, x_{2n+2}\right).\left(\beta + \delta\right) + d\left(x_{2n+1}, x_{2n+1}\right). \quad + \lambda + \mu$$

$+\gamma.d\left(x_{2n}, x_{2n+1}\right) + \gamma.d\left(x_{2n+1}, x_{2n+2}\right) \leq d\left(x_{2n}, x_{2n+1}\right).\left(\alpha + \beta + 2.\gamma + \lambda + \mu\right) +$ $d\left(x_{2n+1}, x_{2n+2}\right).\left(\beta + \delta + 2.\gamma + \lambda + \mu\right)$

$$\left[1 - \left(\beta + \delta + 2.\gamma + \lambda + \mu\right)\right] d\left(x_{2n+1}, x_{2n+2}\right) \leq \left(\alpha + \beta + 2.\gamma + \lambda + \mu\right) d\left(x_{2n}, x_{2n+1}\right)$$

$$d\left(x_{2n+1}, x_{2n+2}\right) \leq \frac{\left(\alpha + \beta + 2.\gamma + \lambda + \mu\right)}{\left[1 - \left(\beta + \delta + 2.\gamma + \lambda + \mu\right)\right]} d\left(x_{2n}, x_{2n+1}\right).$$

Let

$$m = \frac{\left(\alpha + \beta + 2.\gamma + \lambda + \mu\right)}{\left[1 - \left(\beta + \delta + 2.\gamma + \lambda + \mu\right)\right]}.$$

Clearly, $m < 1$ because $\alpha + 2\beta + 2\gamma + \delta + \lambda + \mu < 1$.

$$d\left(x_{2n+1}, x_{2n+2}\right) \leq md\left(x_{2n}, x_{2n+1}\right)$$

$$d\left(x_{2n}, x_{2n+1}\right) \leq md\left(x_{2n-1}, x_{2n}\right).$$

So

$$d\left(x_{n}, x_{n+1}\right) \leq md\left(x_{n-1}, x_{n}\right).$$

Similarly

$$d\left(x_{n-1}, x_{n}\right) \leq md\left(x_{n-2}, x_{n-1}\right).$$

Thus

$$d\left(x_{n}, x_{n+1}\right) \leq m^{2} d\left(x_{n-2}, x_{n-1}\right).$$

Continuing the same procedure, we have

$$d\left(x_{n}, x_{n+1}\right) \leq m^{n} d\left(x_{o}, x_{1}\right).$$

Now$0 \leq m < 1 as n \to \infty so m^n \to 0$, which shows that $\{x_n\}$ is a Cauchy sequence in X. So for $x \in X$ such that

$$\lim_{n\to\infty} x_n = x.$$

Also the sub-sequences $\{x_{2n}\}$ and $\{x_{2n+1}\}$ converge to $x \in X$.
Since S an$d$ T are continuous so

$$T \lim_{n\to\infty} x_{2n+1} = Tx \Rightarrow Tx = x.$$

Likewise $Sx = x$. Therefore x is the fixed point of S and T.
Uniqueness.
Suppose $x \neq y$ are different fixed points of S and T.
So $Tx = Sx = x, Ty = Sy = y$.
By given rule

$$d(x,y) = d(Sx,Ty) \leq \alpha d(x,y) + \beta[d(x,Sx) + d(y,Ty)] + \gamma[d(y,Sx) + d(x,Ty)]$$
$$+\delta \frac{d(y,Ty)[1 + d(x,Sx)]}{1 + d(x,y)} + \lambda \frac{d(y,Sx)[1 + d(x,Ty)]}{1 + d(x,Sy)} + \mu d(y,Sx)$$

since x and y are fixed points of S and T then

$$d(x,y) \leq \alpha d(x,y) + \beta[d(x,x) + d(y,y)] + \gamma[d(y,x) + d(x,y)]$$
$$+\delta \frac{d(y,y)[1 + d(x,x)]}{1 + d(x,y)} + \lambda \frac{d(y,x)[1 + d(x,y)]}{1 + d(x,y)} + \mu d(y,x) \qquad (1)$$

now x is the fixed point of T and$S$ then by using the given condition we have

$$d(x,x) = d(Sx,Tx) \leq \alpha d(x,x) + \beta[d(x,Sx) + d(x,Tx)] + \gamma[d(x,Sx) + d(x,Tx)]$$
$$+\delta \frac{d(x,Tx)[1 + d(x,Sx)]}{1 + d(x,x)} + \lambda \frac{d(x,Sx)[1 + d(x,Tx)]}{1 + d(x,Sx)} + \mu d(x,Sx)$$
$$d(x,x) \leq \alpha d(x,x) + \beta[d(x,x) + d(x,x)] + \gamma[d(x,x) + d(x,x)]$$
$$+\delta \frac{d(x,x)[1 + d(x,x)]}{1 + d(x,x)} + \lambda \frac{d(x,x)[1 + d(x,x)]}{1 + d(x,x)} + \mu d(x,x)$$
$$d(x,x) \leq (\alpha + 2\beta + 2\gamma + \delta + \lambda + \mu).d(x,x)$$

Now

$$\alpha + 2\beta + 2\gamma + \delta + \lambda + \mu < 1$$

So

$$d(x,x) = 0 \qquad (2)$$

Similarly y is the fixed point of T and$S$ then by using the given condition we have

$$d(y,y) = 0 \qquad (3)$$

Use (2) and (3) in (A) we get

$$d(x,y) \leq d(x,y) \overset{\gamma}{.} (\alpha + \gamma) + d(y,x) . \quad {}^{+\lambda+\mu} \qquad (4)$$

Similarly

$$d(y,x) \leq d(y,x) \overset{\gamma}{.} (\alpha + \gamma) + d(x,y) . \quad {}^{+\lambda+\mu} \qquad (5)$$

Subtracting (5) from (4) we get

$$|d(x,y) - d(y,x)| = |(\alpha + \gamma) - (\gamma + \lambda + \mu)| \, |d(y,x) - d(x,y)|$$

Now

$$|(\alpha+\gamma)-(\gamma+\lambda+\mu)|<1$$

Then

$$|d(x,y)-d(y,x)|=0$$
$$d(x,y)-d(y,x)=0$$
$$d(x,y)=d(y,x) \tag{6}$$

Combining (4), (5) and (6), one can we get $d(x,y)=0$ and $d(y,x)=0$.
Now using (d_2) we have $x=y$.
Which proves the uniqueness. So S and Thave the unique common fixed point in X.

3.2. **Example 3.2.** Let $X=[0,1]$ and complete dq-metric is defined by,

$$d(x,y)=x\vee$$

Where the continuous self-mappings S and T are defined by $Sx=0$ and $Tx=x/5$ for all $x\in X$. $\alpha=\frac{1}{5},\beta=\frac{1}{6},\gamma=\frac{1}{8},\delta=\frac{1}{10},\lambda=\frac{1}{12}\wedge\mu=\frac{1}{14}$.
Then S and T satisfies all the conditions of Theorem 3.1 so $x=0$ is the unique common fixed point of S and T in X.
The following corollaries can be deduced from the above theorem.

3.3. **Corollary.** Suppose (X,d) be a complete dq-metric space, an$d$ T:$X\to X$be two
continuous self-mappings satisfying the succeeding property:

$$d(Tx,Ty)\le\alpha d(x,y)+\beta[d(x,Tx)+d(y,Ty)]+\gamma[d(y,Tx)+d(x,Ty)]$$
$$+\delta\frac{d(y,Ty)[1+d(x,Tx)]}{1+d(x,y)}+\lambda d(y,Tx)+\mu d(y,Tx)$$

$\forall x,y\in X$with$\alpha+2\beta+2\gamma+\delta+\lambda+\mu<1$. Then T have a unique common fixed point in$X$.

3.3.1. *Proof.* Put $S=T$ in theorem 3.1 we will get the desire result.

3.4. **Corollary.** Suppose (X,d) be a complete dq-metric space, an$d$ T:$X\to X$be two
continuous self-mappings satisfying the succeeding property:

$$d(Tx,Ty)\le\alpha d(x,y)$$

$\forall x,y\in X$with$\alpha<1$. Then Thave a unique common fixed point in$X$.

3.4.1. *Proof.* Put $\beta=\gamma=\delta=\lambda=\mu=0$ and $S=T$ in theorem 3.1 we will get the desire result.

3.5. **Corollary.** Suppose (X,d) be a complete dq-metric space, an$d$ T:$X\to X$be two
continuous self-mappings satisfying the succeeding property:

$$d(Tx,Ty)\le\beta[d(x,Tx)+d(y,Ty)]$$

$\forall x,y\in X$with$\beta<1$. Then Thave a unique common fixed point in$X$.

3.5.1. *Proof.* Put $\alpha=\gamma=\delta=\lambda=\mu=0$ and $S=T$ in theorem 3.1 we will get the desire result

3.6. **Corollary.** Suppose (X,d) be a complete dq-metric space, and $T{:}X \to X$be two
continuous self-mappings satisfying the succeeding property:

$$d\left(Tx,Ty\right) \leq \gamma\left[d\left(y,Tx\right)+d\left(x,Ty\right)\right]$$

$\forall x,y \in X$with$\gamma < 1$. Then Thave a unique common fixed point in$X$

3.6.1. *Proof.* Put $\alpha = \beta = \delta = \lambda = \mu = 0$ and $S = T$ in theorem 3.1 we will get the desire result.

3.7. **Corollary.** Suppose (X,d) be a complete dq-metric space, and $T{:}X \to X$be two
continuous self-mappings satisfying the succeeding property:

$$d\left(Tx,Ty\right) \leq \alpha d\left(x,y\right)+\beta\left[d\left(x,Tx\right)+d\left(y,Ty\right)\right]$$

$\forall x,y \in X$with$\alpha+\beta < 1$. Then T have a unique common fixed point in$X$.

3.7.1. *Proof.* Put $\gamma = \delta = \lambda = \mu = 0$ and $S = T$ in theorem 3.1 we will get the desire result.

3.8. **Corollary.** Suppose (X,d) be a complete dq-metric space, and $T{:}X \to X$be two continuous self-mappings satisfying the succeeding property:

$$d\left(Tx,Ty\right) \leq \alpha d\left(x,y\right)+\beta\left[d\left(x,Tx\right)+d\left(y,Ty\right)\right]+\gamma\left[d\left(y,Tx\right)+d\left(x,Ty\right)\right]$$

$\forall x,y \in X$with$\alpha+\beta+\gamma < 1$. Then T have a unique common fixed point in$X$.

3.8.1. *Proof.* Put $\delta = \lambda = \mu = 0$ and $S = T$ in theorem 3.1 we will get the desire result.
By using the same procedure we can deduce many more fixed point theorems.

3.9. **Corollary.** Suppose (X,d) be a complete dq-metric space, and $T{:}X \to X$be two
continuous self-mappings satisfying the succeeding property

$$d\left(Tx,Ty\right) \leq \alpha d\left(x,y\right)+\delta\frac{d\left(y,Ty\right)\left[1+d\left(x,Tx\right)\right]}{1+d\left(x,y\right)}$$

$\forall x,y \in X$with$\alpha+\delta < 1$. Then T have a unique common fixed point in$X$.

3.10. **Proof.** Put $\alpha = \beta = \gamma = \lambda = \mu = 0$ and $S = T$ in theorem 3.1 we will get the desire result.

Remarks

(1) Corollary 3.4 is the result of Banach 1.
(2) Corollary 3.5 is the result of Kannan10.
(3) Corollary 3.6 is the result of Chaterjee9.
(4) Corollary 3.7 is the result of Reich 5.
(5) Corollary 3.8 is the result of Hordy and Roger6.
(6) Corollary 3.9 is the result of Dass and Gupta3.

4. Conclusion

Our derived results extends the results of $1, 2, 7, 9, 11, 12$ and 15 in the setting of dislocated quasi metric spaces.

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MUJEEB UR REHMAN

Department of Mathematics Government P.G Jahanzeb College Swat, KPK, Pakistan.

E-mail address: mujeeb846@yahoo.com

SAFIULLAH MUSSA

Department of Mathematics Government P.G Jahanzeb College Swat, KPK, Pakistan.

E-mail address: safimussakhan@gmail.com