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COMPARATIVE GROWTH PROPERTIES OF ANALYTIC FUNCTION OF SEVERAL COMPLEX VARIABLES IN THE UNIT POLYDISC

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Abstract. In this paper, we introduced the idea of relative Nevanlinna $L^\star$-order and relative Nevanlinna $L^\star$-lower order of an analytic function of several complex variables with respect to an entire function of several complex variables in the unit polydisc. Also we study some growth properties of composition of two analytic functions of several complex variables in the unit polydisc on the basis of their relative Nevanlinna $L^\star$-order and relative Nevanlinna $L^\star$-lower order.

1. Introduction

A function f, analytic in the unit disc $U = \{z : \mid z| < 1\}$, is said to be of finite Nevanlinna order [2], if there exist a number μ such that the Nevanlinna characteristic function

$$T(r, f) = \frac{1}{2\pi}\int_0^{2\pi} \log^+ |f(re^{i\theta})|d\theta$$

satisfies, $T(r, f) < (1-r)^{-\mu}$ for all r in $0 < r_0 < r < 1$.

The greatest lower bound of all such number μ, is called Nevanlinna order of f, Thus the Nevanlinna order ρ_f of f is given by

$$\rho_f = \limsup_{r\to 1} \frac{logT(r, f)}{-log(1-r)}.$$

Similarly, Nevanlinna lower order λ_f of f, is given by

$$\lambda_f = \liminf_{r\to 1} \frac{logT(r, f)}{-log(1-r)}.$$

In sec.([3]) introduced the notion of Nevanlinna L - order for an analytic function f in the unit disc $U = \{z :\mid z| < 1\}$, where $L = L(\frac{1}{1-r})$ is a positive continuous

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function in the unit disc U increasing slowly i.e $L(\frac{a}{1-r}) \sim L(\frac{1}{1-r})$ as $r \to 1$ for every positive constant 'a' in the following manner.

Definition 1 If f be analytic in U, then the Nevanlinna L - order ρ_f^L and the Nevanlinna L - lower order λ_f^L of f are defined as

$$\rho_f^L = \limsup_{r\to 1} \frac{logT(r,f)}{log\left(\frac{L(\frac{1}{1-r})}{1-r}\right)},$$

and

$$\lambda_f^L = \liminf_{r\to 1} \frac{logT(r,f)}{log\left(\frac{L(\frac{1}{1-r})}{1-r}\right)}.$$

Now we introduce the concepts of relative Nevanlinna L^* - order and relative Nevanlinna L^* - lower order of an analytic function f with respect to another analytic function g in the unit disc U which are as follows.

Definition 2 If f be analytic in U and g be entire then the relative Nevanlinna L^* - order of f with respect to g, denoted by $\rho_g^{L^*}(f)$ is defined by

$$\rho_g^{L^*}(f) = \inf\{\mu > 0 : T_f(r) < T_g\left(\frac{exp\{L(\frac{1}{1-r})\}}{(1-r)}\right)^{\mu} \quad for \quad all \quad 0 < r_0(\mu) < r < 1\}.$$

Similarly, relative Nevanlinna L^* - lower order f with respect to g denoted by $\lambda_g^{L^*}(f)$ is given by

$$\lambda_g^{L^*}(f) = \liminf_{r\to 1} \frac{logT_g^{-1}T_f(r)}{log\left(\frac{exp\{L(\frac{1}{1-r})\}}{(1-r)}\right)}.$$

When $g(z) = exp(z)$, the definition coincides with the definition of the Nevanlinna L^* - order and the Nevanlinna L^* - lower order.

Now we are extending the notion of single variable to several complex variables in ([3], [4]). let f be an non - constant analytic function of several variables $z_1, z_2, \ldots, z_n$ in the unit polydisc,

$$U = \{(z_1, z_2, \ldots, z_n) : |z| \leq 1, i = 1, 2, \ldots, n\}.$$

Definition 3 Let $T_f(r_1, r_2, \ldots, r_n)$ denote the Nevanlinna's characteristic function of several complex variables and g be an other entire function of several complex variables. Then the relative Nevanlinna L^* - order of f with respect to g , denote by $\rho_g^{L^*}(f)$ is defined by

$$\rho_g^{L^*}(f) = \limsup_{r_1,r_2,\ldots,r_n\to 1} \frac{logT_g^{-1}T_f(r_1, r_2, \ldots, r_n)}{log\left(\frac{exp\{L(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n})\}}{(1-r_1)(1-r_2)\ldots(1-r_n)}\right)}.$$

Similarly , the relative Nevanlinna L^* - lower order of f with respect to g, denoted by $\lambda_g^{L^*}(f)$ is given by

$$\lambda_g^{L^*}(f) = \liminf_{r_1,r_2,\ldots,r_n\to 1} \frac{logT_g^{-1}T_f(r_1, r_2, \ldots, r_n)}{log\left(\frac{exp\{L(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n})\}}{(1-r_1)(1-r_2)\ldots(1-r_n)}\right)},$$

where $L \equiv L(\frac{1}{1-r_1}, \frac{1}{1-r_2}, \ldots, \frac{1}{1-r_n})$ is a positive continuous function in the unit polydisc U increasing slowly i.e., $L(\frac{a}{1-r_1}, \frac{a}{1-r_2}, \ldots, \frac{a}{1-r_n}) \sim L(\frac{1}{1-r_1}, \frac{1}{1-r_2}, \ldots, \frac{1}{1-r})$ as $r_1, r_2, ...r_n \to 1$, for every positive constant $'a'$. When $g(z_1, z_2, \ldots, z_n) = exp(z_1, z_2, \ldots, z_n)$, then the definition coincides with the definition of the Nevanlinna L^* - order and the Nevanlinna L^*- lower order.

In this paper, we study some growth properties of composition of two analytic function of several complex variables in the unit polydisc $U = \{(z_1, z_2, \ldots, z_n) : |z_i| < 1, i = 1, 2, \ldots, n\}$, on the basis of their relative Nevanlinna L^* - order(relative Nevanlinna L^* - lower order). We do not explain the standard definitions and notations in the theory of entire function of several complex variables as those are available in [1],[6]and [7].

In this section we present the main results of this paper.

Theorem 1 Let f, g be any two analytic functions in U and h be any entire function of several complex variables such that $0 < \lambda_h^{L^*}(f \circ g) \leq \rho_h^{L^*}(f \circ g) < \infty$ and $\lambda_h^{L^*}(f) \leq \rho_h^{L^*}(f) < \infty$.If $L(\frac{1}{1-r_1}, \frac{1}{1-r_2}, ..., \frac{1}{1-r_n}) = 0\{logT_h^{-1}T_f(r_1, r_2, ..., r_n)\}$ as $r_1, r_2, ..., r_n \to 1$, then

$$\frac{\lambda_h^{L^*}(f \circ g)}{\rho_h^{L^*}(f)} \leq \liminf_{r_1, r_2, \ldots, r_n \to 1} \frac{logT_h^{-1}T_{f\circ g}(r_1, r_2, \ldots, r_n)}{logT_h^{-1}T_f(r_1, r_2, \ldots, r_n) + L(\frac{1}{1-r_1}, \frac{1}{1-r_2}, \ldots, \frac{1}{1-r_n})} \leq$$

$$\frac{\lambda_h^{L^*}(f \circ g)}{\lambda_h^{L^*}(f)} \leq \limsup_{r_1, r_2, \ldots, r_n \to 1} \frac{logT_h^{-1}T_{f\circ g}(r_1, r_2, \ldots, r_n)}{logT_h^{-1}T_f(r_1, r_2, \ldots, r_n) + L\left(\frac{1}{1-r_1}, \frac{1}{1-r_2}, \ldots, \frac{1}{1-r_n}\right)}$$

$$\leq \frac{\rho_h^{L^*}(f \circ g)}{\lambda_h^{L^*}}.$$

Proof. From the definition of relative Nevanlinna L^* - order and relative Nevanlinna L^* -lower order of an analytic function of several complex variables in the unit polydisc U. we have for arbitrary positive ϵ and for all sufficiently large values of $\frac{1}{1-r_1}, \frac{1}{1-r_2}, \ldots, \frac{1}{1-r_n}$ that

$$logT_h^{-1}T_{f\circ g}(r_1, r_2, \ldots, r_n) \geq (\lambda_h^{L^*}(f \circ g) - \epsilon) log\left(\frac{exp\{L(\frac{1}{1-r_1}, \frac{1}{1-r_2}, \ldots, \frac{1}{1-r_n})\}}{(1-r_1)(1-r_2)\ldots(1-r_n)}\right),$$

i.e.,

$$\frac{logT_h^{-1}T_{f\circ g}(r_1, r_2, \ldots, r_n)}{(\lambda_h^{L^*}(f \circ g) - \epsilon)} \geq log\left(\frac{1}{1-r_1}.\frac{1}{1-r_2}.\cdots.\frac{1}{1-r_n}\right) + L\left(\frac{1}{1-r_1}, \frac{1}{1-r_2}, \ldots, \frac{1}{1-r_n}\right), \quad (1)$$

and

$$logT_h^{-1}T_f(r_1, r_2, \ldots, r_n) \leq (\rho_h^{L^*}(f) + \epsilon) log\left(\frac{exp\{L(\frac{1}{1-r_1}, \frac{1}{1-r_2}, \ldots, \frac{1}{1-r_n})\}}{(1-r_1)(1-r_2)\ldots(1-r_n)}\right),$$

i.e.

$$logT_h^{-1}T_f(r_1,r_2,\ldots,r_n) \le (\rho_h^{L^*}(f)+\epsilon)\{log\left(\frac{1}{1-r_1}.\frac{1}{1-r_2}.\ldots.\frac{1}{1-r_n}\right) + L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)\}. \tag{2}$$

Now from equation (1) and (2), it follows for all sufficiently large values of $\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}$ that,

$$logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n) \ge \frac{(\lambda_h^{L^*}(f\circ g)-\epsilon)}{(\rho_h^{L^*}(f)+\epsilon)} logT_h^{-1}T_f(r_1,r_2,\ldots,r_n),$$

i.e.

$$\frac{logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)+L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)} \ge \frac{(\lambda_h^{L^*}(f\circ g)-\epsilon)}{(\rho_h^{L^*}(f)+\epsilon)} \times \frac{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)+L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)},$$

i.e.,

$$\frac{logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)+L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)} \ge \frac{(\lambda_h^{L^*}(f\circ g)-\epsilon)}{(\rho_h^{L^*}(f)+\epsilon)} \times \frac{1}{1+\frac{L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)}}. \tag{3}$$

Since $L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-_n}\right) = 0\{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)\}$ as $r_1,r_2,\ldots,r_n \to 1$, it follows from equation(3) that

$$\liminf_{r_1,r_2,\ldots,r_n\to 1} \frac{logT_h^{-1}T_{f\circ}(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)+L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)} \ge \frac{\lambda_h^{L^*}(f\circ g)-\epsilon}{\rho_h^{L^*}(f)+\epsilon}. \tag{4}$$

As $\epsilon > 0$, is arbitrary we get from equation (4) that from which we have

$$\liminf_{r_1,r_2,\ldots,r_n\to 1} \frac{logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)+L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)} \ge \frac{\lambda_h^{L^*}(f\circ g)}{\rho_h^{L^*}(f)}. \tag{5}$$

Again for a sequence of values of $\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}$ tending to infinity

$$logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n) \le (\lambda_h^{L^*}(f\circ g)+\epsilon)log\left(\frac{exp\{L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)\}}{(1-r_1)(1-r_2)\ldots(1-r_n)}\right),$$

i.e.,

$$logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n) \le (\lambda_h^{L^*}(f\circ g)+\epsilon)\{log\left(\frac{1}{1-r_1}.\frac{1}{1-r_2}.\ldots.\frac{1}{1-r_n}\right) + L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)\}. \quad (6)$$

And

$$logT_h^{-1}T_f(r_1,r_2,\ldots,r_n) \ge (\lambda_h^{L^*}(f)-\epsilon)log\left(\frac{exp\{L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)\}}{(1-r_1)(1-r_2)\ldots(1-r_n)}\right),$$

i.e.,

$$logT_h^{-1}T_f(r_1,r_2,\ldots,r_n) \ge (\lambda_h^{L^*}(f)-\epsilon)\{log\left(\frac{1}{1-r_1}.\frac{1}{1-r_2}.\ldots.\frac{1}{1-r_n}\right) + L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)\}. \quad (7)$$

From equation (6) and (7), we get for a sequence of values of $\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}$ tending to infinity that

$$logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n) \le \frac{(\lambda_h^{L^*}(f\circ g)+\epsilon)}{(\lambda_h^{L^*}(f)-\epsilon)}logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)$$

i.e.,

$$\frac{logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)+L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)} \le \frac{(\lambda_h^{L^*}(f\circ g)+\epsilon)}{(\lambda_h^{L^*}(f)-\epsilon)} \times\frac{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)+L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)},$$

i.e.,

$$\frac{logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)+L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)} \le \frac{(\lambda_h^{L^*}(f\circ g)+\epsilon)}{(\lambda_h^{L^*}(f)-\epsilon)} \times\frac{1}{1+\frac{L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)}}. \quad (8)$$

As $L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right) = 0\{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)\}$ as $r_1,r_2,\ldots,r_n\to 1$, we get from equation (8) that

$$\liminf_{r_1,r_2,\ldots,r_n\to 1}\frac{logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)+L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)} \le \frac{(\lambda_h^{L^*}(f\circ g)+\epsilon)}{(\lambda_h^{L^*}(f)-\epsilon)}. \quad (9)$$

Since $\epsilon > 0$, is arbitrary it follows from equation (9) that

$$\liminf_{r_1,r_2,\ldots,r_n\to 1} \frac{logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)+L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)} \le \frac{\lambda_h^{L^*}(f\circ g)}{\lambda_h^{L^*}(f)}. \tag{10}$$

Again, for a sequence of values of $\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}$ tending to infinity

$$logT_h^{-1}T_f(r_1,r_2,\ldots,r_n) \le (\lambda_h^{L^*}(f)+\epsilon)log\left(\frac{exp\{L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)\}}{(1-r_1)(1-r_2)\ldots(1-r_n)}\right),$$

i.e.

$$\begin{aligned} logT_h^{-1}T_f(r_1,r_2,\ldots,r_n) \le (\lambda_h^{L^*}(f)+\epsilon)\{log\left(\frac{1}{1-r_1}.\frac{1}{1-r_2}.\ldots.\frac{1}{1-r_n}\right) \\ +L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)\}. \end{aligned} \tag{11}$$

Now, from equation(1) and (11), we obtain for a sequence of values of $\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots$, $\frac{1}{1-r_n}$ tending to infinity that

$$logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n) \ge \frac{(\lambda_h^{L^*}(f\circ g)-\epsilon)}{(\lambda_h^{L^*}(f)+\epsilon)} logT_h^{-1}T_f(r_1,r_2,\ldots,r_n),$$

i.e.

$$\begin{aligned} \frac{logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)+L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)} \ge \frac{(\lambda_h^{L^*}(f\circ g)-\epsilon)}{(\lambda_h^{L^*}(f)+\epsilon)} \\ \times \frac{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)+L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)}, \end{aligned}$$

$$\begin{aligned} \frac{logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)+L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)} \ge \frac{(\lambda_h^{L^*}(f\circ g)-\epsilon)}{(\lambda_h^{L^*}(f)+\epsilon)} \\ \times \frac{1}{1+\frac{L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)}}. \end{aligned} \tag{12}$$

In view of the condition $L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right) = 0\{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)\}$, as $r_1,r_2,\ldots,r_n \to 1$, we obtain from (12) that

$$\limsup_{r_1,r_2,\ldots,r_n\to 1} \frac{logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)+L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)} \ge \frac{(\lambda_h^{L^*}(f\circ g)-\epsilon)}{(\lambda_h^{L^*}(f)+\epsilon)}. \tag{13}$$

Since $\epsilon > 0$, is arbitrary it follows from (13) that

$$\limsup_{r_1,r_2,\ldots,r_n\to 1} \frac{logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)+L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)} \geq \frac{\lambda_h^{L^*}(f\circ g)}{\lambda_h^{L^*}(f)}. \tag{14}$$

Also, for all sufficiently large values of $\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}$,

$$logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n) \leq (\rho_h^{L^*}(f\circ g)+\epsilon)log\left(\frac{exp\{L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)\}}{(1-r_1)(1-r_2)\ldots(1-r_n)}\right),$$

i.e

$$logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n) \leq (\rho_h^{L^*}(f\circ g)+\epsilon)\{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n) + L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)\}. \tag{15}$$

So from equation (7) and (15), it follows for all sufficiently large values of $\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}$ that

$$logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n) \leq \frac{(\rho_h^{L^*}(f\circ g)+\epsilon)}{(\lambda_h L^*(f)-\epsilon)} logT_h^{-1}T_f(r_1,r_2,\ldots,r_n),$$

i.e.

$$\frac{logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)+L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)} \leq \frac{\rho_h^{L^*}(f\circ g)+\epsilon}{\lambda_h L^*(f)-\epsilon} \times \frac{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)+L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)},$$

$$\frac{logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)+L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)} \leq \frac{\rho_h^{L^*}(f\circ g)+\epsilon}{\lambda_h L^*(f)-\epsilon} \times \frac{1}{1+\frac{L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)}}. \tag{16}$$

Using $L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right) = 0\{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)\}$ as $r_1,r_2,\ldots,r_n \to 1$, we obtain from (16) that

$$\limsup_{r_1,r_2,\ldots,r_n\to 1} \frac{logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)+L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)} \leq \frac{(\rho_h^{L^*}(f\circ g)+\epsilon)}{(\lambda_h L^*(f)-\epsilon)}. \tag{17}$$

As $\epsilon > 0$ is arbitrary, it follows from (17) that

$$\limsup_{r_1,r_2,\ldots,r_n\to 1} \frac{logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)+L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)} \leq \frac{\rho_h^{L^*}(f\circ g)}{\lambda_h L^*(f)}. \tag{18}$$

Thus, the theorem follows from (5),(10),(14) and (18).
Similarly, in view Theorem 1, we may state the following theorem without proof for the right g of the composite function $f \circ g$.
Theorem 2 Let f, g be any two analytic function of several complex variables in U and h be any entire function with $0 < \lambda_h^{L^*}(f \circ g) \leq \rho_h^{L^*}(f \circ g) < \infty$, and $0 < \lambda_h^{L^*}(g) \leq \rho_h^{L^*}(g) < \infty$. If $L\left(\frac{1}{1-r_1}, \frac{1}{1-r_2}, \ldots, \frac{1}{1-r_n}\right) = 0\{logT_h^{-1}T_g(r_1, r_2, \ldots, r_n)\}$, as $r_1, r_2, \ldots, r_n \to 1$, then

$$\frac{\lambda_h^{L^*}(f \circ g)}{\rho_h^{L^*}(g)} \leq \liminf_{r_1, r_2, \ldots, r_n \to 1} \frac{logT_h^{-1}T_{f \circ g}(r_1, r_2, \ldots, r_n)}{logT_h^{-1}T_g(r_1, r_2, \ldots, r_n) + L\left(\frac{1}{1-r_1}, \frac{1}{1-r_2}, \ldots, \frac{1}{1-r_n}\right)} \leq$$

$$\frac{\lambda_h^{L^*}(f \circ g)}{\lambda_h^{L^*}(g)} \leq \limsup_{r_1, r_2, \ldots, r_n \to 1} \frac{logT_h^{-1}T_{f \circ g}(r_1, r_2, \ldots, r_n)}{logT_h^{-1}T_g(r_1, r_2, \ldots, r_n) + L\left(\frac{1}{1-r_1}, \frac{1}{1-r_2}, \ldots, \frac{1}{1-r_n}\right)}$$

$$\leq \frac{\rho_h^{L^*}(f \circ g)}{\lambda_h^{L^*}(g)}.$$

Theorem 3 Let f, g be any two analytic functions of several complex variables in U and h be an entire function of several complex variables such that $0 < \rho_h^{L^*}(f \circ g) < \infty$, and $0 < \rho_h^{L^*}(f) < \infty$. If

$$\liminf_{r_1, r_2, \ldots, r_n \to 1} \frac{logT_h^{-1}T_{f \circ g}(r_1, r_2, \ldots, r_n)}{logT_h^{-1}T_f(r_1, r_2, \ldots, r_n) + L\left(\frac{1}{1-r_1}, \frac{1}{1-r_2}, \ldots, \frac{1}{1-r_n}\right)} \leq \frac{\rho_h^{L^*}(f \circ g)}{\rho_h^{L^*}(f)}$$

$$\leq \limsup_{r_1, r_2, \ldots, r_n \to 1} \frac{logT_h^{-1}T_{f \circ g}(r_1, r_2, \ldots, r_n)}{logT_h^{-1}T_f(r_1, r_2, \ldots, r_n) + L\left(\frac{1}{1-r_1}, \frac{1}{1-r_2}, \ldots, \frac{1}{1-r_n}\right)}.$$

Proof. From the definition of $\rho_h^{L^*}(f)$, the relative Nevanlinna L^*-order of an analytic function f of several complex variables in the unit polydisc U with respect to an entire function h of several complex variables we get for a sequence of values of $\frac{1}{1-r_1}, \frac{1}{1-r_2}, \ldots, \frac{1}{1-r_n}$ tending to infinity that

$$logT_h^{-1}T_f(r_1, r_2, \ldots, r_n) \geq (\rho_h^{L^*}(f) - \epsilon) log\left(\frac{exp\{L\left(\frac{1}{1-r_1}, \frac{1}{1-r_2}, \ldots, \frac{1}{1-r_n}\right)\}}{(1-r_1)(1-r_2)\ldots(1-r_n)}\right),$$

i.e.

$$\frac{logT_h^{-1}T_f(r_1, r_2, \ldots, r_n)}{(\rho_h^{L^*}(f) - \epsilon)} \geq log\left(\frac{1}{1-r_1}.\frac{1}{1-r_2}.\cdots.\frac{1}{1-r_n}\right) + L\left(\frac{1}{1-r_1}, \frac{1}{1-r_2}, \ldots, \frac{1}{1-r_n}\right). \quad (19)$$

Now from equation (15) and (19), it follows for a sequence of values of $\frac{1}{1-r_1}, \frac{1}{1-r_2}, \ldots,$ $\frac{1}{1-r_n}$ tending to infinity that

$$logT_h^{-1}T_{f \circ g}(r_1, r_2, \ldots, r_n) \leq \frac{(\rho_h^{L^*}(f \circ g) + \epsilon)}{(\rho_h^{L^*}(f) - \epsilon)} logT_h^{-1}T_f(r_1, r_2, \ldots, r_n),$$

i.e.,

$$\frac{logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)+L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)} \leq \frac{(\rho_h^{L^*}(f\circ g)+\epsilon)}{(\rho_h^{L^*}(f)-\epsilon)}$$

$$\times\frac{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)+L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)},$$

i.e.,

$$\frac{logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)+L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)} \leq \frac{(\rho_h^{L^*}(f\circ g)+\epsilon)}{(\rho_h^{L^*}(f)-\epsilon)} \times\frac{1}{1+\frac{L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)}}. \quad (20)$$

Using $L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right) = 0\{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)\}$ as $r_1,r_2,\ldots,r_n \to 1$, we obtain from (20) that

$$\liminf_{r_1,r_2,\ldots,r_n\to 1}\frac{logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)+L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)} \leq \frac{(\rho_h^{L^*}(f\circ g)+\epsilon)}{(\rho_h^{L^*}(f)-\epsilon)}. \quad (21)$$

As $\epsilon > 0$, is arbitrary it follows from equation(21) that

$$\liminf_{r_1,r_2,\ldots,r_n\to 1}\frac{logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)+L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)} \leq \frac{\rho_h^{L^*}(f\circ g)}{\rho_h^{L^*}(f)}. \quad (22)$$

Again for a sequence of values of $\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}$ tending to infinity,

$$logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n) \geq (\rho_h^{L^*}(f\circ g)-\epsilon)log\left(\frac{exp\{L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)\}}{(1-r_1)(1-r_2)\ldots(1-r_n)}\right)$$

i.e.,

$$\frac{logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)}{(\rho_h^{L^*}(f\circ g)-\epsilon)} \geq log\left(\frac{1}{1-r_1}.\frac{1}{1-r_2}.\ldots.\frac{1}{1-r_n}\right) + L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right). \quad (23)$$

So combining (2) and (23), we get for a sequence of values of $\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}$ tending to infinity that

$$logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n) \geq \frac{(\rho_h^{L^*}(f\circ g)-\epsilon)}{(\rho_h^{L^*}(f)+\epsilon)}logT_h^{-1}T_f(r_1,r_2,\ldots,r_n),$$

i.e.,

$$\frac{logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)+L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)}\geq\frac{(\rho_h^{L^*}(f\circ g)-\epsilon)}{(\rho_h^{L^*}(f)+\epsilon)}$$
$$\times\frac{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)+L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)},$$

i.e.,

$$\frac{logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)+L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)}\geq\frac{(\rho_h^{L^*}(f\circ g)-\epsilon)}{(\rho_h^{L^*}(f)+\epsilon)}$$
$$\times\frac{1}{1+\frac{L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)}}. \quad (24)$$

Since, $L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)=0\{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)\}$ as $r_1,r_2,\ldots,r_n\to 1$, it follows from equation (24) that

$$\limsup_{r_1,r_2,\ldots,r_n\to1}\frac{logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)+L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)}\geq\frac{(\rho_h^{L^*}(f\circ g)-\epsilon)}{(\rho_h^{L^*}(f)+\epsilon)}. \quad (25)$$

As $\epsilon>0$, is arbitrary we get from equation(25) that

$$\limsup_{r_1,r_2,\ldots,r_n\to1}\frac{logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)+L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)}\geq\frac{\rho_h^{L^*}(f\circ g)}{\rho_h^{L^*}(f)}. \quad (26)$$

Thus, the theorem follows from equation (22) and (26).

Theorem 4 Let f, g be any two analytic function of several complex variables in U and h be an entire function of several complex variables with $0<\rho_h^{L^*}(f\circ g)<\infty$, and $0<\rho_h^{L^*}(f)<\infty$. If $L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)=0\{logT_h^{-1}T_g(r_1,r_2,\ldots,r_n)\}$ as $r_1,r_2,\ldots,r_n\to1$, then

$$\liminf_{r_1,r_2,\ldots,r_n\to1}\frac{logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_g(r_1,r_2,\ldots,r_n)+L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)}\leq\frac{\rho_h^{L^*}(f\circ g)}{\rho_h^{L^*}(g)}\leq$$
$$\limsup_{r_1,r_2,\ldots,r_n\to1}\frac{logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_g(r_1,r_2,\ldots,r_n)+L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)}.$$

Theorem 5 Let f, g be any two analytic function of several complex variables in the unit polydisc U and h be an entire function of several complex variables such that $0<\lambda_h^{L^*}(f\circ g)\leq\rho_h^{L^*}(f\circ g)<\infty$, and $0<\lambda_h^{L^*}(f)\leq\rho_h^{L^*}(f)<\infty$. If $L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)=0\{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)\}$ as $r_1,r_2,\ldots,r_n\to1$, then

$$\liminf_{r_1,r_2,\ldots,r_n\to1}\frac{logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)+L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)}\leq$$

$$min\left\{\frac{\lambda_h^{L^*}(f\circ g)}{\lambda_h^{L^*}(f)},\frac{\rho_h^{L^*}(f\circ g)}{\rho_h^{L^*}(f)}\right\}\leq max\left\{\frac{\lambda_h^{L^*}(f\circ g)}{\lambda_h^{L^*}(f)},\frac{\rho_h^{L^*}(f\circ g)}{\rho_h^{L^*}(f)}\right\}\leq$$

$$\limsup_{r_1,r_2,\ldots,r_n\to 1}\frac{logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)+L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)}.$$

Theorem 6 Let f, g be any two analytic function of several complex variables in the unit polydisc U and h be an entire function of several complex variables with $0<\lambda_h^{L^*}(f\circ g)\leq\rho_h^{L^*}(f\circ g)<\infty$, and $0<\lambda_h^{L^*}(g)\leq\rho_h^{L^*}(g)<\infty$ .If $L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)=0\{logT_h^{-1}T_g(r_1,r_2,\ldots,r_n)\}$, as $r_1,r_2,\ldots,r_n\to 1$, then

$$\liminf_{r_1,r_2,\ldots,r_n\to 1}\frac{logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_g(r_1,r_2,\ldots,r_n)+L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)}\leq$$

$$min\left\{\frac{\lambda_h^{L^*}(f\circ g)}{\lambda_h^{L^*}(g)},\frac{\rho_h^{L^*}(f\circ g)}{\rho_h^{L^*}(g)}\right\}\leq max\left\{\frac{\lambda_h^{L^*}(f\circ g)}{\lambda_h^{L^*}(g)},\frac{\rho_h^{L^*}(f\circ g)}{\rho_h^{L^*}(g)}\right\}\leq$$

$$\limsup_{r_1,r_2,\ldots,r_n\to 1}\frac{logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_g(r_1,r_2,\ldots,r_n)+L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)}.$$

Theorem 7 Let f, g be any two analytic function of several complex variables in the unit polydisc U and h be an entire function of several complex variables such that $\rho_h^{L^*}(f)<\infty$, also let g be analytic function of several complex variables in U. If $\lambda_h^{L^*}(f\circ g)<\infty$, that

$$\liminf_{r_1,r_2,\ldots,r_n\to 1}\frac{logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)}=\infty.$$

Proof. Let us suppose that the conclusion of the theorem do not hold.
Then, we can find a constant $\beta>0$, such that for a sequence of values of $\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}$ tending to infinity,

$$logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)\leq\beta logT_h^{-1}T_f(r_1,r_2,\ldots,r_n).\tag{27}$$

Again from the definition of $\rho_h^{L^*}(f)$, it follows that for all sufficiently large values of $\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}$ that,

$$logT_h^{-1}T_f(r_1,r_2,\ldots,r_n)\leq(\rho_h^{L^*}(f)+\epsilon)log\left(\frac{exp\{L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)\}}{(1-r_1)(1-r_2)\ldots(1-r_n)}\right).\tag{28}$$

Thus, from (27) and (28), we have for a sequence of values of $\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}$ tending to infinity that,

$$logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)\leq\beta(\rho_h^{L^*}(f)+\epsilon)log\left(\frac{exp\{L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)\}}{(1-r_1)(1-r_2)\ldots(1-r_n)}\right),$$

i.e.,

$$\frac{logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)}{log\left(\frac{exp\{L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)\}}{(1-r_1)(1-r_2)\ldots(1-r_n)}\right)} \le \frac{\beta(\rho_h^{L^*}(f)+\epsilon)log\left(\frac{exp\{L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)\}}{(1-r_1)(1-r_2)\ldots(1-r_n)}\right)}{log\left(\frac{exp\{L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)\}}{(1-r_1)(1-r_2)\ldots(1-r_n)}\right)},$$

$$\liminf_{r_1,r_2,\ldots,r_n}\frac{logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)}{log\left(\frac{exp\{L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)\}}{(1-r_1)(1-r_2)\ldots(1-r_n)}\right)} = \lambda_h^{L^*}(f\circ g) < \infty,$$

this is a contradiction,
This prove the theorem.
Remark. Theorem 7 is also valid with "limit superior" instead of "limit" if $\lambda_h^{L^*}(f\circ g)=\infty$, is replaced by $\rho_h^{L^*}(f\circ g)=\infty$, and the other conditions are remaining the same.
Corollary 1 Under the assumptions of Theorem 2.7

$$\limsup_{r_1,r_2,\ldots,r_n}\frac{T_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)}{T_h^{-1}T_f(r_1,r_2,\ldots,r_n)} = \infty.$$

Proof. From Theorem 7, we obtain for all sufficiently large values of $\frac{1}{1-r_1}$, $\frac{1}{1-r_2}$, $\ldots$, $\frac{1}{1-r_n}$ and $K>0$, that

$$T_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n) > KT_h^{-1}T_f(r_1,r_2,\ldots,r_n),$$

i.e.,

$$T_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n) > \{T_h^{-1}T_f(r_1,r_2,\ldots,r_n)\}^K,$$

from which the corollary follows.
Theorem 8 Let f, g be any two analytic functions of several complex variables in the unit polydisc U and h be any entire function of several complex variables such that $\rho_h^{L^*}(g)<\infty$, and $\lambda_h^{L^*}(f\circ g)=\infty$, then

$$\lim_{r_1,r_2,\ldots,r_n}\frac{logT_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)}{logT_h^{-1}T_g(r_1,r_2,\ldots,r_n)} = \infty.$$

Remark. Theorem 8 is also valid with "limit superior"instead of "limit" if $\lambda_h^{L^*}(f\circ g)=\infty$, is replaced by $\rho_h^{L^*}(f\circ g)=\infty$, and the other conditions are remaining the same.
In the line of corollary 1, we may easily verify the following.
Corollary 2 Under the assumptions of Theorem 8

$$\limsup_{r_1,r_2,\ldots,r_n}\frac{T_h^{-1}T_{f\circ g}(r_1,r_2,\ldots,r_n)}{T_h^{-1}T_f(r_1,r_2,\ldots,r_n)} = \infty.$$

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